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Optimal choice of prevention and cure under uncertainty on disease effect and cure effectiveness

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Abstract

This paper studies optimal prevention and cure when an agent copes with two different sources of uncertainty: uncertainty on disease effect and uncertainty on cure effectiveness. We first analyze how optimal choices are affected by uncertainty when prevention and cure do not interact. Under both types of uncertainty, we obtain that the optimal level of prevention rises. Furthermore, we characterize for each source of uncertainty the conditions for the optimal level of cure to increase. We show that these conditions are related to different measures of prudence in health and cross-prudence in wealth. Lastly, we generalize our results to the case where prevention and cure interact and characterize for each source of uncertainty the conditions for the optimal level of prevention and cure to jointly increase. These conditions are similar to those obtained in the case without uncertainty but, in this context, Edgeworth-Pareto complementarity is also required.

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1 Introduction

People usually make health decisions while coping with uncertainty. This uncertainty has different dimensions. When an agent is healthy she first has to deal with the risk of becoming sick in the future. In order to reduce the probability that this will occur, the agent performs prevention activities. If, despite prevention, the disease occurs, the agent can reduce its negative effects by means of cure. A recent paper by Menegatti (2014) analyzes the optimal choice of prevention and cure in a context where an agent chooses both of them.[1]

However, uncertainty about the possibility of becoming sick is only one of the risks that the agent has to cope with. Once the risk of contracting a disease is considered, a second source of uncertainty arises, which refers to the strength of the negative effects that the disease produces. It is well-known that the same disease may have very different consequences for different patients. Examples can be provided for both severe and common diseases, for instance, stroke and influenza respectively. In the case of stroke, we have wide variability of outcomes ranging from minor impairment to patient death, according to the location and the extent of the lesion in the brain.[2] An even wider variability is observed in a much less serious disease like influenza, where the infection effects may be asymptomatic, generate mild consequences or cause extremely serious outcomes and even death.[3] As noted above, the variability of disease effect determines the presence of a second type of risk alongside the risk of contracting disease.

Moreover, as emphasized above, once a disease is diagnosed, the agent can try to cure it. However, cure effectiveness is also characterized by a wide variability stemming from several sources. First, the same disease can be treated in different ways, and in many cases it is not clear which treatment is the best.[4] Second, the effects of a treatment, or in general of a cure, can be different in different contexts (different places or different medical structures) because of problems related to cure method transferability.[5] Furthermore, even in the same context, the same cure may have different degrees of effectiveness as well as different side effects on different patients according to how

[1] A similar issue, although in a completely different framework, was studied by Hennessy (2008).

[2] According to the U.S. National Stroke Association "*10% of stroke survivors recover almost completely, 25% recover with minor impairments, 40% experience moderate to severe impairments that require special care, 10% require care in a nursing home or other long-term facility and 15% die shortly after the stroke*" (www.stroke.org). The psychological effect of stroke also varies according to personal characteristics since "*every stroke is unique [...] and each story teller leaves [...] a unique experience.*" (Klein 2007, p.265).

[3] For instance Carrat et al. (2008) documents that, in volunteer challenge studies, influenza is asymptomatic in a one third of cases and that only 40% of cases exhibit fever.

[4] On this issue see for instance Han et al. (2011) and Sox and Greenfield (2009). Sometimes it may also be the case that the uncertainty concerns the need for treatment. This need in fact depends on the degree of medicalization of a specific disease in a particular socio-economic context, as pointed out by Bell (2016), and on the physical and economic repercussions of the treatment, as in the case of the antiretroviral therapy for HIV studied by Zhou (2016).

[5] See for instance Antoñanzas et al. (2012) and Antoñanzas et al. (2009).

each patient reacts to the treatment.[6]

Uncertainty on both disease effects and cure effectiveness are relevant elements in agent's choice of prevention and cure. Despite this, they have been until now neglected in the literature, as discussed in Section 2.

The aim of this work is to fill the gap in the literature on prevention and cure by introducing the new sources of uncertainty described above. These sources of uncertainty are first studied in a basic model where prevention and cure activities do not interact. In this context we analyze if and how optimal levels of prevention and cure change when the effects of a potential disease are not certainly known and become random. Similarly, we then analyze if and how the optimal levels of prevention and cure change when cure effectiveness in improving the health status of the patient cannot be certainly established and is risky.

We then enrich our analysis to take into account possible interactions between prevention and cure. In particular, the basic model described above assumes that prevention has the sole effect of reducing the probability of contracting the disease. In many cases, however, prevention activity also improves agent's health status in case she becomes sick. This circumstance typically occurs in the case of the influenza vaccine, where vaccination reduces the probability of contracting influenza but also dampens its effects if it is contracted.[7] Similarly, preventive treatment on children suffering from seasonal allergic rhino-conjunctivitis generates not only a reduction in the probability of asthma occurrence but also a lower severity if the disease occurs.[8] We therefore study a second model where the activity of prevention reduces the probability of contracting the disease and improves the health status of the agent if the agent becomes sick. In this framework, we analyze how the optimal levels of prevention and cure change in the presence of uncertainty on the effects of the diseases and in the presence of uncertainty on cure effectiveness.

Lastly, following a recent strand of literature starting from the seminal paper by Ekern (1980), we also examine in both the basic model and in the model with interactions, changes in uncertainty as described by nth-order risk changes. These changes are related to modifications in high-order moments of the distribution of the risky variable and have been studied in many different economic fields. In the last part of this paper we apply these changes to our sources of uncertainty in the analysis of prevention and cure.

The paper proceeds as follows. Section 2 briefly summarizes the related literature.

[6]As emphasized by Eddy (1984), this occurs since "*there is a natural variation in the way people respond to a medical procedure. [...] Because of this natural variation we can only talk about [...] the probability that a treatment will yield a certain result.*" On the issue of different patient's reactions to different cures see also Antoñanzas et al. (2014). On side effects see Waters et al. (2009) and Courbage and Rey (2016).

[7]According to the American Society for Microbiology: *The findings of [...] suggest that annual receipt of influenza vaccine, while not 100% effective in preventing disease, may be of some benefit by lessening the severity of symptoms among those who do get influenza.* Similarly Wang et al. (2007) show that influenza vaccination is strongly associated with reducing major cause-specific mortality.

[8]See Niggemann et al., 2006.

Section 3 presents the basic model without interactions between prevention and cure activities, and studies three different settings: a first setting with certainty on both disease effect and cure effectiveness, a second setting with uncertainty on disease effect and a third setting with uncertainty on cure effectiveness. Section 4 introduces a different model, which allows for interactions between prevention and cure activities, and studies the same three settings described above. Section 5 studies the extension to nth-degree risk changes. Section 6 briefly presents some policy implications. Section 7 concludes.

2 Related literature

As is well known, pioneering analysis of health economics models is performed by Arrow (1963) and Grossman (1972). In particular, Arrow (1963) compares for the first time the optimal choices made under certainty and under uncertainty by an agent dealing with a health problem. Moreover Grossman (1972), analyzing an investment in cure which reduces health deterioration, is the first to introduce the idea of demand for health.

In literature where financial variables are studied, Ehrlich and Becker (1972) and Menegatti (2009) introduce the analysis of prevention in a one-period setting and in a two-period setting respectively. Prevention is also studied in a one-period health framework by Courbage and Rey (2006). In this paper the authors focus on prevention alone and do not consider cure, analyzing a model which does not introduce any kind of uncertainty on disease effects.[9]

To our knowledge, Hennessy (2008) and Menegatti (2014) are the only works dealing with models where prevention and cure are analyzed contemporaneously. In fact, both these papers introduce prevention as an activity implemented in order to reduce the probability of becoming sick and study its determination together with a form of cure. Hennessy (2008) introduces cure as an instrument to increase the probability of restoring health while Menegatti (2014) introduces cure as a means to reduce the negative effects of illness. Both these papers, however, assume certainty on the effects of the disease and on cure effectiveness, thus neglecting the sources of uncertainty analyzed in the present work.[10]

Uncertainty on cure effectiveness and on side effects are however introduced by Courbage and Rey (2016) in a model where the agent has to choose whether to undergo preventive treatment or not. In that model, however, the agent does not choose the level of effort exerted in prevention (which is given) and there is no cure.[11]

[9] In Courbage and Rey (2006) uncertainty only concerns the risk that the agent becomes sick.

[10] Note that Hennessy (2008) assumes that cure affects the probability of regaining health when sick. Although the agent is not sure of being able to recover, the effects of cure on this probability are certainly known. So, in fact, cure effectiveness is given in Hennessy (2008).

[11] In more detail, this model assumes that the agent has a binary choice: undergo preventive treatment (which requires a given level of effort), or not. This means that the agent does not choose either a level of prevention or a level of cure. Moreover, the preventive treatment does not affect the probability

Moreover, a kind of uncertainty similar to disease uncertainty was introduced by Eeckhoudt et al. (2007), Denuit et al. (2011) and Liu e Menegatti (2017), who study decision problems where agent's future health status is random. All these papers, however, focus on health investment and not on prevention and cure.

Lastly, our analysis is related to the literature on nth-order risk changes, which starts from Ekern (1980). The effects of these changes have been analyzed in many different economic problems. Applications to health economics are provided by Courbage and Rey (2012), who study priority setting in health and Courbage and Rey (2016), who examine preventive treatments. Other applications in economics include precautionary saving (Eeckhoudt and Schlesinger, 2008 and Menegatti 2015), optimal labour supply (Chiu and Eeckhoudt, 2010), indexes of risk attitude (Denuit and Eeckhoudt, 2010), environmental policy (Baiardi and Menegatti, 2011) and financial prevention (Crainich et al., 2016).

3 The basic model

The basic model builds on the model of prevention and cure by Menegatti (2014). Following his approach we do not consider here possible interactions between the two instruments. We study a two-period framework, where an agent has in both periods the same two-argument utility function $u(W, H)$ where W is wealth and H is health status.[12] As usual in utility theory, we assume that $\partial u/\partial W = u_1 > 0$ (non-satiation in wealth), $\partial u/\partial H = u_2 > 0$ (non-satiation in health), $\partial^2 u/\partial W^2 = u_{11} < 0$ (risk aversion in wealth) and $\partial^2 u/\partial H^2 = u_{22} < 0$ (risk aversion in health).

In the first period the agent is endowed with a level of wealth, w and a level of health, h. In the second period, the agent has a probability p of contracting a disease and a probability $1 - p$ of not contracting it. In the case of disease, the health status becomes $h - d$ (with $h > d > 0$) while it is equal to h in the case where the agent remains healthy.

The agent can reduce the probability of contracting disease by making prevention effort e. So probability is a decreasing function of e, such that $p'(e) < 0$. As usual in prevention literature, we also introduce the regularity assumption $p''(e) > 0$.[13] We assume that the effort chosen by the agent is paid in period 0, is bounded above ($e \in [0, \overline{e}]$) and cannot reduce to zero the probability of contracting the disease ($p(\overline{e}) \neq 0$).

If, despite prevention, the agent contracts the disease, she can try to cure it. So the level of cure $c \geq 0$ is the second choice variable for the agent. The cure is costly and has a unit cost equal to κ. Moreover, after the cure, the agent's health status improves

of becoming sick, but it has a positive effect on health status in case of disease.

[12] For simplicity, we assume that the utility function is the same in the two periods and that the intertemporal subjective discount factor is null. Removing these two simplifying assumptions does not significantly alter the results presented.

[13] $p''(e) > 0$ has the plausible implication that the reduction in probability is smaller when effort increases.

and becomes $h - d + \alpha c$, where $\alpha > 0$ measures cure effectiveness. [14]

Lastly, although we examine a two-period model, we choose to neglect saving in our analysis. This is because our focus is on decisions made in order to deal with a possible future disease, which involve cure and prevention. All the decisions made are thus in the field of health, even though they obviously have some financial cost. As also suggested by the literature on choice bracketing (see Read et al., 1999), agents tend to bracket choices of the same type. This means that a financial decision, such as saving choice, lies outside the range of our analysis. The same approach, moreover, is adopted in both models of prevention and cure quoted above (Hennessy, 2008 and Menegatti, 2014) and is typical also in models of health investment (e.g. Eeckhoudt et al, 2007 and Denuit et al., 2011).[15] Also note that, similarly, the choice on variables affecting health status is usually not studied in models examining financial decisions, such as saving decisions.

In the framework described above, the maximization problem of the agent is

$$\begin{aligned} \max_{e,c} V(e,c,d,\alpha) = u(w-e,h) + (1-p(e))u(w,h) \\ + p(e)u(w-\kappa c, h-d+\alpha c) \end{aligned} \tag{1}$$

In this problem the optimal levels of cure and effort, c^* and e^* respectively, satisfy the following first-order conditions. The first-order condition with respect to c is

$$p(e^*)\alpha u_2(w-kc^*, h-d+\alpha c^*) - p(e^*)\kappa u_1(w-\kappa c^*, h-d+\alpha c^*) = 0 \tag{2}$$

This condition can be opportunely simplified as follows by dividing both sides of (2) by $p(e^*)$:

$$\begin{aligned} V_c(c^*, d, \alpha) = \alpha u_2(w-kc^*, h-d+\alpha c^*) \\ - \kappa u_1(w-\kappa c^*, h-d+\alpha c^*) = 0 \end{aligned} \tag{3}$$

The first-order condition with respect to e is

$$\begin{aligned} V_e(e^*, c^*, d, \alpha) = p'(e^*)[u(w-\kappa c^*, h-d+\alpha c^*) - u(w,h)] \\ - u_1(w-e^*, h) = 0 \end{aligned} \tag{4}$$

For Condition (4) to hold, the quantity $[u(w-\kappa c^*, h-d+\alpha c^*) - u(w,h)]$ must be negative. Note now that this has a simple and clear interpretation: agent's utility must be larger when the agent is healthy than when the agent is sick.[16]

[14]It should be noted that, in general, cure produces a partial recovery in this framework. This is consistent with the idea that, even in the case of complete removal of the disease, the cure takes time to produce its effects, implying, in turn, that the health status remains lower for a period.

[15]Both these papers present models either where health investment is the choice variable or where saving is the choice variable. In both cases, however, each choice is taken in isolation.

[16]Also note that Condition (4) must hold in the point of maximum, i.e. for the optimal levels of prevention and of cure chosen by the agent. However, it holds for every value of prevention an cure under the plausible assumption that marginal utility of cure is null when the agent is healthy.

Second-order conditions require $\frac{\partial^2 V}{\partial e^2} < 0$ and $\frac{\partial^2 V}{\partial e^2} \cdot \frac{\partial^2 V}{\partial c^2} - (\frac{\partial^2 V}{\partial e \partial c})^2 > 0$. Condition $\frac{\partial^2 V}{\partial e^2} < 0$ is satisfied under our assumptions. Moreover, since $\frac{\partial^2 V}{\partial e \partial c} = 0$, condition $\frac{\partial^2 V}{\partial e^2} \cdot \frac{\partial^2 V}{\partial c^2} - (\frac{\partial^2 V}{\partial e \partial c})^2 > 0$ requires $\frac{\partial^2 V}{\partial c^2} < 0$. We explicitly assume that this condition is satisfied everywhere.[17]

Menegatti (2014) specifically studies the features of this equilibrium without introducing the sources of uncertainty we will analyze in the two subsection below. Menegatti first showed that, in this equilibrium, optimal levels of prevention and cure are independently chosen. Moreover he showed that an increase in the certain effect of the disease and an increase in the certain level of cure effectiveness respectively raises and reduces optimal prevention. The same changes have instead ambiguous effects on optimal cure, which depend in some cases on the sign of the cross-derivative u_{12}.

Starting from this general framework, in the subsections below, we study two different problems where different sources of uncertainty are introduced: uncertainty on disease effect and uncertainty on cure effectiveness.

3.1 Uncertainty on disease effect

In this subsection we examine the impact of an uncertain disease effect on the optimal levels of effort and cure. We thus suppose now that disease effect is a random variable $\tilde{d}$ with a generic distribution over the support $[\underline{d}, \overline{d}]$ where $\overline{d} < h$, and with $E[\tilde{d}] = d$.

Under these new assumptions, the maximization problem is given by

$$\max_{e,c} E[V(e, c, \tilde{d}, \alpha)] = u(w - e, h) + (1 - p(e))u(w, h) + p(e)E[u(w - \kappa c, h - \tilde{d} + \alpha c)] \quad (5)$$

In this problem, the optimal levels of cure and effort, c^{**} and e^{**} respectively, satisfy the following first-order conditions. The first-order condition with respect to c is

$$p(e^{**})\alpha E[u_2(w - kc^{**}, h - \tilde{d} + \alpha c^{**})] - p(e^{**})\kappa E[u_1(w - \kappa c^{**}, h - \tilde{d} + \alpha c^{**})] = 0 \quad (6)$$

This condition can be opportunely simplified as follows:

$$E[V_c(c^{**}, \tilde{d}, \alpha)] = \alpha E[u_2(w - kc^{**}, h - \tilde{d} + \alpha c^{**})] - \kappa E[u_1(w - \kappa c^{**}, h - \tilde{d} + \alpha c^{**})] = 0 \quad (7)$$

The first-order condition with respect to e is

$$E[V_e(e^{**}, c^{**}, \tilde{d}, \alpha)] = p'(e^{**})\big(E[u(w - \kappa c^{**}, h - \tilde{d} + \alpha c^{**})] - u(w, h)\big) - u_1(w - e^{**}, h) = 0 \quad (8)$$

As above, we assume that second-order conditions are satisfied everywhere.

In order to analyze the effects of a stochastic disease on optimal cure we compare (3) with (7). We obtain the following results.

[17] The condition $\frac{\partial^2 V}{\partial c^2} < 0$ is satisfied if and only if $\alpha^2 u_{22} + k^2 u_{11} < 2\alpha k u_{12}$

Proposition 1. *Uncertainty on disease effect raises optimal cure ($c^{**} > c^*$) if and only if*

$$\alpha u_{222}(w - kc^*, h - d + \alpha c^*) > \kappa u_{122}(w - kc^*, h - d + \alpha c^*) \tag{9}$$

and reduces optimal cure ($c^{**} < c^*$) if and only if

$$\alpha u_{222}(w - kc^*, h - d + \alpha c^*) < \kappa u_{122}(w - kc^*, h - d + \alpha c^*) \tag{10}$$

Proof. We write the proof for the case where optimal cure increases. The proof for the case where it decreases is similar. Given second-order condition $\frac{\partial^2 V}{\partial c^2} < 0$ and comparing (3) with (7), we can easily verify that $c^{**} > c^*$ if and only if $E[V_c(c^*, \tilde{d}, \alpha)] > V_c(c^*, d, \alpha)$. We can rewrite this inequality as

$$\begin{aligned} E\big[\alpha[u_2(w - kc^*, h - \tilde{d} + \alpha c^*)] - \kappa[u_1(w - \kappa c^*, h - \tilde{d} + \alpha c^*)]\big] > \\ \alpha u_2(w - kc^*, h - d + \alpha c^*) - \kappa u_1(w - \kappa c^*, h - d + \alpha c^*) \end{aligned} \tag{11}$$

By Jensen's inequality, we know that (11) is verified if and only if the second derivative of $V_c(c^*, d, \alpha)$ with respect to d is positive which occurs if and only if $\alpha u_{222}(w - kc^*, h - d + \alpha c^*) - \kappa u_{122}(w - kc^*, h - d + \alpha c^*) > 0$. □

Corollary 1. *a) Conditions $u_{222} > 0$ (prudence in health) and $u_{122} < 0$ (cross-imprudence in wealth) are sufficient to ensure $c^{**} > c^*$;*
b) Conditions $u_{222} < 0$ (imprudence in health) and $u_{122} > 0$ (cross-prudence in wealth) are sufficient to ensure $c^{**} < c^*$.

Proof. The two statements directly follow from the necessary and sufficient condition in Proposition 1. □

Consider now the effects of a stochastic disease on the optimal prevention effort. In order to perform this analysis we compare (4) with (8). We obtain the following results.

Proposition 2. *Uncertainty on disease effect raises optimal prevention ($e^{**} > e^*$).*

Proof. Given the second-order condition $\frac{\partial^2 V}{\partial e^2} < 0$ and since $V_e(e^*, c^*, d, \alpha) = 0$, we have that $e^{**} > e^*$ if and only if

$$E[V_e(e^*, c^{**}, \tilde{d}, \alpha)] > V_e(e^*, c^*, d, \alpha) \tag{12}$$

Obviously (12) holds if

$$E[V_e(e^*, c^{**}, \tilde{d}, \alpha)] > V_e(e^*, c^{**}, d, \alpha) \tag{13}$$

and

$$V_e(e^*, c^{**}, d, \alpha) > V_e(e^*, c^*, d, \alpha) \tag{14}$$

Notice that, since $p'(e) < 0$, (13) is equivalent to

$$E[u(w - \kappa c^{**}, h - \tilde{d} + \alpha c^{**})] < u(w - \kappa c^{**}, h - d + \alpha c^{**}) \tag{15}$$

By Jensen's inequality, (15) is always satisfied under our assumption of risk aversion in health ($u_{22} < 0$). So (13) is always satisfied too.

Consider now (14). Since $p'(e) < 0$, this inequality is equivalent to

$$u(w - \kappa c^{**}, h - d + \alpha c^{**}) < u(w - \kappa c^{*}, h - d + \alpha c^{*}) \tag{16}$$

By (3) and second-order condition $\frac{\partial^2 V}{\partial c^2} < 0$, we get that function $f(c) = u(w - \kappa c, h - d + \alpha c)$ is maximized for $c = c^*$. This entails that (16) is always satisfied, implying in turn that (14) is satisfied too. □

Proposition 1 and Corollary 1 show that the impact of an uncertain disease effect on optimal cure depends on u_{222} and u_{122}. Starting from Eeckhoudt et al. (2007), these two features of agent's preferences have been called "prudence in health" and "cross-prudence in wealth", respectively.[18]

In order to provide an interpretation of this result it is useful to refer to the interpretation of prudence in precautionary saving literature. In this literature, Eeckhoudt and Schlesinger (2006) and Menegatti (2007) show that prudence can be seen as the desire of an agent to increase financial wealth in the period where the agent bears an additive financial risk (typically an income risk).[19] This result is derived in a context where the utility depends only on wealth and not on health status.

In the same way, in the present framework, prudence in health can be interpreted as the desire of an agent to improve her health status when she bears an additive health risk (i.e. uncertainty on disease effect). Analogously, cross-prudence in wealth can be interpreted as the desire to increase financial wealth in the period where the agent bears the health risk.

Given these interpretations, we can easily understand the meaning of Corollary 1. When an agent is prudent in health and cross-imprudent in wealth, she desires a higher health status and a lower level of wealth in the period when she bears the health risk. As a consequence, since more cure improves health and reduces wealth, the agent chooses a higher level of cure. Obviously, in the opposite case, where the agent is imprudent in health and cross-prudent in wealth she desires a lower health status and a higher level of wealth in the period when she bears health risk. Hence, she chooses to reduce the level of cure.

The same reasoning can be used for the changes in marginal benefit and marginal cost of cure when uncertainty on the effect of disease is introduced. Prudence in health ensures that marginal benefit of cure (measured in terms of health status) is larger under uncertainty than under certainty. Similarly, cross-imprudence in wealth implies that marginal cost of cure (measured in terms of wealth) is lower under uncertainty

[18] These two features of agent's preferences have been proved to be relevant in several problems in health economics when a background risk is introduced (Eeckhoudt et al., 2007; Jokung et al., 2015), in the study of priority setting in health care (Courbage and Rey, 2012) and in models studying health investment (Denuit et al., 2011 and Liu and Menegatti, 2017).

[19] Eeckhoudt and Schlesinger (2006) and Menegatti (2007) emphasize the role of the utility premium in this mechanism. On this index see also Menegatti (2011).

than under certainty. These two facts together imply that, under prudence in health and cross-imprudence in wealth, the presence of uncertainty raises marginal benefit and reduces marginal cost of cure, determining, in turn, an increase in its optimal level.

This interpretation also sheds light on the meaning of Proposition 1. This proposition simply shows that, in the presence of uncertainty on disease effect, the agent chooses a higher or a lower level of cure than in the case with certainty depending on the relative strength of the changes in marginal benefit and marginal cost of cure. As suggested above, the change in marginal benefit depends on prudence/imprudence in health multiplied by the cure effectiveness coefficient α while the change in the marginal cost depends on cross-prudence/imprudence in wealth multiplied by cure unit cost κ. When the difference in the change in marginal benefit and the change in marginal cost is positive, the optimal level of cure increases while, when the sign of this difference is negative, the optimal level of cure decreases.

Lastly, Proposition 2 shows that uncertainty on disease effect always increases prevention. This result is crucially related to risk aversion. Indeed, in the absence of uncertainty on disease effect, the agent carries out prevention only in order to reduce the probability of contracting a disease. Uncertainty on disease effect provides the risk-averse agent with a further incentive to carry out prevention: the desire to escape the additional health risk generated by this second source of uncertainty. As a consequence prevention effort increases.

3.2 Uncertainty on cure effectiveness

In this subsection we examine the effects of an uncertain cure effectiveness on the optimal levels of effort and cure. We thus suppose that cure effectiveness is now a random variable $\tilde{\alpha}$ with a generic distribution over the support $[\underline{\alpha}, \overline{\alpha}]$ where $E[\tilde{\alpha}] = \alpha$. Under these new assumptions the maximization problem is given by

$$\max_{e,c} E[V(e,c,d,\tilde{\alpha})] = u(w-e,h) + (1-p(e))u(w,h) + p(e)E[u(w-\kappa c, h-d+\tilde{\alpha}c)] \tag{17}$$

In this problem the optimal levels of cure and effort, c^{***} and e^{***} respectively, satisfy the following first-order conditions. The first-order condition with respect to c is

$$p(e^{***})E[\tilde{\alpha} + u_2(w-kc^{***}, h-d+\tilde{\alpha}c^{***})] - p(e^{***})\kappa E[u_1(w-\kappa c^{***}, h-d+\tilde{\alpha}c^{***})] = 0 \tag{18}$$

This condition can be opportunely simplified as follows:

$$E[V_c(c^{***}, d, \tilde{\alpha})] = E[\tilde{\alpha} u_2(w-kc^{***}, h-d+\tilde{\alpha}c^{***})] - \kappa E[u_1(w-\kappa c^{***}, h-d+\tilde{\alpha}c^{***})] = 0 \tag{19}$$

The first-order condition with respect to e is

$$E[V_e(e^{***}, c^{***}, d, \tilde{\alpha})] = p'(e^{***})\big(E[u(w-\kappa c^{***}, h-d+\tilde{\alpha}c^{***})] - u(w,h)\big) - u_1(w-e^{***}, h) = 0 \tag{20}$$

As in previous subsections, we assume that second-order conditions are satisfied everywhere.

In order to analyze the effects of a stochastic disease on optimal cure we compare (3) with (19). We obtain the following results.

Proposition 3. *Uncertainty on cure effectiveness raises optimal cure if and only if* $\eta - \zeta > 2$ *and reduces optimal cure if and only if* $\eta - \zeta < 2$ *where* $\eta = -c^* \alpha \frac{u_{222}(w-kc^*, h-d+\tilde{\alpha}c^*)}{u_{22}(w-kc^*, h-d+\tilde{\alpha}c^*)}$ *and* $\zeta = -c^* \kappa \frac{u_{122}(w-kc^*, h-d+\tilde{\alpha}c^*)}{u_{22}(w-kc^*, h-d+\tilde{\alpha}c^*)}$.

Proof. We write the proof for the case where optimal cure increases. The proof for the case where it decreases is symmetric. Given second-order condition $\frac{\partial^2 V}{\partial c^2} < 0$ and comparing (3) with (19), we may easily verify that $c^{***} > c^*$ if and only if $E[V_c(c^*, d, \tilde{\alpha})] > V_c(c^*, d, \alpha)$. We can rewrite this inequality as

$$E\big[\tilde{\alpha}[u_2(w - kc^*, h - d + \tilde{\alpha}c^*)] - \kappa[u_1(w - \kappa c^*, h - d + \tilde{\alpha}c^*)]\big] > \\ \alpha u_2(w - kc^*, h - d + \alpha c^*) - \kappa u_1(w - \kappa c^*, h - d + \alpha c^*) \tag{21}$$

By Jensen's inequality, we know that (21) is verified if and only if the second derivative of $V_c(c^*, d, \alpha)$ with respect to α is positive which occurs if and only if[20]

$$2u_{22} + c^*(\alpha u_{222} - \kappa u_{122}) > 0 \tag{22}$$

or

$$c^* \left(\kappa \frac{u_{122}}{u_{22}} - \alpha \frac{u_{222}}{u_{22}} \right) > 2 \tag{23}$$

□

Consider now the effects of a stochastic effectiveness of cure on the choice of the agent with regard to the optimal prevention effort. In order to perform this analysis we compare (4) with (20). We obtain the following results.

Proposition 4. *Uncertainty on cure effectiveness raises optimal prevention* ($e^{***} > e^*$).

Proof. Analogous to the proof of Proposition 2. □

As in Subsection 2.1, we can use precautionary saving literature to shed light on Proposition 3. Starting from the seminal paper by Rothschild and Stiglitz (1970), this literature shows that, in the presence of uncertainty on the return on saving (i.e. a multiplicative risk), we have positive precautionary saving if the index of partial relative prudence[21] exceeds the threshold of two.[22] This threshold indicates that prudence

[20] For the sake of brevity, we omit the argument of the functions in (22) and (23).

[21] In a general framework with a multiplicative risk and an one-argument utility function $u(.) = u(w + x\tilde{\beta})$ where w is given, $\tilde{\beta}$ is a random variable with $E[\tilde{\beta}] = \beta$ and x is the choice variable, the index of partial relative prudence is defined as $-x\beta \frac{\frac{d^3 u(w+x\tilde{\beta})}{dt^3}}{\frac{d^2 u(w+x\tilde{\beta})}{dt^2}}$.

[22] For an interpretation of the value two of the threshold see Magnani (2017).

(which pushes the agent to increase saving) needs to be high enough to sufficiently counterbalance risk aversion (which, in this context, pushes the agent to reduce saving).[23]

In the same way, in our framework, uncertainty on the effectiveness of cure is a multiplicative risk. This means that an increase in the level of cure has two contemporaneous effects. On the one hand, it increases the expected health status of the agent. Since the agent is prudent in health, she likes this effect. On the other hand, a higher level of cure also rises the level of uncertainty on future health status since cure is an instrument whose effect is uncertain. Since the agent is risk averse, she dislikes this effect. The presence of these two opposite incentives for the agent requires the level of the index $-c\alpha\frac{u_{222}}{u_{22}}$ to be sufficiently high in order to have that the incentive due to prudence is sufficiently larger than the incentive due to risk aversion.

In this framework, however, there is a further effect. The choice of the level of cure also affects utility via wealth, since more cure reduces wealth in the second period. This effect is summarized in our condition by the term $-c\kappa\frac{u_{122}}{u_{22}}$. So, our necessary and sufficient condition depends on the difference between $-c\alpha\frac{u_{222}}{u_{22}}$ and $-c\kappa\frac{u_{122}}{u_{22}}$.

As in Subsection 2.1, the reasoning above may be reformulated in terms of marginal benefit and marginal cost. In order to do this, we refer to (22) in the proof of Proposition 3 where the different elements determining the changes in marginal benefit and marginal cost due to the introduction of uncertainty are explicitly shown. In the present context, the variation in the marginal benefit of cure depends on the term $c^*\alpha u_{222}$, which is the same as in the case of uncertainty on disease effect. The variation in the marginal cost of cure depends instead on two elements: $-c^*ku_{122}$ and $2u_{22}$. The former element is the same as in the case of uncertainty on disease effect. The latter element only appears in the present context and is related to risk aversion. As suggested above, this element captures the fact that cure is now an instrument with an uncertain effect on the agent's health status. The condition $\eta - \zeta > 2$, which is derived from (22), is thus the condition ensuring that the difference in the change in marginal benefit and in the change in marginal cost due to uncertainty is positive, which implies, in turn, that the agent chooses a higher level of cure under uncertainty.

Lastly, Proposition 4 shows that also uncertainty on cure effectiveness always raises the optimal level of prevention. The explanation for this is the same as in the case of uncertainty on disease effect: the presence of an uncertain effect of cure on the health status provides an additional reason for a risk averse agent to reduce the probability of becoming sick by means of prevention.

3.3 A comparison of results under the two types of uncertainty

In previous Subsections 2.1 and 2.2 we analyzed the implications of uncertainty on disease effect and of uncertainty on cure effectiveness on optimal cure and optimal prevention. With regard to prevention, our analysis showed that both kinds of uncertainty

[23]For a discussion of this interpretation see Baiardi et al. (2014) and Baiardi et al. (2015).

alternatively introduced surely raise the optimal level of this instrument. On the contrary, with regard to cure, our results show that cure may either increase or decrease and that the direction of its change depends on conditions on agent preferences which are different in the two cases examined. In this subsection we will show that the conditions derived in the two cases are related, and we will provide significant implications of this linkage.

In order to determine this result we first derive the following lemma.

Lemma 1. *Uncertainty on disease effect raises (reduces) optimal cure ($c^{**} > c^*$) if and only if $\eta - \zeta > 0$ and reduces optimal cure ($c^{**} < c^*$) if and only if $\eta - \zeta < 0$*

Proof. The lemma is easily derived from Proposition 1 by multiplying both sides of (9) and (10) by $\frac{c^*}{u_{22}}$ and remembering that $u_{22} < 0$. □

The result in Lemma 1 is simply a rewriting of the conditions in Proposition 1 and its interpretation will be the same as this proposition, discussed in Subsection 2.1. The lemma is however useful in order to compare the conditions determining the agent's optimal choices on cure under the different sources of uncertainty studied in this work. This comparison shows that:

Proposition 5. *a) If an agent raises cure in the presence of uncertainty on cure effectiveness then she will also raise cure in the presence of uncertainty on disease effect.*
b) If an agent reduces cure in the presence of uncertainty on disease effect then she will also reduce cure in the presence of uncertainty on cure effectiveness.

Proof. Proposition 3 shows that if an agent raises cure in the presence of uncertainty on cure effectiveness then $\eta - \zeta > 2$. Condition $\eta - \zeta > 2$ implies $\eta - \zeta > 0$, which, by Lemma 1, implies in turn that the agent raises cure in the presence of uncertainty on disease effect. This proves statement a).
Similarly, Lemma 1 shows that if an agent reduces cure in the presence of uncertainty on disease effect then $\eta - \zeta < 0$. Condition $\eta - \zeta < 0$ implies $\eta - \zeta < 2$, which, by Proposition 3, implies in turn that the agent reduces cure in the presence of uncertainty on cure effectiveness. This proves statement b). □

The conclusions in Proposition 3 show that the choices on optimal cure in the two cases of uncertainty on cure effectiveness and uncertainty on disease effect are not completely distinct. In fact, it may be the case that the direction of agent's reaction under one kind of uncertainty also predicts the direction of her reaction under the other kind of uncertainty. It must however be emphasized that this is true only in some cases, and that the prediction goes in one case from uncertainty on cure effectiveness to uncertainty on disease effect, and in the other case in the opposite direction.

The interpretation of the linkages described in Proposition 5 are related to the discussion presented in the previous subsections. When uncertainty on cure effectiveness is introduced, the agent has a specific disincentive to increase cure: cure is an instrument whose effects are not certain. This disincentive is clearly absent in case of uncertainty on disease effect. This implies in turn that, when an agent is willing to raise cure in the first case then she will be willing to do it also in the second. The opposite reasoning holds when the agent reduces cure in the presence of uncertainty on disease effect. If she reduces cure when its effectiveness is certain then she will surely do the same choice in the case where the effectiveness of cure is random.

4 A model with interactions between prevention and cure

In this section we change our basic model to allow for interactions between the effects of prevention and cure activities. In particular, we consider the circumstance where prevention effort improves the health status of the agent when she contracts the disease. In order to account for this effect we change the setting presented in the previous section. In particular we introduce the additive term γe in the health status of the agent in the state of the world where she contracts the disease.

The maximization problem of the agent becomes thus

$$\max_{e,c} V(e,c,d,\alpha) = u(w-e,h) + (1-p(e))u(w,h) + p(e)u(w-\kappa c, h-d+\alpha c+\gamma e)$$

In this problem the optimal levels of cure and effort, c^+ and e^+ respectively, satisfy the following first-order conditions. The first-order condition with respect to c is

$$\begin{aligned} &p(e^+)\alpha u_2(w-kc^+, h-d+\alpha c^+ + \gamma e^+)+ \\ &-p(e^+)\kappa u_1(w-\kappa c^+, h-d+\alpha c^+ + \gamma e^+) = 0 \end{aligned}$$

This condition can be opportunely simplified as follows:

$$\begin{aligned} V_c(c^+, e^+ d, \alpha) &= \alpha u_2(w-kc^+, h-d+\alpha c^+ + \gamma e^+) + \\ &-\kappa u_1(w-\kappa c^+, h-d+\alpha c^+ + \gamma e^+) = 0 \end{aligned} \tag{24}$$

The first-order condition with respect to e is

$$\begin{aligned} V_e(e^+, c^+, d, \alpha) = p'(e^+)[u(w-\kappa c^+, h-d+\alpha c^+ + \gamma e^+) - u(w,h)] + \\ -u_1(w-e^+, h) + \gamma p(e^+) u_2(w-\kappa c^+, h-d+\alpha c^+ + \gamma e^+) = 0 \end{aligned} \tag{25}$$

As in the previous section, we assume that second-order conditions are satisfied everywhere. This allows us to exclude the case where multiple equlibria arises.[24]

We study now when the optimal levels of prevention and cure change if, in this new setting, we introduce either uncertainty on disease effect or uncertainty on cure effectiveness.

[24]As in Section 2, $V_{ee} < 0$ is always satisfied. $V_{cc} < 0$, which we assume to be satisfied, require the same condition shown in Footnote 17.

4.1 Uncertainty on disease effect

Consider the circumstance where there is uncertainty on disease effect. As in Subsection 2.1, we assume that the disease effect is described by the random variable $\tilde{d}$ with a generic distribution over the support $[\underline{d}, \overline{d}]$ where $\overline{d} < h$ and with $E[\tilde{d}] = d$. Under this hypothesis, the maximization problem of the agent becomes

$$\max_{e,c} V(e, c, \tilde{d}, \alpha) = u(w - e, h) + (1 - p(e))u(w, h) + p(e)E[u(w - \kappa c, h - \tilde{d} + \alpha c + \gamma e)]$$

In this problem the optimal levels of cure and effort, c^{++} and e^{++} respectively, satisfy the following first-order conditions. The first-order condition with respect to c is

$$p(e^{++})\alpha E[u_2(w - kc^{++}, h - \tilde{d} + \alpha c^{++} + \gamma e^{++})] + \\ -p(e^{++})\kappa E[u_1(w - \kappa c^{++}, h - \tilde{d} + \alpha c^{++} + \gamma e^{++})] = 0 \tag{26}$$

This condition can be opportunely simplified as follows:

$$V_c(c^{++}, e^{++}\tilde{d}, \alpha) = \alpha E[u_2(w - kc^{++}, h - \tilde{d} + \alpha c^{++} + \gamma e^{++})] + \\ -\kappa E[u_1(w - \kappa c^{++}, h - \tilde{d} + \alpha c^{++} + \gamma e^{++})] = 0 \tag{27}$$

The first-order condition with respect to e is

$$V_e(e^{++}, c^{++}, d, \alpha) = p'(e^{++})[E[u(w - \kappa c^{++}, h - \tilde{d} + \alpha c^{++} + \gamma e^{++})] - u(w, h)] \\ -u_1(w - e^{++}, h) + \gamma p(e^{++})E[u_2(w - \kappa c^{++}, h - \tilde{d} + \alpha c^{++} + \gamma e^{++})] = 0 \tag{28}$$

In this context, we obtain the following result.

Proposition 6. *In the presence of uncertainty on disease effect, if $V_{ce}(e, c, d, \alpha) > 0$, $u_{222} > 0$ (prudence in health) and $u_{122} < 0$ (cross-imprudence in wealth), are sufficient conditions for optimal prevention and cure to increase ($e^{++} > e^{+}$ and $c^{++} > c^{+}$).*

Proof. We first consider the two curves describing (24) and (25) in a Cartesian diagram, and we label them as CC and EE curves respectively. Note that the slope of the CC curve is given by $-\frac{V_{cc}(e,c,d,\alpha)}{V_{ce}(e,c,d,\alpha)}$ and the slope of the EE curve is given by $-\frac{V_{ce}(e,c,d,\alpha)}{V_{ee}(e,c,d,\alpha)}$. The assumption $V_{ce}(e, c, d, \alpha) > 0$ ensures thus that both curves are upward sloping.

Analyze now the second-order condition of the maximization problem

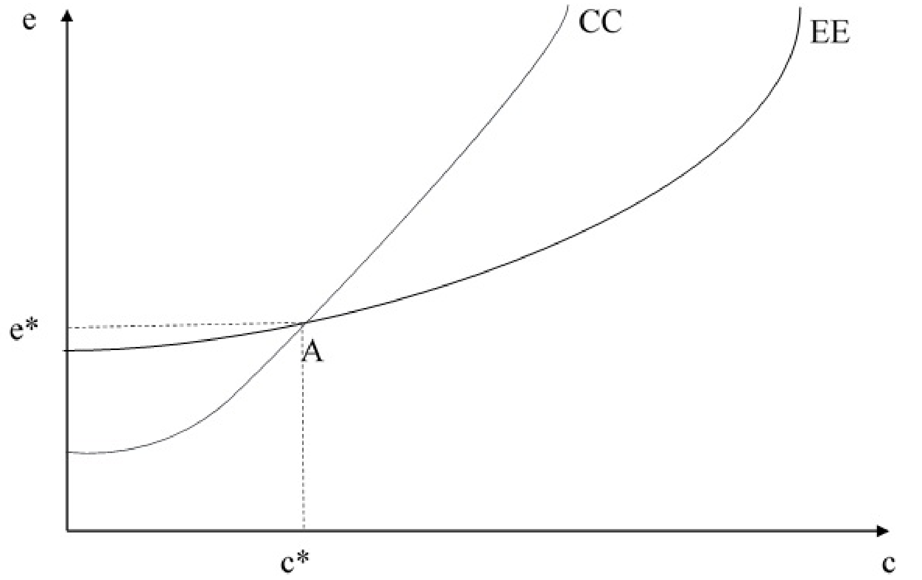
$$V_{ee}(e, c, d, \alpha)V_{cc}(e, c, d, \alpha) - (V_{ce}(e, c, d, \alpha))^2 > 0$$

and note that this implies

$$-\frac{V_{cc}(e, c, d, \alpha)}{V_{ce}(e, c, d, \alpha)} < -\frac{V_{ce}(e, c, d, \alpha)}{V_{ee}(e, c, d, \alpha)}.$$

This last inequality ensures that CC curve is everywhere steeper than the EE curve.[25] The representation of the CC and EE curves is thus that shown in Figure 1. Point

Figure 1: The CC curve and the EE curve in the equilibrium without uncertainty



A in Figure 1, where the two curves intercept, is the equilibrium for the case without uncertainty.

Define now similar curves for the case where we introduce uncertainty in our setting. We thus label the two curves CC' and EE' as the curves describing respectively the two first-order conditions in the presence of uncertainty, (27) and (28). By the same reasoning above, these two curves are upward sloping and the CC' curve is steeper than the EE' curve.

Note now that, by Jensen's inequality, conditions $u_{222} > 0$ and $u_{122} < 0$ ensure that

$$\begin{aligned} &\alpha E[u_2(w - kc, h - \tilde{d} + \alpha c + \gamma e)] + \\ &-\kappa E[u_1(w - \kappa c, h - \tilde{d} + \alpha c + \gamma e)] > \\ &\alpha u_2(w - kc^+, h - d + \alpha c + \gamma e) + \\ &-\kappa u_1(w - \kappa c, h - d + \alpha c + \gamma e) \end{aligned}$$

Since $V_{cc} < 0$, this inequality ensures that the level of c on the CC' curve is lower than on the CC curve for every level of e, which implies in turn that the CC' curve is shifted to the right with respect to the CC curve.

Similarly, again by Jensen's inequality, risk aversion ($u_{22} < 0$) and the condition

[25] Note that this holds everywhere because of the assumption that the second-order conditions are satisfied everywhere, which, in turn, ensures that a unique equilibrium exists.

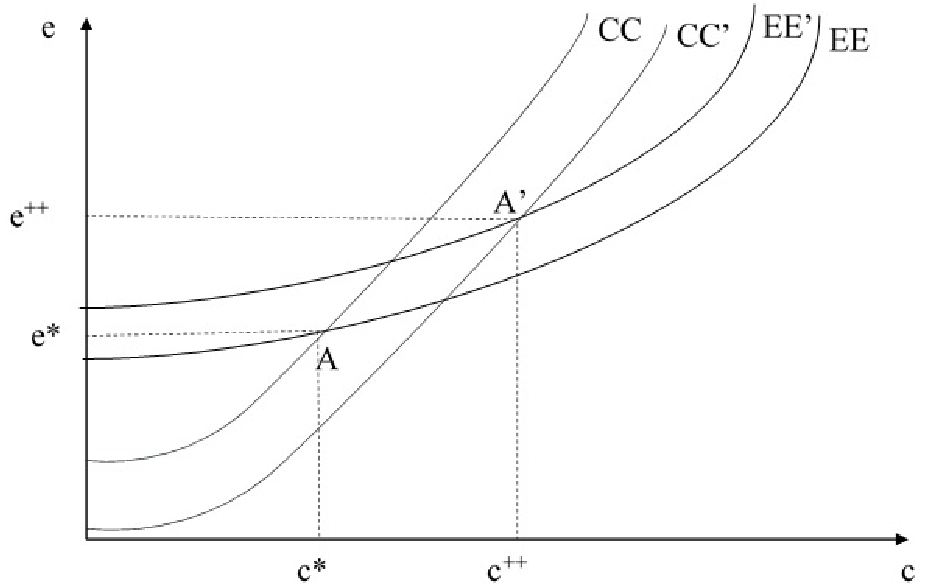
$u_{222} > 0$ ensure that

$$\begin{aligned}
&p'(e)[E[u(w-\kappa c, h-\tilde{d}+\alpha c+\gamma e)] - u(w,h)] + \\
&+\gamma p(e)E[u_2(w-\kappa c, h-\tilde{d}+\alpha c+\gamma e)] - u_1(w-e,h) > \\
&p'(e)[u(w-\kappa c, h-d+\alpha c+\gamma e) - u(w,h)] + \\
&+\gamma p(e)u_2(w-\kappa c, h-d+\alpha c+\gamma e) - u_1(w-e,h)
\end{aligned}$$

Since $V_{ee} < 0$, this inequality ensures that the level of e on the EE' curve is higher than on the EE curve for every level of c, which implies in turn that the EE' curve is shifted upward with respect to the EE curve.

Figure 2 shows the curves CC' and EE' together with the curves CC and EE. The comparison between the equilibrium under uncertainty A', and the equilibrium without uncertainty A shows that, under the assumptions of Proposition 6, the interception between the curves CC' and EE' must lie at a point where both cure and prevention are higher than in the interception between the curves CC and EE. □

Figure 2: The CC' curve and the EE' curve in the equilibrium in the presence of uncertainty



Proposition 6 identifies a set of three sufficient conditions for prevention and cure to increase in the context where prevention and cure interact. These conditions are: prudence in health ($U_{222} > 0$), cross-imprudence in wealth ($U_{122} < 0$) and $V_{ce} > 0$. The first two of these conditions are the same as in the model without interactions between prevention and cure, although their interpretation is partially different. The third condition is completely new and is specifically related to the present setting where cure and prevention interact.

In order to interpret the first two conditions, note that prudence in health and cross-imprudence in wealth affect marginal cost and benefit of cure, implying that marginal benefit is larger under uncertainty than under certainty while marginal cost is lower. This pushes the agent to increase the level of cure in a similar way to that in Section 2.1. Prudence, however, has here a further effect on prevention, which is absent in the model without interactions. In fact, in the present framework, the level of prevention is relevant not only for the probability of contracting the disease (as in Section 2) but also for the health status of the agent when she is sick. This implies that, unlike in Section 2, risk aversion alone does not ensure that prevention increases under uncertainty. In order for this to happen we need also prudence in health, which implies that marginal benefit of prevention is larger under uncertainty, pushing the agent to increase it.

Lastly, the third sufficient condition $V_{ce} > 0$ is specifically related to the interactions between prevention and cure. In particular, following Samuelson (1974, p. 1270), when considering a multi-argument utility function, a positive cross-derivative with respect to two arguments indicates Edgeworth-Pareto complementarity between these two arguments. Similarly, in our case, the positive cross-derivative V_{ce} indicate a kind of Edgeworth-Pareto complementarity between cure and prevention.[26] This complementarity implies that, when uncertainty is introduced, the agent wishes to change the optimal levels of prevention and cure in the same direction. This ensures, in turn, that the interactions between cure and prevention strengthen the incentives provided by prudence in health and cross-imprudence in wealth to increase both cure and prevention.[27]

4.2 Uncertainty on cure effectiveness

Consider now the circumstance where there is uncertainty on cure effectiveness. As in Subsection 2.2, we assume that cure effectiveness is described by the random variable $\tilde{\alpha}$ with a generic distribution over the support $[\underline{\alpha}, \overline{\alpha}]$ where $E[\tilde{\alpha}] = \alpha$. In the present framework, the maximization problem of the agent becomes

$$\max_{e,c} V(e, c, d, \tilde{\alpha}) = u(w - e, h) + (1 - p(e))u(w, h) + p(e)E[u(w - \kappa c, h - d + \tilde{\alpha}c + \gamma e)]$$

In this problem the optimal levels of cure and effort, c^{+++} and e^{+++} respectively, satisfy the first-order conditions below. The first-order condition with respect to c is

$$\begin{aligned} &p(e^{+++})E[\tilde{\alpha}u_2(w - kc^{+++}, h - d + \tilde{\alpha}c^{+++} + \gamma e^{+++})] \\ &-p(e^{+++})\kappa E[u_1(w - \kappa c^{+++}, h - d + \tilde{\alpha}c^{+++} + \gamma e^{+++})] = 0 \end{aligned} \tag{29}$$

[26]A similar kind of Edgeworth-Pareto complementarity has recently been shown to be relevant in the setting studied by Liu and Menegatti (2017).

[27]A negative cross-derivative indicates on the other hand Edgeworth-Pareto substitutability. However, in this case, prudence in health and cross-imprudence in wealth would not be sufficient for cure and prevention to increase. In fact the agent would be pushed to change the levels of cure and prevention in opposite directions and this would counteract the effects of prudence in health and cross-prudence in wealth described above.

This condition can be opportunely simplified as follows:

$$V_c(c^{+++}, e^{+++}, d, \tilde{\alpha}) = E[\tilde{\alpha} u_2(w - kc^{+++}, h - d + \tilde{\alpha} c^{+++} + \gamma e^{+++})] \\ -\kappa E[u_1(w - \kappa c^{+++}, h - d + \tilde{\alpha} c^{+++} + \gamma e^{+++})] = 0 \quad (30)$$

The first-order condition with respect to e is

$$V_e(e^{+++}, c^{+++}, d, \tilde{\alpha}) = p'(e^{+++})[E[u(w - \kappa c^{+++}, h - d + \tilde{\alpha} c^{+++} + \gamma e^{+++})] - u(w, h)] \\ -u_1(w - e^{+++}, h) + \gamma p(e^{+++}) E[u_2(w - \kappa c^{+++}, h - d + \tilde{\alpha} c^{+++} + \gamma e^{+++})] = 0 \quad (31)$$

In this context, we obtain the following result.

Proposition 7. *In the presence of uncertainty on cure effectiveness, if* $V_{ce}(e, c, d, \alpha) > 0$, $u_{222} > 0$ *(prudence in health) and* $\eta' - \zeta' > 2$ *where* $\eta = -c'^{+} \alpha \frac{u_{22}(w-kc^{+}, h-d+\tilde{\alpha}c^{+}+\gamma e^{+})}{u_{22}(w-kc^{+}, h-d+\tilde{\alpha}c^{+}+\gamma e^{+})}$ *and* $\zeta = -c^{+} \kappa \frac{u_{122}(w-kc^{+}, h-d+\tilde{\alpha}c^{+}+\gamma e^{+})}{u_{22}(w-kc^{+}, h-d+\tilde{\alpha}c^{+}+\gamma e^{+})}$, *are sufficient conditions for optimal prevention and cure to increase* ($e^{+++} > e^{+}$ *and* $c^{+++} > c^{+}$).

Proof. The proof is analogous to the proof of Proposition 6. □

Proposition 7 shows a set of three sufficient conditions for uncertainty on cure effectiveness to increase the optimal levels of cure and prevention: $\eta' - \zeta' > 2$, prudence in health ($u_{222} > 0$) and Edgeworth-Pareto complementarity ($V_{ce} > 0$). The first of these conditions is the same as in the model without interactions. In fact, as in Subsection 2.2, the condition $\eta' - \zeta' > 2$ ensures that the difference between the change in marginal benefit and the change in marginal costs due to uncertainty is positive.

The two other conditions (prudence in health and Edgeworth-Pareto complementarity) are instead new and specific to the framework with interactions between cure and prevention. These conditions were relevant also for the case studied in Subsection 3.1 and their interpretation is similar. Prudence in health is related to the increase in prevention, since, in the model with interactions, when the agent is prudent in health, the marginal benefit of prevention becomes larger in the presence of uncertainty. Lastly, Edgeworth-Pareto complementarity pushes the agent to move cure and prevention in the same direction, strengthening the incentive to raise the levels of both instruments, determined by the other conditions.

4.3 A comparison of results under the two types of uncertainty

As in Section 2, we can easily derive some relationships between the results on optimal cure obtained under uncertainty on disease effect and under uncertainty on cure effectiveness.

First it is easy to show that:

Lemma 2. *Assuming* $u_{222} > 0$ *(prudence in health) and* $V_{ce}(e, c, d, \alpha) > 0$, *uncertainty on disease effect raises cure and prevention if* $\eta' - \zeta' > 0$

Proof. Similar to the proof of Lemma 1. □

Using Lemma 3 together with Proposition 7 we get:

Proposition 8. *If an agent raises cure and prevention in the presence of uncertainty on cure effectiveness then she will also raise cure and prevention in the presence of uncertainty on disease effect.*

Proof. Similar to the proof of Proposition 5. □

It is clear that the result in Proposition 8 is similar to that in Proposition 5 with only two differences. The first difference is that the comparison involves here both cure and prevention since because of the interactions between the two instruments their optimal levels are here related. The second difference is that the prediction about agent's behaviour described in Proposition 8, goes here in one direction only: from uncertainty on cure effectiveness to uncertainty on disease effect. This is due to the fact that in the case with interactions we derive conditions only for the case where cure and prevention jointly increase.

5 A generalization to nth-degree risk changes

In this section, we consider a generalization of the results obtained in Sections 2 and 3 and we examine the effects on optimal cure and prevention of an nth-degree increase in risk on the disease effect or on cure effectiveness. The concept of n-th degree risk increase is related to changes in the probability density function of a random variable which cause variations in the high degree moments of the distribution. In particular, a n-th degree risk increase implies that the n-th moment of the distribution increases while the moments from degree 1 to degree $n-1$ are unchanged.[28] This means that, for instance, a third-degree change in risk implies a change in the skewness of the distribution flanked by unchanged variance and expected value. Similarly a fourth-degree change in risk implies a change in the kurtosis of the distribution flanked by unchanged skewness, variance and expected value.

Following Ekern (1980) and Eeckhoudt and Schlesinger (2009), we consider two random variables $\tilde{y}$ and $\tilde{z}$ defined over the interval $[a,b]$. Assume that F and G denote the cumulative distribution functions for these random variables. Define $F^{(0)}(x) \equiv F(x)$ and $F^{(j)}(x) \equiv \int_a^x F^{(j-1)}(t)dt$ for $j \geq 1$ and similarly define $G^{(0)}(x)$ and $G^{(i)}(x)$ for $i \geq 1$. We now have that:

Definition 1. *The random variable $\tilde{z}$ is an nth-degree increase in risk over the random variable $\tilde{y}$ if $F^{(k)}(b) = G^{(k)}(b)$ for $k = 1, 2, ..., n-1$ and $F^{(n-1)}(x) \leq G^{(n-1)}(x)$ $\forall x \in [a,b]$*

[28]Notice that the reverse is not true: an increase in the n-th moment of the distribution with a unchanged moments of lower degrees does not imply that n-th degree risk increase occurred.

Given this framework the following lemma states the well-known relationship between nth-degree increase in risk and expected utility. In particular, following Ekern (1980) and Eeckhoudt and Schlesinger (2009) we have:

Lemma 3. *$E[f(\tilde{y})] \geq E[f(\tilde{z})]$ for every $f(x)$ such that $(-1)^{n+1}\frac{d^n f(x)}{dx^n} > 0$ if and only if $\tilde{z}$ is an nth-degree increase in risk over $\tilde{y}$.*[29]

Lastly, we now introduce a specific class of preferences which will be useful for the next results. Generalizing the analysis of Brockett and Golden (1987) and Caballé and Pomansky (1996) to the case of the two-argument utility function, we define mixed risk aversion in health as follows:

Definition 2. *An agent is mixed risk averse in health when $(-1)^{j+1}\frac{\partial^j u}{\partial H^j} > 0$ for $j = 1, 2 \ldots$*

In the analysis which follows, we assume that the agent are mixed risk averse in health.

5.1 The basic model

We now reconsider Problem (5) in Section 2.1. We assume that $\tilde{d}_2$ is an nth-degree increase risk over $\tilde{d}_1$ and we compare the optimal choice of the agent in the presence of $\tilde{d}_1$ and $\tilde{d}_2$. We call the optimal pairs of values for c and e in the two cases c_1 and e_1 and c_2 and e_2, respectively.

We get the following results.

Proposition 9. *An nth-degree increase in risk over $\tilde{d}$ (from $\tilde{d}_1$ to $\tilde{d}_2$) weakly increases optimal cure ($c_2 \geq c_1$) if and only if*

$$\alpha \frac{\partial^{n+1} u(w - kc_1, h - d + \alpha c_1)}{\partial H^{n+1}} > \kappa \frac{\partial^{n+1} u(w - kc_1, h - d + \alpha c_1)}{\partial W \partial H^n} \tag{32}$$

Proof. By second-order condition $\frac{\partial^2 V}{\partial c^2} < 0$, we have that $c_2 \geq c_1$ if and only if

$$\begin{aligned} E\big[\alpha[u_2(w - kc_1, h - \tilde{d}_2 + \alpha c_1)] - \kappa[u_1(w - \kappa c_1, h - \tilde{d}_2 + \alpha c_1)]\big] \geq \\ E\big[\alpha[u_2(w - kc_1, h - \tilde{d}_1 + \alpha c_1)] - \kappa[u_1(w - \kappa c_1, h - \tilde{d}_1 + \alpha c_1)]\big] \end{aligned} \tag{33}$$

We now define $g(d) = \alpha[u_2(w - kc_1, h - d + \alpha c_1)] - \kappa[u_1(w - \kappa c_1, h - d + \alpha c_1)]$. We apply Lemma 3 in the case where $E[f(\tilde{y})] \leq E[f(\tilde{z})]$ to function $g(d)$ and we get that (33) holds if and only if

$$(-1)^{n+1}\frac{d^n g}{dd^n} = -\alpha \frac{\partial^{n+1} u}{\partial H^{n+1}} + \kappa \frac{\partial^{n+1} u}{\partial W \partial H^n} < 0.$$

which is equivalent to (32). □

[29] In particular, for this presentation of the lemma see Eeckhoudt and Schlesinger (2009, p.996).

Corollary 2. *Sufficient conditions to weakly increase optimal cure* ($c_2 \geq c_1$) *are* $\frac{\partial^{n+1}u}{\partial H^{n+1}} > 0$ *and* $\frac{\partial^{n+1}u}{\partial W \partial H^n} < 0$.

Proof. Straightforwardly follows from Proposition 9. □

Proposition 10. *An nth-degree increase in risk over* $\tilde{d}$ *(from* $\tilde{d}_1$ *to* $\tilde{d}_2$*) weakly increases the optimal level of effort* ($e_2 \geq e_1$) *if*

$$\frac{\partial^n u}{\partial H^n} < 0. \tag{34}$$

Proof. By following the same steps as in Proposition 2, we have that $e_2 \geq e_1$ if and only if

$$E[u(w - kc_2, h - \tilde{d}_2 + \alpha c_2)] \leq E[u(w - kc_1, h - \tilde{d}_1 + \alpha c_1)] \tag{35}$$

which always occurs if

$$E[u(w - \kappa c_2, h - \tilde{d}_2 + \alpha c_2)] \leq E[u(w - \kappa c_2, h - \tilde{d}_1 + \alpha c_2)] \tag{36}$$

and

$$E[u(w - \kappa c_2, h - \tilde{d}_1 + \alpha c_2)] \leq E[u(w - \kappa c_1, h - \tilde{d}_1 + \alpha c_1)] \tag{37}$$

By the same argument used in Proposition 2 for Inequality (16), (37) is always satisfied. We now define $h(d) = u(w - \kappa c_2, h - d + \alpha c_2)$. We apply Lemma 3 in the case where $E[f(\tilde{y})] \geq E[f(\tilde{z})]$ on function $h(d)$ and we get that (36) holds if and only if

$$(-1)^{n+1}\frac{d^n h}{dd^n} = (-1)^{n+1}\frac{\partial^n u}{\partial d^n} = -\frac{\partial^n u}{\partial H^n} > 0.$$

□

We now reconsider Problem (17) in Section 2.2 and we examine an nth-degree increase in risk over cure effectiveness. We assume that $\tilde{\alpha}_2$ is an nth-degree increase risk over $\tilde{\alpha}_1$ and we compare the optimal choice of the agent in the presence of $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$. We call the optimal pairs of values for c and e in the two cases c_1' and e_1' and c_2' and e_2', respectively.[30]

We obtain the following results.

Proposition 11. *If the agent is mixed risk-averse in health, an nth-degree increase in risk over* $\tilde{\alpha}$ *(from* $\tilde{\alpha}_1$ *to* $\tilde{\alpha}_2$*) weakly increases optimal cure* ($c_2' \geq c_1'$) *if and only if*

$$\eta_n - \zeta_n > n \tag{38}$$

where

$$\eta_n = -c_1'\alpha\frac{\frac{\partial^{n+1}u(w-kc_1', h-d+\alpha c_1')}{\partial H^{n+1}}}{\frac{\partial^n u(w-kc_1', h-d+\alpha c_1')}{\partial H^n}} \text{ and } \zeta_n = -c_1'\kappa\frac{\frac{\partial^{n+1}u(w-kc_1', h-d+\alpha c_1')}{\partial W \partial H^n}}{\frac{\partial^n u(w-kc_1', h-d+\alpha c_1')}{\partial H^n}}.$$

[30] An example of an increase in the second-degree increase in risk is presented in Section 4 since the random variable $\tilde{\alpha}$ and the degenerate random variable α have the same mean but different variance.

Proof. By following the same steps as Proposition 3, we have that $c'_2 \geq c'_1$ holds if and only if

$$\begin{aligned} E\big[\tilde{\alpha_2}[u_2(w - kc'_1, h - d + \tilde{\alpha_2}c'_1)] - \kappa[u_1(w - \kappa c'_1, h - d + \tilde{\alpha_2}c'_1)]\big] \geq \\ E\big[\tilde{\alpha_1}[u_2(w - kc'_1, h - d + \tilde{\alpha_1}c'_1)] - \kappa[u_1(w - \kappa c'_1, h - d + \tilde{\alpha_1}c'_1)]\big] \end{aligned} \tag{39}$$

We now apply Lemma 3 in the case where $E[f(\tilde{y})] \leq E[f(\tilde{z})]$ to function $q(\alpha) = \alpha[u_2(w - kc'_1, h - d + \alpha c'_1)] - \kappa[u_1(w - \kappa c'_1, h - d + \alpha c'_1)]$ and we get that (39) holds if and only if

$$(-1)^{n+1}\frac{d^n q}{d\alpha^n} = (-1)^{n+1}\left[n\frac{\partial^n u}{\partial H^n} + c'_1\left(\alpha\frac{\partial^{n+1} u}{\partial H^{n+1}} - \kappa\frac{\partial^{n+1} u}{\partial W \partial H^n}\right)\right] \leq 0 \tag{40}$$

After some algebra, by the assumption of mixed risk aversion, we get that Inequality (40) is equivalent to Inequality (38). □

Proposition 12. *An nth-degree increase in risk over $\tilde{\alpha}$ (from $\tilde{\alpha_1}$ to $\tilde{\alpha_2}$) weakly increases the optimal level of effort ($e'_2 \geq e'_1$) if*

$$(-1)^{n+1}\frac{\partial^n u}{\partial H^n} > 0 \tag{41}$$

Proof. Analogous to the proof of Proposition 10. □

Corollary 3. *If the agent is mixed risk-averse in health, an nth-degree increase in risk over $\tilde{\alpha}$ (from $\tilde{\alpha_1}$ to $\tilde{\alpha_2}$) always weakly increases the optimal level of effort ($e'_2 \geq e'_1$).*

Proof. Straightforwardly follows from Proposition 12 and Definition 2. □

5.2 A model with interactions between prevention and cure

Consider now the model with interactions between prevention and cure; in this context it is possible to extend the results obtained in Section 3 to the case of nth-degree increase in risk. We analyze initially with the case where there is uncertainty on disease effect and reconsider Problem (17) in Section 3.1. In particular, we compare the optimal choice of the agent in the presence of $\tilde{d}_1$ and $\tilde{d}_2$, where we recall that $\tilde{d}_2$ is an nth-degree increase risk over $\tilde{d}_1$. We call the optimal pairs of values for c and e in the two cases c''_1 and e''_1 and c''_2 and e''_2, respectively. We obtain the following results.

Proposition 13. *In the presence of uncertainty on disease effect, if $V_{ce}(e, c, d, \alpha) > 0$,*

$$\frac{\partial^{n+1} u}{\partial H^{n+1}} > 0 \tag{42}$$

and

$$\frac{\partial^{n+1} u}{\partial W \partial H^n} < 0 \tag{43}$$

are sufficient conditions for optimal prevention and cure to increase ($e''_2 > e''_1$ and $c''_2 > c''_1$).

Proof. Analogous to the proof of Proposition 6. □

We now reconsider Problem (17) in Section 3.2 and we compare the optimal choice of the agent in the presence of $\tilde{\alpha_1}$ and $\tilde{\alpha_2}$, where we recall that $\tilde{\alpha_2}$ is an nth-degree increase risk over $\tilde{\alpha_1}$. We call the optimal pairs of values for c and e in the two cases c_1''' and e_1''' and c_2''' and e_2''', respectively. We obtain the following results.

Proposition 14. *In the presence of uncertainty on cure effectiveness, if* $V_{ce}(e, c, d, \alpha) > 0$*,*

$$(-1)^{n+1}\frac{\partial^{n+1}u}{\partial H^{n+1}} > 0 \tag{44}$$

and

$$\eta_n' - \zeta_n' > n \tag{45}$$

where

$$\eta_n' = -c_1'''\alpha\frac{\frac{\partial^{n+1}u(w-kc_1''',h-d+\tilde{\alpha}c_1'''+\gamma e_1''')}{\partial H^{n+1}}}{\frac{\partial^{n}u(w-kc_1''',h-d+\tilde{\alpha}c_1'''+\gamma e_1''')}{\partial H^{n}}} \tag{46}$$

and

$$\zeta_n' = -c_1'''\kappa\frac{\frac{\partial^{n+1}u(w-kc_1''',h-d+\tilde{\alpha}c^{+}+\gamma e_1''')}{\partial W\partial H^{n}}}{\frac{\partial^{n}u(w-kc_1''',h-d+\tilde{\alpha}c_1'''+\gamma e_1''')}{\partial H^{n}}} \tag{47}$$

are sufficient conditions for optimal prevention and cure to increase ($e_2''' > e_1'''$ *and* $c_2''' > c_1'''$*).*

Proof. Analogous to the proof of Proposition 6. □

5.3 Results interpretation

In the present section we provide a generalization of the results previously obtained both in the basic model and in the model with interactions between prevention and cure. Consider initially the basic model. Propositions 9 and 11 examine the effects of an nth-degree risk increase on the optimal level of cure and provide a generalization of the results in Propositions 1 and 3. Indeed, it is clear that Inequality (32) is simply the nth-degree version of Inequality (9) in Proposition 1. Similarly Inequality (38) is the nth-degree version of $\eta - \zeta > 2$ in Proposition 3.

With regard to the changes in the optimal level of prevention, our results in Sections 2 show that the introduction of uncertainty (either on disease effect or on cure effectiveness) always raises the optimal level of prevention. The generalization to the nth-degree risk changes requires further conditions on agent's preferences which involve the signs of higher-order derivatives of the utility function. This is due to the fact that, in the cases examined in Sections 2.1 and 2.2, the required conditions on preferences are automatically satisfied because of the assumption of risk aversion.

Analogously if we consider the model where prevention and cure interact, it is easy to see that the sufficient conditions for the level of prevention and cure to jointly increase derived in Propositions 13 and 14 are the counterpart of Propositions 6 and 7 for the case of an nth-degree risk increase. Moreover, as in the basic model, when the conditions for an increase in optimal prevention are considered, the sufficient conditions for both instruments to increase include some additional elements. First, as in the case of Propositions 9 and 11, we introduce a condition on the signs of the higher-order derivatives of the utility function, which plays the same role as the assumption of risk aversion in the model of Section 4. Second, when interactions are considered, we must introduce the further assumption of Edgeworth-Pareto complementarity between cure and prevention

It should be noted that the conditions provided in Propositions from 9 to 14 involve high-order derivatives of agent's utility function. This result is typical when high-degree risk changes are introduced. In particular, similar conditions are obtained in health economics literature by Courbage and Rey (2012) when introducing nth-degree risk changes in the study of priority setting in health care, and by Courbage and Rey (2016) in the study of preventive treatment. Similar conditions are derived in other fields of economics such as precautionary saving theory (Eeckhoudt and Schlesinger, 2008), the determination of labor supply (Chiu and Eeckhoudt, 2010), the analysis of indexes of risk attitude (Denuit and Eeckhoudt, 2010) and the exam of environmental policy (Baiardi and Menegatti, 2011).

Some of our results are obtained under the assumption of mixed risk aversion in health. Mixed risk averse preferences were first introduced by Brockett and Golden (1987)[31] and Caballé and Pomansky (1996) with reference to the one-argument utility function.[32] Some recent papers provide applications of these preferences (e.g. Crainich et al., 2013) and of their generalization to a two-argument utility function (e.g. Denuit et al., 2011).

In the present framework what is required is a specific form of mixed risk aversion which only involves the second argument of the utility function (i.e. health) which implies that what we seek is mixed aversion in health.

6 Policy implications

It is important to emphasize that our analysis clearly shows that optimal levels of prevention and cure change when uncertainty on disease effects and on cure effectiveness are taken into account. This implies that a mistaken perception of these kinds of uncertainty, particularly underestimating them, determines suboptimal choices for the agent. In particular, with regard to prevention, which we find tends, in many cases, to rise under uncertainty, the outcome may be a level of preventive action which is too low. This conclusion has implications for communications between patient and physician, and clearly suggests that the physician should provide the best available information on the variability of disease effects and possible cure effectiveness.

[31]Brockett and Golden (1987) called these preferences completely monotone preferences.

[32]For a new characterization of this class of function see Menegatti (2015).

It is also important to note that just as misinformation can make prevention levels too low, suboptimal prevention levels can also be too high. In fact, recent literature has noted the significant increase in false information on treatment risks available online and on social networks (e.g. Basch et al., 2017). Our analysis shows that this could mislead patients into overestimating uncertainty on cure effectiveness, thus generating an excess of prevention.

These findings also indicate that there is a role for public intervention to support patients in their decision-making processes. On one hand, there is a need for clear and common protocols for the provision of information about disease effects and cure effectiveness, and on the other, intervention against misinformation on treatment risks is required.

Lastly, the risk that the lack of information determines excessively low levels of preventive action also has important implications for the probability of contracting diseases. In fact, awareness of uncertainty on the effects of either disease or cure may imply an increase in prevention, and thus a decrease in the probability of being ill. As noted by Hennessy (2008), if we consider our representative decision maker as an atomic entity representing a continuum of homogeneous agents, better awareness of uncertainty implies that the amount of population hit by disease will decrease. This point is significant for all cases where there is an underprovision of cure in equilibrium, which suggests the need for public subsidies (e.g. Cremer et al. 2016).[33] In these cases, in fact, better information on disease effects and cure effectiveness may reduce the share of the population hit by disease, reducing in turn the total amount of subsidies.

7 Conclusions

The present paper studies how the optimal choice of prevention and cure changes when uncertainty on disease effect and uncertainty on cure effectiveness are alternatively introduced. We examined two different settings: a basic model where cure and prevention do not interact and a more general model where we allow for interactions.

In the case without interactions we obtain the following main results. In the case of uncertainty on the effect of disease, we derive necessary and sufficient conditions for the optimal level of cure to increase involve prudence/imprudence in health and cross-prudence/cross-imprudence in wealth. Similarly, in the case of uncertainty on cure effectiveness, we have conditions on the index of partial relative prudence in health and the index of partial relative cross-prudence in wealth. In all these results, the direction of the change in the level of cure depends on the difference between the strength of prudence in health and the strength of cross-prudence in wealth, weighted respectively for the effectiveness of cure and the cost of cure. With regard to prevention we find that both uncertainty on disease effect and uncertainty on cure effectiveness increase

[33] Cremer et al. (2016) analyse a model where parents incur the risk of becoming dependent and needing long-term care, and children are not fully altruistic. They show that the laissez-faire is not efficient, and that social outcome can be improved by taxation to subsidize long-term care.

the optimal prevention effort exerted by the agent. This occurs since the presence of an additional source of uncertainty pushes the agent, who is risk averse, to reduce the probability of contracting disease.

In the case with interactions between prevention and cure, we analyze the sufficient conditions ensuring that both instruments jointly increase. These conditions include all the conditions for cure to increase derived in the basic model, which are now relevant also for prevention, and a further condition specifically related to the interactions between the two instruments. This condition is Edgeworth-Pareto complementarity between prevention and cure, which implies that the agent wishes to move the two instruments in the same direction, strengthening the incentive to raise both of them, which is determined by the other conditions.

Lastly, we generalize the previous results to the case of nth-order risk change. We derive necessary and sufficient conditions for cure and for prevention to increase respectively, which involves high-order derivatives of the utility function.

This work introduces into the analysis of prevention and cure two new sources of uncertainty previously neglected in the literature. This also provides the possible basis for future research lines.

First, in this paper we analyzed uncertainty on disease effect and uncertainty on cure effectiveness separately. This was because, as shown by recent literature where additive and multiplicative risks are contemporaneously introduced,[34] results in this framework can be derived only by introducing specific assumptions either on risk size or on the joint distribution of the two risks. This requires a specific analysis which is beyond the scope of the present paper but could be fruitfully developed in a future work.

Second, as emphasized in the introduction, uncertainty in health decisions is a multifaceted issue which has many dimensions. In this paper, we focus on the most important ones. However, the case where other elements influencing agent's decision involve risk could also be a promising subject for future research.

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