Fatigue assessment of notched specimens by means of a critical plane-based criterion and energy concepts

Submitted to Special Issue “**Notch Mechanics**”
Theoretical and Applied Fracture Mechanics – December 2015
Revised version - March 2016

FATIGUE ASSESSMENT OF NOTCHED SPECIMENS BY MEANS OF A CRITICAL PLANE-BASED CRITERION AND ENERGY CONCEPTS

Andrea Carpinteri[1], Filippo Berto[2,3], Alberto Campagnolo[2], Giovanni Fortese[1], Camilla Ronchei[1], Daniela Scorza[1], Sabrina Vantadori[1]

[1]Dept. of Civil-Environmental Engineering and Architecture, University of Parma, Parco Area delle Scienze 181/A
43124 Parma – Italy
[2]Dept. of Management and Engineering, University of Padua, Stradella San Nicola 3, 36100, Vicenza - Italy
[3]NTNU, Department of Engineering Design and Materials, Richard Birkelands vei 2b, 7491, Trondheim - Norway

Corresponding author:
Andrea Carpinteri, e-mail: andrea.carpinteri@unipr.it

Abstract
A strain-based multiaxial fatigue criterion previously proposed for fatigue assessment of unnotched specimens is here extended to the case of notched ones. Such a criterion is a reformulation of the stress-based multiaxial High-Cycle Fatigue (HCF) criterion by Carpinteri and Spagnoli. The extension herein presented considers as the critical point, where to perform the fatigue assessment, the material point located at a certain distance from the notch tip,

depending such a distance on both the biaxiality ratio (defined as the ratio between the applied shear stress amplitude and the normal stress amplitude) and the control volume radii R_1 (related to Mode I) and R_3 (related to Mode III). Some multiaxial fatigue data related to specimens made of titanium grade 5 alloy (Ti-6Al-4V) and weakened by sharp notches are examined to validate the extended criterion.

KEYWORDS: strain-based criterion; critical-plane approach; fatigue lifetime, control volume radius

NOMENCLATURE

d	notch depth
e_1	notch geometry parameter related to Mode I
e_3	notch geometry parameter related to Mode III
I	error index
N_a	reference number of loading cycles to failure
N_f	theoretical fatigue life
$N_{f,\exp}$	experimental fatigue life
$\mathrm{P}uvw$	reference system attached to the critical plane
$Prtz$	fixed frame
$P123$	principal strain axes frame
$P\hat{1}\hat{2}\hat{3}$	principal strain axes frame at time instant for which the maximum principal strain, ε_1, attains its peak value over the loading cycle
R_m	mean control volume radius
R_1	control volume radius related to Mode I
R_3	control volume radius related to Mode III
t	time
$\mathbf{w}$	unit vector normal to the critical plane
γ_a	Manson-Coffin shear strain amplitude
δ	angle between the averaged direction 1 and the normal **w** to the critical plane
ΔK_{1A}	Notch Stress Intensity Factor under Mode I
ΔK_{3A}	Notch Stress Intensity Factor under Mode III
ΔW	Strain Energy Density range
$\Delta\sigma_{1A}$	High-Cycle Fatigue strength of smooth specimens under Mode I
$\Delta\tau_{3A}$	High-Cycle Fatigue strength of smooth specimens under Mode III
$\boldsymbol{\varepsilon}$	strain tensor at material point P
$\varepsilon_{eq,a}$	equivalent normal strain amplitude
ε_a	Manson-Coffin normal strain amplitude

ϕ	phase angle between tension and torsion loading
ν	elastic Poisson ratio
ν_{eff}	effective Poisson ratio
$\boldsymbol{\eta}$	displacement vector at material point P, related to the critical plane
$\boldsymbol{\eta}_N$	normal displacement vector component of $\boldsymbol{\eta}$, acting on the critical plane
$\boldsymbol{\eta}_C$	tangential displacement vector component of $\boldsymbol{\eta}$, acting on the critical plane
λ	biaxiality ratio, defined as the ratio between the applied shear stress amplitude and the normal stress amplitude
λ_1	eigenvalues for Mode I
λ_3	eigenvalues for Mode III
ρ	notch root radius
σ_{af}	normal stress fatigue limit
τ_{af}	shear stress fatigue limit

Subscript

a	amplitude

1. INTRODUCTION

The problem of the multiaxial fatigue behaviour of metallic materials is very challenging, still open and worth of investigation.

Reviews of different fatigue criteria have recently been published by both Fatemi - Shamsaei [1] and Nieslony - Sonsino [2], who have analysed a large number of experimental data related to notched specimens, by using the most promising fatigue criteria available in the literature.

Among these, the critical plane-based criteria perform fatigue failure assessment on a specific plane (critical plane) within specimen or structural component. The common aspect of all criteria based on the critical plane approach is that fatigue assessment is carried out by employing a combination of stresses and/or strains related to such a plane.

On the other hand, the above criteria are characterized by different methods to define the critical plane orientation. More precisely, several researchers define the critical plane as the plane where some stress/strain components (or a combination of them) exhibit a maximum value. For instance, the critical plane is assumed to be the maximum shear strain amplitude plane according to Fatemi et al. [3,4] and Li et al. [5], whereas it is assumed to be the plane experiencing the maximum shear stress according to Kluger [6]. The critical plane can also be defined as that containing the direction along which the variance of the resolved shear stress reaches its maximum value, by taking full advantage of the Shear

Stress-Maximum Variance Method (τ-MVM) recently proposed by Susmel et al. [7].

Alternatively, the critical plane orientation may be connected with the principal stress directions, by using both suitable weight functions and an off-angle, the latter depending on the shear-to-normal stress fatigue ratio [8,9].

The above definitions represent only some of those reported in the literature [10], and a comparison of different methods to determine the critical plane orientation is reported in Ref.[11].

Within the framework of the critical plane approach, the concepts presented above are further elaborated to take into account the influence of stress/strain gradient on the fatigue strength. In particular, specific critical plane-based models have recently been proposed for notched structural components, where the effect of multiaxial stress/strain gradient is significant [12], as well as for welded joints [13].

Regarding multiaxial fatigue assessment, energy-based criteria also find remarkable application [14]. The first researcher to use an energy-based parameter was Jasper in a pioneering work dated 1923 [15] where fatigue strength of ferrous metallic specimens under tension–compression loading was analysed. Worth mentioning is the contribution by Ellyin [16,17], who proposed an approach based on a combination of both plastic and elastic strain energy density (SED). A review of several available energy-based multiaxial fatigue life formulations can be found in Ref.[18]. The deviatoric strain energy density evaluated at the notch tip was used by Park and Nelson [19] to assess the fatigue behaviour of specimens weakened by blunt notches under multiaxial stresses. A deviatoric interpretation of the Neuber's rule and the Smith-Watson-Topper (SWT) parameter has recently been presented by Kujawski in Refs [20,21], where the criterion is successfully applied to multiaxial fatigue assessment of different metallic materials.

Note that pointwise criteria as those reported above cannot be employed in the case of sharp notches. In order to overcome such an issue, a volume-based SED approach originally proposed for sharp V-notches and cracks under Mode I [22] has been extended to multiaxial fatigue loading [23–28].

In the present paper, the fatigue life estimation of severely notched specimens under low-cycle fatigue [28] is carried out by extending to notched structural components the strain-based criterion recently proposed by Carpinteri et al. for plain specimens [29–31]. Such an extension is formulated by implementing the concept of the control volume related to the Strain Energy Density (SED) criterion proposed by Lazzarin et al. [23–28]. More precisely, the material point P, where the fatigue strength assessment is carried out by employing the Carpinteri et al. criterion, is assumed to be at a distance which is function of the SED control volume radius provided by the SED criterion.

The paper is structured as follows. *Section 2* summarises the analytical basis of the stain-based criterion here employed, with reference to both smooth and notched structural components. *Section 3* briefly summarises the experimental work reported in Ref. [28], as far as specimen geometry, testing procedure and experimental results are concerned, such results being used to validate the criterion herein proposed. Finally, a comparison between experimental data and results deduced through the above criterion is reported in terms of fatigue life in *Section 4*.

2. STRAIN-BASED MULTIAXIAL FATIGUE CRITERION

2.1 Formulation for smooth structural components

The strain-based criterion, recently proposed by Carpinteri et al. [29–31] in order to estimate the fatigue lifetime under low-cycle fatigue (LCF) regime, is a reformulation of the stress-based multiaxial High-Cycle Fatigue (HCF) criterion named Carpinteri-Spagnoli (C-S) criterion [32–34], proposed in terms of strain since LCF is strain-controlled. The aim of each of the above criteria, based on the critical plane approach, is to reduce the multiaxial stress/strain state to an equivalent uniaxial one. Then, the fatigue strength evaluation is performed by employing an equivalent stress/strain amplitude together with a unique material reference curve, that is: either a Wöhler curve (for HCF) or a Manson-Coffin curve (for LCF).

The main features of the C-S criterion are:

(i) The orientation of the critical plane is linked to averaged directions of the principal stress axes;

(ii) The fatigue strength evaluation is performed by employing the amplitude of an equivalent stress related to the critical plane.

Note that the fatigue strength estimation capabilities of the C-S criterion have recently been improved by varying the procedure to determine the orientation of the critical plane [35].

Taking the above concepts as a starting point, Figure 1 shows the framework of the strain-based criterion here proposed.

Figure 1.

By considering the strain state at a material point P of a generic structural component, the averaged directions $\hat{1}, \hat{2}, \hat{3}$ of the principal strain axes can be determined on the basis of the instantaneous ones. In particular, the reference system $P\hat{1}\hat{2}\hat{3}$ corresponds to the instantaneous one at time instant for which the maximum principal strain, ε_1, attains its peak value over the loading cycle. Then, the normal vector, $\mathbf{w}$, to the critical plane is defined through the off-angle, δ, by performing a counter-clockwise rotation in the principal plane $\hat{1}\hat{3}$:

$$\delta = \frac{3}{2}\left[1 - \left(\frac{1}{2\left(1+\nu_{eff}\right)}\frac{\gamma_a}{\varepsilon_a}\right)^2\right]45° \tag{1}$$

where ν_{eff} is the effective Poisson ratio, and ε_a and γ_a are defined by the well-known Manson-Coffin equations

$$\varepsilon_a = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c \tag{2a}$$

$$\gamma_a = \frac{\tau'_f}{G}\left(2N_f\right)^{b_0} + \gamma'_f\left(2N_f\right)^{c_0} \tag{2b}$$

being σ'_f, ε'_f, b, c and τ'_f, γ'_f, b_0, c_0 material constants, and N_f the number of loading cycles to failure.

Note that the above δ expression is able to take into account the nature of fracture and, consequently, the degree of the ductility of the material [31].

When the fracture is *extremely ductile* ($N_f \rightarrow 0$) the ratio γ_a/ε_a is equal to:

$$\lim_{N_f \rightarrow 0} \frac{\gamma_a}{\varepsilon_a} = \frac{\gamma'_f}{\varepsilon'_f} \tag{3}$$

Considering the above ratio equal to $\sqrt{3}$ [36] and $\nu_{eff} = 0.5$, the off-angle δ results to be equal to $45°$ (Eq.(1)).

On the other hand, when the fracture is *brittle* ($N_f \rightarrow 2(10)^6$, being $2(10)^6$ the number of loading cycles in correspondence to the material fatigue limits, independent of the loading applied), the ratio γ_a/ε_a is given by:

$$\lim_{N_f \rightarrow 2(10)^6} \frac{\gamma_a}{\varepsilon_a} = \frac{\tau_{af}}{G}\frac{E}{\sigma_{af}} = \frac{\tau_{af}\, 2\left(1+\nu_{eff}\right)}{E}\frac{E}{\sigma_{af}} = \frac{2\left(1+\nu_{eff}\right)\, \tau_{af}}{\sigma_{af}} \tag{4}$$

where σ_{af} and τ_{af} are the fatigue limits for normal stress and shear stress, respectively. In the limit case of *extremely brittle* fracture, characterized by a ratio between the above fatigue limits equal to 1 (very hard metals), the off-angle δ is equal to $0°$.

For brittle fracture, the expression of δ (Eq.(1)) agrees with the expression employed in the C-S criterion for HCF regime [32–34], while it is different from that proposed in Refs [29,30], where δ is assumed to be constant and equal to $45°$.

Now let us consider the local reference system, $Puvw$, attached to the critical plane: the u-axis is the intersection line between the critical plane and the plane defined by the normal vector $\mathbf{w}$ and the z-axis, and the v-axis is such as to form a right-handed system together with the u-axis and the w-axis.

The strain tensor related to both the material point, P, (Fig.1) and the above critical plane orientation can be computed and, consequently, the corresponding displacement vector, $\boldsymbol{\eta}$, is determined. Such a vector is decomposed into:

(1) a normal vector $\boldsymbol{\eta}_N$, with a norm represented by the normal strain ε_w;

(2) a tangential vector $\boldsymbol{\eta}_C$, with a norm which is a function of the shear strains γ_{uw} and γ_{vw}

$$\eta_C = \|\boldsymbol{\eta}_C\| = \sqrt{(\eta_{Cu})^2 + (\eta_{Cv})^2} = \sqrt{\left(\frac{1}{2}\gamma_{uw}\right)^2 + \left(\frac{1}{2}\gamma_{vw}\right)^2} \tag{5}$$

Since the direction of the normal vector, $\boldsymbol{\eta}_N$, is time-invariant, the corresponding amplitude in a loading cycle ($\eta_{N,a}$) can easily be computed [31].

On the other hand, the direction of the tangential vector, $\boldsymbol{\eta}_C$, changes during a loading cycle, and the definition of the corresponding amplitude is not unique. Therefore, several methods to evaluate such an amplitude are available in the literature: for instance, the Prismatic Hull method, recently implemented in the C-S criterion [37,38]. A min-max procedure is implemented in the present paper by using the Minimum Circumscribed Circle (MCC) method [39].

Note that, during a cycle of synchronous out-of-phase sinusoidal biaxial normal and shear stress loading, the tip of the tangential vector, $\boldsymbol{\eta}_C$, describes an elliptical path, $\mathbf{s}$, on the critical plane so that the above amplitude coincides with the major semi-axis $\eta_{C,a}$ of the above ellipse (Fig.2), and is given by:

$$\eta_{C,a} = \sqrt{\frac{f^2+g^2+p^2+q^2}{2} + \sqrt{\left(\frac{f^2+g^2+p^2+q^2}{2}\right)^2 (fq-gp)^2}} \tag{6}$$

where functions f, g, p and q depend on both the strain tensor component signals (in terms of amplitudes and phase angles) and the normal vector $\mathbf{w}$ orientation.

Figure 2.

Then, the fatigue strength is assessed through an equivalent normal strain amplitude, $\varepsilon_{eq,a}$, expressed by a quadratic combination of the norm amplitudes of both normal vector ($\eta_{N,a}$) and tangential vector ($\eta_{C,a}$, Eq.(6)):

$$\varepsilon_{eq,a} = \sqrt{(\eta_{N,a})^2 + \left(\frac{\varepsilon_a}{\gamma_a}\right)^2 (\eta_{C,a})^2} \tag{7}$$

being ε_a and γ_a given by Eqs (2).

All terms in Eq.(7) depend on the number N_f of loading cycles. Therefore, by equating Eq.(7) with Eq.(2a), the value of N_f can be found through an iterative procedure.

The above criterion has successfully been used to evaluate the fatigue lifetime of plain specimens under low/medium-cycle biaxial loading [29-31].

2.2 Formulation for notched structural components

In the present paper, an extension of the above criterion is formulated by implementing the concept of the control volume of the strain energy density criterion proposed by Lazzarin et al. [23-28]. In particular, the radii R_1 and R_3 of such a control volume, related to the loading conditions of Mode I and Mode III, respectively, can be computed by means of the following expressions:

$$R_1 = \left(\sqrt{2e_1} \cdot \frac{\Delta K_{1A}}{\Delta\sigma_{1A}}\right)^{\frac{1}{1-\lambda_1}} \tag{8a}$$

$$R_3 = \left(\sqrt{\frac{e_3}{1+\nu}} \cdot \frac{\Delta K_{3A}}{\Delta\tau_{3A}}\right)^{\frac{1}{1-\lambda_3}} \tag{8b}$$

where e_1 and e_3 are two parameters that summarise the dependence on the notch geometry [28], ν is the elastic Poisson ratio, and λ_1 and λ_3 are the eigenvalues of Mode I and Mode III, respectively. Moreover, the control radii are functions of the mean values of Mode I and Mode III Notch Stress Intensity Factors (NSIFs) ranges (ΔK_{1A} and ΔK_{3A}, respectively) and of the high-cycle fatigue strengths of smooth specimens ($\Delta\sigma_{1A}$ for Mode I and $\Delta\tau_{3A}$ for Mode III), all referred to the same reference number of loading cycles to failure (N_A).

According to the control volume radii concept applied to the above criterion, the fatigue strength assessment is carried out at point P (verification point), which is distant r from the notch tip. Such a distance r, measured along the notch bisector line, is here proposed to be directly linked to the mean control volume radius, R_m, computed by averaging the control volume radius related to Mode I, R_1, and that related to Mode III, R_3:

$$r = -(0.221)^{\lambda - 1.484} \cdot R_m + 11.3 R_m \tag{9}$$

where λ is the biaxiality ratio, defined as the ratio between the remote applied shear stress amplitude and the normal stress amplitude.

Then, the steps discussed in the previous Sub-Section have to be followed to perform the fatigue life estimation.

3. FATIGUE EXPERIMENTAL CAMPAIGN

The strain-based multiaxial fatigue criterion formulated for notched specimens is here applied to a set of data, recently published in the literature [28]. In particular, a large bulk of uniaxial and multiaxial tests have been carried out on V-notched round bars made of grade 5 titanium alloy (Ti-6Al-4V). Figure 3a displays the geometry of the notched specimens tested.

Each cylindrical notched specimen presented a V-notch depth equal to 6 mm and an opening angle equal to 90 degrees, whereas the notch root radius, ρ, was lower than 0.1 mm (Figure 3).

Figure 3.

As is reported in Ref. [28], the above tests were performed by using a MTS 809 servo-hydraulic biaxial machine, with a 100 kN axial load cell and a torsion load cell of 1100 Nm. All tests were conducted under load control at a frequency from 10 to 15 Hz, as a function of the applied load. The details of the tests are reported in the original work [28]. The experimental data numerically examined in the following are shown in Tables 1 to 4.

Tables 1 - 4.

According to Eq. (8), the values of the volume control radii are evaluated by taking:

(a) The parameters e_1 and e_3, which are linked to the integral of the stress functions inside the control volumes, are equal to 0.146 and 0.310, respectively (details on the values of e_1 and e_3 are provided in [23]);

(b) The elastic Poisson ratio $\nu = 0.3$;

(c) The eigenvalues λ_1 and λ_3 equal to 0.545 and 0.667, respectively;

(d) The mean values of the NSIFS, $\Delta K_{1A} = 452$ MPa mm$^{0.445}$ (Mode I) and $\Delta K_{3A} = 1216$ MPa mm$^{0.333}$ (Mode III) (with the number of loading cycles to failure N_A equal to 2 $(10)^6$);

(e) The high-cycle fatigue nominal stress range of smooth specimens, $\Delta\sigma_{1A} = 950$ MPa (Mode I) and $\Delta\tau_{3A} = 776$ MPa (Mode III) (with the number of loading cycles to failure N_A equal to 2 $(10)^6$) [28].

According to these parameter values, R_1 is equal to 0.051 mm, whereas R_3 is equal to 0.837 mm. These differences in the control volume can be explained by the different behaviour in the crack propagation life under tension and torsion loading, but also by the fact that plasticity under torsion is usually much higher around the notch tip as is discussed in Ref. [25]. Further investigations are needed to better explain this behaviour by monitoring more precisely the crack initiation and propagation phases.

Data related to notched specimens and examined in the present work are shown in terms of SED in Figure 4. The scatter band includes all the data under pure tension, pure torsion and multiaxial loading, regardless the phase angle. The unifying capacity of the SED approach can easily be observed: as a matter of fact, such an approach is capable of synthesize all the fatigue strength data in a single quite-narrow scatter band regardless of the loading mode.

Figure 4.

4. CRITERION VALIDATION

Now the strain-based multiaxial fatigue criterion is employed for fatigue life estimation of the V-notched specimens made of Ti-6Al-4V titanium alloy, whose experimental campaign is examined in Section 3.

The data series here discussed are characterised by both nominal load ratio equal to -1.0 and experimental fatigue life lower than 400,000 loading cycles (LCF regime). More precisely, we examine:

(1) Two series of tests on V-notched specimens under pure tension and pure torsion fatigue loading (Tables 1 and 2);

(2) Two series of tests under combined tension and torsion loading, with a constant biaxiality ratio $\lambda = 0.6$ and phase angle $\Phi =$ 0° (in-phase loading) or $\Phi =$ 90° (out-of-phase loading) (Tables 3 and 4).

Different values of the distance r to determine the position of the verification point P are computed through Eq.(9):

(a) for pure tension loading, that is $\lambda = 0$, r is equal to $1.9 \cdot R_m$;

(b) for pure torsion loading, that is $\lambda = \infty$, r is equal to $11.3 \cdot R_m$;

(c) for combined in- and out-of-phase tension and torsion loading, that is for the data examined ($\lambda = 0.6$), r is equal to $7.5 \cdot R_m$.

Note that Eq.(9) here proposed, describing as the distance r increases with the biaxiality ratio λ, has been obtained from a best-fit procedure. The data-points to be interpolated have been determined (for some values of λ related to the experimental tests reported in Ref. [28]) by optimising the following error index, I:

$$I = \frac{\sqrt{(\eta_{N,a})^2 + \left(\frac{\varepsilon_a}{\gamma_a}\right)^2 (\eta_{C,a})^2} - \varepsilon_a}{\varepsilon_a} \tag{10}$$

ε_a and γ_a being defined by Eq.(2).

In order to estimate the fatigue lifetime of the above V-notched specimens, the strain tensor at the verification point P is obtained from a linear finite element analysis, by employing the commercial software Straus7®. In more detail, the strain state at point P is determined through a tridimensional model for each investigated fatigue test. The effective Poisson ratio, ν_{eff}, is assumed to be equal to the elastic Poisson ratio ($\nu = 0.3$).

The material parameters of the Manson-Coffin curves are shown in Table 5 [40,41]. Note that the parameters of the tensile curve are reported in Ref. [41], whereas the corresponding ones for the torsional curve are evaluated by using the von Mises criterion [42].

Table 5.

After the strain state and the above material parameters are deduced, the proposed strain-based criterion can directly be applied to the experimental data in order to estimate the fatigue lifetime.

Figure 5 shows the diagram of the experimental, $N_{f,exp}$, against the theoretical, N_f, fatigue life. Note that 88% of the theoretical results fall within the scatter band 3, and this holds true independent of the applied loading conditions (that is, for both uniaxial and multiaxial fatigue tests).

Figure 5.

Figure 6 shows the experimental fatigue life plotted against the equivalent strain amplitude $\varepsilon_{eq,a}$ (see Eq.(7)). The solid curve corresponds to the experimental Manson-Coffin equation related to tensile loading (Eq. (2a)). The satisfactory accuracy level of the proposed strain-based criterion in estimating notch fatigue lifetime can be deduced from Fig.6, where the theoretical data lie very close to the experimental curve.

Figure 6.

6. CONCLUSION

A strain-based multiaxial fatigue criterion previously proposed for smooth specimens has been here extended to notched specimens. This criterion is a reformulation of the stress-based multiaxial high-cycle fatigue (HCF) criterion by Carpinteri and Spagnoli.

The extension herein formulated is connected to energy concepts: as a matter of fact, it considers as the critical point (where to perform the fatigue assessment) the material point located at a certain distance from the notch tip, depending such a distance on the control volume radii under Mode I and Mode III, related to the strain energy density criterion proposed by Lazzarin and co-authors.

Some multiaxial fatigue data available in the literature have been here analysed by employing the proposed formulation. In particular, specimens made of titanium grade 5 alloy (Ti-6Al-4V) and weakened by a sharp notch have been examined to validate the proposed criterion.

The agreement between experimental data and estimated fatigue lives is quite satisfactory and promising for application to experimental tests characterised by fatigue ratio different from -1.0.

REFERENCES

[1] A. Fatemi, N. Shamsaei, Multiaxial fatigue: An overview and some approximation models for life estimation, Int. J. Fatigue. 33 (2011) 948–958.

[2] A. Nieslony, C.M. Sonsino, Comparison of some selected multiaxial fatigue assessment criteria, 2008.

[3] A. Fatemi, D.F. Socie, A critical plane approach to multiaxial fatigue damage including out-of-phase loading, Fatigue Fract. Eng. Mater. Struct. 11 (1988) 149–165.

[4] A. Fatemi, P. Kurath, Multiaxial Fatigue Life Predictions Under the Influence of Mean-Stresses, J. Eng. Mater. Technol. 110 (1988) 380.

[5] J. Li, C. Li, Y. Qiao, Z. Zhang, Fatigue life prediction for some metallic materials under constant amplitude multiaxial loading, Int. J. Fatigue. 68 (2014) 10–23.

[6] K. Kluger, Fatigue life estimation for 2017A-T4 and 6082-T6 aluminium alloys subjected to bending-torsion with mean stress, Int. J. Fatigue. 80 (2015) 22–29.

[7] L. Susmel, R. Tovo, D.F. Socie, Estimating the orientation of Stage I crack paths through the direction of maximum variance of the resolved shear stress, Int. J. Fatigue. 58 (2014) 94–101.

[8] A. Carpinteri, A. Karolczuk, E. Macha, S. Vantadori, Expected position of the fatigue fracture plane by using the weighted mean principal Euler angles, Int. J. Fract. 115 (2002) 87–99.

[9] K. Walat, T. Łagoda, Lifetime of semi-ductile materials through the critical plane approach, Int. J. Fatigue. 67 (2014) 73–77.

[10] A. Karolczuk, E. Macha, A Review of Critical Plane Orientations in Multiaxial Fatigue Failure Criteria of Metallic Materials, Int. J. Fract. 134 (2005) 267–304.

[11] Z. Marciniak, D. Rozumek, E. Macha, Verification of fatigue critical plane position according to variance and damage accumulation methods under multiaxial loading, Int. J. Fatigue. 58 (2014) 84–93.

[12] V. Anes, L. Reis, B. Li, M. Freitas, C.M. Sonsino, Minimum

Circumscribed Ellipse (MCE) and Stress Scale Factor (SSF) criteria for multiaxial fatigue life assessment, Theor. Appl. Fract. Mech. 73 (2014) 109–119.
[13] A. Bolchoun, J. Wiebesiek, H. Kaufmann, C.M. Sonsino, Application of stress-based multiaxial fatigue criteria for laserbeam-welded thin aluminium joints under proportional and non-proportional variable amplitude loadings, Theor. Appl. Fract. Mech. 73 (2014) 9–16.
[14] D. Ye, O. Hertel, M. Vormwald, A unified expression of elastic–plastic notch stress–strain calculation in bodies subjected to multiaxial cyclic loading, Int. J. Solids Struct. 45 (2008) 6177–6189.
[15] T.M. Jasper, The value of the energy relation in the testing of ferrous metals at varying ranges of stress and at intermediate and high temperatures, Philos. Mag. Ser. 6. 46 (1923) 609–627.
[16] F. Ellyin, Cyclic strain energy density as a criterion for multiaxial fatigue failure, Biaxial an, EGF Publication, London, 1989.
[17] F. Ellyin, Fatigue damage, crack growth and life prediction, Edmonton: Chapman and Hall; 1997.
[18] E. Macha, C.M. Sonsino, Energy criteria of multiaxial fatigue failure, Fatigue Fract. Eng. Mater. Struct. 22 (1999) 1053–1070.
[19] J. Park, D. Nelson, Evaluation of an energy-based approach and a critical plane approach for predicting constant amplitude multiaxial fatigue life, Int. J. Fatigue. 22 (2000) 23–39.
[20] D. Kujawski, A deviatoric version of the SWT parameter, Int. J. Fatigue. 67 (2014) 95–102.
[21] D. Kujawski, P.C.R. Sree, On deviatoric interpretation of Neuber’s rule and the SWT parameter, Theor. Appl. Fract. Mech. 71 (2014) 44–50.
[22] P. Lazzarin, R. Zambardi, A finite-volume-energy based approach to predict the static and fatigue behavior of components with sharp V-shaped notches, Int. J. Fract. 112 (2001) 275–298.
[23] P. Lazzarin, C.M. Sonsino, R. Zambardi, A notch stress intensity approach to assess the multiaxial fatigue strength of

welded tube-to-flange joints subjected to combined loadings, Fatigue Fract. Eng. Mater. Struct. 27 (2004) 127–140.

[24] B. Atzori, F. Berto, P. Lazzarin, M. Quaresimin, Multi-axial fatigue behaviour of a severely notched carbon steel, Int. J. Fatigue. 28 (2006) 485–493.

[25] F. Berto, P. Lazzarin, J.R. Yates, Multiaxial fatigue of V-notched steel specimens: A non-conventional application of the local energy method, Fatigue Fract. Eng. Mater. Struct. 34 (2011) 921–943.

[26] F. Berto, P. Lazzarin, C. Marangon, Fatigue strength of notched specimens made of 40CrMoV13.9 under multiaxial loading, Mater. Des. 54 (2014) 57–66.

[27] F. Berto, P. Lazzarin, R. Tovo, Multiaxial fatigue strength of severely notched cast iron specimens, Int. J. Fatigue. 67 (2014) 15–27.

[28] F. Berto, A. Campagnolo, P. Lazzarin, Fatigue strength of severely notched specimens made of Ti-6Al-4V under multiaxial loading, Fatigue Fract. Eng. Mater. Struct. 38 (2015) 503–517.

[29] A. Carpinteri, A. Spagnoli, C. Ronchei, S. Vantadori, A Strain-based Multiaxial Fatigue Criterion Connected to the Critical Plane Approach, Procedia Eng. 74 (2014) 317–320.

[30] A. Carpinteri, C. Ronchei, A. Spagnoli, S. Vantadori, Lifetime estimation in the low/medium-cycle regime using the Carpinteri–Spagnoli multiaxial fatigue criterion, Theor. Appl. Fract. Mech. 73 (2014) 120–127.

[31] A. Carpinteri, C. Ronchei, D. Scorza, S. Vantadori, Fatigue life estimation for multiaxial low-cycle fatigue regime: The influence of the effective Poisson ratio value, Theor. Appl. Fract. Mech. 79 (2015) 77–83.

[32] A. Carpinteri, A. Spagnoli, Multiaxial high-cycle fatigue criterion for hard metals, Int. J. Fatigue. 23 (2001) 135–145.

[33] A. Carpinteri, A. Spagnoli, S. Vantadori, Multiaxial fatigue assessment using a simplified critical plane-based criterion, Int. J. Fatigue. 33 (2011) 969–976.

[34] A. Carpinteri, A. Spagnoli, S. Vantadori, C. Bagni, Structural integrity assessment of metallic components under multiaxial

fatigue: the C-S criterion and its evolution, Fatigue Fract. Eng. Mater. Struct. 36 (2013) 870–883.

[35] C. Ronchei, A. Carpinteri, G. Fortese, A. Spagnoli, S. Vantadori, M. Kurek, et al., Life estimation by varying the critical plane orientation in the modified Carpinteri-Spagnoli criterion, Frattura ed Integrità Strutturale 34 (2015) 74-79.

[36] L. Susmel, Multiaxial notch fatigue, Woodhead publishing, Cambridge, 2009.

[37] J.A. Araújo, A. Carpinteri, C. Ronchei, A. Spagnoli, S. Vantadori, An alternative definition of the shear stress amplitude based on the Maximum Rectangular Hull method and application to the C-S (Carpinteri-Spagnoli) criterion, Fatigue Fract. Eng. Mater. Struct. 37 (2014) 764–771.

[38] A. Carpinteri, C. Ronchei, A. Spagnoli, S. Vantadori, On the use of the Prismatic Hull method in a critical plane-based multiaxial fatigue criterion, Int. J. Fatigue. 68 (2014) 159–167.

[39] I. Papadopoulos, Critical plane approaches in high-cycle fatigue: on the definition of the amplitude and mean value of the shear stress acting on the critical plane, Fatigue Fract. Eng. Mater. Struct. 21 (1998) 269–285.

[40] R. Basan, M. Franulović, I. Prebil, N. Črnjarić-Žic, Analysis of strain-life fatigue parameters and behaviour of different groups of metallic materials, Int. J. Fatigue. 33 (2011) 484–491.

[41] N.G. Bourago, A.B. Zhuravlev, I.S. Nikitin, Models of multiaxial fatigue fracture and service life estimation of structural elements, Mech. Solids. 46 (2012) 828–838.

[42] K. Kim, X. Chen, C. Han, H. Lee, Estimation methods for fatigue properties of steels under axial and torsional loading, Int. J. Fatigue. 24 (2002) 783–793.