Integration of a Levee Breach Erosion Model in a GPU-Accelerated 2D Shallow Water Equations Code

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Key Points:

- The paper presents a 2D SWE model able to simulate the opening of a levee breach due to overtopping, and the subsequent flooding.
- The physically based erosion model allows predicting the breach evolution in levees made of cohesive or non-cohesive material.
- The model can be applied to real cases, thanks to the fast execution times guaranteed by the GPU-parallelization of computations.

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Abstract

This paper presents a two-dimensional (2D) shallow water equations code coupled with a physically-based erosion model, able to predict the opening and evolution of breaches forming in levees built with either cohesive or non-cohesive material. The bottom elevation change is evaluated using an excess shear-stress equation, which accounts for the hydrodynamic conditions and for the material characteristics. The proposed model modifies the local topography at runtime wherever the levee is overtopped without having to predefine the position and shape of the breach. The model is implemented in CUDA programming language, so that simulations can be run on Graphics Processing Units (GPU), guaranteeing fast execution times even for high-resolution meshes and large domains. The validation is performed based on several experimental tests, and numerical predictions are in good agreement with the measurements. The strengths and weaknesses of the proposed approach are also discussed by comparison with a sediment transport model based on the Exner equation: while the latter gives good results only for breaches forming in levees built with non-cohesive material, the proposed model can also be applied to cohesive embankments. The application to a historical flood event is also presented, showing that the model can effectively be employed for real field simulations also in the case of multiple breaches.

1 Introduction

Many lowland areas are protected from flooding by levees, whose main purpose is to confine the flow inside the river region. Levees are usually built with erodible material (clay, silt or sand), and their height is designed to contain a specific hydrograph. However, in the case of an extreme event exceeding their design return period, overtopping occurs, often causing the embankment collapse. Land use modifications and climate change might also increase the discharge for a given return period, making the levees no longer adequate. Moreover, earthen embankments may experience breaching for piping and internal erosion processes, even before the water surface elevation reaches the levee crown. The dens of burrowing animals (e.g. porcupine, badger, nutria) have been recently identified as another cause for levee collapse in Northern Italy (Orlandini et al., 2015; Sofia et al., 2017; Viero et al., 2013).

Since embankment failures can have damaging consequences and lead to huge economic and human losses, the design of flood hazard maps and emergency plans is particularly important, and numerical modeling represents a powerful tool for these analyses. The simplest approach to model the flood propagation caused by a levee breach is to adopt a one-dimensional (1D) model for the channel, and a two-dimensional (2D) model for the floodable area outside the river region (e.g. Masoero et al., 2013; Vorogushyn et al., 2010). The breaches are included in the 1D model as lateral spillway structures (Mazzoleni et al., 2014), and the outflow from these structures is imposed as upstream boundary condition to the 2D model, often neglecting possible backwater effects caused by the presence of road or railway embankments in the floodable region. Moreover, the 1D model cannot accurately describe the markedly 2D flow near the breach inside the river. On the contrary, when both the river and the floodable area are simulated using fully 2D Shallow Water Equations (SWE) models (e.g. Teng et al., 2017), these features can inherently be taken into account. In the past, the choice of using 1D-2D models was often necessary to reduce the computational effort, but the same outcome can now be achieved by using modern High Performance Computing clusters and/or adopting parallelization techniques, such as implementing codes able to run on Graphics Processing Units (GPUs) (Dazzi et al., 2018; Vacondio et al., 2014, 2017).

The breach opening must be somehow included in the 2D modeling. The detailed simulation of the breach process is actually rather difficult, due to its 3D nature and to the complex interactions between hydrodynamic conditions, bank stability, sediment transport and infiltration processes; however, for practical use in the field of flood simulation, simplified approaches are often introduced. The gradual breach opening can be modelled by a time-varying topography with predefined geometric characteristics (e.g. Dewals et al., 2011), or by coupling the SWE with a 2D sediment transport model (e.g. Faeh, 2007).

For example, Vacondio et al. (2016) recently simulated a breach-generated flood with a GPU-accelerated fully 2D model, adopting a purely geometric approach to describe the breach evolution (specifying the breach position and assuming a trapezoidal shape, with final width and failure time defined a priori). In this way, however, the levee material characteristics are completely neglected from the breach modeling, possibly resulting in inaccurate predictions. The hydrodynamic conditions, such as the upstream inflow in the river, the presence of river bends and floodplains, possible backwater effects that reduce the velocity of outflowing water (and consequently the erosion), can also severely affect the breach evolution (Viero et al., 2013). Moreover, for (lateral) fluvial breaches the choice of "geometric" parameters cannot be assisted by results provided by the parametric (e.g. Froehlich, 2008; Xu & Zhang, 2009) or simplified (e.g. Chen & Anderson, 1987; Fread, 1988; Macchione, 2008; Mohamed et al., 2002; Visser, 1999; Wu, 2013) models available in the literature (ASCE/EWRI, 2011), which were developed for (frontal) dam breach configurations. In fact, recent experimental investigations (Elalfy et al., 2017; Kakinuma & Shimizu, 2014; Michelazzo et al., 2018; Rifai et al., 2017; Wei et al., 2016) show that, apart from the very first stages, the evolution of a lateral breach is quite different from what observed in the frontal configuration, due to the different direction of the flow (parallel/tilted vs perpendicular to the levee crest).

The best option for simulating riverine levee breaches would be the use of detailed physically-based multi-dimensional models, based on the integration of hydrodynamic and morphodynamic equations (e.g. Canelas et al., 2013; Li & Duffy, 2011; Murillo & Garcia-Navarro, 2010). In this way, the flow along the river, through the breach and in the inundated region can be simulated simultaneously without the need of introducing internal boundary conditions at the breach location. Attempts of applying 2D sediment transport models to simulate breaches in dams built with non-cohesive material include Cao et al. (2011), Evangelista (2015), Guan et al. (2014), Juez et al. (2014), Van Emelen et al. (2015), Volz et al. (2012), Wang & Bowles (2006), and Wu et al. (2012), while Faeh (2007) performed a fluvial breach test. Nevertheless, applications to real cases are limited by the heterogeneity and scarcity of data on levee materials, and by the prohibitive computational time required for running these models (unless parallelization techniques are introduces, see Juez et al., 2016). Moreover, the application of these models to breach erosion may be questionable, since most sediment transport equations were derived in uniform flow conditions, for small slopes, and with non-cohesive materials, while the breach development is highly unsteady and often involves high slopes and cohesive sediments. For this reason, Morris et al. (2009) suggested that the employment of erosion laws would be more consistent with the breach process. Erosion laws have been applied to dam breach modeling by Chen & Anderson (1987), Morris et al. (2009), and Wang & Bowles (2006), and have the advantage of including specific erodibility parameters in the computations, and of being applicable also to cohesive embankments. In fact, the erosion process of dams/levees built with non-cohesive and cohesive material is quite different (e.g. Morris et al., 2007). In the latter case, headcut erosion is observed: one or more rills develop into a series of overfalls, which form a headcut (i.e. a vertical or nearly vertical drop on the bed); the headcut migrates upstream and reduces the dam crest height; this phase is then followed by a breach widening stage. Clearly, this 3D process cannot be adequately simulated using a 2D depth-averaged model and sediment transport equations, hence a headcut migration rate is often introduced to model this type of breaches (e.g. Hanson et al., 2005; Wu, 2013). However, since this quantity depends on the same erodibility coefficients that appear in simple erosion laws, such laws can arguably be used to model the general breach process, at least as regards the failure time and the final width, avoiding a detailed description of the headcut migration process.

The present work aims at introducing an efficient numerical tool for the simulation of inundations generated by levee breaches, including a physically-based prediction of the breach evolution (instead of a geometric approach) and avoiding the necessity of defining its characteristics a priori. The GPU-accelerated 2D SWE numerical code PARFLOOD (Vacondio et al., 2014, 2017) was coupled with an erosion model, in which an excess shear-stress law is employed to predict the time evolution of the bottom elevation at the levee breach site as a function of the local hydrodynamic conditions and of the material characteristics. The model can be used for either non-cohesive or cohesive embankments. For comparison purposes, a simple but robust bedload transport model (Juez et al., 2014) was implemented as well. Both models are enriched by a bank failure algorithm, which simulates the sudden failure of blocks of material due to slope instability. Validation is performed based on four experimental tests, and an example of application to the real levee-breach event occurred on the Enza River (Italy) in December 2017 is also presented.

The paper is structured as follows. Section 2 presents the main features of the proposed model. Section 3 is dedicated to the description of all the test cases used for the validation and application of the model, while in Section 4 a discussion on the advantages and disadvantages of the model is presented. In the last Section, conclusions are drawn.

2 Model description

The proposed model couples three modules: a hydrodynamic model (already presented in previous works, see Vacondio et al., 2014, 2017), and two newly developed models for erosion and bank failure simulations. Moreover, a sediment transport model was implemented for comparison purposes (see Appendix A).

2.1 Hydrodynamic model

The hydrodynamic model (Vacondio et al., 2014, 2017) is based on the 2D SWEs written in integral form (Toro, 2001) as follows:

$$\frac{\partial}{\partial t}\int_A \mathbf{U}\,\mathrm{d}A + \int_C \mathbf{H}\cdot\mathbf{n}\,\mathrm{d}C = \int_A \left(\mathbf{S}_0+\mathbf{S}_f\right)\mathrm{d}A, \qquad (1)$$

where A and C are the integration volume area and boundary, respectively, $\mathbf{U}$ is the vector of conserved variables, $\mathbf{H} = (\mathbf{F},\mathbf{G})$ is the tensor of fluxes in the x- and y-directions, $\mathbf{n}$ is the outward unit vector normal to C, $\mathbf{S}_0$ and $\mathbf{S}_f$ are the bed and friction slope source terms, respectively, and t is the time. The well-balanced form of the SWEs, proposed by Liang & Marche (2009), is adopted:

$$\mathbf{U} = \begin{bmatrix}\eta \\ uh \\ vh\end{bmatrix},\ \ \mathbf{F} = \begin{bmatrix} uh \\ u^2h+\frac{1}{2}g(\eta^2-2\eta z) \\ uvh\end{bmatrix},\ \ \mathbf{G} = \begin{bmatrix} vh \\ uvh \\ v^2h+\frac{1}{2}g(\eta^2-2\eta z)\end{bmatrix}, \qquad (2a)$$

$$\mathbf{S}_0 = \begin{bmatrix} 0 \\ -g\eta\frac{\partial z}{\partial x} \\ -g\eta\frac{\partial z}{\partial y}\end{bmatrix},\ \ \mathbf{S}_f = \begin{bmatrix} 0 \\ -gh\frac{n_f^2 u\sqrt{u^2+v^2}}{h^{4/3}} \\ -gh\frac{n_f^2 v\sqrt{u^2+v^2}}{h^{4/3}}\end{bmatrix}, \qquad (2b)$$

In Eq. (2), $\eta = h + z$ is the water surface elevation, h is the flow depth, and z is the bed elevation; u and v are the velocity components along the x- and y-directions respectively, n_f is Manning's roughness coefficient, and g is the acceleration due to gravity.

An explicit FV scheme is used to discretize the equations; both first-order and second-order accurate approximations in space and time are implemented, but only the first-order approximation is here recalled for the sake of brevity. The following equation is used to update the conserved variables in time:

$$\mathbf{U}_{i,j}^{n+1} = \mathbf{U}_{i,j}^{n} - \frac{\Delta t}{\Delta x}\left(\mathbf{F}_{i+\frac{1}{2},j} - \mathbf{F}_{i-\frac{1}{2},j}\right) - \frac{\Delta t}{\Delta y}\left(\mathbf{G}_{i,j+\frac{1}{2}} - \mathbf{G}_{i,j-\frac{1}{2}}\right) + \Delta t\left(\mathbf{S}_0+\mathbf{S}_f\right). \qquad (3)$$

Subscripts *i,j* represent the cell position, while superscript *n* refers to the time level; Δx and Δy are the cell dimensions in the *x* and *y* directions respectively, and Δt is the time step size. The HLLC approximate Riemann solver (Toro, 2001) is used to compute intercell fluxes, and the correction proposed by Kurganov & Petrova (2007) is implemented to avoid non-physical velocity values at wet/dry fronts. The slope source term is discretized with a centered approximation (Vacondio et al., 2014), while the friction source term is discretized using the implicit formulation proposed by Caleffi et al. (2003). The minimum allowable time step is computed according to the CFL stability condition (Toro, 2001). The model is implemented in a CUDA/C++ code, which exploits the intrinsic parallelization of computations on GPU devices, thus guaranteeing fast execution times compared to serial codes. More details on the scheme and implementation can be found in Vacondio et al. (2014, 2017).

2.2 Erosion model

The breach evolution is predicted according to the erosion model described in the following. The bottom elevation change is not allowed in the whole domain, but only along the erodible levees, where potential breaches might occur. In these selected regions, the morphodynamic evolution is described by the following equation:

$$\frac{\partial z}{\partial t} = -\frac{E}{1-p}, \qquad (4)$$

where *p* is the bed porosity, and *E* is the bed erosion rate (eroded volume per unit area per unit time). This latter quantity can be estimated according to a linear erosion law, also referred to as excess stress equation (e.g. Hanson & Simon, 2001; Partheniades, 1971):

$$E = \begin{cases} k_d(\tau - \tau_c) & \text{if } \tau > \tau_c \\ 0 & \text{if } \tau \le \tau_c \end{cases}, \qquad (5)$$

In Eq. (5), k_d represents the erodibility coefficient of the embankment material, while τ and τ_c are the bed shear stress and its critical value for sediment motion, respectively. The bed shear stress is computed as follows:

$$\tau = \gamma \frac{n_f^2(u^2+v^2)}{h^{1/3}} \qquad (6)$$

where γ is the specific weight of water. Eq. (4) is simply discretized as follows:

$$z_{i,j}^{n+1} = z_{i,j}^{n} - \frac{E(\tau_{i,j})}{1-p}\Delta t. \qquad (7)$$

In essence, Eq. (7) modifies the bottom elevation of each cell (belonging to an erodible levee) at runtime according to the local value of the bed shear stress, and non-negligible erosion is only obtained in the practice whenever and wherever the levee is overtopped. The opening of a breach, and its subsequent deepening and widening, is then predicted automatically, without the need to predefine the breach position and dimensions as input data. A minimum bed elevation can also be specified in order to avoid erosion below a non-erodible foundation. From Eq. (7), it can be noticed that only scour is allowed, while deposition of sediments is not included in the computations: the eroded material is supposed to be washed away by the flow, in line with other simplified approaches. Conversely, the global water volume is conserved by maintaining the original water depth of each cell where sediment is eroded; hence, the water surface elevation must be adjusted in these cells by subtracting the scour computed during the current update.

By means of Eqs. (4) and (5), the breach erosion process depends both on the flow field characteristics (via τ) and on the embankment material (via k_d, τ_c, and p). The erodibility parameters k_d and τ_c control the erosion process, and thus must be accurately calibrated. Despite the fact that specific experimental tests for their determination were developed, such as the Jet Erosion Test (JET) (Hanson & Cook, 2004), reliable correlations with the sediment characteristics are not available yet. Different test configurations and interpretations of results lead to different estimates for k_d and τ_c (Khanal et al., 2016), and even the use of a linear erosion law is still debated (Walder, 2015). Moreover, these parameters are observed to be quite sensitive not only to the type of material, in particular to soil texture and plasticity, but also to the compaction effort and water content (Fell et al., 2013; Nguyen et al., 2017; Wahl et al., 2009), with a variability up to 2-3 orders of magnitude (Hanson & Hunt, 2007). Therefore, the choice of erodibility parameters must be carefully considered, and the uncertainty in their values must be properly taken into account by means of a sensitivity analysis when data for calibration are not available. The diagram and tables reported by Hanson et al. (2010) can provide guidance for defining the range of variability of these parameters. Moreover, some authors suggest empirical relationships between τ_c and k_d (e.g. Hanson & Simon, 2001; Nguyen et al., 2017); Wu (2013) also reports an empirical formula for computing the erodibility coefficient (based on the clay content and the dry specific weight of the soil), which can help in the choice of reasonable values when specific erosion tests cannot be performed.

2.3 Bank failure algorithm

While the breach triggering is mainly due to the erosion following levee overtopping, its enlargement is also due to the sudden collapse of the lateral banks, which lose stability as long as the breach deepens (Hunt et al., 2005). Numerical models must consider this mechanism for a correct prediction of the breach development. For this reason, different bank failure algorithms were presented in the literature (e.g. Evangelista et al., 2015; Swartenbroekx et al., 2010; Volz et al., 2012), usually based on the idea of reducing the local slope of each cell when it exceeds a critical value φ_c. Obviously, all these models ignore cantilever failures, which are sometimes observed in experimental tests (e.g. Wei et al., 2016), but cannot be described in a depth-averaged model. In this work, the scheme of Guan et al. (2014) for structured grids was adapted for guaranteeing efficiency on GPUs.

Two different values are specified for the critical angles, depending on the fact that the cell is above or below the water surface level: $\varphi_{c,wet}$ for wet cells, and $\varphi_{c,dry}$ for dry cells; moreover, φ_{dep} is the angle that the deposited material forms after collapse. These values are often assumed equal to the angle of repose of the material. Let us consider cell (*i*,*j*) with bottom elevation $z_{i,j}$ in a Cartesian grid, with four neighbor cells (*i*+1,*j*), (*i*−1,*j*), (*i*,*j*+1), (*i*,*j*−1). The local slope φ_k in the k^{th} direction can be computed as:

$$\tan\varphi_k = \left(z_k - z_{i,j}\right)/l_k, \tag{8}$$

where z_k is the bottom elevation of the neighboring cell in the k^{th} direction (i.e. cell (*i*+1,*j*) to the east, cell (*i*,*j*+1) to the north, etc.), and l_k is the grid size in the same direction (i.e. Δx to the east/west, and Δy to the south/north). If $|\varphi_k| > \varphi_c$ (wet or dry, depending on the cell state), then the bottom is considered locally unstable, and a correction Δz_k can be calculated as:

$$\Delta z_k = 0.5\, l_k \left(\tan|\varphi_k| - \tan\varphi_{dep}\right) \operatorname{sign}(\varphi_k). \tag{9}$$

An equal and opposite correction will be computed in the neighboring cell, so that the total sediment mass is conserved in this procedure, and the bed slope is simply tilted from φ_k to φ_{dep}. Finally, the updated value for the bottom elevation becomes:

$$z_{i,j}^{new} = z_{i,j} + \sum_{k=1}^{4} \Delta z_k. \tag{10}$$

These operations are performed in a specific CUDA kernel, and threads (i.e. the basic work unit in CUDA, corresponding to one computational cell) are processed in parallel. Differently from Guan et al. (2014), that processed all cells sequentially, and applied the correction Δz_k both to the current cell and to its neighbor (with proper sign), in this implementation each thread computes its own corrections, even at the cost of repeating calculations twice in two different threads. As already discussed in Vacondio et al. (2014) as regards intercell fluxes, accepting this small computational overhead makes the code more efficient than storing an extra array for the values of Δz_k and accessing it later.

Obviously, local changes in the bed slope may in turn affect the stability of other neighboring cells. The algorithm previously described, then, must be repeated iteratively until no more corrections are necessary. In order to reduce the computational time, this recursive procedure is not performed at every time step Δt dictated by the CFL condition, but at a larger pace Δt_{stab}; in particular, preliminary tests showed that checking and correcting the slope stability with Δt_{stab}=500-1000Δt provides the same results as with Δt_{stab}=Δt.

3 Numerical tests

In this Section, five test cases are presented for the validation of the erosion model. The first case is a 1D frontal dam breach experiment, and was chosen for its simplicity in order to highlight the differences in the predictions of the erosion model compared to a sediment transport model. Moreover, two 2D frontal breach test cases, which differ in the type of material used for building the dam (cohesive vs. non-cohesive), were considered in order to assess the capability of predicting the breach enlargement, and to study the influence of the model parameters on the simulation results. Then, since the erosion model is conceived for real field applications to fluvial breaches, a large-scale experimental test case concerning a lateral breach in a channel was privileged over available small-scale laboratory experiments as the fourth validation test, and it still involves levees built with non-cohesive material. Finally, the applicability of the model to a real event is assessed by means of the simulation of the levee breach on the Enza River (Northern Italy) and the subsequent flooding occurred in December 2017.

3.1 1D frontal dam breach

The experimental test concerning dam erosion due to overtopping, reported by Tingsanchali & Chinnarasri (2001) (Test C-2), was simulated in order to compare the results of the erosion and the sediment transport models. A 0.8 m-high dam, with crest width equal to 0.3 m and upstream and downstream slopes equal to 1V:3H and 1V:2.5H respectively, was built in a 35 m-long, 1 m-wide rectangular flume. The dam was built with sand with the following characteristics: d_{50} = 0.86 mm, d_{30} = 0.52 mm, d_{90} = 3.80 mm, d_m = 1.13 mm, and ρ_s = 2650 kg/m^3. A constant inflow (1.23 l/s) was supplied at the upstream end of the channel; a vertical plate was held at the dam crest until the reservoir was filled, and the water depth in the reservoir was 3 cm higher than the dam crest; then, the sudden removal of the plate allowed overflow to start.

The domain was discretized by means of a uniform grid with $\Delta x = \Delta y$ = 0.05 m. The following parameters were common to both erosion and sediment transport models: bed porosity p = 0.4, critical angles $\varphi_{c,wet} = \varphi_{dep}$ = 30°, $\varphi_{c,dry}$ = 50°. Manning's coefficient and the erosion model parameters, τ_c and k_d, were subjected to calibration, and a sensitivity analysis on their values was performed.

First, Manning's coefficient was determined by simulating the experiment with the sediment transport model, which did not require any other calibration coefficient. Figures 1a-1b compare the experimental dam profiles along the centerline of the breach at selected times (t = 30 s and t = 60 s) and the results from the numerical simulations performed assuming n_f = 0.016 m$^{-1/3}$s, 0.018 m$^{-1/3}$s, and 0.020 m$^{-1/3}$s. An increase in this coefficient leads to a slightly more rapid erosion. However, there is no clear best fit value; in the first phases (Fig. 1a), the smallest roughness value seems to mimic the dam erosion better, while it underestimates the erosion at the dam crest after some time (Fig. 1b). The intermediate value (0.018 $m^{-1/3}s$) is hence selected for describing the general process.

The erosion model was tested next. As a starting point for choosing the erodibility parameters, the work by Hanson et al. (2010) was considered: for soils with low clay content, the suggested values for k_d are in the range 50-800 $cm^3/N/s$ (depending on compaction); the corresponding τ_c range is 10^{-3}-10^{-1} Pa.

Preliminary simulations were performed varying τ_c (10^{-3}, 10^{-2}, and 10^{-1} Pa) with different values of k_d, and results of these tests show that changes in the values of τ_c have negligible effects in the process, because the bed shear stress exceeds the critical value by 1-2 orders of magnitude ($\tau >> 1$). Then, τ_c was assumed equal to 10^{-2} Pa, and the erodibility coefficient was varied: the best fit was obtained for $k_d = 500$ $cm^3/N/s$. In Figures 2a-2b, the simulated and measured dam profiles along the centerline at selected times for different values of k_d (300, 400, 500, and 600 $cm^3/N/s$) are reported, and stress how much the choice of this parameter can influence the breach evolution. The fact that the proposed model can only reproduce the erosion process can be clearly noticed from this comparison: while experimentally the downstream slope flattens as long as the dam crest is eroded because sediments are deposited at the dam toe, numerically the downhill slope simply retreats due to erosion; nevertheless, the calibrated value for k_d guarantees that the model predicts the eroded dam crest height correctly. The outflow discharge is reproduced reasonably, and the peak value is well predicted, as can be observed in Figure 2c, even if the model, probably due to the differences in the bathymetry towards the end of the simulation (caused by the exclusion of the deposition processes), overestimates the falling limb of the hydrograph. The outflow discharge predicted by the sediment transport model is also reported in Figure 2c for the sake of comparison; the dam erosion is probably too fast in the first stages, and the reservoir emptying is anticipated compared to experimental observations, leading to a lower discharge peak.

3.2 2D frontal breaches

In this section, a sensitivity analysis on the erosion model parameters is undertaken by simulating two 2D frontal breaches experimental test cases performed at the HR Wallingford laboratory for the IMPACT project (Morris et al., 2005). All tests were carried out in a 50 m long and 10 m wide flume, where an erodible dam was built roughly 36 m downstream from the channel entrance (Figure 3a). Water was allowed into the flume until the reservoir upstream of the dam was filled and the water level exceeded the elevation of a pilot channel carved in the central portion of the dam, thus triggering the breach opening. Tests labelled #2 and #10 were selected, and the main features of these experiments are reported in Table 1. Notably, in Test #2 the dam was built with non-cohesive material (nearly uniform sand with d_{50} = 0.25 mm), while for Test #10 a cohesive material (clay) was employed; the breach evolution was hence different for the two tests. For the non-cohesive dam, the first phase was similar to the 1D case, and was characterized by uniform erosion on the downhill slope, which retreated and became milder at the dam section corresponding to the pilot channel. On the other hand, for the test with a cohesive dam, headcut erosion was observed on the downhill slope. Then, in both cases, the breach side slopes started to lose stability and bank failures occurred, so that the breach enlarged in time symmetrically; the process ended when the upstream reservoir was almost empty.

For both tests, the domain was discretized with square cells of size $\Delta x = \Delta y = 0.05$ m, and Manning's coefficient was set equal to 0.018 $m^{-1/3}s$ for Test #2 (as suggested by Wu et al., 2012), and to 0.016 $m^{-1/3}s$ for Test #10 (according to Wu, 2013). The time series for the inflow discharge were imposed as upstream boundary condition, while a free outlet condition was set downstream.

Test #2 is analyzed first. As in the previous test case, the values of the erodibility parameters were chosen in the range suggested by Hanson et al. (2010) for low clay content soils. In particular, the best fit was obtained with $\tau_c = 10^{-2}$ Pa, and $k_d = 150$ $cm^3/N/s$. The other model parameters were set as follows: $p = 0.4$, $\varphi_{c,wet} = \varphi_{dep} = 30°$, $\varphi_{c,dry} = 45°$. Figures 3b-3e show the bottom elevation contour maps at selected times, together with the velocity vectors. The absence of deposition downstream can be noticed, but the "hourglass" shape in the first phase of the breach opening and the final top width are well reproduced.

The measured and simulated breach top widths in time are reported in Figure 3f. Results obtained with three different values of the erodibility coefficient are compared (k_d = 100, 150, and 200 $cm^3/N/s$, while all the other parameters are kept constant). The model with $k_d = 150$ $cm^3/N/s$ is able to reproduce the breach evolution in time quite well. However, regardless of the selected erodibility coefficient, the model underestimates the outflow discharge, as can be observed in Figure 3g. The model slightly anticipates the beginning of the breach enlargement, which is more rapid in the experiments than in the numerical simulations, and this may influence the outflow discharge and the reservoir emptying process. Another possible cause of these discrepancies can be the uncertainty in the position and technique of the discharge measurements (the dam centerline is used as cross-section for the discharge extraction in the numerical simulations, while no information is available as regards the experimental setup).

Additional sensitivity analyses were performed on Manning's coefficient, on the critical shear stress, and on the critical angles for slope stability, which are expected to influence the breach enlargement. The values 0.016 and 0.020 $m^{-1/3}s$ for Manning's coefficient were investigated. The critical shear stress τ_c was varied to 10^{-3} and 10^{-1} Pa, maintaining all the other parameters constant. The critical angle for slope stability in wet conditions $\varphi_{c,wet}$ (and the angle of deposition φ_{dep}) was changed from 30° to 25° and to 35°; as regards $\varphi_{c,dry}$, the values 40°, 50°, 60°, 70° were examined. For all these simulations, the main breach characteristics (final top width, peak discharge) are reported in Table 2, and the relative error with reference to the experimental measurement and to the best-fit simulation is computed. The critical shear stress has negligible influence on the results, as already noticed in the previous test case.

Moreover, results are observed not to be much dependent on the “wet” critical angle for slope stability. On the other hand, an increase in the value of $\varphi_{c,dry}$ can reduce the final breach width and the peak discharge. Finally, Manning’s coefficient does not particularly affect the main breach features. In fact, despite the fact that τ increases quadratically with n_f (Equation 6), at the same time velocity magnitude decreases, thus limiting the variation of the bed shear stress.

For simulating Test #10, erodibility parameters for a cohesive soil had to be set. Following Hanson et al. (2010), and also considering the formula for the estimation of k_d reported by Wu (2013), the range 0.1-10 $cm^3/N/s$ can be considered adequate for the erodibility coefficient value of this kind of material. The critical shear stress τ_c should be set in the range 0.01-1 Pa. The following parameters were selected for the reference simulation: $p = 0.4$, $\tau_c = 0.1$ Pa, $k_d = 5$ $cm^3/N/s$, $\varphi_{c,wet} = \varphi_{dep} = 30°$, $\varphi_{c,dry} = 45°$.

The dynamics of the breach evolution predicted by the model, though much slower because of the reduced erodibility of the dam material, is similar to Test #2. Due to the limitation of the SWE assumptions, the model is not able to predict the headcut erosion observed experimentally; in spite of this, the final top width is well reproduced (Figure 4a), even if the enlargement process is faster than in the measurements. In this case, since the dynamics is much slower than in the case of Test #2, this discrepancy does not influence the reservoir emptying very much. In fact, the outflow discharge fits the one registered during the experiments well (Figure 4b), in both shape and peak discharge.

When values of the erodibility coefficient smaller than 5 $cm^3/N/s$ were adopted, the breach formation process was initially too slow, and the increase in the water level upstream caused the overtopping of the whole dam (not limited to the pilot channel) and the consequent widespread erosion, which was never observed in the experiments. This probably happens because the initial evolution of the breach is generated by headcut erosion, which cannot be simulated with SWE models. Hence, only one larger value of k_d was considered in the sensitivity analysis ($k_d = 10$ $cm^3/N/s$), and numerical results for this simulation are compared with the experiments and with the reference simulation in Figure 4. The breach opening is faster than in the reference simulation, but the final top width is still well caught; the overflow discharge presents a slightly different trend, with a sudden initial increase due to the rapid breach erosion and an underestimated peak value. Also for this test case, a sensitivity analysis on the critical shear stress revealed that this parameter does not influence the model predictions (results not shown).

3.3 Experimental levee breach

The aim of the present test case is to investigate how the erosion model can reproduce the opening of lateral breaches. In particular, one of the field-scale levee breach experimental tests presented by Kakinuma & Shimizu (2014) is considered (Case 4). A 176 m-long and 8 m-wide stretch of the floodway channel of the Tokachi River (Japan), with bottom slope equal to 1/500, was set up by inserting a vertical wall on the left and by substituting a portion of the existing right levee with a 3 m-high erodible dyke, made of sand with d_{50} = 0.7 mm, $d_{30} \approx$ 0.2 mm, $d_{90} \approx$ 40 mm, ρ_s = 2650 kg/m^3, p = 0.4. The levee crest width was equal to 6 m, while the side slopes were both 1V:2H. The inflow discharge was increased until the levee was overtopped just at the location where a notch (with length 3 m and depth 0.5 m) had been previously carved to trigger the breach; the outflow discharge inundated a floodable area specifically arranged (Figure 5a).

The grid size was set at 0.5 m, while Manning's coefficient was assumed equal to 0.023 m$^{-1/3}$s, as suggested by the experimenters. The measured inflow discharge was set as upstream boundary condition, and a rating curve was imposed downstream far enough to avoid disturbances in the water level at the breach site. The following parameters were assumed for the levee material for the erosion model: τ_c = 0.5 Pa, k_d = 80 cm^3/N/s, $\varphi_{c,wet} = \varphi_{dep}$ = 30°, $\varphi_{c,dry}$ = 40°. The same critical angles were used for the sediment transport model.

Experimental observations show that, initially, overflow water starts eroding the downhill slope of the levee, until the top of the front slope is reached, and erosion proceeds downward to the bottom of the levee; then, the breach begins to widen to both sides. However, soon the breach is observed to widen at a much higher rate in the downstream direction than in the upstream direction; this is due to the development of a high-velocity flow band near the downstream end of the breach, and of a dead water area near the upstream end, where sedimentation occurs. The asymmetry of the breach final width with reference to the initial notch position is a typical feature of lateral breaches, in contrast with what is usually observed in frontal dam breach test cases. The sediment transport model is able to capture this process, as can be noticed in Figure 5b, where the bed elevation contour maps at selected times are reported to show the breach evolution. On the other hand, the erosion model, which neglects deposition, is not able to reproduce this process in detail; nevertheless, the asymmetry of the breach widening is still predicted, especially in the first stages (see Figure 5c), even if in a less pronounced way than in the experimental observations towards the end of the process. Despite these differences in the simulation of the erosion process, the two models predict a similar trend for the total breach width, which is slightly underestimated with reference to the experimental data, as can be noticed in Figure 5d. Breach widening seems to start somewhat late in the sediment transport model simulation, but then evolves at a higher rate than in the erosion model simulation. A similar trend can be observed in the outflow discharge time series, reported in Figure 5e. The peak discharge is underestimated by only 4-5% by both models with reference to the measured value.

A sensitivity analysis on the erodibility parameters was performed. First, the erodibility coefficient was analyzed, and simulations were repeated assuming k_d = 40, 60, 80, 100, and 120 cm^3/N/s

(τ_c = 0.5 Pa). The final breach width is observed to increase with erodibility (simulated values are 8, 33, 62, 80, and 127 m, respectively). The smallest erodibility value is probably not representative of the levee material, since overtopping does not generate appreciable erosion; on the other hand, the highest erodibility values overestimate the final breach width, also because erosion is predicted along the inner riverbank (upstream of the breach), and this fact was not observed experimentally. As regards the predicted discharges, Figure 5f compares the outflow hydrographs: the arrival time is slightly anticipated for the highest erodibility values, but the final peak discharge is very similar for $k_d \geq 80$ cm^3/N/s.

The sensitivity to the critical shear stress was also analyzed, and τ_c was varied from 0.1 to 0.5 and 1 Pa (maintaining k_d = 80 cm^3/N/s). The outflow hydrographs obtained from the three simulations are compared in Figure 5g, showing that the arrival time and the peak discharge are only slightly dependent on the critical shear stress.

3.4 Levee breach on the Enza River

The model was finally employed to simulate the recent flood event that took place on the Enza River (Northern Italy), a tributary of the Po River, in order to verify its applicability to real test cases. A severe flood event followed the prolonged heavy rainfall occurred on the river basin on December 10-11, 2017, resulting in the highest water levels ever recorded at all the gauging stations along the river. On December 12 at 05:30 a.m. water started to overtop the right levee near Lentigione di Brescello (Reggio Emilia), initially triggering three very close breaches, which almost merged into a single large one in time. The overtopped part of the levee was 250 m long, and the total final breach width was approximately 160 m, while the widening took about 4 hours. The total flooded area was about 6.3 km^2, restricted by the levees of the Enza and Po Rivers, by a road embankment and a channel levee (see Figure 6a).

The terrain elevation was obtained from a digital terrain model (DTM) with resolution equal to 1 m, based on a LiDAR survey of the area. The domain was then discretized with square cells of size 2 m × 2 m (approximately 5 million active cells), but the levee crest elevations were preserved after the down sampling of the original DTM. The roughness coefficient was set equal to 0.05 m$^{-1/3}$s, after a calibration procedure. The upstream boundary condition is the discharge time series obtained from the conversion of measured water levels at the level gauge station of Sorbolo (whose position is reported in Figure 6a). An experimental rating curve was available, but the presence of the breach is expected to influence the level measurements due to the generation of a drawdown profile, thus "invalidating" the rating curve after the breach opening. For this reason, two different stage-discharge relations (before and after the breach) were used to convert the water levels, following the same procedure described by Vacondio et al. (2016). The discharge hydrograph is shown in Figure 6b, together with the water levels in the Po River, imposed downstream (these were relatively low, and backwater effects were not observed during the event).

The levee is built with silt loam with the following texture: sand 15-39%, silt 49-67%, clay 12-18%. The material porosity is equal to 0.4, and the dry specific weight of soil is approximately 1.55 Mg/m^3. Considering these characteristics, Hanson et al. (2010) suggest an erodibility coefficient in the range 0.5-10 cm^3/N/s, while the critical shear stress should be assumed in the range 0.1-1 Pa. Moreover, the formula reported by Wu (2013) for estimating k_d would lead to 2-3 cm^3/N/s. The following parameters were then assumed for the levee material for the erosion model: τ_c = 1 Pa, k_d = 5 cm^3/N/s, p = 0.4, $\varphi_{c,wet} = \varphi_{dep}$ = 30°, $\varphi_{c,dry}$ = 50°. The erosion equation was applied only in the cells representing the levee (hence the riverbed is never modified), and the bottom elevation of the levee foundation (assumed equal to the local terrain elevation outside the river) was also specified, in order to prevent erosion below the ground level. Note that erosion can potentially occur anywhere on the levees; the exact position of the breach does not have to be defined a priori, because the opening occurs where the levee is overtopped. Figure 6c reports the longitudinal profile of the (right) levee crest elevation for the 8 km-long stretch of the river downstream of Sorbolo, together with the profiles of the maximum water surface elevations along the river obtained from two different simulations. In the first one the levees are assumed to be non-erodible (bathymetry constant in time in the whole domain), whereas in the second one the bathymetry can change accordingly to Equation (7) and to the bank failure algorithm, leading to the breach formation where the levee is overtopped. In the second simulation, the maximum water levels are lowered due either to the drawdown effect induced by the breach upstream or to the reduced discharge downstream. The maximum observed water levels surveyed after the event are also reported at selected locations, and confirm that the flood propagation along the river is correctly reproduced by the model (Figure 6c). Please note that the water levels (both simulated and surveyed ones) are quite close to the levee crest in different locations, but the levee is actually overtopped only where breaches were observed in the field.

Figure 7 shows the breach evolution in terms of contour maps of the bottom elevation at selected times (velocity vectors are also reported). Initially, erosion is not concentrated in a restricted area (as in the previous test case), because water overtops the levee crest along roughly 270 m, and the erosion process appears scattered along this length. However, the levee crest elevation is not regular in this area, hence some low points appear more vulnerable to erosion, and give origin to the development of multiple small breaches, which enlarge in time. The most vulnerable point is just downstream of the levee bend, and the highest velocities are observed there. The breach takes about 3 hours from the beginning of overtopping to reach its final extension, with only small further modifications in the following 2 hours. The total final width is approximately 150 m, separated into five segments along roughly 250 m, the largest of which is 60 m wide. Actually, only three breaches were observed in the field; however, given the uncertainties in boundary conditions, material parameters and terrain elevations, the model correctly predicts the opening of multiple breaches, and captures the overall process quite well. Clearly, any model that only employs geometric relations or internal links between river and floodplain to simulate the breach opening would hardly capture this complex behavior.

The actual total flooded area is reported in Figure 8, compared with the simulation results in terms of maximum water depths reached at 07:00 p.m. (December 12); by this time, operations for draining the flooded volume, which are not included in the simulation, had just started.

The model sensitivity to erodibility parameters was also analyzed. First, the simulation was repeated with a fixed critical shear stress (τ_c = 1 Pa), and the erodibility coefficient was doubled (10 $cm^3/N/s$), halved (2.5 $cm^3/N/s$), and further reduced to 1 $cm^3/N/s$, in order to explore the whole range of variability of this parameter for the given material. The breach evolution for these simulations is compared in Figures 9a-9i. The erodibility coefficient has a significant impact on both the failure time and the breach evolution. When the value of k_d is reduced, the erosion process is slower and less pronounced. When k_d assumes the smallest value (1 $cm^3/N/s$), the breach evolution takes 6 hours, and is characterized by a generalized erosion along the whole overtopped length. Results of the simulation with k_d = 2.5 $cm^3/N/s$ are similar to the reference case, even if the opening time slightly increases (4 hours). Surprisingly, while an increase in the value of k_d to 10 $cm^3/N/s$ reduces the failure time to only 1.5 hours, the total width does not increase compared to the reference simulation. In fact, a single breach is generated: the most vulnerable portion of the levee is eroded very rapidly, and the consequent drop in the water level in the river stops the overtopping and erosion processes along the rest of the levee. A comparison of the breach outflow hydrographs is reported in Figure 9j for these simulations, and shows that, apart from the case with k_d = 1 $cm^3/N/s$ which highly underestimates the outflow discharge (and the total flooded area), for the other values the peak discharge is underestimated by less than 10% compared to the reference simulation, and the peak is observed within ±0.5 hours. The differences in the total outflow volume are always below 10% compared to the reference simulation, except for the case with the lowest value of the erodibility coefficient, for which the volume difference is over 30%.

A sensitivity analysis to the critical shear stress was also performed, changing its value from 1 Pa to 0.1 Pa, and to 10 Pa, maintaining k_d = 5 $cm^3/N/s$, in order to evaluate its influence on the simulation results. The results confirm, similarly to the previous test cases, that the critical shear stress does not influence the breach evolution significantly, especially in the case of a reduced value assigned to this parameter. The adoption of the highest value, on the other hand, results in a slightly slower erosion process and reduced overflown volume of water (−15%), due to the fact that the predicted bed shear stresses and the critical value are of the same order of magnitude (10^1 Pa). Finally, the sensitivity to the critical angles for slope stability was also analyzed, but their influence on the simulation results is not evident for this test case (results not shown).

The flood event was also simulated using the sediment transport model described in Appendix A. The following parameters were assumed for the levee material: d_{50} = 0.04 mm, d_{90}/d_{30} = 10, ρ_s = 2650 kg/m^3. Sediment transport was allowed only in a wide region around the breach, not only to reduce the computational burden, but also to better compare the two models. A single breach develops rapidly after levee overtopping: the failure time is less than 1 hour, and the final width is 65-75 m (a map of the breach site is reported in Figure 10). Actually, a similar behaviour can be obtained from the erosion model if the erodibility coefficient is increased to 50 cm^3/N/s, which however is no longer representative of the levee material. In fact, it must be stressed that a bedload transport model is not expected to describe the complex behavior of the erosion process in a levee built with cohesive material.

With regard to the simulation time, the erosion model takes 1.3 h to simulate 2 days of physical time on a P100 Tesla® GPU, resulting in a ratio of physical to computational time equal to 37. Compared to an analogous simulation where the levee breach on the Enza River is modelled using a geometric approach (similarly to Vacondio et al., 2016), the computational overhead is negligible (3%). The sediment transport model takes 2.8 h on the same device (physical/computational time = 17). The good performance of GPU-accelerated models for high-resolution simulations is thus confirmed, and this makes the application of these models to complex real field test cases particularly convenient.

4 Discussion

The main aim of the present work is to introduce a tool able to automatically handle the possible opening of one or more levee breaches due to overtopping in a state-of-the-art 2D numerical code for flood propagation. The need to define the breach dimensions and position as input parameters is avoided by introducing an equation that predicts the bottom erosion wherever the local value of the bed shear stress exceeds a critical value (in the practice, this coincides with the occurrence of levee overtopping). The model was validated by simulating experimental test cases and one real flood event, and the numerical results were in good agreement with the measurements. In particular, the application to the historical test case of December 2017 on the Enza River, presented in Section 3.4, shows that the proposed model can effectively simulate this kind of problems: the breach position is correctly identified, and the opening of multiple breaches can be captured; moreover, the predicted outflow hydrograph from the breach results in a well-reproduced flooded area.

Clearly, the proposed model does not capture the breach process in detail, because some of the many interrelated factors influencing the breach dynamics are neglected. The simplifications introduced in the model must be kept in mind (most of them are common to other simplified approaches). Among these, it is worth recalling that a hydrostatic pressure distribution is assumed in the depth-averaged model, while vertical accelerations and streamline curvatures are often non-negligible at the breach site, especially in the case of headcut erosion in cohesive embankments. Moreover, other factors that may increase the bed shear stress or reduce the levee resistance to erosion, such as sediment concentration, infiltration processes, etc., are not considered in the present model. Despite these simplifications, the final breach width and peak outflow discharge are predicted reasonably even in the case of cohesive embankments, as shown by results from Test #10 in Section 3.2. Finally, the model assumes that the eroded material is washed away by the flow (i.e. sediment deposition is not simulated). This may slightly influence the breach configuration, especially in the case of non-cohesive materials (see for example Figure 5), and the effect is more evident for small-scale experimental test cases. However, for real field simulations, this hypothesis does not probably have much impact on the overall process.

The influence of the levee material on the erosion process is taken into account by means of two model parameters, the erodibility coefficient and the critical shear stress. Although their definition is not straightforward, some indications from the literature allow identifying the plausible range of variability of their values. The sensitivity analyses performed for several test cases show that the critical shear stress does not influence the model results significantly, because the bed shear stresses largely exceed the critical value during overtopping. On the other hand, the erodibility coefficient must be defined more carefully, especially in the case of non-cohesive material, for which its range of variability can be quite large. The analyses conducted in this study show that the breach width is affected by the value assigned to this parameter, while the outflow hydrograph is somewhat less influenced (as long as the value of k_d is representative of the material type), as can be noticed from results reported in Section 3.3. Similar considerations are true for the case of a cohesive material (see results from Section 3.4). This is encouraging, since the outflow hydrograph is the most relevant outcome of the model when the aim of the simulation is the prediction of the levee breach-induced flooding.

In all tests simulated in the present work, the embankment is considered homogeneous, meaning that a unique value of the erosion model parameters is adopted for the whole domain, but different values can easily be assigned to different segments of the levee; scour-dependent parameters could also be used in order to model stratified embankments. In general, however, real levees are heterogeneous, and data concerning the spatial distribution of the material type are often lacking, hence assigning a unique “average” value to the model parameters can be considered reasonable in the practice. A possible enhancement to this approach can be the inclusion of the presence of a grass cover, which protects the levee surface and delays the beginning of the erosion process, reducing the probability of levee failure (Mazzoleni et al., 2017). This effect can be achieved by increasing the critical shear stress, or by adjusting Manning’s coefficient (as done by Viero et al., 2013), during the first stages of overtopping. Finally, a scour-dependent erodibility coefficient could be used to distinguish between the embankment material and the levee foundation material, when this cannot be assumed as non-erodible. These modifications to the proposed model will be considered in future works.

Results obtained with the erosion model were also compared with the predictions of a simple sediment transport model that integrates the Exner equation for bedload transport. The main drawbacks of this latter model were found to be not only the greater complexity and computational effort required, but more importantly its inadequacy for simulating the erosion of cohesive levees. Moreover, even in the case of a non-cohesive material, a sediment transport model may overestimate the embankment erosion process. In fact, experimental observations show that a fine sand is more resistant to erosion than a coarse sand, due to its “apparent cohesion” (Evangelista, 2015; Pickert et al., 2011), while bedload transport laws usually assume that the transport capacity increases when the size of particles decreases. More sophisticated sediment transport models could be employed, but only at the cost of introducing a larger number of model parameters and of further increasing the computational time. For these reasons, the employment of this kind of models does not seem justified for simulating the flooding triggered by levee breaches in very large domains.

Finally, the efficiency of the proposed model must be stressed. Previous works (e.g. Vacondio et al., 2014) already assessed that a speed-up up to two orders of magnitude can be obtained using GPU-accelerated models (as PARFLOOD) instead of serial codes. The implementation of the proposed erosion model does not degrade the computational efficiency compared to the adoption of a simple geometric approach, and requires a lower computational time than a sediment transport model.

5 Conclusions

In this paper, a 2D SWE code was coupled with an erosion model that allows simulating the opening of levee breaches generated by overtopping, without the need to set the breach position and dimensions as input parameters. The breach evolution was predicted correctly for all tests used for model validation, which concerned both cohesive and non-cohesive materials. Therefore, the model can be particularly useful to create flood hazard maps and to support the design and verification of existing levee systems, also thanks to the high computational efficiency of the GPU implementation.

Appendix A. Sediment transport model

A simple bedload transport model was also developed for comparison purposes. The sediment transport model was taken from the literature, and the implementation of Juez et al. (2014) was adopted, also because it can be efficiently coded for GPUs. The model is based on the integration of the Exner equation, hence only bedload transport is considered; moreover, an approach where SWE and the morphodynamic equation are uncoupled is chosen (in order to reduce model complexity). For the hydrodynamic part, the model described in Section 2.1 is adopted. Here, only the main features of the sediment transport model are briefly recalled.

The bed evolution is described by the 2D Exner equation:

$$\frac{\partial z}{\partial t}+\frac{1}{1-p}\frac{\partial q_{s,x}}{\partial x}+\frac{1}{1-p}\frac{\partial q_{s,y}}{\partial y}=0, \tag{A1}$$

where $q_{s,x}$ and $q_{s,y}$ are the bed load discharges in the x and y directions, respectively, which can be expressed by any sediment transport formula; in this work, Smart equation (Smart, 1984) is adopted:

$$\Phi=4(d_{90}/d_{30})^{0.2}FS^{0.1}\theta^{0.5}(\theta-\theta_c^{SM}). \tag{A2}$$

In Eq. (A2), Φ is the dimensionless sediment discharge, computed according to the following expression:

$$\Phi=\frac{|q_s|}{\sqrt{g(s-1)d_m^3}}, \tag{A3}$$

where $s = \rho_s/\rho$ is the ratio of the material density ρ_s to the water density ρ, and d_m is the mean sediment diameter. Moreover, Eq. (A2) contains the Shields parameter θ (i.e. the dimensionless shear stress τ), defined as follows:

$$\theta=\frac{\tau}{g(\rho_s-\rho)d_m}, \tag{A4}$$

and θ_c^{SM} is its critical value according to Smart (1984); d_{90} and d_{30} are the grain sizes at which 90% and 30% by weight of material is finer, F is the Froude number, and S is the friction slope. Equation (A2) is applied only when $\theta > \theta_c^{SM}$ (otherwise $\Phi = 0$).

The bed elevation is updated as follows:

$$z_{i,j}^{n+1}=z_{i,j}^{n}-\frac{\Delta t}{\Delta x}\left(q^*_{s,i+\frac{1}{2},j}-q^*_{s,i-\frac{1}{2},j}\right)-\frac{\Delta t}{\Delta y}\left(q^*_{s,i,j+\frac{1}{2}}-q^*_{s,i,j-\frac{1}{2}}\right), \tag{A5}$$

where q^*_s represents the "sediment" flux at each intercell. Only the computation at the intercell between cells (i,j) and (i+1,j) is reported in the following (for the other three fluxes, similar expressions can be used). Flux is calculated as:

$$q^*_{s,i+\frac{1}{2},j}=\begin{cases}\frac{1}{1-p}q_{s,x,i,j} & \text{if } \tilde{\lambda}_s>0\\ \frac{1}{1-p}q_{s,x,i+1,j} & \text{if } \tilde{\lambda}_s\le 0\end{cases}. \tag{A6}$$

The "sediment" celerity $\tilde{\lambda}_s$ is estimated as:

$$\tilde{\lambda}_s=\frac{\delta q_{s,x,}}{\delta z}, \tag{A7}$$

where $\delta q_{s,x} = q_{s,x,i+1,j} - q_{s,x,i,j}$, and $\delta z = z_{i+1,j} - z_{i,j}$.

The bank failure algorithm presented in Section 2.3 is also added to the model.

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Table 1. Main features for the 2D frontal breach test cases by HR Wallingford (Tests #2 and #10).

Test	**#2**	**#10**
Material type	Sand	Clay
d_{50} (mm)	0.25	0.005
Dam height (m)	0.50	0.60
Crest width (m)	0.20	0.20
Side slopes	1V:1.7H	1V:2H
Initial reservoir level (m)	0	0.58
Pilot channel depth (m)	0.02	0.05
Pilot channel width (m)	0.15	0.50

Table 2. Sensitivity analysis on the model parameters for Test #2 by HR Wallingford. The reference simulation was performed adopting $n_f = 0.018$ m$^{-1/3}$s, $\tau_c = 10^{-2}$ Pa, $k_d = 150$ cm^3/N/s, $\varphi_{c,wet} = \varphi_{dep} = 30°$, $\varphi_{c,dry} = 45°$.

Case	**Peak discharge (m^3/s)**	**% Error (meas.)**	**% Error (ref.)**	**Final top width (m)**	**% Error (meas.)**	**% Error (ref.)**
Measured	0.91	-	-	3.75	-	-
Reference simulation	0.63	-31%	-	3.73	-1%	-
$k_d = 100$ cm^3/N/s	0.54	-41%	-14%	3.23	-14%	-13%
$k_d = 200$ cm^3/N/s	0.68	-25%	8%	4.01	7%	8%
$\tau_c = 10^{-1}$ Pa	0.63	-31%	0%	3.73	-1%	0%
$\tau_c = 10^{-3}$ Pa	0.63	-31%	0%	3.73	-1%	0%
$\varphi_{c,wet} = \varphi_{dep} = 25°$	0.64	-30%	1%	3.65	-3%	-2%
$\varphi_{c,wet} = \varphi_{dep} = 35°$	0.62	-32%	-2%	3.7	-1%	-1%
$\varphi_{c,dry} = 40°$	0.68	-25%	8%	4.15	11%	11%
$\varphi_{c,dry} = 50°$	0.57	-37%	-10%	3.23	-14%	-13%
$\varphi_{c,dry} = 60°$	0.49	-46%	-22%	2.39	-36%	-36%
$\varphi_{c,dry} = 70°$	0.35	-62%	-44%	1.54	-59%	-59%
$n_f = 0.016$ m$^{-1/3}$s	0.60	-34%	-5%	3.64	-3%	-2%
$n_f = 0.020$ m$^{-1/3}$s	0.64	-30%	2%	3.81	2%	2%

Figure 1. 1D frontal dam breach test. Experimental dam profiles and numerical results with the sediment transport model, assuming different values for Manning's coefficient n_f (in $m^{-1/3}s$), at t = 30 s (a), and t = 60 s (b). The dashed line represents the initial dam profile.

Figure 2. 1D frontal dam breach test. Sensitivity analysis on the erodibility coefficient k_d (in cm^3/N/s) with the erosion model: dam profiles at t = 30 s (a), and at t = 60 s (b), and overflow discharge (c). The discharge predicted by the Exner based model is also reported in (c) for comparison. The dashed line represents the initial dam profile.

Figure 3. Test #2 by HR Wallingford: (a) Sketch of the experimental set-up (dimensions for Test #10 in brackets). (b)-(e) Simulated breach evolution in time, represented by the bottom elevation maps at selected times, and velocity vectors map. (f)-(g) Experimental and numerical breach top width evolution in time (f), and discharge exiting through the breach (g); the inflow discharge is also represented in (g); numerical results are reported for three different values of the erodibility coefficient k_d (in cm^3/N/s).

Figure 4. Test #10 by HR Wallingford. Experimental and numerical (a) breach top width evolution in time, and (b) discharge exiting through the breach. The inflow discharge is also represented. Numerical results are reported for two different values of the erodibility coefficient k_d (in cm^3/N/s).

Figure 5. Levee breach experiment. (a) Sketch of the test set-up. (b-c) Breach evolution in time, represented by bottom elevation maps at selected times, as predicted by the sediment transport model (b), and by the erosion model (c). (d) Breach width evolution in time (experimental and numerical). (e) Discharge exiting through the breach (experimental and numerical). (f-g) Discharge exiting through the breach: results of the sensitivity analysis on the erodibility coefficient k_d (in cm^3/N/s) (f), and on the critical shear stress τ_c (in Pa) (g). The inflow and outflow discharge time series in the channel are also reported in panels (e-f-g).

Figure 6. Levee breach on the Enza River. (a) Sketch of the study area: the Enza and Po Rivers (blue), the levees (yellow), the breach location (red), the main residential and industrial areas (black), and the flooded area (cyan) are identified on an orthoimage. (b) Discharge time series imposed as upstream boundary condition, and water levels imposed downstream. (c) Longitudinal profile of the (right) levee crest elevation and of the maximum water surface elevations obtained from simulations with and without the breach erosion. The maximum surveyed water levels at selected locations are also reported for comparison.

Figure 7. Levee breach on the Enza River. Contour maps of the simulated bottom elevation and vector maps of the velocity in the breach zone at selected times: (a) 0.5 h, (b) 1 h, (c) 1.5 h, and (d) 5 h after the beginning of overtopping. Only one vector out of 9 is represented for the sake of clarity. The red contour line identifies the wet/dry front.

Figure 8. Levee breach on the Enza River. Contour map of the simulated maximum water depth (up to 07:00 p.m., December 12), and actual total flooded area (white line).

Figure 9. Levee breach on the Enza River: sensitivity analysis on the erodibility coefficient. (a)-(i) Contour maps of the bottom elevation in the breach zone at selected times for the different

simulations: (a)-(c) k_d = 10 cm^3/N/s; (d)-(f) k_d = 2.5 cm^3/N/s; (g)-(i) k_d = 1 cm^3/N/s. (j) Simulated outflow hydrographs from the breach for different values of the erodibility coefficient k_d (expressed in cm^3/N/s).

Figure 10. Levee breach on the Enza River. Contour map of the final bottom elevation in the breach zone obtained from the sediment transport model.