Goodness-of-fit testing for the Newcomb-Benford law with application to the detection of customs fraud

Supplementary Material for the paper:

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Experimental results for the MAD statistic

We provide experimental results for the MAD conformity measure that complement those for the χ^2 goodness-of fit statistic given in §4. Specifically, in Table 1 we give Monte carlo estimates of the marginal and conditional quantiles of the exact distribution of the MAD statistic. These quantiles are defined in our approach as

$$q^*_{T_{\{l_1,\ldots,l_m\}}}(\alpha) \tag{1}$$

and

$$q^*_{T_{\{l_1,\ldots,l_m\}}|\Psi_{\{1,\ldots,k\}}(\beta)}(\alpha), \tag{2}$$

respectively. We take $\alpha = \beta = 0.99$.

We also report Monte Carlo estimates of the proportion of false rejections (Table 2), and comparison with the same sequential multiple-testing procedures considered in the manuscript (Table 3), using the MAD conformity measure as our test statistic.

Table 1: Monte Carlo estimates $q^*_{T_{\{l_1,\ldots,l_m\}}}(0.99)$ (Marg) and $q^*_{T_{\{l_1,\ldots,l_m\}}|\Psi_{\{1,\ldots,k\}}(0.99)}(0.99)$ (Cond) of the quantiles of the MAD statistic of conformity under the NB model, for $k = 1, 2, 3, 4$ and all possible index selections $l_1, \ldots, l_m$. $B = 1,000,000$ independent replications for each sample size.

		$n = 20$	$n = 100$	$n = 500$
k	$\{l_1, \ldots, l_m\}$	Marg Cond	Marg Cond	Marg Cond
2	$\{1\}$	1.7853; 2.1988	4.0485; 4.7811	9.0506; 10.7876
	$\{2\}$	1.7055; 1.9443	3.9035; 4.6473	8.7577; 10.4095
	$\{1, 2\}$	0.3588; –	0.9520; –	2.1303; –
3	$\{1\}$	1.7853; 2.1988	4.0485; 4.8546	9.0506; 9.9028
	$\{2\}$	1.7055; 1.7870	3.9035; 4.0755	8.7577; 9.2008
	$\{3\}$	1.6141; 1.7937	3.8666; 4.0275	8.7597; 9.2928
	$\{1, 2\}$	0.3588; 0.3772	0.9520; 1.0130	2.1303; 2.2450
	$\{1, 3\}$	0.3591; 0.3771	0.9514; 1.0050	2.1301; 2.2333
	$\{2, 3\}$	0.3395; 0.3419	0.8994; 0.9303	2.0889; 2.2105
	$\{1, 2, 3\}$	0.0434; –	0.1933; –	0.6014; –
4	$\{1\}$	1.7853; 2.1959	4.0485; 4.9374	9.0506; 10.7714
	$\{2\}$	1.7055; 1.7510	3.9035; 4.0428	8.7577; 8.9125
	$\{3\}$	1.6141; 1.6160	3.8666; 3.8486	8.7597; 8.9166
	$\{4\}$	1.6015; 1.6013	3.8070; 3.9977	8.7883; 8.9844
	$\{1, 2\}$	0.3588; 0.3771	0.9520; 1.0071	2.1303; 2.2341
	$\{1, 3\}$	0.3591; 0.3769	0.9514; 0.9946	2.1301; 2.2063
	$\{1, 4\}$	0.3592; 0.3766	0.9507; 0.9998	2.1305; 2.2175
	$\{2, 3\}$	0.3395; 0.3406	0.8994; 0.9049	2.0889; 2.1076
	$\{2, 4\}$	0.3394; 0.3401	0.8995; 0.9085	2.0888; 2.1204
	$\{3, 4]$	0.3400; 0.3400	0.8824; 0.8830	2.0752; 2.0906
	$\{1, 2, 3\}$	0.0434; 0.0436	0.1933; 0.1959	0.6014; 0.6120
	$\{1, 2, 4\}$	0.0434; 0.0436	0.1933; 0.1959	0.6015; 0.6112
	$\{1, 3, 4\}$	0.0434; 0.0436	0.1936; 0.1961	0.6046; 0.6150
	$\{2, 3, 4\}$	0.0393; 0.0393	0.1819; 0.1821	0.6213; 0.6248
	$\{1, 2, 3, 4\}$	0.0044; –	0.0219; –	0.1028; –

Table 2: Monte Carlo estimates of the proportion of false rejections in testing $H_0^{\{l_1,\dots,l_m\}}$ through the MAD conformity measure when $q^*_{T_{\{l_1,\dots,l_m\}}}(0.99)$ is used instead of $q^*_{T_{\{l_1,\dots,l_m\}}|\Psi_{\{1,\dots,k\}}(0.99)}(0.99)$, for $k = 1, 2, 3, 4$ and all possible index selections $l_1, \dots, l_m$ ($m = 1, \dots, k-1$). For each sample size, $B = 1,000,000$ independent replications are run under the NB model and only those in which $H_0^{\{1,\dots,k\}}$ is rejected at level 0.01 are retained.

k	$\{l_1, \dots, l_m\}$	$n = 20$	$n = 100$	$n = 500$
2	$\{1\}$	0.150	0.066	0.066
	$\{2\}$	0.056	0.084	0.084
3	$\{1\}$	0.187	0.105	0.028
	$\{2\}$	0.018	0.019	0.022
	$\{3\}$	0.014	0.016	0.023
	$\{1, 2\}$	0.356	0.085	0.051
	$\{1, 3\}$	0.316	0.080	0.050
	$\{2, 3\}$	0.023	0.039	0.068
4	$\{1\}$	0.193	0.154	0.105
	$\{2\}$	0.015	0.016	0.014
	$\{3\}$	0.011	0.009	0.013
	$\{4\}$	0.009	0.012	0.013
	$\{1, 2\}$	0.333	0.072	0.042
	$\{1, 3\}$	0.288	0.061	0.036
	$\{1, 4\}$	0.297	0.058	0.041
	$\{2, 3\}$	0.015	0.013	0.013
	$\{2, 4\}$	0.014	0.014	0.019
	$\{3, 4\}$	0.011	0.012	0.014
	$\{1, 2, 3\}$	0.791	0.413	0.090
	$\{1, 2, 4\}$	0.791	0.425	0.090
	$\{1, 3, 4\}$	0.664	0.370	0.082
	$\{2, 3, 4\}$	0.022	0.028	0.029

Table 3: Monte Carlo estimates of the empirical sizes of our test using the conditional quantile $q^*_{T_{\{l_1,\ldots,l_m\}}|\Psi_{\{1,\ldots,k\}}(0.99}(0.99)$ (Cond), of the serial gatekeeping (Gate) and the Bonferroni-adjusted (Bonf) tests of $T_{\{1\}}$ and $T_{\{2\}}$. The MAD conformity measure is adopted and the nominal test size is 0.01. The empirical sizes are computed by conditioning on rejection of $H_0^{\{1,2\}}$, based on the estimated quantile $q^*_{T_{\{1,2\}}}(0.99)$. $B = 1,000,000$ independent replications are taken for each value of n.

	$n = 20$			$n = 100$			$n = 500$		
	Cond	Gate	Bonf	Cond	Gate	Bonf	Cond	Gate	Bonf
$H_0^{\{1\}}$	0.010	0.093	0.094	0.011	0.041	0.042	0.010	0.042	0.042
$H_0^{\{2\}}$	0.010	0.035	0.036	0.009	0.054	0.055	0.010	0.051	0.052