The theory of precautionary saving: an overview of recent developments

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Abstract

This work reviews recent developments in the literature analyzing precautionary saving. After a description of traditional precautionary saving theory, which considers labor income risk and interest rate risk, we present different research lines which introduce a wide range of extensions and generalizations of the classical model: the contemporaneous presence of multiple risks, changes in risks of different types, multiple variables affecting household utility, preferences non-featuring risk aversion and joint decisions on many choice variables. For each of these issues, we provide specific highlights which summarize the main results obtained in the literature. Lastly, we briefly discuss the analyzes beyond the classical model.

Keywords Precautionary saving, prudence, labor income risk, interest rate risk, background risk, high order risk changes.

1 Introduction

Among the many decisions made every day by households, the choice of how much to save is probably one of the most significant. In fact, saving decisions directly affect

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the well-being of households because they define the intertemporal allocation of wealth and, ultimately, the household consumption profile. Wealth allocation over time may further involve an intergenerational transfer, and in this case, savings are the main tool for realizing a bequest to members of the household who have yet to be born.

On the other hand, not only are savings crucial for household well-being, but they are also crucial for the level of the aggregate income of a country. In fact, the share of consumption on total income is on average about 60 per cent in OECD countries in the years 1970-2017. Therefore, gaining a deeper insight into how the level of saving is determined is of major interest for economists and for economic policy authorities as well expressed by the words of Milton Friedman (1957, p.1) '*The relation between aggregate consumption or aggregate savings and aggregate income [...] has occupied a major role in economic thinking ever since Keynes made it a keystone of his theoretical structure in The General Theory*'.

Recognizing the prevalent role that uncertainty, which usually characterizes the environment where households operate, plays in saving decisions is a necessary first step. **Preliminarily acknowledging the previous circumstance is even more important, as in recent years, in many OECD countries, the increasing trend in wage dispersion due to the process globalisation and digitalization, which is well documented by Berlingieri et al. (2017), has probably scaled up the level of labor income risk and thus overall uncertainty faced by households.** In this context, the effects of uncertainty are relevant not only from a positive perspective but also from a normative perspective. In fact, for individual households and for the economic system as whole, savings represent a useful buffer against negative idiosyncratic random shocks and possible downturns of the economic cycle.

The earliest understanding that the presence of uncertainty has complex effects on saving dates back to the paper by Leland (1968), where the choice of saving in a certainty framework is compared with the same choice made in a context where future income is random. Leland's well-known finding is that an increase in saving due to the presence of a future random income occurs when the third-order derivative of the utility function is positive. Leland calls this extra-saving 'precautionary demand for saving', giving rise to the expression 'precautionary saving'. In the same period, Sandmo (1970) and Rothschild and Stiglitz (1971) examine the consequences of the presence of a random interest rate on the saving choice, again highlighting the role of the third-order derivative of the utility function. Precautionary saving thus captures the effects of uncertainty on intertemporal wealth allocation. Therefore, it is one of the main drivers of saving decisions and has originated a specific and broad literature.

In particular, a novel insight into precautionary saving theory was provided twenty years after these first contributions in the comprehensive analysis by Kimball (1990). This paper introduced for the first time a new feature of household preferences, the concept of 'prudence', related to the convexity of marginal utility. The seminal papers described

above provided the foundations for the modern study of precautionary saving. Over time, the simple framework examined in early works has become more complex. In recent years, the use of new concepts and tools in risk theory has stimulated significant developments in different directions.

The aim of this paper is to provide an overview of the latest developments in precautionary saving theory.[1] In doing so, we specifically focus on works that adopt a classical approach to the problem using the standard expected utility framework, which is the natural and prevailing setting for the analysis of precautionary saving. We present these many contributions and their findings in detail, highlighting the lines of research that appear most significant and promising. However, the last section of the paper provides a short summary of contributions that adopt a non-expected utility framework.

Given this premise, our work proceeds as follows. Section 2 provides a review of basic precautionary saving theory when only future income is uncertain. Section 3 provides a similar review of precautionary saving theory when only the future interest rate is uncertain. Section 4 then examines both these kinds of uncertainty together by assuming that both income and the interest rate are risky. Section 5 generalizes the study of the impact of the introduction of uncertainty by considering the effects of different kinds of modification in the distribution of risk faced by the household, called high-order risk changes. Section 6 examines the case where the utility function of the household depends not only on wealth but also on other non-financial variables, such as the health status of the household or the quality of the environment, which may be uncertain in the future. Section 7 studies precautionary saving in the case of risk loving. Section 8 analyzes the situation where the choice of precautionary saving is not made in isolation but, rather, is made together with the choices of other variables, such as insurance, prevention, labor supply and health investment. In each of these sections, we provide specific highlights that summarize the main results obtained in the literature. Section 9 briefly reviews the literature that uses non-classical approaches to precautionary saving analysis. Lastly, Section 10 discusses some possible future research lines in the precautionary saving literature and concludes.

2 Uncertainty on labor income

Labor income risk is the main source of uncertainty in saving choice, and its effects are studied by a vast literature starting with the seminal papers by Leland (1968), Sandmo (1970) and Dréze and Modigliani (1972). These works analyze the conditions under which precautionary saving arises in a two-period model by comparing the saving choice when there is no uncertainty to the saving choice in the presence of a random labor income in the second period. In this context, Leland (1968) was the first to show that risk aversion

[1] This paper focuses on the theoretical literature on precautionary saving. For a recent survey on the main empirical findings on this issue, see Lugilde et al. (2018).

alone is not a sufficient condition for a precautionary motive for saving to arise. Other conditions need to be satisfied by household preferences, which we describe in detail in this section.

To do so, we consider a simple two-period framework in which a household has a Von Neumann-Morgenstern additively separable utility function, which depends on wealth. We further assume that the utility function is $u(x)$ in period 0 and $v(x)$ in period 1.[2] We denote by u_1, u_{11} and u_{111}, (v_1, v_{11} and v_{111}, respectively) the first, second and third derivatives of u (v respectively). Functions u and v are assumed to be strictly increasing and strictly concave ($u_1 > 0$, $v_1 > 0$, $u_{11} < 0$, $v_{11} < 0$), and, at least, three times continuously differentiable.

We start by considering the case where the household has a certain labor income y_0 in period 0 and an uncertain labor income $\tilde{y_1} = y_1 + \tilde{\epsilon}$ in period 1, where $\tilde{\epsilon}$ is a random variable with a known variance, such that $\mathbb{E}[\tilde{\epsilon}] = 0$, further implying $\mathbb{E}[\tilde{y_1}] = y_1$. To study precautionary saving, we compare the optimal choice of the household in the case described above with the optimal choice in the case where the labor income in period 1 is certain and equal to y_1.

The following is a simple formulation of the household decision problem in the certainty case:

$$\max_s u\left(y_0 - s\right) + \frac{1}{1+\rho} v\left(y_1 + Rs\right) \tag{1}$$

where s denotes saving, the parameter ρ is the intertemporal discount factor, and R is the return on saving.

The optimal level of saving $s^\star$ in this problem is defined by the following first order condition:

$$u_1\left(y_0 - s^\star\right) = \frac{1}{1+\rho} R v_1\left(y_1 + Rs^\star\right). \tag{2}$$

On the other hand, when labor income in period 1 is assumed to be uncertain, the household decision problem becomes the following:

$$\max_s u\left(y_0 - s\right) + \frac{1}{1+\rho} \mathbb{E}\left[v\left(\tilde{y_1} + Rs\right)\right] \tag{3}$$

In this risky framework, the optimal level of saving $s^{\star\star}$ is defined by the following first order condition:

$$u_1\left(y_0 - s^{\star\star}\right) = \frac{1}{1+\rho} R \mathbb{E}\left[v_1\left(\tilde{y_1} + Rs^{\star\star}\right)\right]. \tag{4}$$

The traditional approach to the study of precautionary saving involves a comparison between Equations (**??**) and (**??**). Since $v_{11} < 0$, it is clear that $s^{\star\star} > s^\star$ if and only if

$$\mathbb{E}\left[v_1\left(\tilde{y_1} + Rs^{\star\star}\right)\right] - v_1\left(y_1 + Rs^\star\right) > 0 \tag{5}$$

[2]Pioneering analyses of precautionary saving (i.e., Leland, 1968 and Sandmo, 1970) considered non-separable utility. However, we focus here on the case of a separable utility function, which is the setting typically used in the subsequent precautionary saving literature.

By Jensen's inequality, Condition (**??**) holds when marginal utility v_1 is convex.

The convexity of marginal utility ($v_{111} > 0$) is thus the characteristic of the household preferences responsible for precautionary saving to arise. Beyond providing a comprehensive analysis of it, Kimball (1990) was the first to interpret this feature of the utility function as a specific aspect of household preferences, distinct from others, and he called it 'prudence'. This implies the following well-known result:

Highlight 1. *Prudence is a necessary and sufficient condition for precautionary saving when future labor income is uncertain (Leland, 1968 and Kimball, 1990).*

Kimball (1990) further shows an isomorphism between the theory of precautionary saving, described by prudence, and the theory of risk attitude, described by risk aversion, which allows to apply some concepts and indexes concerning risk aversion to prudence and precautionary saving.[3] Starting from this idea, he derived a measure of the intensity of the precautionary saving motive based on the 'equivalent precautionary premium' concept, which indicates a certain reduction in wealth required for the household to choose the same optimal level of saving in the certainty case as in the case where the household bears a future labor income risk. The parallel with the equivalent risk premium concept by Pratt (1964) is clear.

Furthermore, it can be shown that the equivalent precautionary premium can be approximated by the following:

$$\psi(\tilde{y_1}, s^{\star\star}) \cong -\frac{v_{111}(y_1 + Rs^{\star\star})}{v_{11}(y_1 + Rs^{\star\star})} \cdot \frac{var[\tilde{y_1}]}{2} \tag{6}$$

Here, the isomorphism with risk aversion theory is also clear. Pratt (1964) defines the index of absolute risk aversion as $-\frac{v_{11}(y_1+Rs^{\star\star})}{v_1(y_1+Rs^{\star\star})}$. The index $-\frac{v_{111}(y_1+Rs^{\star\star})}{v_{11}(y_1+Rs^{\star\star})}$ in Equation (**??**) is thus the index of absolute prudence.

On the basis of Equation (**??**), we conclude the following:

Highlight 2. *The intensity of the precautionary saving motive can be measured by the index of absolute prudence (Kimball, 1990).*

The findings described thus far are the classical results of precautionary saving under labor income risk. These results have recently been developed in different directions to either weaken the conditions or extend applicability. In the first direction, Menegatti (2001) reconsiders Highlight **??** and shows that in the case of an unbounded domain of the utility function, risk aversion and the assumption that the sign of the third derivative of the utility function is invariant (either positive, negative or null for every value of wealth) are sufficient assumptions to ensure precautionary saving without explicitly assuming a positive third derivative.

[3] This is done simply by 'augmenting' the order of each derivative involved in them by one degree.

In the second direction, since Kimball's index of absolute prudence (like Pratt's index of absolute risk aversion) is a local measure, Eisenhauer (2006) proposes an extended version of this measure that is applicable to both small and large risks. A different extension of Kimball's analysis is provided by Hau (2002), who derives a version of Kimball's precautionary premium for the case of a non-additively separable utility function and shows that the comparison between the precautionary saving chosen by different households depends on both their absolute prudence and their intertemporal substitution rates. Lastly, Light (2018) extends the result that prudence implies precautionary saving to Markovian earnings dynamics.

The previous findings clearly suggest that prudence is the feature of preferences that ensures that a household has a precautionary saving motive and that it also defines this motive. This has led the literature to enrich the set of possible interpretations of prudence using different tools: specific changes in the risk distribution, a comparison between lotteries, and the utility premium concept.

The first interpretation of prudence considers the effects of specific changes in risk. In this context, the concept of an increase in downside risk describes a change in the distribution of a risky variable that increases its left skewness, leaving the expected value and the variance unchanged. Menezes et al. (1980) show that if the marginal utility is convex, the household likes this change. Thus, prudence can be interpreted as the feature of preferences that causes a household to appreciate an increase in downside risk.[4]

The second interpretation of prudence refers to the idea of risk apportionment of Eeckhoudt and Schlesinger (2006). This idea concerns a household whose utility function presents subsequent derivatives that alternate in sign, being positive in the case of odd derivatives and negative in the case of even derivatives. This means a positive first derivative, a negative second derivative, a positive third derivative, a negative fourth derivative, and so on.[5] These preferences are called 'mixed risk aversion', and they were first studied by Brockett and Golden (1987) and Caballé and Pomanski (1996).

By comparing different pairs of lotteries, Eeckhoudt and Schlesinger (2006) show that if a household exhibits this kind of preferences, it desires to combine things that are perceived as good with things perceived as bad instead of combining good things with other good things and bad things with other bad things. This feature of preferences is called a preference for harm disaggregation and reflects the household desire to separate different harms. In this context, prudence can be interpreted as the feature of the household utility function that defines a specific kind of preference for harm disaggregation that is closely related to precautionary saving. In fact, in a two-period saving model, the household bears the labor income risk in the second period, and due to risk aversion, this

[4]Similarly, Eeckhoudt et al. (1995) analyze the concept of an upward shift of an increase in risk as a change in the distribution of a risky variable inducing more risk in the wealthier states and less risk elsewhere, and they demonstrate that a prudent household likes this shift.

[5]Note also that a negative fourth derivative of the utility function is called 'temperance' by Kimball (1992) and that a positive fifth derivative is called 'edginess' by Lajeri-Chaherli (2004).

uncertainty is perceived as bad. Therefore, the household desires to combine a random labor income with a good. This good is a higher level of wealth in the second period that is obtained by increasing savings. Thus, household savings are larger under uncertainty than under certainty.

The third interpretation of prudence is based on the utility premium concept, defined as the difference between the expected utility when the household bears the labor income risk and the utility when this risk is absent.[6] This quantity measures the disutility caused by the presence of a random labor income. Note now that when labor income is random, if no specific insurance is available, the household cannot escape this risk. The only thing that the household can do is to increase its utility by changing its level of wealth in the period where there is uncertainty. This is done by means of saving. Accordingly, Menegatti (2007) shows that prudence can be interpreted as the feature of preferences that makes the household seek to increase its wealth when labor income is risky to reduce the disutility due to uncertainty.

To conclude, the three interpretations described above can be summarized as follows:

Highlight 3. *In the presence of labor income risk, a household that saves more under uncertainty:*

1. *likes changes in the distribution of risks that increase left skewness, given expected value and variance (Menezes et al., 1980);*
2. *wants to undertake risk apportionment by combining the 'bad' of bearing labor income risk with the 'good' of having higher wealth (Eeckhoudt and Schlesinger, 2006);*
3. *has a lower reduction in its utility due to the presence of labor income risk when it is wealthier (Menegatti, 2007).*

3 Uncertainty on the interest rate

When choosing its level of saving, a household may face a further type of uncertainty other than labor income risk. This second kind of uncertainty, first studied by Sandmo (1970) and Rothschild and Stiglitz (1971), concerns the return on saving and is known as interest rate risk.

To study this issue, we consider a setting where the labor income in period 1 is certain, while uncertainty involves the return on saving R. As in the previous section, we denote the uncertain level of the return on saving by $\tilde{R} = R + \tilde{\eta}$, where $\tilde{\eta}$ is a random variable with known variance, such that $\mathbb{E}[\tilde{\eta}] = 0$, implying $\mathbb{E}[\tilde{R}] = R$. In this case, the maximization problem of the household is as follows:

$$\max_{s} u\left(y_0 - s\right) + \frac{1}{1+\rho}\mathbb{E}\left[v\left(y_1 + \tilde{R}s\right)\right] \tag{7}$$

[6] The general definition of the utility premium was first provided by Friedman and Savage (1948).

and the optimal level of saving $s^{\star\star\star}$ is defined by the following first order condition:

$$u_1\left(y_0 - s^{\star\star\star}\right) = \frac{1}{1+\rho}\mathbb{E}\left[\tilde{R}v_1\left(y_1 + \tilde{R}s^{\star\star\star}\right)\right]. \tag{8}$$

By comparing Equation (**??**) and Equation (**??**), which defines the optimal level of saving in the certainty case, it is clear that $s^{\star\star\star} > s^{\star}$ if and only if

$$\mathbb{E}\left[\tilde{R}v_1\left(y_1 + \tilde{R}s^{\star\star\star}\right)\right] - Rv_1\left(y_1 + Rs^{\star}\right) > 0. \tag{9}$$

Following Rotschild and Stiglitz (1971) and Eeckhoudt and Schlesinger (2008) we study this condition under the assumption $y_1 = 0$.[7] In this case, Condition (**??**) holds if

$$-\frac{v_{111}\left(Rs\right)}{v_{11}\left(Rs\right)}Rs > 2 \tag{10}$$

where the left-hand side of Condition (**??**) is defined as the index of relative prudence.[8]

Thus, we have the following:

Highlight 4. *An index of partial relative prudence larger than* 2 *is a necessary and sufficient condition for precautionary saving when the return of saving is uncertain (Rothschild and Stiglitz, 1971).*

By comparing the present setting with the setting where labor income is risky, from Highlights **??** and **??**, it is easy to see that in the presence of an interest rate risk, the simple convexity of marginal utility does not ensure that a precautionary motive for saving emerges. When the return on saving is random, the necessary and sufficient condition ensuring precautionary saving becomes more restrictive and requires not only the household to be prudent but also that its preferences display additional specific features.

A reason for this is provided by Rothschild and Stiglitz (1971), who show that Condition (**??**) summarizes two opposite effects: an income effect and a substitution effect. The income effect is also at work when a labor income risk is present and pushes the household to increase savings to increase its wealth in the period where it faces uncertainty. The substitution effect, on the other hand, emerges only in the presence of an interest rate risk and pushes the household to reduce its level of saving. This happens because by

[7] **The generalization to the case where y_1 is different from zero is studied by Jouini et al. (2013). On this case see also Chiu et al. (2012).**

[8] For further analytical details on the index of partial relative prudence, see Choi et al. (2001) and Eichner and Wagener (2004 a,b). Additionally, note that in this case, there are parallels with risk aversion theory. In fact, Fishburn and Porter (1976) show that an increase in the risk-free interest rate in a static portfolio problem generates an increase in savings if the index of partial relative risk aversion is larger than one.

saving and investing in an asset whose interest rate is random, the household increases its exposure to risk.[9]

The income effect is related to the term $Rsv_{111}(y_1 + Rs)$, which includes prudence, while the substitution effect is related to the term $-2v_{11}(y_1 + Rs)$, which includes risk aversion. Condition (**??**) ensures that the first effect prevails over the second effect.[10]

Highlight **??** can also be interpreted by means of lotteries by appealing to the concept of risk apportionment. As explained in the previous section, this concept, introduced by Eeckhoudt and Schlesinger (2006) and generalized by Eeckhoudt et al. (2009a), relates different aspects of preferences to the household desire to disaggregate different kinds of harms. In particular, Eeckhoudt et al. (2009b) study a setting where the household chooses between two lotteries involving a sure loss and a multiplicative zero-mean risk. The loss and the risk occur simultaneously in one lottery and separately in the other lottery.

In this context, different features of household preferences have opposing effects on the choice over the combination of these two elements. On the one hand, prudence pushes the household to disaggregate the two harms by choosing the lottery where the sure loss and the zero-mean multiplicative risk occur separately. On the other hand, risk aversion pushes the household to scale down the level of risk by choosing the lottery where the multiplicative risk and the sure loss jointly occur. In fact, in this case, the zero-mean risk is applied to lower wealth, and this decreases the variance of wealth in the state of the world where the loss and the risk are present. It follows that the household chooses harm disaggregation only when prudence is sufficiently strong (i.e., the index of partial relative prudence is larger than 2). Chiu et al. (2012) provide a similar result in a more general context and study an application to a specific problem of precautionary saving.[11]

In conclusion, the interpretations described above can be summarized as follows:

Highlight 5. *In the presence of interest rate risk, a household saves more under uncertainty when:*

1. *the elasticity of household risk aversion with respect to saving is larger than the elasticity of the variance of second-period income with respect to saving (Rothschild and Stiglitz, 1971);*

[9] Rothschild and Stiglitz (1971, p.69) explain this last effect, suggesting that for a risk-averse household, '*a bird in the hand is worth two in the bush*'.

[10] Magnani (2017) interprets Condition (**??**) in terms of a comparison between elasticities: the household saves more under uncertainty if the elasticity of its risk aversion with respect to saving is larger than the elasticity of the variance of second-period income with respect to saving, which is the constant threshold 2.

[11] In the same research line, Denuit and Rey (2014) again compare different lotteries and show that an elementary correlation increasing the transformation in risk, i.e., a simultaneous increase in the probability of obtaining extreme outcomes and a corresponding decrease in the probability of obtaining intermediate outcomes, causes a decrease in the expected utility of a household when the partial relative prudence index is larger than 2.

2. *the household desire to disaggregate harms (risk apportionment) is stronger than its desire to avoid the scaling up of risk induced by saving (Eeckhoudt et al., 2009b).*

4 Uncertainty on both labor income and the interest rate

The two previous sections study cases where the household saving choice is affected by a single source of uncertainty related to either labor income (Section 2) or the interest rate (Section 3). A natural generalization is to consider these two sources of uncertainty jointly. This problem was not examined in the literature for a long period of time due to its analytical complexity. Li (2012) was the first to address the issue of precautionary saving in the presence of labor income and interest rate risks together, giving rise to a debate that has recently been enriched by the contributions of Baiardi et al. (2014, 2015). This section reviews the main results obtained in these works.

When labor income and interest rate risks are simultaneously considered,[12] the household problem becomes the following:

$$\max_s u\left(y_0 - s\right) + \frac{1}{1+\rho}\mathbb{E}\left[v\left(\tilde{y_1} + \tilde{R}s\right)\right] \tag{11}$$

The optimal level of saving s° in this setting is determined by the following first-order condition:

$$u_1\left(y_0 - s^\circ\right) = \frac{1}{1+\rho}\mathbb{E}\left[\tilde{R}v_1\left(\tilde{y_1} + \tilde{R}s^\circ\right)\right]. \tag{12}$$

The conditions ensuring positive precautionary saving are obtained by comparing the optimal choice when labor income and the interest rate are certain with the optimal choice when these variables are random (Equations **??** and **??**, respectively). It is clear that $s^\circ > s^\star$ holds if and only if:

$$\mathbb{E}\left[\tilde{R}v_1\left(\tilde{y_1} + s^\circ\tilde{R}\right)\right] - Rv_1\left(y_1 + Rs^\circ\right) > 0 \tag{13}$$

Starting from Condition (**??**), it is possible to obtain different sufficient conditions for precautionary saving. These conditions depend on some specific assumptions involving the correlation between the two risks, risk size and household preferences.

In particular, Li (2012) considers a kind of dependence between the two risks, termed 'positive quadrant dependence'.[13] Given the two generic random variables X and Z this

[12] We assume that the analytical features of the two random variables describing the two risks are the same as in Sections 2 and 3.

[13] Formally, given the two generic random variables X and Z, with $F(X,Z)$ being the joint distribution of X and Z, and $F_X(X)$ and $F_Z(Z)$ the marginal distribution of X and Z, (X,Z) is positively quadrant dependent if $F(X,Z) \geq F_X(X)F_Z(Z)$, $\forall X, Z$.

assumption implies that '*knowledge of X being small increases the probability of Z being small*' (Lehmann, 1966 p.1143). Li (2012) shows that if labor income and interest rate risks are positive quadrant dependent, then Condition (**??**), indicating a threshold 2 for the partial relative prudence index, is sufficient for precautionary saving. As a consequence, the finding by Rothschild and Stiglitz (1971) obtained when only the interest rate is risky, also holds when labor income and the interest rate are jointly risky and positive quadrant dependent.[14]

Baiardi et al. (2014) re-examine this problem in a setting that does not introduce restrictions on the joint distribution of the two risks but the risks are small. They derive a necessary and sufficient condition for precautionary saving that involves a variable threshold for the index of partial relative prudence. This threshold is a function of the variances of the two risks and of their covariance. Both the results of Li (2012) and those of Baiardi et al. (2014) can be read in light of the reasoning presented in Section 3. Their interpretation is thus analogous to that provided for Condition (**??**): precautionary saving arises when the income effect of saving is larger than the substitution effect of saving.[15]

More recently, Baiardi et al. (2015) demonstrate that, for any risk size, the following is a sufficient condition for a prudent household to save more in the presence of risk:

$$cov[\tilde{y_1}, \tilde{R}] < -s^{\circ} var[\tilde{R}] \tag{14}$$

where $cov[\tilde{y_1}, \tilde{R}]$ is the covariance between random labor income ($\tilde{y_1}$) and the random interest rate ($\tilde{R}$), while $s^{\circ} var[\tilde{R}]$ is the variance of the random return on saving. The authors provide three different interpretations of Condition (**??**). The first interpretation shows that a prudent household saves more in the presence of risk whenever a marginal increase in saving (meaning a larger investment in the risky asset) reduces the variance of total income, where total income is the sum of labor income and the return on saving. This idea is also at the basis of the second interpretation, which shows that a prudent household saves more in the presence of risk when the covariance between total income and the interest rate is negative. Lastly, the third interpretation is based on the utility premium concept described in Section 2 and shows that as in the presence of labor income risk only, precautionary saving is related to the desire of a prudent household to reduce the pain caused by uncertainty.

To conclude, the results described above can be summarized as follows:

Highlight 6. *In the presence of labor income and interest rate risks, a household saves more under uncertainty when one of the following sets of conditions holds:*

[14]Generalizing the analysis by Gunning (2010) made under the assumption of a Constant Relative Risk Aversion (CRRA) utility function, Vergara (2017) obtains a similar result in the simpler case where the two risks are perfectly positively correlated.

[15]Notably, in this setting, a precautionary motive for saving may arise even if the household is imprudent, which is not the case in the rest of the literature.

1. *the two risks are positive quadrant dependent and the partial relative prudence index is greater than* 2 *(Li, 2012);*

2. *the two risks are small and the partial relative prudence index is greater than a variable threshold depending on the variances of the two risks and on their covariance (Baiardi et al., 2014);*

3. *the covariance between the two risks is smaller than the opposite of the variance of the random returns on saving, and the household is prudent (Baiardi et al., 2015).*

5 Changes in risks of different orders

The literature on precautionary saving presented in the previous sections studies the effects of the introduction of some kind of risk in the decision process of a household that chooses its level of saving. This analysis is typically implemented by comparing the choice under certainty with the choice under uncertainty. A generalization of this approach has been adopted by another strand of the literature that considers the comparison between two risky situations, where changes in high-order moments of the distribution of a risk occur. Hereafter, we refer to these kinds of changes in risk as 'high-order changes in risk'.

The seminal contribution in this field was provided by Eeckhoudt and Schlesinger (2008), who were the first to study how different changes in high-order changes in risk affect the demand for saving. More recently, other authors, including Chiu et al. (2012), Denuit and Rey (2014), Liu (2014), and Menegatti (2015), have contributed to the debate.

These analyses study possible changes in risk that rely on the concepts of Nth-order stochastic dominance (NSD) and an Nth-degree risk increase. Both of these concepts are widely used in the theory of finance and in the analysis of investment choices. Since a complete analysis falls beyond the scope of this work, we provide here only a short presentation of these concepts, and we refer to Ekern (1980), Ingersoll (1987) and Levy (2006) for further details.

To introduce NSD and the Nth-degree risk increase, consider two generic random variables X and Z. We denote the cumulative distribution function of X by $F_X(x)$. Define further $F_X^1(x) = F_X(x)$ and $F_X^{n+1}(x) = \int_a^x F_X^n(t)dt$ for all $x \geq \alpha$ and all $n \in \{1, 2, ...\}$. We similarly define $F_Z(x)$, $F_Z^1(x)$ and $F_Z^{n+1}(x)$. In this context we have that:

Definition 7. *Z dominates X via NSD if both $F_Z^N(x) \leq F_X^N(x)$ for all $x \geq \alpha$, and $F_Z^n(M) \leq F_X^n(M)$ for $n = 2, ..., N$ hold with at least a strict inequality, where $M > 0$ is such that $F_Z^1(M) = F_X^1(M) = 1$.*

From the definition of NSD, as a special case, it is possible to define the concept of an Nth-degree risk increase.

Definition 8. *X is an Nth degree risk increase over Z if $F_Z^N(x) \leq F_X^N(x)$ for all $x \geq \alpha$, where the inequality is strict for some x and $F_Z^n(M) = F_X^n(M)$ for $n = 2, ..., N$, where $M > 0$ is such that $F_Z^1(M) = F_X^1(M) = 1$.*

As noted by Ekern (1980), an Nth-degree risk increase is equivalent to the case where Z dominates X via NSD and the first $N-1$ moments of Z and X are equal. The concept of an Nth-degree risk increase applies to many changes in risk that are commonly studied in the literature. For instance, mean-preserving spreads, which were first studied by Rothschild and Stiglitz (1970), can be described by a second-degree increase in risk and are changes in the distribution of a risky variable that cause its variance to vary while leaving the mean unchanged. Similarly, increases in downside risk, studied by Menezes et al. (1980), can be described by a third-degree increase in risk and, as mentioned in the previous section, are changes in the distribution of a random variable that cause its skewness to vary while leaving the mean and the variance unchanged.

Consider now the saving decision of a household. A change in risk of the types described above may occur either in the case where labor income is random or in the case where the interest rate is random. In all these circumstances, the level of the uncertainty faced by the household increases because the distribution of the random variable(s) entails 'more risk' in some sense. As in the models presented in the previous sections, where there is a transition from a certain situation to an uncertain situation, in this setting, the reaction of the household to a change in risk can entail an increase in savings, implying that a precautionary motive for saving may arise. To analyze the conditions necessary for this to happen, we refer to the maximization problems of Equations (**??**) and (**??**) and to the first-order conditions (**??**) and (**??**) presented in the previous sections.

In particular, in the presence of labor income risk, the analysis of the precautionary motive for saving involves a comparison between the optimal levels of saving when the household faces either the risky income $\tilde{y}_1^a$ or the risky income $\tilde{y}_1^b$. In this setting, when shifting from $\tilde{y}_1^a$ to $\tilde{y}_1^b$, the household commits to precautionary saving, i.e. $s_a^{\star\star} \leq s_b^{\star\star}$ (where $s_i^{\star\star}$ denotes the optimal level of saving in the presence of risky income $\tilde{y}_1^i$ with $i = a, b$) if and only if

$$\mathbb{E}\left[v_1\left(\tilde{y}_1^b + Rs_a^{\star\star}\right)\right] - \mathbb{E}\left[v_1\left(\tilde{y}_1^a + Rs_a^{\star\star}\right)\right] \geq 0 \tag{15}$$

As shown by Eeckhoudt and Schlesinger (2008), the conditions for Inequality (**??**) to be satisfied vary according to the type of risk change faced by the household. When $\tilde{y}_1^a$ dominates $\tilde{y}_1^b$ via NSD, $s_a^{\star\star} \leq s_b^{\star\star}$ if

$$(-1)^{n+1}\frac{d^n v(x+z)}{dx^n} \geq 0 \quad for\ n = 1, 2, ..., N. \tag{16}$$

Similarly, when $\tilde{y}_1^b$ is an Nth-degree risk increase over $\tilde{y}_1^a$, $s_a^{\star\star} \leq s_b^{\star\star}$ if

$$(-1)^{N+1}\frac{d^N v(x+z)}{dx^N} \geq 0 \tag{17}$$

Clearly, this second condition is less restrictive, since an Nth-degree risk change is a special case of NSD. Additionally, note that Condition (**??**) requires that the derivatives of the utility function alternate in sign until order $N+1$, being positive for odd orders and negative for even orders. As noted in Section 2, this kind of preferences is called 'mixed risk aversion'.

Considering again the case of high-order changes in labor income risk, Liu (2014) generalizes the precautionary premium concept by Kimball (1990) to the case of Nth-degree deteriorations in future income. Moreover, Menegatti (2015) shows that in some circumstances, the response of saving in case of increases in risk of a given order can be inferred by the response in case of an increase in risk of a different order. He also shows that under the same assumptions, Condition (**??**) implies Condition (**??**).

The second kind of risk to be analyzed in this context is the interest rate risk. This analysis involves a comparison between the optimal levels of saving when the household faces either the risky interest rate $\tilde{R}_1^a$ or the risky interest rate $\tilde{R}_1^b$. In this setting, the household commits to precautionary saving, i.e., $s_a^{\star\star\star} \leq s_b^{\star\star\star}$ (where $s_i^{\star\star\star}$ denotes the optimal level of saving in the presence of the interest rate $\tilde{R}^i$ with $i = a, b$) if and only if

$$\mathbb{E}\left[\tilde{R}^b v_1\left(y_1 + \tilde{R}^b s_a^{\star\star\star}\right)\right] - \mathbb{E}\left[\tilde{R}^a v_1\left(y_1 + \tilde{R}^a s_a^{\star\star\star}\right)\right] \geq 0 \tag{18}$$

As in Section 3, where interest rate risk is introduced, we study this condition under the assumption $y_1 = 0$ (see Eeckhoudt and Schlesinger, 2008).[16]Additionally, in this case, the conditions for Inequality (**??**) to hold change in the cases of NSD or Nth-degree risk changes. As shown by Eeckhoudt and Schlesinger (2008), in case $\tilde{R}^a$ dominates $\tilde{R}^b$ via NSD, $s_a^{\star\star\star} \leq s_b^{\star\star\star}$ if

$$-x\,\frac{\frac{d^{n+1}v(x+z)}{dx^{n+1}}}{\frac{d^n v(x+z)}{dx^n}} \geq n \quad for\ n = 1, 2, ..., N \tag{19}$$

Similarly, when $\tilde{R}^b$ is an Nth-degree risk increase over $\tilde{R}^a$, $s_a^{\star\star\star} \leq s_b^{\star\star\star}$ if

$$-x\,\frac{\frac{d^{N+1}v(x+z)}{dx^{N+1}}}{\frac{d^N v(x+z)}{dx^N}} \geq N \tag{20}$$

so long as $\frac{d^N v(x+z)}{dx^N} \neq 0$. The left-hand side of Inequality (**??**) is called N+1th partial relative risk aversion (see Eeckhoudt and Schlesinger, 2008). Additionally, in this case, the condition for the case of an Nth-degree risk increase is less restrictive than that required for NSD.

In a recent paper, Wong (2018) generalizes the previous analyses of high-order changes in interest rate risk to the case where the increase in interest rate risk is characterized

[16]**For the case where y_1 is different from zero, see again Jouini et al. (2013).**

by a general notion of (M, N)th-order stochastic dominance (where N $\geq$ M $\geq$ 1).[17] He further provides an interpretation of the conditions for precautionary saving that extends the interpretation by Magnani (2017) presented in Section 3. Lastly, note that the conditions derived above can be interpreted as a kind of generalization of those presented in Highlights (**??**) and (**??**). In this line, the insight by Chiu et al. (2012) and Denuit and Rey (2014), presented in Section 3, can be generalized to the case of changes in risk studied in this section.

To conclude, the results described above can be summarized as follows:

Highlight 9. *When considering high-order changes in risk, the household saves more under uncertainty if:*

1. *the derivatives of the utility function until order N alternate in sign, being positive for odd orders and negative for even orders, in the case of NSD changes in labor income risk (Eeckhoudt and Schlesinger, 2008);*

2. *the Nth derivative of the utility function is positive if N is odd and negative if N is even in the case of an Nth-degree increase in labor income risk (Eeckhoudt and Schlesinger, 2008);*

3. *each index of relative risk aversion of order from* 2 *to* $N+1$ *is larger than a threshold equal to the order of the index minus* 1 *in the case of NSD changes in interest rate risk (Eeckhoudt and Schlesinger, 2008);*

4. *the N+1th relative risk aversion index is larger than the threshold N in the case of an Nth-degree increase in interest rate risk (Eeckhoudt and Schlesinger, 2008).*

6 Precautionary saving and background risks

Saving typically affects the intertemporal allocation of household wealth, moving wealth from one period to another. For this reason, the analysis of saving choice, and thus of precautionary saving, focuses on the effects of wealth on utility. This usually implies considering a utility function where the only relevant dimension is wealth, i.e., a univariate utility function.

Household utility, however, is affected by many other factors. For instance, the wellness of a household is influenced by its health status or by the quality of the environment where the household lives. Taking this into account means considering a multivariate utility function, where dimensions other than wealth are introduced. In this context, it can be the case that these variables, which affect utility, will have uncertain levels in the future. For instance, future health status may be uncertain because of possible health deterioration

[17] On the concept of (M, N)th-order stochastic dominance, see Liu (2014) and Ebert et al. (2018).

or possible disease contagion. Similarly, uncertainty on environmental quality may arise because of pollution and climate change.

The presence of these kinds of uncertainty generates other types of risk different from labor income risk and interest rate risk that may influence the level of precautionary saving. These other risks involve elements related to the *background* conditions in which household decisions on saving are made. For this reason, they are called 'background risks'.[18]

The introduction of background risks in saving analysis was first considered by Eeckhoudt et al. (2007), who studied a problem similar to that presented in the previous sections but moved from the *one-argument* (or *univariate*) utility framework to a model including a *two-argument* (or *bivariate*) utility function, where the second argument represents variables such as health status or environmental quality.

In this context, we analyze precautionary saving in a setting where the consumer has a utility function $u(x, h)$ in period 0 and $v(x, h)$ in period 1, where x is wealth and h is a non-financial variable. The functions u (v respectively) are at least three times continuously differentiable in both arguments. We denote $u_1 = \partial u/\partial x$, $u_2 = \partial u/\partial h$, $u_{11} = \partial^2 u/\partial x^2$, $u_{22} = \partial^2 u/\partial h^2$, $u_{12} = \partial^2 u/\partial x \partial h$, $u_{111} = \partial^3 u/\partial x^3$, $u_{222} = \partial^3 u/\partial h^3$ and so on. The same notation is also adopted for $v(x, h)$. In the same way as in the univariate case, functions $u(x, h)$ and $v(x, h)$ are assumed to be strictly increasing and strictly concave with regard to each argument ($u_1 > 0$, $v_1 > 0$, $u_{11} < 0$, $v_{11} < 0$ and $u_2 > 0$, $v_2 > 0$, $u_{22} < 0$, $v_{22} < 0$).

In the certainty case, the household problem is as follows:

$$\max_s u\left(y_0 - s, h_0\right) + \frac{1}{1+\rho}\left[v\left(y_1 + Rs, h_1\right)\right] \tag{21}$$

The optimal level of saving $\bar{s}$ is defined by the following first-order condition:

$$u_1\left(y_0 - \bar{s}, h_0\right) = \frac{1}{1+\rho} R v_1\left(y_1 + R\bar{s}, h_1\right). \tag{22}$$

When a background risk is present, the household problem becomes the following:

$$\max_s u\left(y_0 - s, h_0\right) + \frac{1}{1+\rho}\mathbb{E}\left[v\left(y_1 + Rs, \tilde{h}_1\right)\right] \tag{23}$$

where $\tilde{h}_1$ is a random variable with known variance, such that $\mathbb{E}[\tilde{h}_1] = h_1$. In the latter case, the household chooses its optimal level of saving in a context where its future income and the interest rate are certain but there is, for instance, uncertainty on its future health status or on future environmental quality. The optimal level of saving $\check{s}$ is now determined

[18]Note that the background risk concept was first introduced in a setting with a univariate utility function as an exogenous risk (typically uninsurable) different from the main endogenous risk (e.g., Kimball, 1993; Eeckhoudt et al., 1996; Gollier and Pratt, 1996).

by the following first-order condition:

$$u_1\left(y_0 - \check{s}, h_0\right) = \frac{1}{1+\rho} R\mathbb{E}\left[v_1\left(y_1 + R\check{s}, \tilde{h}_1\right)\right]. \tag{24}$$

By comparing Equations (**??**) and (**??**), Eeckhoudt et al. (2007) show that the household saves more under uncertainty when the condition $v_{122} > 0$ is satisfied. This feature of household preferences is defined by these authors as 'cross-prudence in wealth'. Its interpretation is again related to the idea of harm disaggregation described in Section 2. In the case of univariate utility, prudence is associated with the desire to disaggregate the harm of having low wealth and the harm of bearing risk on wealth. Similarly, in the case of a bivariate utility, cross-prudence in wealth is the desire to disaggregate the harm of having low wealth and the harm of bearing risk on the background variable. By the reasoning presented in Section 2, this desire pushes the household to increase its savings, thus generating precautionary saving.

We now consider the case where both labor income and the background variable are assumed to be risky in period 1. The maximization problem becomes the following:

$$\max_s u\left(y_0 - s, h_0\right) + \frac{1}{1+\rho}\mathbb{E}\left[v\left(\tilde{y}_1 + Rs, \tilde{h}_1\right)\right] \tag{25}$$

and the optimal level of saving $\hat{s}$ is defined by the following first-order condition:

$$u_1\left(y_0 - \hat{s}, h_0\right) = \frac{1}{1+\rho} R\mathbb{E}\left[v_1\left(\tilde{y}_1 + R\hat{s}, \tilde{h}_1\right)\right]. \tag{26}$$

This problem has been investigated in many papers. Courbage and Rey (2007) and Menegatti (2009a) provide some results under different assumptions on the joint distribution of the two risks. Their main result makes use of a feature of household preferences, $v_{112} > 0$, which we call 'cross-prudence in the background variable'.[19] This feature of household preferences is easily interpreted by analogy with the concept of cross-prudence in wealth. In fact, it is the desire to disaggregate the harm of having a low level of the background variable (for instance, health) and the harm of bearing risk on wealth. The above-mentioned authors show that when the two risks are Bernoulli distributed, the household saves more under uncertainty when one of the two following different sets of conditions is satisfied:

- prudence ($v_{111} > 0$), cross-prudence in the background variable ($v_{112} > 0$) and positive correlation between the two risks;
- prudence ($v_{111} > 0$), cross-imprudence in the background variable ($v_{112} < 0$) and negative correlation between the two risks.

[19] Eeckhoudt et al. (2007) define this concept by studying a model of tertiary prevention, where the second argument of the utility function is health. For this reason, they call this condition 'cross-prudence in health'.

Furthermore, Menegatti (2009b) considers the case of small risks without making any assumption on their distribution and derives a sufficient condition for precautionary saving that involves prudence, cross-prudence in wealth, cross-prudence in the background variable, the variances of the two risks and their covariance.

These results highlight the importance for the analysis of the cross-derivatives of the household utility function. This importance is also confirmed by the findings of Denuit et al. (2011), who examine background risks without introducing any assumptions on risk size. They show that under different types of positive dependence between the two risks, a sufficient condition to have precautionary saving is that all the derivatives and the cross-derivatives of the utility function until order three in the wealth argument and until order two in the background variable alternate in sign, being positive in the case of an odd total order of differentiation and negative in the case of an even total order of differentiation.[20]

A generalization of some of the results presented in Section 5 to this context is provided by Jouini et al. (2013), who consider a model where the household faces labor income risk and a background risk. In this case, the condition ensuring precautionary saving involves high-order partial and cross-derivatives of the utility function.

The case where interest rate risk is added to labor income risk and to background risk is analyzed by Baiardi et al. (2014), although only in the case of small risks. This paper shows that the sufficient condition for a household to save more under uncertainty is a complex combination of different partial and cross-derivatives of the utility function of second and third orders, the variances of the three risks and their covariances.

Notably, when the household utility function is bivariate and a financial risk and a background risk are jointly present, the covariances between the risks play a significant role in the condition for a precautionary motive for saving to emerge. Beyond this, the covariance between financial and background risks is also important for assessing the possible effects of different background variables. Indeed, thus far, we generically referred to background risk as every risk other than financial risk, involving elements related to the background conditions in which household decisions on saving are made. As a consequence, in the presentation of the literature, we treated symmetrically risks such as health risk and environmental risk whose presence, though, is likely to quite differently affect real-world saving decisions.

In fact, it is clear that while changes in environmental conditions do not have a significant impact on household income, it is probable that changes in health status significantly affect the households ability to generate earnings and, thus, the level of household wealth.[21] These differences can easily be accounted for by means of the covariance term. In fact, a positive covariance between health and financial risks may capture the negative

[20]Here, as the total order of differentiation, we consider the sum of the order of differentiation with respect to wealth and the order of differentiation with respect to the background variable.

[21]Although this effect is well documented in the health economics literature (e.g., Bertoni et al. 2018), to date, it has not been directly introduced in saving models.

effect on income of a shock deteriorating household health status. On the other hand, a nil or possibly very small covariance between environmental and financial risks may capture the fact that these two risks are substantially independent.

The results described above can be summarized as follows:

Highlight 10. *In the presence of a background risk, a household saves more under uncertainty under:*

1. *cross-prudence in wealth when there are no other risks (Eeckhoudt et al., 2007);*
2. *different conditions involving positive derivatives and cross-derivatives of odd orders and negative derivatives and cross-derivatives of even orders when there is a positive dependence between labor income risk and background risk and no interest rate risk (Courbage and Rey, 2007; Menegatti, 2009a,b; Denuit et al., 2011);*
3. *complex conditions involving different second- and third-order derivatives and cross-derivatives of the utility function, risk variances and risk covariances when risks are small and the background risk is flanked by either labor income risk alone or by labor income risk and interest rate risk together (Baiardi et al., 2014).*

7 Precautionary saving and risk loving

In the previous sections, we described the conditions that imply that a household has a precautionary saving motive. These conditions are derived in settings where risk aversion is included among the preliminary assumptions of the analysis and reveal that precautionary saving also depends on aspects of preferences other than risk aversion itself. Given this, it is now reasonable to wonder whether a risk lover can also save more under uncertainty.

Crainich et al. (2013) address this issue in a simplified version of the saving model described in Section 2. In particular, in their setting, the household has the same utility function in the two periods, and the intertemporal discount rate and the interest rate are nil. Lastly, income in the second period is equal to the sum of savings and a random shock; thus, it is uncertain. The household maximization problem becomes the following:

$$\max_{s} v\left(y_0 - s\right) + \mathbb{E}\left[v\left(s + \tilde{\epsilon}\right)\right] \tag{27}$$

In this context, if the household is a risk lover (and thus $v_{11} > 0$), a corner solution emerges: either savings are zero or the optimal level of saving is equal to first period income y_0. The household total utility in the first case is equal to $v(y_0) + \mathbb{E}\left[v\left(\tilde{\epsilon}\right)\right]$, while in the second case, it is equal to $v(0) + \mathbb{E}\left[v\left(y_0 + \tilde{\epsilon}\right)\right]$. Crainich et al. (2013) show that the total utility in the second case is larger than the total utility in the first case if the household is prudent. Therefore, prudence is again the sufficient condition for precautionary saving.

The intuition for this result is simple and refers again to the risk apportionment concept of Eeckhoudt and Schlesinger (2006). In their analysis, the authors consider a utility function whose derivatives alternate in sign, i.e., they assume mixed risk aversion and show that a household with these preferences desires to 'combine the good with the bad'. Similarly, in the present context, we have a household whose derivatives all have a positive sign (termed 'mixed risk lover' by Crainich et al., 2013) and that desires to 'combine the good with the good'. Since larger wealth is a good for the household (because of non-satiation) and facing a zero-mean risk is a good as well (because of risk loving), the household wants to be wealthier in the second period when it faces the risk, and as a consequence, it saves more under uncertainty.[22]

New results along the same research line are provided by Nocetti (2016) and Wang et al. (2015). In particular, Nocetti (2016) studies the general properties of comparative statics in the case of risk changes under a set of assumptions that does not include risk aversion. Applying his findings to saving, he shows that the condition $v_{111} > 0$ ensures that there is positive precautionary saving without introducing any assumption on the sign of v_{11}, i.e., without assuming that the household is risk averse, risk neutral or a risk lover. Nocetti (2016) also generalizes this result to the case of high-order changes in risk studied in Section 5.[23]

Lastly, Wang et al. (2015) reconsider the model where a household faces labor income risk and background risk together studied in Section 6. They show that the condition of cross-prudence in wealth ($v_{122} > 0$) ensures that the household saves more under uncertainty without assuming that it is risk averse.

These conclusions can be summarized as follows:

Highlight 11. *When the household is a risk lover, prudence and cross-prudence in wealth are the conditions that ensure that the household saves more under uncertainty in the case of labor income risk (Crainich et al., 2013) and in the case of background risk (Wang et al., 2015).*

8 Precautionary saving and other choice variables

The above analysis considers the effect of uncertainty on saving under the assumption that the household makes its saving choice in isolation. Another strand of the literature,

[22]Ebert (2013) suggests a limitation to the analysis by Crainich et al. (2013), showing that the convex power utility function (the most natural choice for the utility function of a risk lover) never implies mixed risk loving. However, Menegatti (2014) shows that preferences cannot exhibit mixed risk loving when the domain of the utility function is unbounded since, in this case, non-satiation and prudence imply risk aversion.

[23]Nocetti (2016) also shows that, using this approach, a prudent household will also save more if it can only purchase discrete monetary amounts of the asset yielding the interest rate when saving has the form of a 'take it or leave it project' or when the returns on capital are increasing.

however, examines different frameworks where the choice of saving is made contemporaneously with some other choices. In this section, we summarize the main results of this literature.

The first choice that is natural to consider together with saving decisions in a risky framework is the insurance choice. A typical formalization of the maximization problem when the household contemporaneously chooses saving s and insurance b is the following:[24]

$$\max_{s,b} u\left(y_0 - s - \pi b\right) + pv\left(y_1 + Rs - L + b\right) + (1-p)v\left(y_1 + Rs\right) \tag{28}$$

Specifically, this formalization assumes that in the second period, the household bears the risk of a loss L, which may occur with probability p, in a context where there are only two states of the world. The cost of insurance is the insurance premium π, which may either be a fair premium (equal to the expected loss) or include a factor loading.

The contemporaneous choices of insurance and saving were first examined by Moffet (1975) and Dionne and Eeckhoudt (1984). Two main results are obtained. First, comparative statics analysis shows that under plausible assumptions on preferences (typically, decreasing absolute risk aversion),[25] saving and insurance are Hicksian substitutes. Second, when the insurance premium is equal to the fair premium, there is a kind of 'separation' between the two instruments since the household buys full insurance to remove the risk and uses saving for the purpose of consumption smoothing.

Hofmann and Peter (2016) consider the case of the optimal contemporaneous choices of saving and self-insurance, where self-insurance can be defined as an effort made by the household to reduce the size of the loss suffered by the household in the bad state of the world.[26] Their results show that saving and self-insurance are substitutes as well.

A second instrument typically used to deal with risk is prevention (also called self-protection), which can be defined as a cost to be paid or an effort to be exerted in the present to reduce the probability of incurring future losses.[27] When the level of this instrument is chosen together with the level of saving, the household maximization problem becomes the following:

$$\max_{s,e} u\left(y_0 - s - e\right) + p\left(e\right) v\left(y_1 + Rs - L\right) + (1-p(e))v\left(y_1 + Rs\right) \tag{29}$$

where e is the effort exerted in prevention and the probability of incurring the loss L is now a decreasing function of it (i.e., $\frac{dp(e)}{de} < 0$).[28]

[24]For this case, as for the other cases examined in this section, the formalization shows significant differences between papers. We thus present a simple unifying framework, including the main elements of the problem under investigation.

[25]The result by Dionne and Eeckhoudt (1984) presents a more complex condition under the assumption that the utility function is not additively separable.

[26]Thus, self-insurance has an effect similar to that of market insurance, but it is self-made by the household and not purchased on the market.

[27]For instance, buying a house alarm in the present reduces the probability of incurring a loss due to a burglary in the future.

[28]These models also usually introduce the regularity assumption that function $p(e)$ is convex.

In line with the papers studying saving and insurance, the literature on saving and prevention also analyzes the substitutability between the two instruments. In partially different frameworks, Menegatti and Rebessi (2011) and Steinorth (2011) show that under the usual assumption that in the case of an increase in the interest rate, the substitution effect prevails over the income effect (thus, saving increases), saving and prevention are substitutes.[29]

Menegatti and Rebessi (2011) also reconsider the separation result by Dionne and Eeckhoudt (1984) by examining the optimal choices of saving, insurance and prevention under the assumption that the insurance premium is fair. They obtain an extended separation result where the household purchases full insurance to remove the risk, uses saving to smooth consumption between the two periods and uses prevention to reduce the cost of market insurance.[30,31]

Peter (2017) reconsiders the interaction between saving and prevention by investigating the effect of interest rate risk on the optimal choices of the two instruments. Comparing the case with certainty to the case with uncertainty, Peter shows that prudence and a value of the index of relative prudence lower than 2 imply that the presence of interest rate risk increases prevention and reduces the optimal level of saving. This result is consistent with that derived in the standard model with interest rate risk described in Section 3, and it provides additional insight into this issue by also showing the effect on prevention. Lastly, the opposite signs of the variation in the levels of saving and prevention also confirm that the two instruments are substitutes.[32]

A further strand of research investigates the mutual relationship between precautionary saving and labor supply (Low, 2005; Floden, 2006; Nocetti and Smith, 2011). In the certainty case, we here adopt the following formalization of the household maximization problem:

$$\max_{s,l_0,l_1} u\left((1-l_0)w_0 - s, l_0\right) + v\left(k_1 + (1-l_1)w_1 + s, l_1\right) \tag{30}$$

where u and v depend on wealth and leisure, l_0 is leisure in the first period, l_1 is leisure in the second period, w_0 is the wage rate in the first period, w_1 is the wage rate in the second period and k_1 is non-labor income in the second period. Since the household has an endowment of one unit of time in each period, $1 - l_0$ is the labor supply in the first period, and $1 - l_1$ is the labor supply in the second period. In this framework, we introduce a stochastic wage rate and a stochastic non-labor income in the second

[29]The substitution effect prevails over the income effect under the assumption that the partial relative risk aversion is lower than one.

[30]The fair premium paid by the household depends on the probability of incurring the loss. Greater prevention reduces this probability, thus reducing the premium.

[31]The results by Menegatti and Rebessi (2011) are generalized to the analysis of saving and health prevention by Liu and Menegatti (2019a).

[32]Peter (2017) also shows that the presence of saving has significant consequences for the role of prudence in determining optimal prevention. However, this issue is not related to precautionary saving analysis, and thus, it falls beyond the scope of this survey.

period, and we study the effects of uncertainty on precautionary saving and on the total labor supply. Under plausible assumptions on household preferences,[33] a precautionary motive for saving emerges. Moreover, in the presence of precautionary saving, the current supply of labor increases, and the expected future supply of labor decreases. In a sense, precautionary saving is thus a complement to the current labor supply and a substitute for the future labor supply.

A problem involving multiple choice variables also emerges when a household can make investments affecting different dimensions of its utility. In this context, saving is an investment because it implies a current cost in terms of wealth that is sustained to obtain a higher level of wealth in the future. Analogously, other kinds of investment may involve present costs in exchange for future benefits in other dimensions different from wealth. This is the case, for instance, of 'health investment', where a household pays a financial cost and/or sustains a health cost today to improve its future health status.[34] In particular, the choice of the optimal level of health investment may be made either in isolation or together with the choice of saving. In this second case, the household problem becomes the following:

$$\max_{s,H} u\left(y_0 - s - H, H_0 - m_0 H\right) + \frac{1}{1+\rho} v\left(y_1 + Rs, H_1 + m_1 H\right) \tag{31}$$

where u and v depend on wealth and health, H_0 is the baseline health status in period 0, H_1 is the baseline health status in period 1, H is health investment, m_0 is the unit cost of health investment in the health dimension, and m_1 is the return on health investment.

This problem has recently been analyzed by Liu and Menegatti (2019b), who examine the effect of the introduction of uncertainty either in the return on financial investment R, in the return on health investment M, or in both returns together. These authors show that saving and health investment can be either substitutes or complements depending on certain features of household preferences.[35] Moreover, the effects of the introduction of an interest rate risk and of a random return on health investment depend on the interaction of substitutability/complementarity with four aspects of preferences: the values of partial relative prudence in wealth and partial relative prudence in health as well as the signs of cross-prudence in wealth and cross-prudence in health. The first two aspects describe the usual direct effect of each risky return on 'its' type of investment, while the other two aspects capture the cross-effects of each risky return on the other investment.

A last case where saving is chosen jointly with other variables is studied by the financial literacy literature, which reports a large amount of empirical evidence on the fact that

[33] In particular, in the case of non-labor income uncertainty, the main condition required is decreasing absolute risk aversion.

[34] An example is preventive surgery, which may require a financial payment and may determine a negative side-effect in the present but will improve future health status.

[35] Substitutability/complementarity depends on a comparison between two elasticities: the elasticity of the expected future marginal utility of wealth investment with respect to the return on health investment and the elasticity of the present marginal utility of wealth with respect to the cost of health investment.

different groups of people in different countries exhibit a significant lack of knowledge and competence with regard to financial concepts and financial instruments. This can have an important impact on household financial decisions, biasing the choice of the optimal values of specific variables (Lusardi and Mitchell, 2014).

In this context, the interaction of financial literacy and the saving choice has been studied in the literature in two alternative ways. On the one hand, a strand of literature (Lusardi et al. 2017) considers the choice of the optimal level of saving together with the choice of a possible investment in financial literacy. This approach assumes the existence of a one-to-one mapping between knowledge and the return on saving by conjecturing that more financial literacy implies a better ability to choose the investment strategy and, thus, larger returns. Under this assumption, the household optimization problem becomes the following:

$$\max_{s,R} u\left(y_0 - s - kR\right) + \frac{1}{1+\rho} v\left(y_1 + Rs\right) \tag{32}$$

where k is the unit cost of the investment in financial literacy. Lusardi et al. (2017) show that in this setting, there is complementarity between the desire of a household to save and its willingness to invest in knowledge about financial instruments. Moreover, this complementarity is strengthened by uncertainty on future income.

The second approach analyzing the interaction between financial literacy and the saving choice assumes that households are not sufficiently 'sophisticated' to correctly understand the features of all financial instruments they can purchase. This lack of sophistication is described by Neumuller and Rothschild (2017) as a friction in information that causes unsophisticated households to be unable to perfectly recognize the prices and the rates of return on the available financial instruments. In this model, the authors consider households that are heterogeneously affected by the friction according to their level of sophistication. Their decisions on the level of saving and on its optimal allocation between a risk-free investment and a risky asset are then studied. The focus is on portfolio composition, and the results show that beyond determining lower returns, less sophistication also implies lower participation in the market of risky assets.

9 Precautionary saving beyond 'the classical approach'

As stated in the Introduction, the goal of this work is to present recent developments in precautionary saving analysis within the classical approach, founded on the expected utility framework. The choice of limiting our review to this field of the literature looks quite natural to us, as the expected utility framework is the most commonly used for the analysis of precautionary saving. Beyond this, the breadth of the contributions in this field also suggested restricting the scope of the present review and excluding from the

main body of the paper models where household preferences are characterized differently from the framework presented in the previous sections.

Nonetheless, there are many works that have developed different approaches to the analysis of precautionary saving with the aim of examining issues and answering questions that cannot be considered in the classical approach and, in particular, using expected utility. Neglecting these contributions would have meant providing the reader an incomplete overview of the literature; therefore, in this section, we present a general picture of the most important of these departures from the 'standard' model. In particular, we provide a sketch of the results in this field without entering into analytical detail and without providing specific highlights of them since we prefer to leave a more in-depth study to papers devoted to this topic.

We first consider a departure from the classical approach that removes one of its key assumptions: the hypothesis that the household knows the objective probabilities of a future event relevant for its choice or, at least, that it has a clear subjective opinion on them. However, Knight (1921) noted that in some circumstances, this clear opinion does not form and that there is a kind of uncertainty on the *a priori* elements, which are relevant for the household decision. In the literature, this situation is called 'ambiguity', and the preferences of a household that does not like this uncertainty are said to exhibit 'ambiguity aversion' (e.g., Klibanoff et al., 2005).

The effects of ambiguity on the saving decision have recently been studied in different contributions (Berger, 2014; Baillon, 2017; Wang and Li, 2018 and Peter, 2019). In some cases, ambiguity aversion alone, as defined above, is sufficient to generate extra-saving due to the presence of uncertainty (Peter, 2019). In other cases, more complex conditions, such as 'decreasing ambiguity aversion', are necessary (Wang and Li, 2018). Lastly, in a further group of circumstances, conditions involving the concept of 'ambiguity prudence' are required. This concept has been defined either 'à la Kimball', i.e., with reference to a specific derivative of a function describing some aspects of preferences (Berger, 2014), or 'à la Eeckhoudt and Schlesinger', i.e., by using preferences over lotteries (Baillon, 2017).

A different departure from the expected utility approach changes the way households perceive probabilities and adopts the so-called rank-dependent utility model (Quiggins, 1982). In this setting, the probabilities attached to the different states of the world are transformed by weights determined by household attitude. Pessimistic households assign larger weights to the probabilities that bad outcomes will occur, while optimistic households assign larger weights to the probabilities that good outcomes will occur. By means of rank-dependent utility, Bleichrodt and Eeckhoudt (2005) study the effect of pessimism and optimism, as described above, on precautionary saving. They show that the direction of these effects depends on prudence and temperance and that, in the most plausible case where the equivalent precautionary premium is decreasing in wealth, the impact of pessimism on saving is positive.

A further aspect of the standard structure of the utility function that has been modified is related to the formalization of household preferences for risk and time. In fact,

in the classical approach, the concavity of the utility function contemporaneously measures two different aspects of preferences: the strength of the household desire to smooth consumption (related to the intertemporal elasticity of substitution) and the household dislike for the presence of uncertainty (related to risk aversion).[36] Kreps and Porteus (1978) and Selden (1978) independently study a way of removing this 'double role' by proposing a class of preferences that disentangles household attitudes toward risk and toward time.

Adopting this approach, Kimball and Weil (2009) study household consumption and saving choices and, thus, also precautionary savings. Their analysis considers how household preferences for risk and time interact in determining the precautionary premium and, thus, the strength of the precautionary saving motive. In particular, they show that for large risks, decreasing absolute risk aversion (a condition requiring prudence) ensures that the precautionary saving motive is stronger than risk aversion, regardless of the level of the intertemporal elasticity of substitution.

An extension of Kreps and Porteus (1978) preferences proposed by Epstein and Zin (1989) has recently been used by Bommier and Le Grand (2018) to study precautionary saving in a multiperiod framework.[37] The main result obtained is that larger precautionary saving is implied by greater risk aversion á la Yaari (1969).[38]

A different departure from standard expected utility models is represented by models exhibiting loss aversion. The concept of loss aversion was introduced in the so-called prospect theory by Kahneman and Tversky (1979) and is founded on the idea that there is an asymmetry in evaluating the increment and decrement in consumption with respect to a 'reference point', often represented by past consumption. This asymmetry implies that losses relative to the reference point are more important for households than gains.

The introduction of loss aversion in consumption-saving models clearly influences the saving path, making households more reluctant to reduce consumption and increase saving when this reduction implies falling below the reference point. When a random future income is introduced in this framework, a similar effect affects precautionary saving since a household tends to resist reducing consumption below its reference point, even when it is dealing with uncertainty on the future (Bowman et al. 1999). Moreover, in a similar context, Aizenman (1998) shows that in the presence of loss aversion, prudence is again a determinant of precautionary saving.

Lastly, behavioral economics also argues for the relevance for intertemporal decision

[36]For instance, when using a CRRA utility function, the parameter measuring constant relative risk aversion is also a measure of the constant intertemporal elasticity of substitution.

[37]Van der Ploeg (1993) and Weil (1993) also study the issue of precautionary saving in multiperiod frameworks but under specific fully parametrized income processes and closed-form solutions and introducing specific forms for the instantaneous utility function (quadratic and CRRA in Van der Ploeg (1993) and Weil (1993), respectively).

[38]Yaari (1969) defines greater risk aversion as follows: a household is at least as risk averse as another household if it is willing to accept every risk that is acceptable to the other household.

making of psychological mechanisms such as 'present bias' or 'mental accounting', which may affect household decisions on wealth allocation and, thus, on saving. In particular, the idea of present bias suggests that in discounting future utility with respect to present utility, there is a bias that causes a household to significantly reduce the weight attached to future utility. This bias, which, in principle, does not depend on uncertainty on the future, may be even larger when risk on future income or on future rewards is introduced in the framework (Marzilli Ericson and Laibson, 2019). In contrast, mental accounting is related to the conjecture that when making choices, households consider just some of the elements affecting their wealth, treating other elements as 'out of bounds' of their analysis (Beshears et al., 2018).

Both present bias and mental accounting have an effect on saving decisions, generating a consumption path closer to the income path than that suggested by the usual theory of intertemporal choice. This implies a reduction in consumption smoothing and, potentially, a reduction in precautionary saving that typically generates deviations from the income path. Similarly, the use of hyperbolic discounting, which implies discount rates that decline over time, can determine a reduction in precautionary saving (Laibson, 1998).

10 Conclusions

The debate on precautionary saving, which dates back to the 1960s, has been revived by a significant new literature. New research lines have developed the traditional model, which studies the saving choice under uncertainty, by introducing a wide range of additional elements: the contemporaneous presence of multiple risks, complex changes in risks, multiple variables affecting household utility or multiple choice variables, and preferences non-featuring risk aversion.

These contributions provide a detailed description of the precautionary saving motive and cover most of the relevant issues related to it. Nonetheless, in this field of analysis, some aspects remain underinvestigated, revealing the most promising directions for future research. In particular, we consider that there is room for further development in precautionary saving theory along the following lines.

A very recent literature focuses on the analysis of precautionary saving when the household does not necessarily perceive uncertainty as a bad, for instance, in the case of risk loving. In this context, the literature shows that prudence remains the key element for precautionary saving. In a sense, this is in line with the origins of the precautionary saving literature since it confirms the conjecture by Leland (1968), who opened up the field of precautionary saving analysis precisely by stating that '*we must reject the association of precautionary demand of saving with simple risk avoidance*' (Leland, 1968, p.467). A peculiar aspect of this analysis is the unusual sequence in the signs of the derivatives of the utility function. Although this opens up to the study of many different kinds of household behavior in the presence of risk, also in contexts different from precautionary saving, a

challenge emerges because the meaning of these preferences has yet to be completely understood and requires further investigation.

A further promising strand of literature is related to the idea of coupling the saving choice to other types of choice in a risky framework. This approach has the clear advantage of considering the interaction between different household decisions, removing the simplification associated with the assumption that the saving level is set in isolation. A question that remains open in this framework is the definition of the bundle of choices that the household makes together. The way in which households bracket choices is still a debated issue,[39] and involves elements pertaining to different branches of knowledge that go beyond economics to reach other disciplines, such as sociology and psychology.

As shown in the previous sections, the analysis of precautionary saving also evolved in the direction of considering different kinds of risk changes. However, in our opinion, further developments along this line are possible. Taking as a basis the large impact on household well-being of the subprime crisis and, more generally, of all the disruptive events that, in the current opinion, are included in the category of 'black swan' circumstances, we believe that a deeper investigation of the effects of rare events on precautionary saving could prove to be very useful. **In fact, although one of the purposes of saving is to build a buffer that limits the negative effects of these events, a common characteristic of rare events is that decision-makers are usually unprepared for them. This may happen for instance, because in a context where agents can process only a finite amount of information and cannot prepare perfectly for all contingencies, thinking more about optimal actions in unusual times implies less thinking about optimal actions in normal times. This issue studied by Maćkowiak and Wiederholt (2015) is likely to affect saving decision in a new way, substantially changing agent preferences on the intertemporal distribution of income.**

In conclusion, a last new research line may include in the picture the effects of policies affecting saving decisions. In this context, particular relevance is assumed by all public interventions that provide a form of insurance against specific risks, for instance, unemployment, health and longevity risks.[40] Assessing the impact on the precautionary motive for saving of the different structures included in the welfare state of a country, which range from the sanitary system to social security, is warranted for two main reasons. On the one hand, such an assessment would help us understand how it is possible to affect the level of precautionary saving and thus, indirectly, the overall level of saving. On the other hand, it would supply useful insights into the effects of policy uncertainty, for instance, in the case of the recent distress in the pension systems of several developed countries. This fact introduced uncertainty on the actual payment of pension benefits, an element that is

[39] See, for instance, Read et al. (1999).

[40] To the best of our knowledge, there exist only some sporadic examples of papers examining these issues (e.g., Abel, 1985; Hubbard and Judd, 1987; Engen and Gruber, 2001), and no systematic literature to date has developed.

likely to deeply affect household behavior, especially with regard to saving decisions.

References

[1] Abel, A.B., (1985). Precautionary saving and accidental bequest. *American Economic Review* 75, 777-791.

[2] Aizenman, J., (1998). Buffer stocks and precautionary savings with loss aversion. *Journal of International Money and Finance* 17, 931-947.

[3] Arcand, J., L., Berkes, E., Panizza, U., (2015). Too much finance? *Journal of Economic Growth* 20, 105-148.

[4] Baiardi, D., Magnani, M., Menegatti, M., (2014). Precautionary saving under many risks. *Journal of Economics* 113, 211-228.

[5] Baiardi, D., Magnani, M., De Donno, M., Menegatti, M., (2015). New results on precautionary saving under two risks *Economics Letters* 130, 17-20.

[6] Baillon, A., (2017). Prudence with respect to ambiguity. *The Economic Journal* 127 (September), 1731-1755. *Doi* : 10.1111/*ecoj*.12358.

[7] Berger, L., (2014). Precautionary saving and the notion of ambiguity prudence. *Economics Letters* 123, 248-251.

[8] Berlingieri G., Blanchenay P., Criscuolo, C., (2017). The great divergence(s). OECD Science, Technology and Industry Policy Papers, May 2017.

[9] Bertoni, M., Brunello, G., Mazzarella, G., (2018). Does postponing minimum retirement age improve healthy behaviors before retirement? Evidence from middle-aged Italian workers. *Journal of Health Economics* 58, 215-227.

[10] Beshears J, Choi J.J., Laibson, D., Madrian, B.C., (2108). Behavioral Household Finance. In: Bernheim D, Laibson D, DellaVigna S *Handbook of Behavioral Economics - Foundations and Applications 1.* Elsevier.

[11] Bleichrodt, H., Eeckhoudt, L., (2005). Saving under rank-dependent utility. *Economic Theory* 25, 505-511.

[12] Bommier, A., Le Grand, F., (2018). Risk Aversion and Precautionary Savings in Dynamic Settings. *Management Science*, in press.

[13] Bowman, D., Minehart, D., Rabin, M., (1999). Loss aversion in a consumption-savings model. *Journal of Economic Behavior & Organization* 38, 155-178.

[14] Brochett, P.L., Golden, L.L., (1987). A class of utility functions containing all the common utility functions. *Management Science* 33, 955-964.

[15] Caballé, J., Pomanski, A., (1996). Mixed risk aversion. *Journal of Economic Theory* 71, 485-513.

[16] Chiu, H.W., Eeckhoudt, L., Rey, B., (2012). On relative and partial risk attitudes: theory and implications. *Economic Theory* 50, 151-167.

[17] Choi, G., Kim, I., Snow, A., (2001). Comparative statics predictions for changes in uncertainty in the portfolio and savings problems. *Bulletin of Economics Research* 53, 61-72.

[18] Courbage, C., Rey, B., (2007). Precautionary saving in the presence of other risks. *Economic Theory* 32, 417-424.

[19] Crainich, D., Eeckhoudt, L., Trannoy, A., (2013). Even (mixed) risk lovers are prudent. *American Economic Review* 103, 1529-1535.

[20] Denuit, M.M., Eeckhoudt, L., Menegatti, M., (2011). Correlated risks, bivariate utility and optimal choices, *Economic Theory* 46, 39-54.

[21] Denuit, M., Rey, B., (2014). Benchmark values for higher degree coefficients of relative risk aversion, *Theory and Decision* 76, 81-94.

[22] Dionne, G., Eeckhoudt, L., (1984). Insurance and saving: some further results. *Insurance: Mathematics and Economics* 3, 101-110.

[23] Drèze, J., Modigliani, F., (1972). Consumption decision under uncertainty, *Journal of Economic Theory* 5, 308-335.

[24] Ebert, S., (2013). Even (mixed) risk lovers are prudent: Comment. *American Economic Review* 103, 1536-1537.

[25] Ebert, S., Nocetti, D.C., Schlesinger, H., (2017). Greater mutual aggravation. *Management Science*, 64, 2809-2811.

[26] Eeckhoudt, L., Gollier, C., Schneider, T., (1995). Risk-aversion, prudence and temperance: A unified approach. *Economics Letters* 48: 331-336.

[27] Eeckhoudt, L., Gollier, C., Schlesinger, H., (1996) Changes in Background Risk and Risk Taking Behavior *Econometrica* 64, 683-689.

[28] Eeckhoudt, L., Rey, B., Schlesinger, H., (2007). A good sign for multivariate risk taking. *Management Science* 53, 117-124.

[29] Eeckhoudt, L., Schlesinger, H., (2006). Putting risk in its proper place. *American Economic Review* 96, 280-289.

[30] Eeckhoudt, L., Schlesinger, H., (2008). Changes in risk and the demand for savings. *Journal of Monetary Economics* 55, 1329-1336.

[31] Eeckhoudt, L., Schlesinger, H., Tsetlin, I., (2009a). Apportioning of risks via stochastic dominance. *Journal of Economic Theory* 144, 994-1003.

[32] Eeckhoudt, L., Etner, J., Schroyen, F., (2009b). The values of relative risk aversion and prudence: A context–free interpretation. *Mathematical Social Science* 58, 1–7.

[33] Ekern, S. (1980). Increasing Nth degree risk. *Economics Letters* 6, 329-333.

[34] Eichner, T., Wagener, A., (2004a). Relative risk aversion, relative prudence, and comparative statics under uncertainty: the case of (μ, σ)-preferences. *Bulletin of Economic Research* 56, 159–170.

[35] Eichner, T., Wagener, A., (2004b). Insurance demand, the elasticity of risk aversion, and relative prudence: a further result. *OR Spectrum* 26, 441–446.

[36] Eisenhauer, J.G., (2006). Risk aversion and prudence in the large. *Research in Economics* 60, 179-187.

[37] Engen, E.M., Gruber, J., (2001). Unemployment insurance and precautionary saving. *Journal of Monetary Economics* 47, 545-579.

[38] Epstein, L.G., Zin, S.E., (1989). Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica*, 57, 937969.

[39] Fishburn, P.C., Porter, R.B., (1976). Optimal Portfolios with One Safe and One Risky Asset: Effects of Changes in Rate of Return and Risk. *Management Science* 22, 1064-1073.

[40] Floden, M., (2006). Labor Supply and Saving Under Uncertainty. *Economic Journal* 116, 721-737.

[41] Friedman, M., (1957). A Theory of the Consumption Function. Princeton University Press.

[42] Friedman, M., Savage, L.J., (1948). The utility analysis of choices involving risk. *Journal of Political Economy* 56, 279-304.

[43] Global Financial Stability Report (October 2017): Is Growth at Risk? International Monetary Fund, October 2017.

[44] Gollier, C., Pratt, J.W., (1996). Risk Vulnerability and the Tempering Effect of Background Risk *Econometrica* 64, 1109-1123.

[45] Gunning, J.W., (2010). Risk and savings: A taxonomy. *Economics Letters* 107, 39-41.

[46] Hau, A. (2002). Prudence, Intertemporal Substitution, and Savings under Uncertainty. *Journal of Economics*, 35-52.

[47] Hofmann, A., Peter, R., (2016). Self-Insurance, Self-Protection, and Saving: On Consumption Smoothing and Risk Management, *Journal of Risk and Insurance* 83, 719-734.

[48] Hubbard, R.G., Judd, K.L., (1987). Social security and individual welfare: precautionary saving, liquidity constraints, and the payroll tax. *American Economic Review* 77, 630-646.

[49] Kahneman, D., Tversky, A., (1979). Prospect theory: An analysis of decision under risk. *Econometrica* 47, 263-291.

[50] Kimball, M.S., (1990). Precautionary savings in the small and in the large, *Econometrica* 58, 53-73.

[51] Kimball, M.S., (1992). Precautionary motives for holding assets. *Econometrica* 61, 589-611.

[52] Kimball, M.S., (1993). Standard risk aversion. Peter Newman, Murray Milgate, John Eatwell (Eds.), The New Palgrave Finance, Mcmillan Press, Ltd., London.

[53] Kimball, M., Weil, P., (2009). Precautionary saving and consumption smoothing across time and possibilities. *Journal of Money, Credit and Banking* 41, 245-284.

[54] Klibanoff, P., Marinacci, M., Mukerji, S., (2005). A smooth model of decision making under ambiguity. *Econometrica* 73, 1849-1892.

[55] Knight, F.H. (1921). *Risk, uncertainty and profit*, Boston, New York, Houghton Mifflin.

[56] Kreps, D.M., Porteus, E.L., (1978). Temporal resolution of uncertainty and dynamic choice theory. *Econometrica* 46, 204-213.

[57] Jouini, E., Napp, C., Nocetti, D., (2013). Economic consequences of Nth-degree risk increases and Nth-degree attitudes. *Journal of Risk Uncertainty* 47, 199-224.

[58] Ingersoll, J.,E., (1987). Theory of financial decision making. Rowman and Littlefield, Totowa.

[59] Laibson, D., (1998). Life-cycle consumption and hyperbolic discount functions. *European Economic Review* 42, 861-871.

[60] Lajeri-Chaherli, F., (2004). Proper prudence, standard prudence and precautionary vulnerability. *Economics Letters* 82, 29-34.

[61] Lehmann, EL., (1966). Some concepts of dependence. *The Annals of Mathematical Statistics* 37, 1137-1153.

[62] Leland, H., (1968). Saving and uncertainty: the precautionary demand for saving. *Quarterly Journal of Economics* 82, 465-473.

[63] Levy, (2006). Stochastic Dominance. Investment decision making under uncertainty. Second Edition, Springer, New York.

[64] Li, J., (2012). Precautionary saving in the presence of labor income and interest rate risks. *Journal of Economics* 106, 251-266.

[65] Light, B., (2018). Precautionary saving in a Markovian earnings environment. *Review of Economic Dynamics* 29, 138-147.

[66] Liu, L., (2014). Precautionary saving in the large: nth degree deteriorations in future income. *Journal of Mathematical Economics* 52, 169-172.

[67] Liu, D., Menegatti, M., (2019a). Optimal saving and health prevention. *The Journal of Economics*, forthcoming.

[68] Liu, D., Menegatti, M., (2019b). Precautionary Investment in Wealth and Health. *The Journal of Risk and Insurance* 86, 237-255.

[69] Low, H., (2005). Self-Insurance in a life-cycle model of labor supply and savings. *Review of Economic Dynamics* 4, 945-975.

[70] Lugilde, A., Bande, R., Riveiro, D., (2018). Precautionary saving: a review of the empirical literature. *Journal of Economic Surveys* Article in press.

[71] Lusardi, A. , Mitchell, O.S., (2014). The economic importance of financial literacy: theory and evidence. *Journal of Economic Literature* 52, 5-44.

[72] Lusardi, A. , Michaud, P.C., Mitchell, O.S., (2014). Optimal financial knowledge and wealth inequality (2017) Journal of Political Economy 125, 431-477.

[73] Maćkowiak, B., Wiederholt, M., (2015). Inattention to rare events. ECB Working Papers, n. 1841, August 2015.

[74] Magnani, M., (2017). A new interpretation of the condition for precautionary saving in the presence of an interest-rate risk. *Journal of Economics* 120, 79-87.

[75] Marzilli Ericson, K., Laibson, D., (2019). Intertemporal choice. *Handbook of Behavioral Economics. Foundations and Applications* 2, Volume 2, Chapter 1, 1st Edition.

[76] Menegatti, M., (2001). On the conditions for precautionary saving, *Journal of Economic Theory* 98, 189-193.

[77] Menegatti, M., (2007). A new interpretation for the precautionary saving motive: a note. *Journal of Economics* 92, 275-280.

[78] Menegatti, M., (2009a). Precautionary saving in the presence of other risks: a comment, *Journal of Economics* 39, 473-476.

[79] Menegatti, M., (2009b). Optimal saving in the presence of two risks, *Journal of Economics* 96, 277-288.

[80] Menegatti, M., (2014). New results on the relationship among risk aversion, prudence and temperance. *European Journal of Operational Research* 232, 613-617.

[81] Menegatti, M., (2015). New results on high-order risk changes. *European Journal of Operational Research* 243, 678-681.

[82] Menegatti, M., Rebessi, F., (2011). On the Substitution Between Saving and Prevention, *Mathematical Social Sciences* 62:, 176-182.

[83] Menezes, C., Geiss, C., Tressler, J., (1980). Increasing downside risk. *American Economic Review* 70, 921-932.

[84] Moffet, D., (1975). Risk bearing and consumption theory. *Astin Bulletin* 3, 342-358.

[85] Nocetti, D.C., Smith, W.T., (2011). Precautionary Saving and Endogenous Labor Supply with and without Intertemporal Expected Utility. *Journal of Money, Credit and Banking* 43, 1475-1504.

[86] Nocetti, D.C., (2016). Robust comparative statics of risk changes. *Management Science* 62, 1381-1392.

[87] Peter, R., (2017). Optimal self-protection in two periods: On the role of endogenous saving. *Journal of Economic Behavior & Organization* 137, 19-36.

[88] Peter, R., (2019). Revisiting precautionary saving under ambiguity. *Economics Letters* 174, 123-127.

[89] Pratt, J.W., (1964). Risk aversion in the small and in the large. *Econometrica* 32, 122-136.

[90] Quiggin, J., (1982). A theory of anticipated utility. *Journal of Economic Behavior and Organization* 3, 323-343.

[91] Read, D., Loewenstein, G., Rabin, M., Keren, G., Laibson, D., (1999). Choice bracketing. *Elicitation of Preferences*, 171-202.

[92] Rothschild, M., Stiglitz J., (1970). Increasing risk I: A definition. *Journal of Economic Theory* 2, 225-243.

[93] Rothschild, M., Stiglitz J., (1971). Increasing risk II: economic consequences. *Journal of Economic Theory* 3, 66-84.

[94] Sandmo, A., (1970). The effect of uncertainty on saving decisions, *Review of Economic Studies* 37, 353-360.

[95] Selden, L., (1978). A new representation of preferences over 'certain x uncertain' consumption pairs: the 'Ordinal certainty equivalent' hypothesis. *Econometrica* 46, 1045-1060.

[96] Steinorth, P., (2011). Impact of Health Savings Accounts on Precautionary Savings, Demand for Health Insurance and Prevention Effort. *Journal of Health Economics* 30, 458-465.

[97] van der Ploeg, F., (1993). A Closed-Form Solution for a Model of Precautionary Saving. *Review of Economic Studies* 60, 385395.

[98] Vergara, M., (2017). Precautionary saving: A taxonomy of prudence. *Economics Letters* 150, 18-20.

[99] Wang, J., Li, J., (2018). Comparative ambiguity aversion in intertemporal decisions. *The Journal of Risk and Insurance* Vol. XX, No. XX, 118.

[100] Wang, H., Wang, J., Li, J., Xia, X., (2015). Precautionary paying for stochastic improvements under background risks. *Insurance: Mathematics and Economics* 64, 180-185.

[101] Weil, P., (1993). Precautionary Savings and the Permanent Income Hypothesis, *Review of Economic Studies* 60, 367383.

[102] Wong, K. P., (2018). An interpretation of the condition for precautionary saving: the case of greater higher-order interest rate risk. *Journal of Economics*, forthcoming.

[103] Yaari, M.E., (1969). Some Remarks on Measures of Risk Aversion and their Uses. *Journal of Economic Theory* 1, 315-329.