Early fretting crack orientation by using the critical plane approach

Submitted to **International Journal of Fatigue**

January 2018

Revised version R1 – April 2018

EARLY FRETTING CRACK ORIENTATION BY USING THE CRITICAL PLANE APPROACH

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ABSTRACT

In the present paper, the novel critical direction method by Araújo et al. and the multiaxial critical plane-based criterion by Carpinteri et al. are combined together to estimate early crack orientation in configurations involving high stress gradients, as fretting fatigue configurations. More precisely, the first method is used to compute the input data for the second one, in terms of normal and shear stresses over a line with a characteristic length. The experimental data herein analysed are related to an Al7050 T7451 aluminium alloy, the fretting tests related to a cylinder-on-plane configuration being performed by the Research Group of Fatigue, Fracture and Materials at the University of

Brasilia. Results show that the estimated values of the crack initiation plane fall in the interval defined by the mean experimental values and the standard deviation for each test configuration examined.

KEYWORDS: Al7050 T7451, critical direction, critical plane, early cracking, fretting, multiaxial fatigue.

NOMENCLATURE

a	contact semi-width
B	static bulk load
C_a	shear stress amplitude
f	coefficient of friction
$h[\ldots]$	Heaviside step function
L	El Haddad intrinsic crack length
N_a	normal stress amplitude
N_m	normal stress mean value
p_0	contact peak pressure
P	normal contact load per unit length
Q	cyclic shear fretting load per unit length
Q_a	amplitude of the cyclic shear fretting load
R	radius of the cylindrical fretting pad
t	time
T	period
$\mathbf{w}$	normal unit vector perpendicular to the critical plane
W	weight function

$1, 2, 3$	principal stress directions
δ	angle between the mean direction $\hat{1}$ and the normal $\mathbf{w}$ to the critical plane
$\Delta K_{I,th}$	threshold range of the stress intensity factor for long cracks
θ_{exp}	experimental crack initiation plane orientation
θ_{anal}	analytical crack initiation plane orientation
σ	standard deviation
$\sigma_1, \sigma_2, \sigma_3$	principal stresses
$\sigma_{af,-1}$	fully reversed normal stress fatigue limit
σ_B	static bulk stress
σ_u	ultimate tensile strength
$\tau_{af,-1}$	fully reversed shear stress fatigue limit
ϕ, θ, ψ	principal Euler angles

1. INTRODUCTION

Many metallic structural components used in both automotive and aircraft industries are subjected to time-varying multiaxial stress states in service, and are generally designed to operate in high-cycle fatigue regime. The fatigue strength of the above components is difficult to be estimated, since they usually have complex geometries and operate under multiaxial loading. Further, such a strength depends on material properties, size effects, stress concentration and stress gradient **[1,2]**.

Most high-cycle fatigue criteria are stress-based, and aim to reduce the actual multiaxial stress state to an equivalent uniaxial one, which is then compared to the uniaxial fatigue strength of the material, at a specified number of loading cycles **[3]**. Some of such criteria are based on the so-called, critical plane approach, which allows us both to perform the fatigue strength (or life) assessment and to determine the orientation of the crack in its early stage **[4-10]**.

In such a context, the Carpinteri et al. criterion **[11]** computes the critical plane orientation by averaging the principal stress directions over a period and, as fatigue damage parameter, employs a nonlinear combination of the equivalent normal stress amplitude and the shear stress amplitude, both of them acting on the so-called critical plane. The shear stress amplitude can be evaluated through the novel Maximum Rectangular Hull (MRH) method proposed by Araújo et al. **[12]**, which is able to capture the effect of non-proportional stress histories projected onto the critical material plane.

The term fretting designates a phenomenon that occurs at the contact interface between two assembly components, under a loading that gives rise to a slight oscillatory relative displacement (typically, of the order of microns) of the contact surfaces. When a cyclic bulk fatigue loading is applied to one or both of the components, the assembly can fail because of fretting fatigue **[13]**.

Note that the cyclic stress history in fretting fatigue is invariably non-proportional due to the non-linear characteristic of the frictional contact configuration. Further, a number of assembly components under fretting conditions operate in high-cycle fatigue regime **[13]**.

Under such a loading condition, crack initiation is greatly encouraged, and may lead to failure of mechanical assemblies, like dovetail connections between disc and blade in aeroengines, suspension clamp/conductor cable arrangements, orthopaedic joint replacement implants, and so on.

Several studies, which aim to estimate the early fretting crack orientation, i.e. the crack initiation plane, are available in the literature. Araújo and Mamiya **[14]** used the Dang Van mesoscopic fatigue criterion (i.e, a deviatoric stress tensor-based criterion) in order to examine crack initiation in specimens made of Aluminum alloy (Al4%Cu) fretted by cylindrical pads made of the same material. Fouvry et al. **[15]** used the Crossland high-cycle multiaxial criterion (i.e. a stress invariant-based criterion) to evaluate the risk of crack initiation in Ti-6Al-4V alloy specimens under cylindrical contact. Ferre et al. **[16]**, keeping the ratio of shear and normal contact loads constant and varying the fatigue stress, studied the influence of both the plasticity and the ratio between the tensile and shear fatigue limits on the crack initiation in a Ti-6V-4Al alloy under fretting conditions.

The crack initiation plane can be also estimated by exploiting the critical plane approach. Araújo and Nowell **[17]** performed tests on flat specimens made of two different aeronautical alloys

(Al4%Cu and Ti-6Al-4V), under cylindrical contact. In that research work, different criteria based on the critical plane approach were employed. Namjoshi et al. **[18]** carried out both experimental and analytical investigations on the crack initiation process. Flat specimens, made of Ti-6Al-4V alloy, were used together with two different pad types (cylindrical and flat center with rounded edges), and several critical plane-based criteria were applied. Vázquez et al. **[19]** performed fretting fatigue tests on Al7075-T651 alloy specimens with cylinder-on-flat contact configuration, and compared the experimental results (in terms of crack initiation plane) with the estimations determined through two different multiaxial criteria based on the critical plane approach.

In the present paper, the novel critical direction method by Araújo et al. **[20]** and the multiaxial critical plane-based criterion by Carpinteri et al. **[11]** are combined to estimate early crack orientation in fretting fatigue configurations. Such a method together with the multiaxial criteria of Smith-Watson-Topper, Fatemi-Socie, and the Modified Wöhler Curve Method was successfully applied in previous works **[20,21]** to estimate the crack initiation plane in specimens made of AISI 1034 and 35NCD16, under cylindrical contact.

The experimental campaign herein examined have been carried out at the University of Brasília **[21]**. The tests have been performed by varying firstly the tangential load, secondly the mean value of the bulk load applied to the specimen, and then the pad radius.

2. THE CARPINTERI ET AL. CRITERION

The multiaxial high-cycle fatigue criterion by Carpinteri et al. **[11]** is based on the so-called critical plane approach. According to such an approach, the fatigue failure analysis consists of two steps detailed in the following: Step I, where the critical plane is theoretically determined; Step II, where the fatigue failure assessment is carried out in such a plane.

2.1 Step I: critical plane determination

Three stages may be distinguished in the fatigue fracture process: crack initiation, crack propagation, and final failure **[22]**.

The crack initiation stage is characterised by irreversible sub-microscopic changes in the metal crystal lattice: such changes are named dislocations, and consist in crystallographic facets caused by cyclic shearing forces. As cyclic loading continues to be applied, such shearing forces produce a dislocation slipping inside the crystal, commonly along a plane oriented at 45° with respect to the loading (crack initiation plane). Note that, when this slipping involves many other crystals, a macroscopic crack forms due to joining of dislocations.

In the crack propagation stage under uniaxial load, the crack orientation changes from 45° to 90° with respect to the maximum principal stress direction (i.e. direction of the applied loading), and such a crack propagation occurs on a plane named final fracture plane. The failure stage occurs because of the cross-section reduction produced by fatigue crack propagation. According to such experimental evidences, Carpinteri et al. **[11, 23-27]** proposed to assume: (a) as *crack initiation plane*, a plane whose orientation is linked to that of the final fracture plane through a function that takes into account the fatigue material properties (that is, the fully reversed normal and shear stress fatigue limits); (b) as *final fracture plane*, a plane whose orientation is linked to averaged principal stress directions **[11, 23-29]**.

Let us consider a generic time instant, t, of fatigue load history ($0 \leq t \leq T$, where T is the period of the loading cycle). At such an instant, the principal stresses σ_1, σ_2, and σ_3 ($\sigma_1 \geq \sigma_2 \geq \sigma_3$) and the corresponding directions 1, 2, and 3 (identified by the principal Euler angles, ϕ, θ, and ψ) can be computed. However, since such quantities are generally time-varying under fatigue loading, Carpinteri et al. **[25, 28-29]** proposed to compute mean principal stresses, $\hat{\sigma}_1$, $\hat{\sigma}_2$, and $\hat{\sigma}_3$. The corresponding directions, $\hat{1}$, $\hat{2}$, and $\hat{3}$, are identified by the mean principal Euler angles, $\hat{\phi}$, $\hat{\theta}$, and $\hat{\psi}$, given by:

$$\hat{\phi} = \frac{1}{W}\int_0^T \phi(t) \cdot W(t)\, dt \qquad \hat{\theta} = \frac{1}{W}\int_0^T \theta(t) \cdot W(t)\, dt \qquad \hat{\psi} = \frac{1}{W}\int_0^T \psi(t) \cdot W(t)\, dt \tag{1}$$

where

$$W = \int_0^T W(t)\, dt \tag{2a}$$

$$W(t) = h[\sigma_1(t) - \sigma_{1,\max}] \tag{2b}$$

being $h[\ldots]$ the Heaviside step function ($h[x]=1,\ x \geq 0;\quad h[x]=0,\ x<0$), and $\sigma_{1,\max}$ is the maximum value achieved by $\sigma_1(t)$ during a loading cycle.

According to Carpinteri et al. criterion **[11, 23-27]**, the normal to the final fracture plane is assumed to coincide with the $\hat{1}$-direction, identified by the mean Euler angles $\hat{\phi}$ and $\hat{\theta}$. Further, the normal $\mathbf{w}$ to the crack initiation plane is assumed to be linked to the above $\hat{1}$-direction through the off-angle, δ, given by this empirical expression:

$$\delta = \frac{3}{2}\left[1 - \left(\frac{\tau_{af,-1}}{\sigma_{af,-1}}\right)^2\right] 45° \tag{3}$$

where $\sigma_{af,-1}$ is the fully reversed normal stress fatigue limit, and $\tau_{af,-1}$ is the fully reversed shear stress fatigue limit.

Such a rotation δ is generally performed in the $\hat{1}\hat{3}$ plane. Note that, for plane problems (as that herein examined), such a rotation has a physical meaning when it is performed in the plane containing the stress tensor components, i.e. in the $\hat{1}\hat{3}$ plane when $\hat{\sigma}_2$ is equal to zero, or in the $\hat{1}\hat{2}$ plane when $\hat{\sigma}_3$ is equal to zero.

The rotation δ is generally performed from $\hat{1}$ to $\hat{3}$. Note that, when the problem examined is characterised by a symmetry involving both the geometry and the loading configuration, the direction of δ is arbitrary, whereas experimental evidence related to crack initiation plane has to be taken into account to define the rotational direction in the case of an anti-symmetry (as that herein examined).

According to the Carpinteri et al. criterion, the critical plane is that of crack initiation.

2.2 Step II: fatigue failure assessment

The fatigue failure assessment is carried out in the critical plane, by defining an equivalent uniaxial stress whose amplitude is expressed as follows:

$$\sigma_{eq,a}=\sqrt{N_{a,eq}^2+\left(\frac{\sigma_{af,-1}}{\tau_{af,-1}}\right)^2 C_a^2} \qquad \textbf{(4)}$$

with

$$N_{a,eq} = N_a + \sigma_{af,-1}\left(\frac{N_m}{\sigma_u}\right) \qquad \textbf{(5)}$$

where N_a and N_m are the amplitude and mean value of the normal stress, respectively, C_a is the shear stress amplitude, and σ_u is the material ultimate tensile strength.

Note that Eq.(5) considers the linear relationship between N_a and N_m, proposed by Goodman, which allows us to take into account the strong decrease of metal resistance when a tensile mean normal stress is superimposed on an alternating normal stress. Therefore, in the case of a compressive mean normal stress, the criterion implements the conservative condition to put the N_m value equal to zero.

The procedure to compute C_a is the algorithm proposed by Araújo et al. **[30]**, named Maximum Rectangular Hull Method, according to which the shear stress amplitude is given by **[30,31]**:

$$C_a(\xi) = \max_{0^\circ \leq \xi < 90^\circ} \left\{ \sqrt{C_{1,a}^2(\xi) + C_{2,a}^2(\xi)} \right\} \qquad \textbf{(6)}$$

where $C_{1,a}$ and $C_{2,a}$ are resolved shear stress amplitudes with respect to a rotated frame system on the critical plane, and ξ is the rotational angle. Such an algorithm is characterised by a simple numerical implementation, because it involves only axes rotations, and is sensitive to the shear stress path shape.

Note that, when σ_u is not available, the yield stress f_y might be used instead of the ultimate tensile strength, especially in the case of mild metals.

The multiaxial fatigue limit condition is achieved when the equivalent uniaxial stress amplitude, $\sigma_{eq,a}$, is equal to the fully reversed normal stress fatigue limit, $\sigma_{af,-1}$.

3. THE CRITICAL DIRECTION METHOD BY ARAÚJO ET AL.

The method by Araújo et al. **[20]** together with any critical plane-based criterion allows us to estimate the orientation of the crack initiation plane, that is to say, the orientation of the critical plane.

Generally, the critical plane is determined by examining the fatigue load history at the critical point, usually located on the surface of the structural component. As a matter of fact, surface cracks are common since both defects and the highest stresses (hot spot) are frequently on the outer boundary of the body.

For high stress gradients, typical of fretting fatigue configurations, it would be more appropriate to analyse suitable quantities summarizing the stress field at the process zone instead of examining the fatigue load history related to a single point (critical point). In such a way, it would be avoided the situation where the critical planes related to adjacent material points can have completely different orientations. As a matter of fact, from a mechanical point of view it looks sensible that the crack initiation direction under a rapidly varying stress field be controlled not by the state of stress in a single material point, but by an average stress condition within a zone **[17]**. In such a contest, the critical direction method by Araujo et al. **[20]** establishes that, for remarkable stress gradients, the concept of a critical direction along a line (in two-dimensional problems) is more appropriate than a critical plane. This means that the critical plane criteria should be applied as critical direction models.

Let us consider the fretting fatigue configuration shown in **Figure 1**, consisting in a cylinder-on-flat contact. The specimen is held between a fixed and a movable jaw. A normal constant contact load per unit length, P, and a cyclic shear fretting load $Q(t)$ per unit length are applied to the pads. A static bulk load that produces a constant tensile stress, σ_B, is applied to the specimen.

Figure 1.

Firstly, by maximizing the fatigue damage, the crack initiation point H is determined on the contact surface ($-a \le x \le +a$, being a the contact semi-width in **Figure 1**). Note that the quantities linked to a given critical plane-based criterion can be employed in the damage evaluation, as for example by means of an equivalent uniaxial stress amplitude.

Then the line emanating from such a point, with orientation θ, is considered (**Figure 1**), and the segment $2L$ is taken into account, where L is the El Haddad intrinsic crack length **[31]**:

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{I,th}}{2 \cdot \sigma_{af,-1}}\right)^2 \quad \textbf{(7)}$$

being $\Delta K_{I,th}$ the threshold range of the stress intensity factor for long cracks.

During the period T, the fatigue load history of normal stress and that of shear stress along such a line can be computed by employing an analytical formulation or a numerical method. The quantities to describe the fatigue stress field along this line are: the maximum and the minimum values of the normal and the shear stresses. Finally, the high-cycle fatigue parameter is computed according to the critical plane-based criterion adopted.

Such a procedure is repeated by varying the orientation θ from -90° to +90° (**Figure 1**) and, therefore, the crack initiation plane is identified by that orientation for which the above fatigue parameter is maximized.

4. THE CRITICAL DISTANCE METHOD COMBINED WITH THE CARPINTERI ET AL. CRITERION

Firstly, the crack initiation point H is determined on the contact surface ($-a \le x \le +a$, **Figure 1**) by maximizing the fatigue damage computed through the equivalent stress amplitudes expressed by Eqs(4) and (5). According to such a maximisation, H coincides with the intersection between specimen and free surface of the cylinder (trailing edge).

Then the segment $2L$ starting from such a point is considered. During the period T, the maximum and minimum value of both normal and shear stress are computed and, finally, the high-cycle fatigue parameter is evaluated according to Eqs **(4)** and **(5)**.

Note that, since the calculations involve quantities related to planes with the fixed orientation θ, the procedure described in Sub-section 2.1 to determine the critical plane does not need to be applied.

When the problem analysed is plane as in the case here examined, the procedure described in Sub-section 2.2 to determine the shear stress amplitude does not need to be applied, because the shear stress path is represented by a segment and, therefore, C_a is computed as the half length of such a segment.

Finally, $\sigma_{eq,a}$ is computed according to Eqs **(4)** and **(5)**, by varying the angle θ.

Different from what is discussed in Section 3, the crack initiation plane is here proposed to be identified by the angle θ that satisfies the fatigue limit condition according to the Carpinteri et al. criterion ($\sigma_{eq,a} = \sigma_{af,-1}$). If more than one orientation satisfies such a limit condition, the crack initiation plane is taken as that maximizing the amplitude of the maximum principal stress, $\sigma_{1,a}$, computed by using the maximum and minimum principal stress values along each of the above orientations.

Note that, the above quantities ($\sigma_{eq,a} = \sigma_{af,-1}$ and $\sigma_{1,a}$) can be related to both different material points along a given line and different time instants during the period.

5. EXPERIMENTAL CAMPAIGN EXAMINED

The experimental tests herein examined have been carried out at the University of Brasilia **[21]** using a cylinder-on-flat contact configuration **(Figure 1)**. Flat-dog bone specimens and cylindrical pads were made of Al7050 T7451 aluminium alloy. The values of the pad radius R are listed in **Table 1** for the six test configurations analysed, whereas the material mechanical and fatigue properties are shown in **Table 2**.

Table 1.

Table 2.

The tests have been performed by employing a custom MTS servo-hydraulic machine (**Figure 2**) equipped with two independent hydraulic actuators, which allow us to separately apply tangential/fretting (i.e. Q in **Figure 1**) and bulk loads (i.e. B; such a loading B divided by the cross-section area provides the static bulk stress σ_B in **Figure 1**). More precisely, the 100 kN upper actuator (No.1 in **Figure 2**) has produced the tangential load Q, whereas the 250 kN servo-hydraulic actuator (No.3 in **Figure 2**) has applied the bulk load B to the specimen.

Figure 2.

The normal static contact load per unit length P has been applied through the ENERPAC hydraulic system (**Figure 3**) composed by a hydraulic hand pump Ultima P-39 (No.1 in **Figure 3**), a coupler

ACBS-202 (No.2 in **Figure 3**), a hydraulic accumulator ACL202 (No.3 in **Figure 3**), and a single acting cylinder RC-51 (No.4 in **Figure 3**).

In order to avoid the offset of the stick regime, the first load applied has been the static bulk load B, then P and finally Q. All experiments have been performed in the partial slip regime, i.e. $Q_a \le f \cdot P$, where f is the coefficient of friction in the slip zone.

The values of P, Q_a, and σ_B are listed in **Table 1.** The computed values of a and p_0 are obtained from the following expressions:

$$a = \sqrt{\frac{4PR}{\pi E}} \quad \textbf{(8a)}$$

$$p_0 = \frac{2P}{\pi a} \quad \textbf{(8b)}$$

are also shown in **Table 1.**

The tests, running at a frequency equal to 10Hz, have been stopped at a number of loading cycles equal to 10^6. Then the specimens have been cut along the centerline of the contact zone, embedded in a phenolic resin, polished, chemically etched. Finally, the orientation of the main crack path observed through a confocal laser microscope Olympus LEXT OLS4100.

6. RESULTS

6.1 Experimental results

The crack path is described by means of the angle θ_{exp} formed between the line perpendicular to the contact surface and the crack orientation. Such a crack orientation angle is considered positive if counterclockwise. An example of a microscopic analysis is shown in **Figure 4** related to the test configuration No.4, consisting of two tests.

The crack initiation point corresponds to point H in **Figure 1**, for each test configuration examined. The θ_{exp} values, mean values $\theta_{exp,m}$, and standard deviations σ are listed in **Table 3**.

6.2 Estimates of crack initiation path

The stress field in the vicinity of the contact zone is directly evaluated by means of analytical closed-form expressions **[32]** according to the solutions by Hertz **[33]** and Mindlin **[34]**, as is detailed below.

The crack initiation point corresponds to point H in **Figure 1**, for each examined test configuration.

In **Figure 5**, the amplitude values of both the equivalent stress and the maximum principal stress are plotted by varying the angle θ. The expected orientation of the crack initiation plane, θ_{anal}, is represented by the abscissa of the intersection point between the curve $\sigma_{eq,a}$ and the straight line $\sigma_{af,-1}$. In the case of more than one intersection point, the solution is represented by the point with the highest value of $\sigma_{1,a}$. The expected orientation values are reported in **Table 3**.

Figure 5.

It can be remarked a quite satisfactory agreement between experimental and analytical results, since θ_{anal} falls in the interval $\theta_{exp,m} \pm \sigma$ for each test examined.

7. CONCLUSIONS

In the present paper, the novel critical direction method by Araújo et al.**[20]** and the multiaxial critical plane-based criterion by Carpinteri et al.**[11]** have been combined together to estimate early crack orientation in the case of high stress gradients.

To validate the analysis, experimental fretting data using a cylinder-on-plane configuration, made of an Al7050 T7451 alloy, have been generated. The test program has varied the tangential load, the mean value of the bulk stress, and the pad radius to capture their influence on the crack initiation path. All tests were performed under the partial slip regime.

The comparison between the orientation of the expected crack initiation plane, θ_{anal}, and that of the experimental one, θ_{exp}, is quite satisfactory since the estimated θ_{anal} value falls in the interval defined by the mean value $\theta_{exp,m}$ and the standard deviation σ ($\theta_{exp,m} \pm \sigma$), for each testing configuration examined.

Therefore, the combination of the methods here proposed seems to be promising not only for fretting fatigue, but also for other fatigue problems involving high stress gradients, in the case of both proportional and non-proportional stress histories. Here it should be noticed that the analysis conducted in this work is essentially based on a Solid Mechanics (stress based) approach; hence a possible influence of the slip amplitude on the fatigue behavior of the Al7050 T7451 is discarded. This hypothesis may initially seem strong, however the superficial damage produced at the surface within the partial slip regime where the tests were carried out is not high. On the other hand, the influence of the rapidly varying stress gradient on the crack initiation path is clearly captured using the critical direction concept.

Note that the tests and validations conducted in the present study are related to loading histories where the bulk load applied to the specimen is static and only the tangential/fretting load is cyclic. Therefore, to draw more firm and general conclusions, further tests and analyses on the effect of a cyclic bulk load on the early crack orientation should be performed.

Acknowledgements

This work was financially supported by Centrais Elétricas do Norte do Brasil S/A (Eletronorte) and INTESA as part of the ANEEL Research and Development Program in Brazil. Professors Vantadori and Araújo also would like to gratefully acknowledge the financial support of the Italian Ministry of Education, University and Research (MIUR) and of the Brazilian Research Council CNPq (309748/2013-5), respectively.

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