Hooping as an Ancient Remedy for Conservation of Large Masonry Domes

Federica Ottoni and Carlo Blasi

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2016, VOL. 10, NO. 2, 1–18
http://dx.doi.org/10.1080/15583058.2015.1113335

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Hooping as an Ancient Remedy for Conservation of Large Masonry Domes

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ABSTRACT
The insertion of metal tie rods all around masonry domes, in order to hoop them and to absorb their horizontal thrusts, is the most ancient traditional strengthening technique for these fascinating structures. This article presents some general considerations for this primeval remedy, starting from the historical and structural analysis of the large 16th-century octagonal dome of Madonna dell'Umiltà in Pistoia (Italy). Several hooping systems had been inserted around this dome in different periods and with different techniques in order to reinforce this weak masonry structure, and their tensile stresses have been measured by means of dynamic tests. The results have shown the large differences in the tensile stresses among the different ties, allowing an understanding of the real contribution of each hooping systems. These data were then used to calibrate a finite element model, which allowed researchers to retrace and quantify from a structural point of view the passage through the centuries of this daring construction and to understand the efforts made to preserve it up to now. The final results presented here are new suggestions on the most efficient way to ensure, once again with hooping ties, the preservation of this monument for the future, starting from empiricism.

ARTICLE HISTORY
Received 4 March 2015
Accepted 14 October 2015

KEYWORDS
Ancient masonry domes; dynamic tests; empiricism; hooping tie rods; Madonna dell'Umiltà in Pistoia

The primeval remedy to the dome "TRAP"

Domes are the highest expression of masonry: Indeed they can cover large spans without intermediate columns and their construction requires the solution of complex technical, geometrical, and static problems. Moreover, domes symbolize the celestial sphere, as their architect was God himself.

Constructively, dome can be considered as the structural evolution of arch and, with its perfect compressive functioning, it surely constitutes the best way to use the potentiality of masonry. Actually, rotational domes, unless particular conditions are present of load and constraint, are always funicular of distributed loads, in virtue of the mutual tension exchange between the elements belonging to two contiguous meridians and parallels. The actions along each parallel are tensile; they decrease progressing upwards until they change, beyond a certain angle, into compressive stresses. Nevertheless, this perfect functioning of the internal actions has a flaw that, in the case of masonry domes, is unavoidable: it shows states of traction at the base, which cannot be eliminated and which, at times, can cause the dome fracturing (Heyman 1995). Moreover, it is well known that in time the extremely low tensile strength of masonry is reduced to practically zero, for creep phenomena.

Over time, vertical fractures have appeared in nearly all the masonry domes, highlighting their fundamental mechanical principle (the "arch") as well as their typical mechanism of instability (Figure 1), their "trap": The droop of the top of the structure under its own weight and the horizontal thrust at its base. The fractures can vary for amplitude and position; regardless, the "trap" is created and the dome has been transformed from a compact and solid body to a sequence of arches with variable sections, mutually contrasting through the compressed parallels.

This theory has been reached after centuries of construction practice, and its equations were certainly unknown to the ancient master-builders. Conversely, these builders knew well the final effect of this mechanism, having observed the cracks. It is a matter of fact that in most of the existing large masonry domes iron or wooden chains have been inserted over the centuries, which avoided their collapse. However, the first solution to the ineradicable dome trap came from the observation of repeated similar collapses, rather than from precise analyses and calculations, and the

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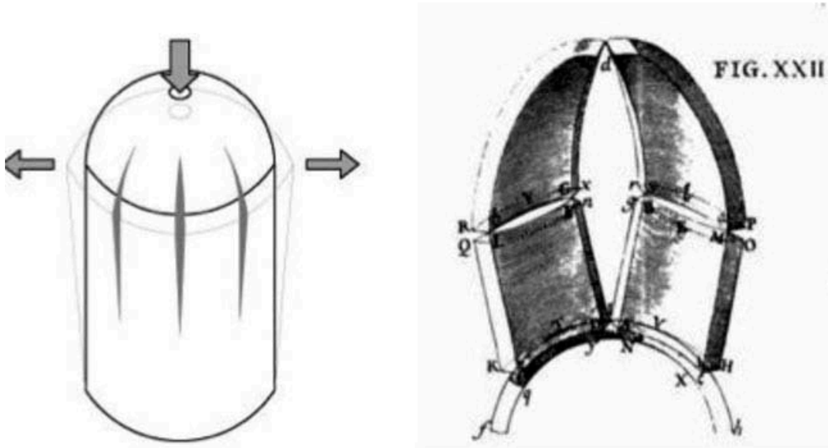
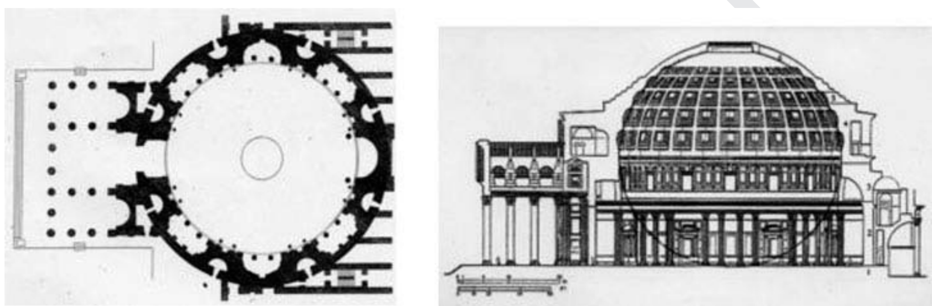


Q31 **Figure 1.** The horizontal thrust of the dome: its "trap" (Poleni 1748). © [rightsholder]. Reproduced by permission of [rightsholder]. Permission to reuse must be obtained from the rightsholder.



Q26 Q32 **Figure 2.** The perfect geometry of the Roman Pantheon, plan and section (Milani 1920). © [rightsholder]. Reproduced by permission of [rightsholder]. Permission to reuse must be obtained from the rightsholder.

encircling tie-rods remained, at least until 18th century, the traditional strengthening method for domes.

Actually, these slender elements of iron (or wood) make the building more stable by tying two or more walls together and not allowing any relative movement among them. These rods, apparently, are quite simple devices. Nevertheless, a complete understanding of the mechanical behavior of the reinforced structures and, in particular, of the real contribution given by these retrofit elements, is not an easy task because it requires to take into consideration the history of the building with all the structurally relevant interventions, the technique adopted to insert the hooping, the plastic behavior of masonry, and the evolution of the crack pattern. All of these aspects can be clarified in the peculiar case study presented herein.

Brief history of dome hooping

As already noted, almost all major ancient masonry domes have hoopings; in some cases, hoopings have been planned in the construction phase, in other domes they were inserted later, and often they have been added to existing chains, less efficient, after some damages. A brief comparison between the hooping systems of several large domes can be useful to better understand their fundamental role in the conservation process of these great structures. In step with this approach, in the following discussion some brief considerations on the most famous ancient masonry domes are reported.

The roman pantheon: The only "masonry" dome without hoopings

It can seem curious, in a brief chronological *excursus* on hoopings, to begin right by a not-encircled dome, perhaps the greatest one—the dome of the Pantheon, in Rome. This great dome has some characteristics that differentiate it from all the others and that explain why it did not need any hooping interventions over time. The height of this dome is equal to its diameter, actually being built around a sphere; therefore, the horizontal thrust of the dome acts at a fairly low altitude and finds adequate contrast in the thick circular tambour. Actually, in the Roman Pantheon the ratio between the thickness of the tambour and the inner diameter (equal to approximately 150 feet) is approximately 1/7.5, far greater than that of the other masonry domes (Pelliccioni 1986). Moreover, the thickness of the dome is variable and reaches only 6 feet at the top. Despite the fact that today it shows a widespread crack pattern, this dome is the only one that had solved the primeval trap of domed structures without hoopings—in virtue of its perfect geometry and of the extraordinary resistance to traction of its materials (*opus caementicium*, not proper "masonry")—and, for this reason, it necessarily represents the reference for any successive dome reinforcement intervention.

The dome of hagia sophia: The solution from empiricism

The perfect dome of Hagia Sophia, in Istanbul, built in order to bid against the Roman Pantheon for dimension and greatness, collapsed three times over the centuries, always after earthquakes, definitively clarifying its inability in fighting the horizontal actions. Following the functional and structural dualism of the church, between plan and elevation, two different solutions have been applied in order to contrast the horizontal thrust: two massive buttresses in north–south direction and two semi-domes in the east–west direction, both organized in a rigorous geometrical structure (Figure 3).

The original dome—with a lowered profile and extremely thrusting (Mainstone 1969)—collapsed soon after its construction, in 557 AC, after an earthquake, although it was equipped with wooden chains at the level of the windows at the impost plane and other partial wooden chains were inserted at lower levels (Sato et al. 1996). The second dome, reconstructed 20 feet higher, collapsed other two times (in 989 AC and in 1346 AC), despite the numerous buttresses and the new iron chains (very thin) that were added in times by Byzantines (Blasi and Bianchini 2001). The four massive pillars that should have passively absorbed the thrust of the dome, have slowly rotated over time until reaching an inclination of 130 cm, with a consequent increasing of transverse diameter of the dome of approximately 260 cm. The remedy arrived from empiricism: In the 16th century, Jusuf Sinan added massive encircling tie-rods to the dome, definitely solving its trap: since then, in fact, the crack pattern and the deformation of the dome have not significantly increased.[1]

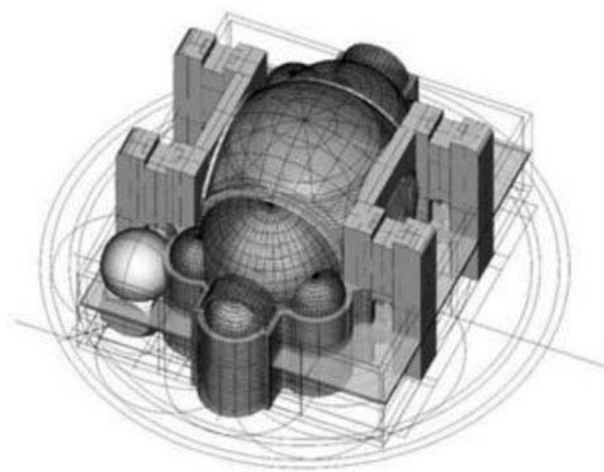


Figure 3. The perfect geometry of Hagia Sophia and its different systems of thrust restraint: The two semi-domes in longitudinal direction, and the two great pillars in the transversal one (Blasi and Bianchini 2001). © [rightsholder]. Reproduced by permission of [rightsholder]. Permission to reuse must be obtained from the rightsholder. Q33

In 1847, the Swiss architect Gaspare Fossati, charged with the restoration of Hagia Sophia, added two more orders of tie-rods, one at the base of the dome and the other one in the tambour. Furthermore, Fossati built some rampant arches, which were subsequently removed (Figure 4). However, the first solution to the dome's ineradicable trap comes from the observation of repeated similar collapses, rather than from precise analyses and calculations, and the encircling tie-rods remained, at least until 18th century, the traditional strengthening method for domes.

The baptistery of san giovanni in florence: From wooden to iron chains

Five century later, another lesson on the importance of hoopings for dome stability came from the Baptistery of Giotto, in Florence. Built around the 12th century, the dome shows, at present, a good structural stability status, reached also thanks to a mindful maintenance, conservation, and strengthening work carried out by the Florentine architects over the centuries (Rocchi Coopmans De Yoldi 1996). Actually, the Baptistery has not always been as stable as it is today, and the last studies on the crack pattern, both past and present, together with a specific historical study, allowed identification of the ancient serious structural disorders that were faced with good restoration interventions (Blasi, Coisson, and Ottoni 2014).

[1]Hagia Sophia collapse mechanism had to be a lesson for Sinan, who then circled with sturdy tie-rods of iron—thanks to the exceptional skill of the Ottoman in steel industry—all his great domes, up to the great one of Edirne Mosque (Blasi 2003).

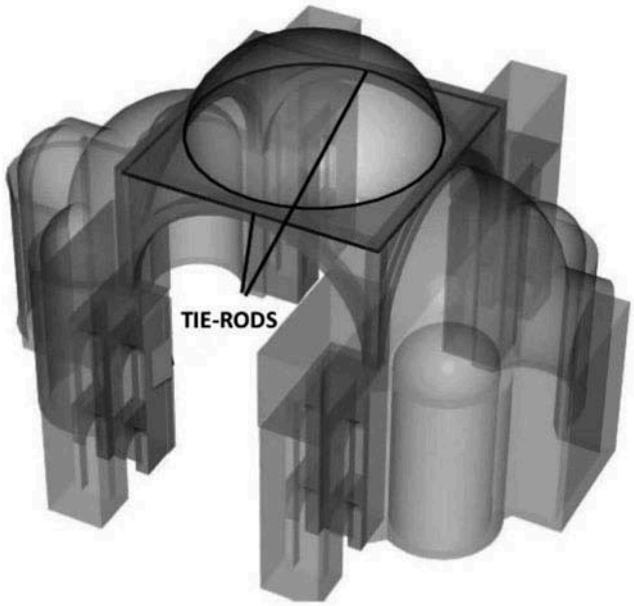


Figure 4. The encircling system in Hagia Sophia (Blasi and Bianchini 2001). © [rightsholder]. Reproduced by permission of [rightsholder]. Permission to reuse must be obtained from the rightsholder.

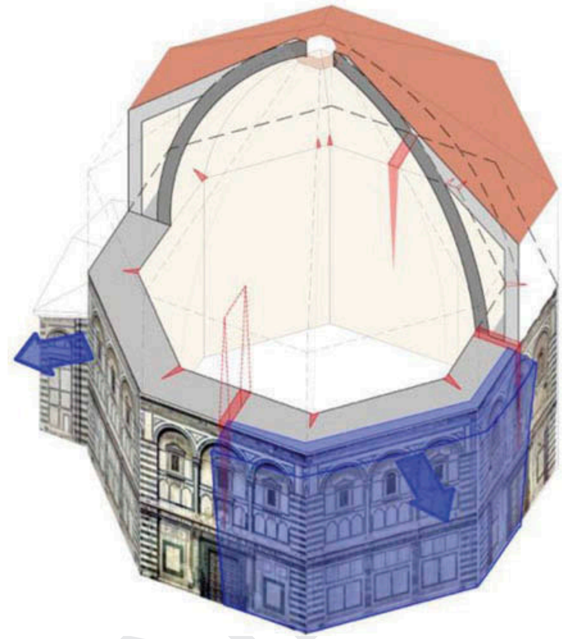


Figure 5. The widespread crack pattern of the Baptistery dome is due a system of structural disorders and deformations which is typical of domed buildings.

The crack pattern of the dome demonstrates the presence, also in this monument, of a system of structural disorders and deformations that is typical of domed buildings (Figure 5), with interesting correspondences to the nearby Brunelleschi dome.[2] In order to contrast this collapse mechanism, chestnut-wood encircling ties—which the recent inspections made by the CNR-IVALSA laboratory proved to be now nearly slack —had been inserted around the dome during the construction phase. The dating (at 1268) of one piece of these ties—clearly in substitution of a previously deteriorated element—shows however the great care given in the 13th century to its maintenance as a sign of the structural role that was ascribed to it.

Despite the numbers of maintenance interventions, the wooden chain resulted in an insufficient intervention, and the width of the cracks on the external and internal surfaces must have reached at the beginning of the 16th century at least 4–5 cm as clearly visible on the marble slabs on the outside and in the gallery. Therefore, these serious structural disorders, in 1514, called for a decisive intervention: the insertion of a sturdy steel tie rods system—which has been found and measured—all around the dome at the level of the second cornice[3] (Figure 6). This hooping system proved to be a decisive and non-invasive restoration for the Florentine Baptistery since the structural disorders have substantially stopped or at least were reduced to very small cracks after this intervention, thus demonstrating its efficacy on the stability of the dome (Figure 7).

The brunelleschi's dome in florence and its wooden chain

The Santa Maria del Fiore dome has remained unchanged during centuries and it currently is—unique among the other masonry domes (apart from the Roman Pantheon)—the only one without an iron hooping. Unlike the Pantheon, however, the dome of Florence has a thickness which is only 1/10 of its internal diameter (about 43 m), and its impost plane starts at a height of about 50 m. Filippo Brunelleschi's tricks in order to achieve the best structural behavior

[2]In particular, the cracks along the corners—which are also present in the Brunelleschi dome—are the physiological consequence of the elasto-plastic deformation of the cylindrical webs, generating tensile stresses in the inner surfaces that are incompatible with the low tensile strength of masonry. In the same way the vertical cracks on the three sides with the portals are the consequence of the well-known tensile stresses that develop in the lower meridians of domes and produced similar cracks in all the masonry domes.

[3]The inserted tie rod has similar shape and dimensions (about 4.5 x 4.5 cm) to the one inserted by Vasari some years later (1570 and 1572) around the dome of the Umiltà church in Pistoia.

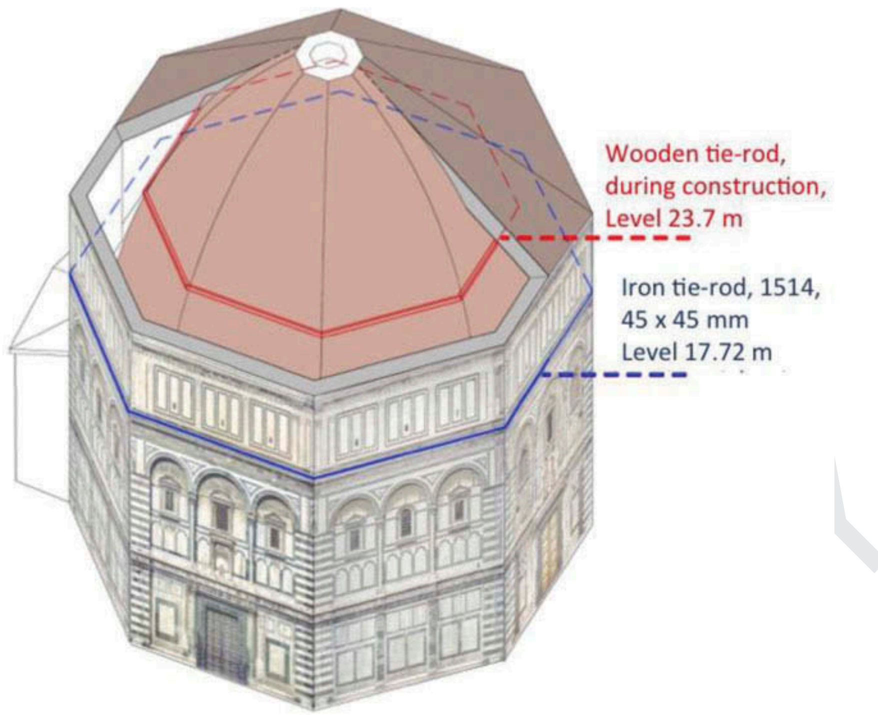


Figure 6. The two hoopings of the dome: The first wooden chains (in red), inserted in construction phase at a level of 23,7 m; and the second iron ones (in blue) inserted in 1514, at 17,7 m.

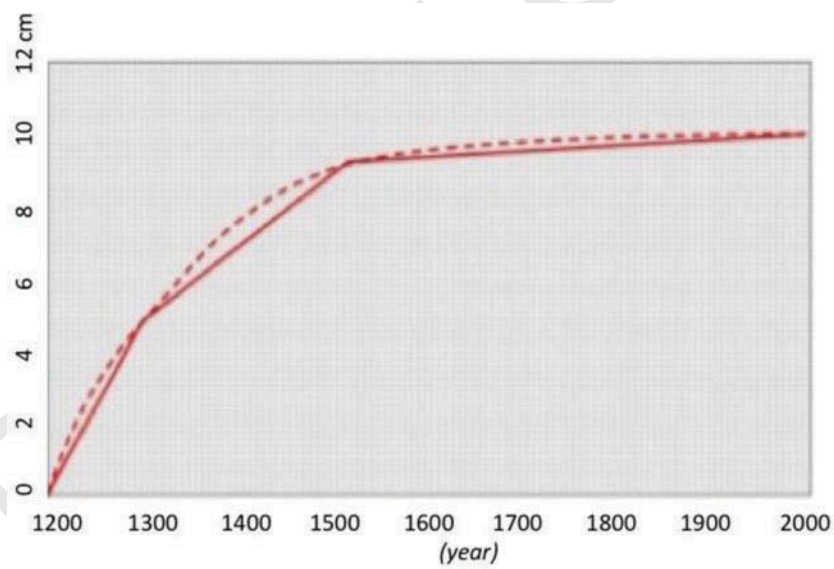


Figure 7. Graph of the hypothesized evolution of the main cracks on sides 1, 3, 5 of the dome, starting from its construction (at the end of XII century): The increasing trend has clearly decreased after the insertion of the two hooping systems.

are well known. Despite some irregularities (Giorgi and Matracchi 2008), they have assured masonry resistance and homogeneity for centuries, transforming the octagonal dome into a rotational one (Blasi and Ottoni 2012) (Figure 8).

Moreover, Brunelleschi must have been certainly familiar, in his design for the dome, with both the structural disorders of the near Baptistery of Giotto and its wooden ties. Actually, Brunelleschi considered the encircling ties system, even if just a wooden one, as essential; indeed in his dome he created one of far greater size and superior craftsmanship. He realized a robust chestnut chain, for which it is difficult to determine the real grade of stiffness and efficiency, composed by 24 segments well connected together in order to achieve a nearly circular hooping (Figure 9).

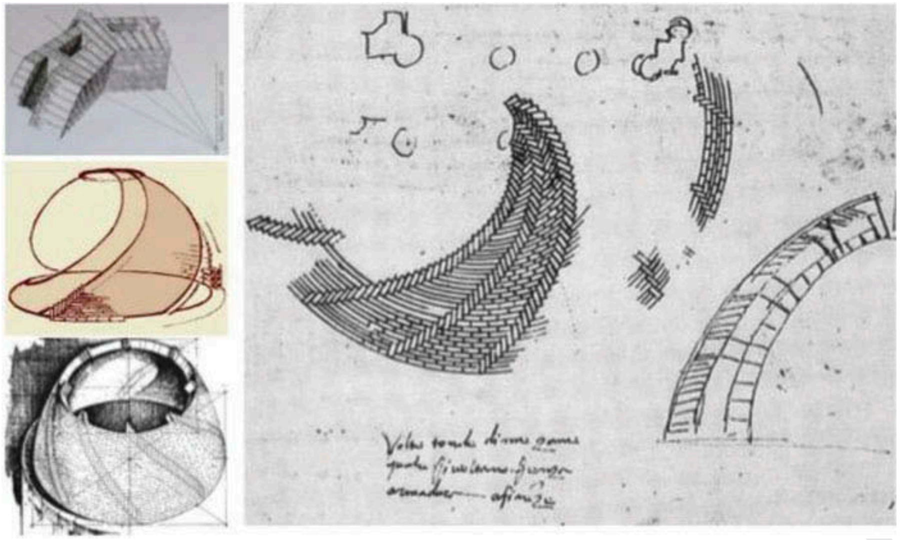


Figure 8. On the right, the Drawing of a "spinapesce" dome (XVI century) by Antonio da Sangallo il Vecchio; on the same "trick" applied by Brunelleschi in Santa Maria del Fiore dome: courses of bricks interrupted at regular intervals (about 1.20 m) from vertical bricks, radially oriented, with the function not only of containing the courses of bricks and of building the dome without centinas, but also of creating a dome of radial propellers in the thickness of the walls. The spirals actually follow, with the normal to the joints of mortar, the isostatic lines inside the shell (which are always tangent to the principal directions of stresses).

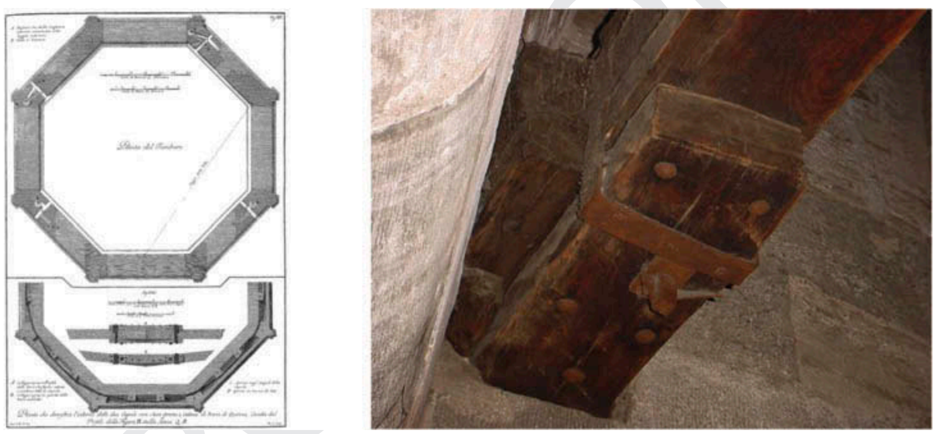


Figure 9. The wooden tie of Santa Maria del Fiore, inserted by Brunelleschi during the construction, in the drawing by S. Sgrilli (a) and one of the iron fixing systems between two wooden ties (b) (Nelli and Sgrilli 1733). © [rightsholder]. Reproduced by permission
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Recent surveys and tests carried out by the CNR group, led by A. Ceccotti, showed that the hooping—although partly defribed and despite today requiring new maintenance—is still active and is subject to a tensile stress rated of about 300–400 KN, a non-negligible value for the
Q28 stability of the monument. Nevertheless, despite all the artifices applied by its architect in order to avoid the dome thrust, the Brunelleschi dome also began to show its weakness, soon cracking just after its completion.

As is well known, its damage has aroused alarm during centuries and its severe crack pattern—large fractures of about 7 cm width (*screpoli*) mainly concentrated on the dome—has been object of different observations and studies by different experts involved in the rough debates on dome stability, already referred in previous studies (Blasi and Ottoni 2012). In particular, at the end of XVII century, the Scientific Commission charged by the Gran Duke to solve the question of dome stability suggested to install four order of iron encircling tie rods around the structure (Figure 10) in order to retain its horizontal thrust, but the subsequent controversy about the causes of cracks—erroneously attributed to insufficient foundation structures—led to cancel the intervention (Galluzzi 1977).

Only three centuries later, in 1988, the last Ministerial Committee—thanks to the results of the first numerical model of the whole dome (Chiarugi, Bartoli, and Bavetta 1995)—finally reaches the

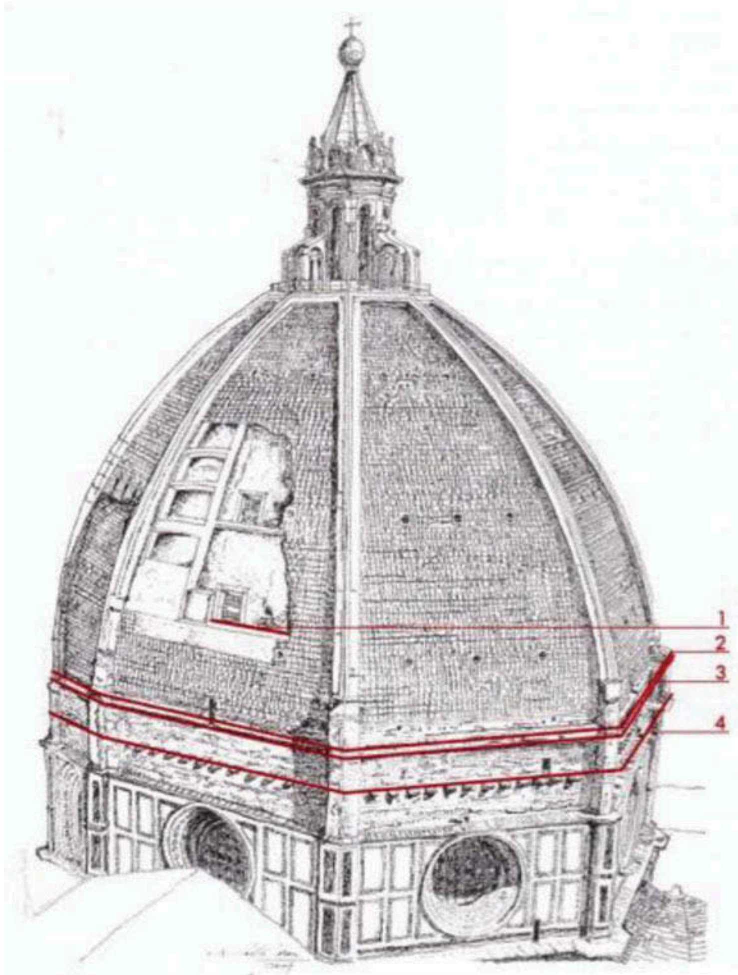

Figure 10. The four orders of iron hoopings, never realized, proposed by Vincenzo Viviani and the Gran Duke Commission at the end of XVII century.

conclusion that the main cause of the crack pattern of the monument was the self weight of the dome, combined to the lack of tensile resistance of masonry, therefore confirming the intuition of Vincenzo Viviani (Chiarugi 1996). Nevertheless, despite the last monitoring data analysis also confirming the beneficent effect of the hooping for the dome stability (Ottoni, Coisson, and Blasi 2010; Ottoni and Blasi 2015), the Santa Maria del Fiore dome remains today without iron hooping.

The vatican dome and the 17th-century debate on hoopings

The traditional remedy of hooping was used also for the great dome of San Pietro in Vatican. The events of this great dome are well known. Built by Michelangelo without chains and on a high tambour with many windows, the dome was too weak to support its horizontal thrust, and it soon fractured.[4] The first cracks on the great dome compared since 1603, but only in 1740, after the precise survey made by Carlo Fontana in 1694 (Fontana 1694), the Pope Benedetto XIV decided to calm the voices about an imminent collapse of the great dome and named a Scientific Commission in order to give an answer, definitive and general, on the stability of the dome. The first commission, composed by three mathematicians, reached the conclusion that the dome was no longer a monolithic system: The parallel continuity was interrupted, from the impost up to the lantern, transforming the dome into a series of arches, connected only at the

[4]Actually, St. Peter's dome was completed by Giacomo della Porta, who partially modified the profile of the dome, originally designed by Michelangelo, who was also the responsible of the high tambour, which represented one of the dome main structural problems. Despite some recent acquisitions that testify the presence of some metal connection elements at certain levels of the dome (Bussi, L., Carusi, M., *Nuove ricerche sulla cupola del Tempio Vaticano*, pp. 61–89, Rome: Pre progetti s.a.s.)—probably inserted by G. Della Porta during the construction—the dome was built without the insertion of proper chains.

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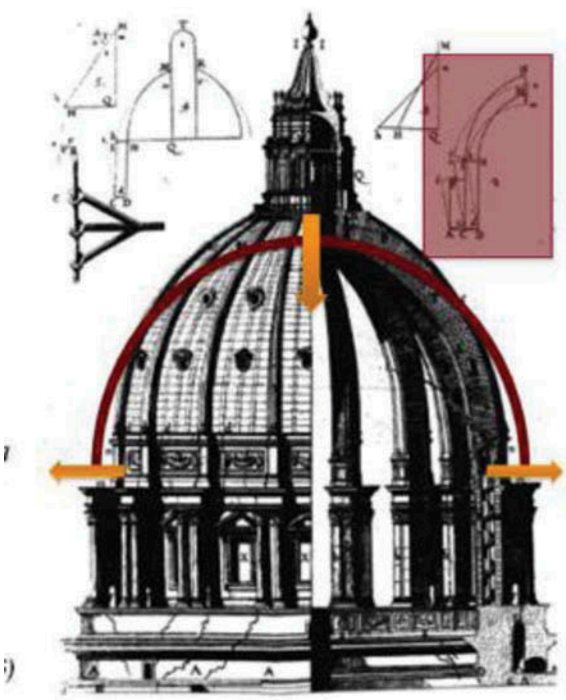


Figure 11. The statical scheme of San Pietro system dome-tambour set up by the Three Mathematicians (Le Seur, T., Jacquier, F. and Boscovich, R.G. 1742). © [rightsholder]. Reproduced by permission of [rightsholder]. Permission to reuse must be obtained from the rightsholder. Q36

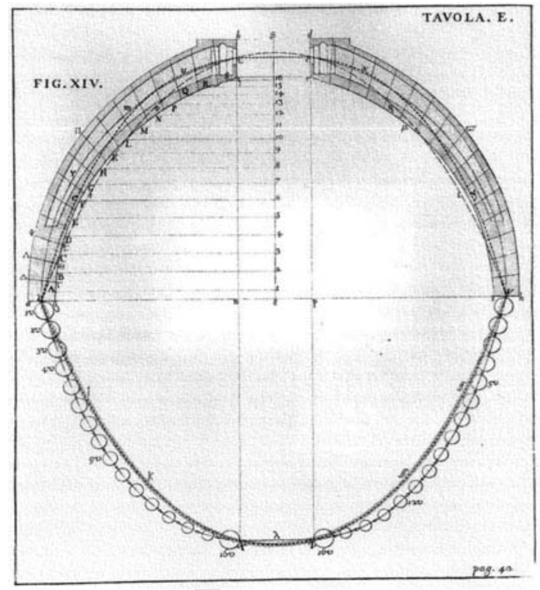


Figure 12. Poleni's famous analysis of San Pietro dome (Poleni 1748). © [rightsholder]. Reproduced by permission of [rightsholder]. Permission to reuse must be obtained from the rightsholder. Q28 Q37

top by a parallel band. Moreover, a correlation between the deformation of the tambour and the vertical propagation of the cracks along the meridians was clear (Niglio 2007). The detected cracks seemed to suggest a rotation of the upper part of the dome, and the mathematicians were very concerned about the horizontal cracks and by the detachment between the two spheres, evident by the cracks along the corridor between the two domes (Figure 11). Q30

In 1748, the physician Giovanni Poleni was charged to study the problem. Following the recent theory of Hooke mixed with the static graphics and experimental tools, Poleni stated that the dome was stable (Figure 11). He divided the dome into 50 arches, each one composed by 16 ashlars, and demonstrated that the funicular line was all contained into the thickness of the arch, thus proving its stability. Nevertheless, he disregarded the slender tambour in his analysis, thereby underestimating the real level of damage (Como 1997).

Only after a century-long rough debate, five encircling tie rods were put in work to contrast the progression of cracks and, between 1743 and 1747, Vanvitelli oversaw the works for the insertion of five encircling tie rods in the dome (Di Stefano 1980). In 1748 the strengthening intervention was completed and the dome was finally stabilized (Figure 13).

The french pantheon in paris and its "hidden" chains

The last episode of this brief *excursus* is the French Pantheon in Paris, which probably represents the last great masonry dome and the first application of the modern membrane theory. Many changes occurred to the first drawing by Soufflot, such as the thickening of the pillars (in 1806) and the closing of side windows (made in 1791 by Rondelet). Moreover, the present dome is much larger than the first one designed by Soufflot. The reasons for the changes have to be retraced both in constructive difficulties and in the controversy raised about it (Patte 1769; Guillerme 1989)

It is not clear if, at the time of the project, Soufflot already knew how to solve the "trap" of the dome or whether Rondelet had introduced the innovation of materials.[5] It is a matter of fact that the three superposed domes, which to date are still in a good state of stability (Blasi and Coisson 2006), had been built by Rondelet in

[5]Certainly, Soufflot knew well that the slender pillars would have not endured an overly thrusting dome, and the first solution to this problem was geometric, designing a truncated cone dome that invoked the tholos and recalled the Wren's design for Saint Paul in London.

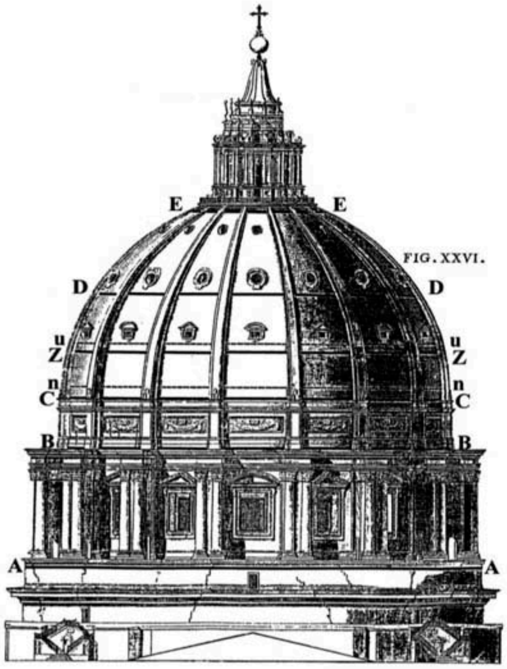


Figure 13. The iron hoopings designed and realized by Vanvitelli in 1748 (Poleni 1748). © [rightsholder]. Reproduced by permission of [rightsholder]. Permission to reuse must be obtained from the rightsholder. Q38

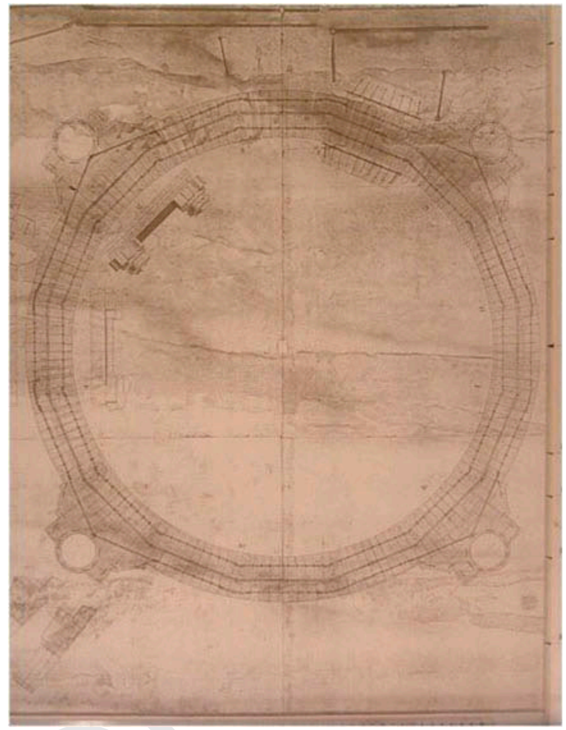


Figure 15. The „hidden" hooping (reinforced stone) inserted by Rondelet in the dome (Rondelet 1797). © [rightsholder]. Reproduced by permission of [rightsholder]. Permission to reuse must be obtained from the rightsholder. Q40

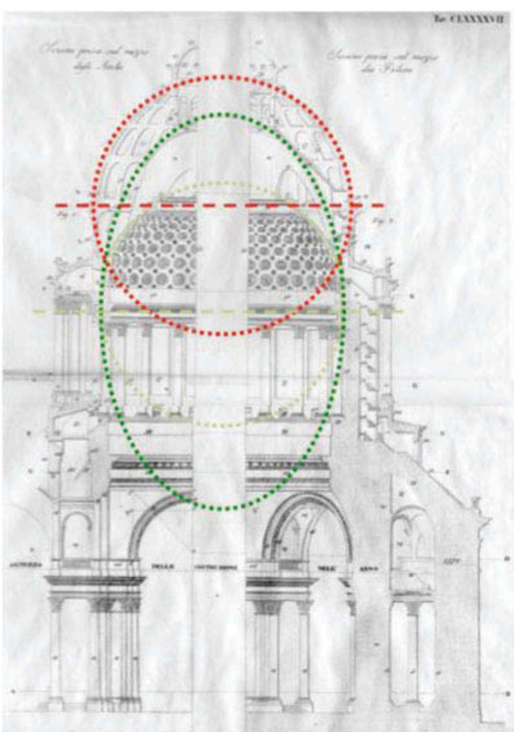


Figure 14. The three superposed domes of the French Pantheon (Rondelet 1797). © [rightsholder]. Reproduced by permission of [rightsholder]. Permission to reuse must be obtained from the rightsholder. Q39

Figure 16. The new iron hooping, realized by two pre-tensioned high resistance cables, recently added around the French Pantheon dome.

1790, after two violent debates (Figure 14). Therefore, in order to ensure that the dome, with the perfect shape of the catenary (corresponding to the minimum thrust), would not have pushed anymore on the pillars (which would have been only vertically loaded), Rondelet transformed the masonry into armed stonework.[6] (Figure 15).

A deep and precise geometric knowledge (*catenaria*), joined with the innovative use of materials (reinforced stone) was thus able—16 centuries after the first Roman Pantheon—to definitively solve the dome trap. In fact, not only have the two visionary architects (Soufflot and Rondelet) reduced the thrusts at a minimum, replicating Wren, but they have also encircled the dome with heavy chains, transforming it into a rigid body that is able to transmit only vertical loads on the pillars, actually decreting the end of the centennial debate on domes and their stability, thus including the traditional remedy of hooping "inside" the dome.[7]

Nevertheless, although the dome is not involved in significant collapse mechanism (because *"les voûtes sphériques n'avoient pas de poussée"*, translated roughly as "it doesn't push anymore" (Rondelet 1797), a new iron hooping, realized by two pre-tensioned high resistance cables (Figure 16), has been recently added in order to guarantee the monument stability.

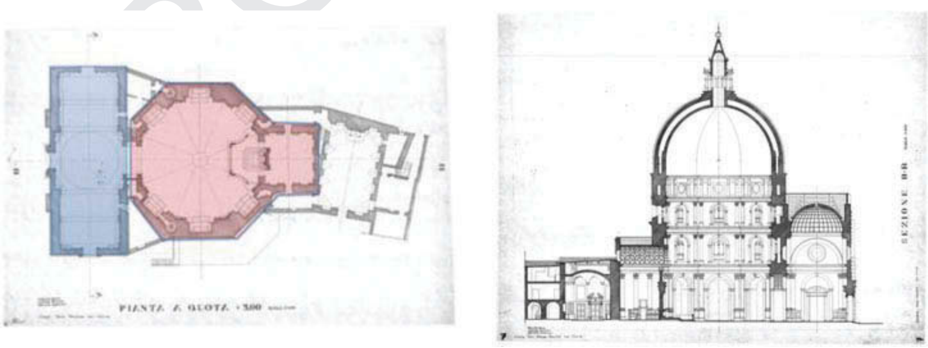
Figure 17. The dome of Madonna dell'Umiltà in Pistoia and its outer iron chains, clearly visible.

Case study: The dome of madonna delL'umiltá in pistoia

The previous brief resume on the main masonry dome hooping interventions has clarified the importance of hooping as traditional remedy to the ineradicable trap of masonry domes. In step with this focus, the large octagonal dome of Madonna dell'Umiltà in Pistoia, designed by Vasari in the 16th century, is a good occasion to make some general consideration of the effects of tie rod insertion as a strengthening intervention. In fact, this masonry structure was so weak that, in different periods and with different techniques, several hooping systems have been inserted on it (Figure 17).

Dome of madonna dell'umiltà

The construction of this church dates back to the end of 1400, following the original design by Giuliano da Sangallo. The complex double system of the church—composed by a large vestibule with a rectangular plan and the dome—was executed by Ventura Vitoni from Pistoia. In the mid-1500s, Giorgio Vasari was commissioned to build the masonry dome double-shell on the octagonal tambour, which is clearly inspired to be larger (twice) than Brunelleschi's dome in Florence (Figure18).



Q41 Figure 18. The structure of the dome: plan (in red, on the left) and section (on the right) (Belluzzi 1993). © [rightsholder]. Reproduced by permission of [rightsholder]. Permission to reuse must be obtained from the rightsholder.

[6]Perhaps also in this operation Rondelet actually followed Soufflot's indication. Actually Gauthey underlined that the iron hooping around the dome would had given a solid consistency to the dome itself, in this way anticipating the modern concept of membrane (Gauthey 1798).

[7]Rondelet could not foresee that the inclusion of iron into the stones would have provoked in time, due to the hygrothermal oxidation, damages to the structures. Indeed, the current crack pattern of the monument testifies two different phenomena: subsidence problems and oxidation phenomena of the iron inserted into the stone blocks (Blasi et al. 2008).

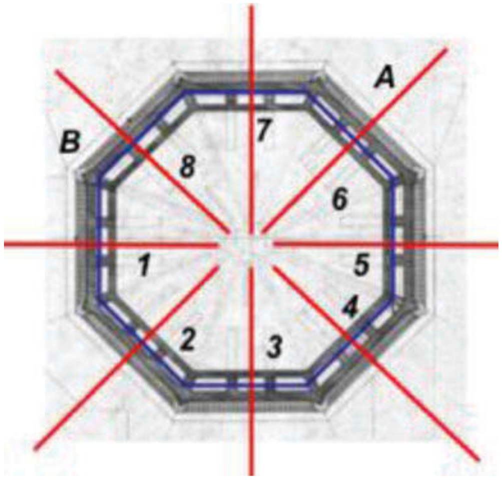


Figure 19. The crack pattern in Madonna dell'Umiltà dome: A
Q42 (in red) and B (in blue).

The architect partially modified the original design, raising the tambour and transforming the profile of the dome from pointed arch to hemispherical one, thus
Q4 creating a more thrusting structure (Belluzzi 1993).

The thickness of the dome varies with its height. At the impost plane both the shells measure approximately 90 cm each, up to 4.5 m where there is an offset. At this level they reach 65 cm thick, and then their thickness progressively decreases towards the top, up to 35 cm. A total of 32 ribs connect the two shells: eight are at the vertices of the octagon, with a single center of curvature; other 16 (2 for each web), orthogonal to the octagon sides, start from 4.5 m and stop at around 10.50 m; moreover eight central ribs, irregular and non-radial, descend from the top of the dome and stop at 10.50 m. Considering the different constructive technique, these last eight ribs have probably been added at a later date. Moreover, there is a detachment between the ribs and radial webs, both internal and external, as if they were three distinct structures (inner dome, outer dome, and ribbed corners). The two shells are linked only through the central ribs, which have also slight connection. In general, the dome does not have a perfect geometry—the octagonal plan of the tambour is irregular (the vestibule side is 50 cm longer than the others)—and masonry apparatus is very poor: the laying of the bricks is approximate (variable thickness and length), with thickness of mortar greater than 1 cm (Tucci, Nobile, and Riemma 2011).

Dome damage and structural behavior

As expected, the great pavilion dome is the main structural element in the whole crack outline of the building which presents an ancient and widespread crack pattern, mainly concentrated on the dome and composed by almost symmetrical cracks. In step with the commonly recognized classification (Tonietti, Ensoli, and Calonaci 1992), the whole crack pattern can be fully described as follows (Figure 19):

- A: Major piercing cracks (on both the domes) with vertical direction, lay at the center of the eight webs, being visible from the inside. They start from the impost level and continue upwards, cutting the tambour throughout its whole thickness, up to around 60°.
- B: Other major cracks are due to the separate movements of the two shells. Even notable detachments are visible between the intrados of angular ribs and the extrados of the inner shell, between 55° and 75°. Some passing cracks cut the median and central ribs, with vertical direction on the median ribs, testifying the relative movement between the two shells.

This complex crack pattern, formed soon after the completion of the dome, evolved during centuries pointing out a substantial symmetry, which seems to confirm the well-known collapse mechanism typical of masonry domes. However, in order to fully understand the structural behavior of this dome, and its damage, it is useful to point out some differences with the Florentine dome that inspired it (Figure 20).

Vasari's dome certainly is reminiscent of the Florentine one, but it is the "bad copy"—surprisingly, the architect did not adopt any of the technical innovation used by Brunelleschi. The main structural problems of this dome are due to specific constructive factors. First, the dome is almost semi-circular; then, with the same span, it pushes more of the pointed arch dome of Florence. The presence of corridors inside the masonry, which turn around the perimeter, greatly weakens the tambour to its entire height. Actually, the tambour is practically made up of two thin walls connected only in some points; over time, the architects attempted to overcome this problem with the insertion of metal brackets connecting the radial direction. Moreover, the tambour is very high, and it is devoid of any contrasting element; in Florence, Brunelleschi had realized a series of perimetral semi-domes, supporting the tambour base, which is sensibly less slender. Furthermore, the Madonna dell'Umiltà dome had not been supported by any encircling system during the construction, which resulted in a structure devoid of efficient contrasting elements. Finally, the lack of cooperation between the two shells, which actually work separately, and the excessive load of the lantern (1/8 of the entire dome weight, very high compared to Santa

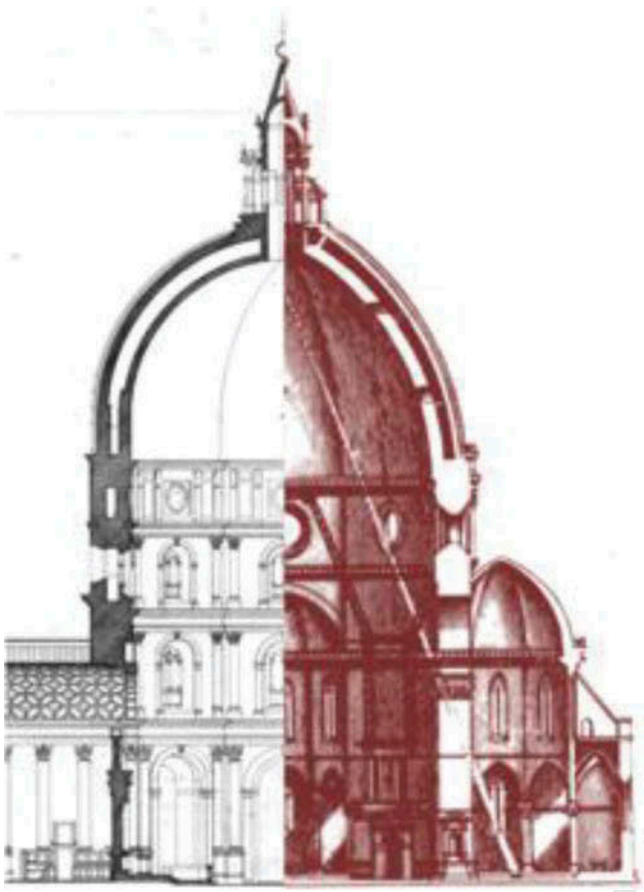


Figure 20. The comparison between the two domed structure: on the left, the Vasari's dome; on the right, the Brunelleschi's one, appropriately scaled.

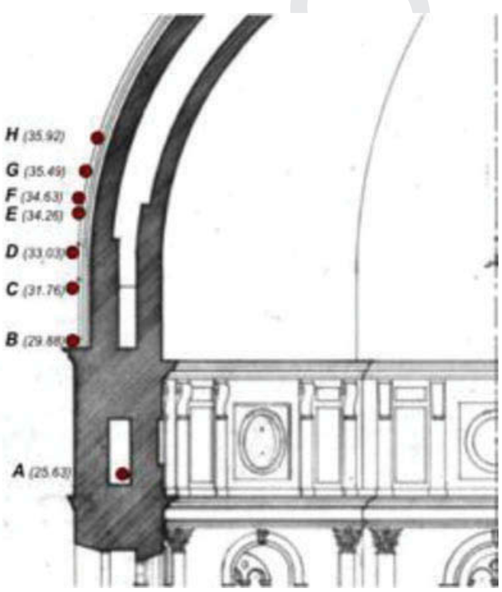


Figure 21. Section of the dome, with the indication of the different tie-rods inserted during centuries.

Table 1. Dating, size and level of the 8 tie-rods (A-H) plotted in Figure 21.

Tie-rod	Age [year]	Architect [name]	Size [mm]	Level [m]
A	1570–72	Vasari	48x48	25.63
B	1570–72	Vasari	40x40	29.88
C	1592 (1840–46)	Baglioni (substitution)	48x48	31.76
D	1592	Baglioni	48x48	33.03
E	1585	Ammannati	44.6x44.6	34.26
F	1966	Lecchi	50x40	34.63
G	1617	Lafri—Meghini	47.8x47.8	35.49
H	1966	Lecchi	50x50	35.92

Maria del Fiore lantern, around 1/25) function as trigger of the well-known "trap" of domes— the different encircling tie rods inserted around the dome over the centuries have tried to solve this problem.

Tie rod insertion over the centuries

The first cracks on the Pistoia dome appeared very soon, just 2 years after the beginning of the construction and in 1570–1572 two iron tie rods were inserted, probably by Vasari himself: the first in the corridor between the two shells (A, at 25 m) and the second around the outer dome (B, at around 30 m), both at the base of the structure (Figure 21, Table1). Despite this intervention, the cracks of the dome were showing a constant increase, and a third tie rod (E) was inserted 10 years later, around 1585, at 4 m over the cornice, by Ammannati and in 1592 two more encircling systems were inserted at 31 (C) and 33 m (D), respectively. The lower one was then substituted in middle 19th century because lightning had broken it.

In 1617, documents refer of some debris falling down from the ancient cracks of the dome and a new tie rod was added by Jacopo Lafri (G, at 35 m), who emphasized the different behavior and movement of the two shells constituting the dome, essentially due to the bad execution of the entire structure. The crack pattern of the dome seems to be steady during the following two centuries, and only after the great earthquake of 1917 the dome showed new cracks and movements. Therefore, two more tie rods were inserted in 1920, again hooping only the outer dome, at around 5 (F) and 6 m (H) over the cornice level.

Today, eight tie rods are present on this dome, seven of which—with the exception of the first one inserted by Vasari in the corridor between the two shells (Figure 22)—are well visible at the extrados of the outer shell (Figure 23). This fact makes unique the dome of Madonna dell'Umiltà which is actually the only one in which we have ancient tie-rods free to vibrate.

Figure 22. Pictures of the first iron chains inserted by Vasari, in 1570, in the corridor between the two shells.

The dynamic tests on tie rods

The dynamic tests carried out on the external tie rods let us to some considerations about the efficiency of these encircling elements on the ancient masonry dome and consequently about the stability of the dome itself. Actually, tie rods are elastic elements able to carry a load when tensioned in displacement, in function of their stiffness, and the stability of the dome essentially depends on the effective tension load in each of these strengthening elements and on the distribution of this tension between them. Moreover, the masonry structures, are subject to relaxation phenomena. They slowly deform, and they are supported, if present, by these elements, which, although more deformable, over time become increasingly more essential. All of this change happens even more so in case of seismic events during which the absence of tensile strength exposes masonry to uncontrolled rotations. Therefore, in order to monitor the dome safety and to fully understand its structural behavior, it is fundamental to measure the distribution of tensile stress on the ties and its evolution in time.

Figure 23. Picture of the iron chains at the extrados of the dome, which are free from rib to rib.

In general, the measurement of axial load of in-service tie rods is not immediate because it cannot be made directly. Moreover, for historic monument, non-destructive, indirect and non-invasive measures have to be used, and some methods have been proposed in literature (Lagomarsino and Calderini 2005; Rebecchi, Tullini, and Laudiero 2013). On Madonna dell'Umiltà tie rods a simple and efficient method (developed by Garziera, Amabili, and Collini 2011; Collini and Garziera 2014) was applied; this method is based on the measure of the vibration of the tie-rod and in the identification of the axial load in it by the matching of a sufficient number of natural frequencies numerically calculated. Different parameters have to be settled in this case: the uncertainties on the constraints; the irregularity of the ancient tie-rod cross-section; the reliable estimation of mass of the tightening wedge system (which is influent on the vibratory behavior of the tie); moreover, connections and local stiffeners increase the difficulties in the measuring process. Particularly influent in this process is the junction system, very different in the eight orders of ties (Figure 24).

Actually, in this particular case, it is possible to compare the efficiency of ties belonging to two distinct ages: Renaissance and middle 20th century. Through their comparison we can obtain interesting information about the effect of time in the relaxation process of masonry.

Experimental measures of vibration were performed by hitting each of the seven external tie rods with an instrumented hammer and the acceleration signals were acquired by two accelerometers fixed in two points of

Figure 24. Two different junction systems present on the dome, belonging to the two different period: (a) 1570 and (b) 1620.

the rod, at about 50 cm from the center. The measured stress goes from 60 to 170 N/mm2, and considering the “allowable stress level” for steel around 160 MPa (as reported in the Italian law, D.M. 1992) at least four tie-rods resulted near the limit and two clearly over-stressed (B and G, around 180 MPa).

Moreover, from the experimental measurements, it was noted that the average stress values regularly decrease clockwise from sector 2 to sector 8, with a maximum variation of about 22%; this slightly non-uniform load distribution in the sectors of the dome, with respect to its axis, reflects the differentiated crack pattern. Anyhow, the load is transmitted quite uniformly between sectors and the constraints at the rod extremities can be well approximated by hinges. Conversely, if we consider the stress values in each order of tie rods, some heterogeneity is registered: the tie rods more loaded are B and G, while the less ones are the tie-rods H and F (as reported in Figure 21). This indicates that tie-rods first placed at support of the structure are the only ones still working and the modern tie-rods, inserted in 1960, are nearly useless to the dome stability. Naturally, the design of the junction and tensioning systems can be one of the probable reasons for this inefficiency.

From strain measurements to numerical modelling

Therefore, in the process of mechanical identification of the structure, a numerical model of the dome has been performed by consecutive steps, in order to obtain a static identification of the structure and to investigate the role of tie-rods. The solid continuum mechanics, especially the finite element method, offers the most suitable and practical models for skeletal structures macro-modeling. The calculation model was created with the software Modest Tecnisoft using the two-dimensional linear elements with 4 nodes with plate-shell behavior (Dvorkin and Bathe 1984).

The dome was modeled with 4328 two-dimensional variable thickness elements (from 80–70 at the tambour to 45 cm at the top), depending on the actual size surveyed by laser scanner.

The structure is composed by three basic elements: the outer dome, the inner one and the connecting ribs between them. The fractures in the dome, which extend also in the underlying structure, were simply modeled by disconnecting the shell, avoiding nodes in common between two-dimensional elements along the crack (Figure 25, left).

The tie-rods, present externally and internally in the dome, were modeled by using linear beam elements, placed at the real level deduced from the laser scanner survey. Their end nodes coincide with those belonging to the ribs, at the vertices of the octagonal dome (Figure 25, right).

The mechanical characteristics of masonry have been properly modified, in order to consider the presence of fracture in masonry and its “relaxation” due to creep phenomena. In particular, the values used in the

Figure 25. On the left, the linear elastic model: (a) the real nodes at the 8 vertices of the dome.

Table 2. Mechanical characteristic of the fractured masonry.

Model	E [MPa]	G [MPa]	w [kN/m3]	v
Initial	750	250	18	0.15
1 step	200	66	18	0.15
2 step	100	33	18	0.15
3 step	50	15	18	0.15

Table 3. Mechanical characteristic of the iron.

E [MPa]	G [MPa]	w [kN/m^3]	v
210000	80000	78.5	0.30

model for the mechanical characteristics of the fractured masonry are reported in the following table (Table 2); while in Table 3 the values assigned to the tie-rods are reported.

Some endoscopic investigations were performed in the two shells of the dome (by the DICEA of Florence, under the guidance of Prof. G. Bartoli) in order to verify the constructive characteristics of masonry, and several flat jacks tests were carried out to determine its real state of stress and deformability. These experimental investigations—far away to be exhaustive for historical buildings—can anyhow be useful in determine the order of magnitude for such important parameters inserted in the model.

Figure 26 shows the results of the axial load of one tie-rod (B), obtained through a linear elastic model of the dome performed with different values of E modulus, compared against the measured stress of the tie-rod (in blue), which resulted remarkably higher (around three times).

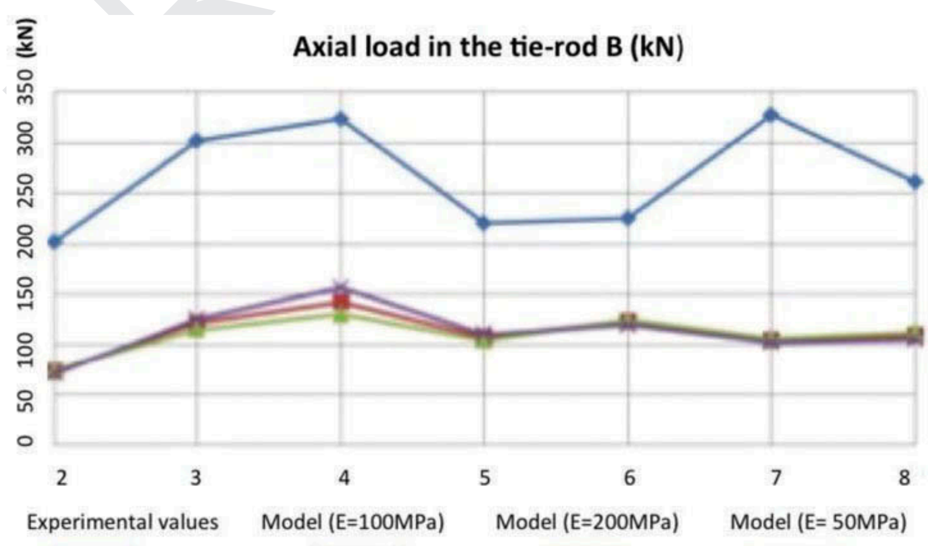
Actually, it is well known that linear static analysis cannot give precise results in case of historic masonry and this is demonstrated by the results shown in the figure. Nevertheless, the elastic linear model, with the empiric insertion of the cracks, could represent a good approximation of the reality, of easy application and control, if the comparison with the empirical method is applied: actually, the insertion of the real cracks of the dome in the model allows to not consider the masonry extremely low resistance to tension and shear strain, by concentrating the non-linearity in the cracks.

In this way, the fractured model simulates the real behavior of the structure, and it confirms that the dome is not able to sustain itself without encircling systems, thus confirming the intuition of the ancient master builders.

Conversely, the approximation used in the simulation of the tie rods (in particular their mechanical characteristics and their connection with masonry) are strong and they were calculated with reference to short-time behavior; therefore we have to consider the measured values of stress as the real **ones**.

In particular, the difference between the measured value and the calculated one **could therefore be explained** considering the different behavior of masonry in time. Actually, the experimental results have clearly shown that masonry, although massive, is subject to “relaxation” phenomena and ancient constraining elements (as the inserted tie-rods), although more deformable, become in time more and more essential to the stability of ancient structures, especially in case of seismic action.

In order to obtain the same values of traction strain measured in the tie-rods with the performed model, the deformability parameter of masonry would have been settled around **500 MPa**, then three times lower than the one proposed by the Italian law. Indeed, the higher



Q44 **Figure 26.** Axial load (KN) in tie-rod B: measured (in blue) and calculated (for each step of simulation).

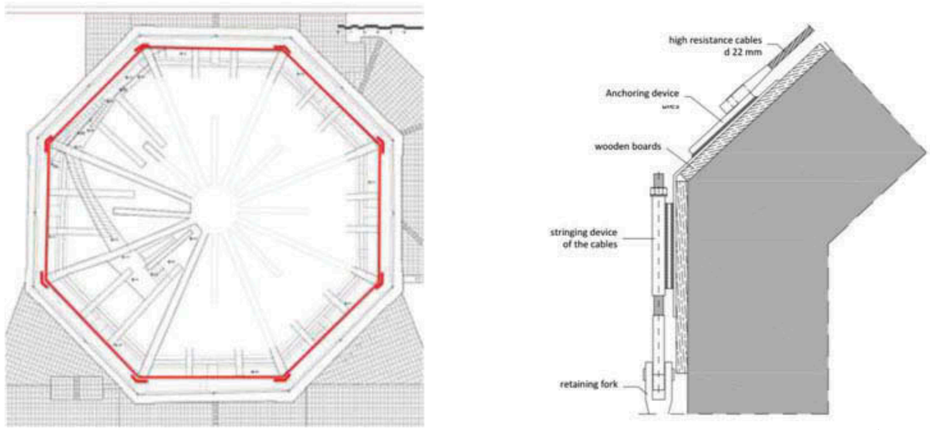


Figure 27. (a) The new tie-rod inserted around the inner dome, in the corridor, at 30 m. (b) A detail of the steel plates—contrasting elements—placed at the vertices.

values of stress have been surveyed in the most ancient tie-rods (B), which are "aged" with the dome, carrying the scars (thermal cycles, earthquakes, etc.) which gradually were added in time: actually, the stress states do not regress and the cycles are never to zero residue.

Hence, the experimental evidences, properly interpreted through a quite simple numerical model (which, although rough, was anyhow useful to provide reliable orders of magnitude) finally have demonstrated that encircling tie-rods are fundamental in dome stability, coming into operation at the proper time (when It is necessary).

The new tie rods as a future remedy: A proposal

The results of dynamic tests, combined with historical analysis and numerical simulation, have confirmed the insertion of tie-rods as the most efficient way to ensure the preservation of domed structures. Moreover, it is interesting to notice that the ancient ones, in this peculiar example, are the only one effective (indeed they result overstressed) and the historical analysis of the different strengthening intervention made on this dome has been fundamental in order to understand the behavior of this structure and to suggest indication for its future conservation.

Therefore, the insertion of a new octagonal tie rod (Figure 27a), at the same level of the existing tie-rod B has been proposed. This new high-resistance element has been recently inserted (in January 2015) in the corridor between the two domes, assuming a design value of stress equal to 40% of the whole level of stress measured in all the existing ties. In this way, if one of the existing tie-rods, already overstressed, would fail, then the new tie rod alone would be able to counteract good part of the thrust of the dome.

As shown by the crack pattern described previously, the two shells are however connected, although slightly (especially in the upper part, the compressed one), and the encircling of the inner one (the more thrusting) works as support of the outer shell, which then also significantly reduces its thrust. It was therefore considered sufficient—also in step with the "minimum intervention" principle (very important in ancient monument restoration)—to insert a single tie between the two caps.

Eight high strength steel tie rods, connected each other in an external polygonal system by means of contrasting elements appropriately shaped—steel plates—have been positioned in correspondence of the eight edges (Figure 27b). Each chain is provided with a tensioning system, reachable from the corridor, which allows recalibrating the action of dome encirclement during time. In step with the experimental nature of this intervention, dynamic tests will be periodically performed on the tie-rods, in order to verify their effective degree of tensioning, which has always to be related to the real response of the dome to eventual external actions (e.g., temperature, wind, seismic events).

This new element, suitably monitored, could give fundamental information on the evolution of displacement of the dome in time. Moreover, it could be considered as real-scale experimentation for other major domes, as Santa Maria del Fiore, which although in better static condition has a similar crack pattern due to what we called the "trap" common to all masonry domes.

Conclusions

The combined dynamic tests and static analysis of the complex dome of Madonna dell'Umiltà in Pistoia has finally allowed reliable knowledge today of the behavior of the dome and its relation to the different tie-rods inserted around it during centuries. Previous studies have already pointed out the difficulty in accurately evaluating the effective state of stress of ancient tie-rods on masonry domes. In this article, new results of the direct measurements carried out on these fundamental elements have been presented, considering not only the well-known encircling effect, but also their relation with the thorny issue of relaxation of masonry in time. Thanks to the variety of technical solutions and joint conformations present in this dome, due to the different periods in which the eight level of tie-rods were inserted in order to reinforce it, some final conclusions are added in relation to their efficacy and a solution for a new intervention is proposed, which recovers the ancient empiricism.

Acknowledgment

The present study has been carried out in the frame of a protocol (signed in March 2008) between the Curia Bishop of Pistoia and the Foundation of "Cassa di Risparmio di Pistoia and Pescia" for the restoration of Madonna dell'Umiltà in Pistoia, in cooperation with the Superintendence for Architectural Heritage and the Tuscan provinces of Florence, Pistoia and Prato.

Within this study, the dynamic tests on the tie-rods have been carried out by the research group leaded by Prof. R. Garziera, Parma University, while the accurate geometrical survey, has been realized by Prof. G. Tucci, Florence University. Heartfelt thanks go to both of them, for their precious work.

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