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# Green and Fast DSL via Joint Processing of Multiple Lines and Time-Frequency Packed Modulation

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## Abstract

In this paper, strategies to enhance the performance, in terms of available data-rate per user, energy efficiency, and spectral efficiency, of current digital subscriber lines (DSL) lines are proposed. In particular, a system wherein a group of copper wires is jointly processed at both ends of the communication link is considered. For such a scenario, a resource allocation scheme aimed at energy efficiency maximization is proposed, and, moreover, time-frequency packed modulation schemes are investigated for increased spectral efficiency. Results show that a joint processing of even a limited number of wires at both ends of the communication links brings remarkable performance improvements with respect to the case of individual point-to-point DSL connections; moreover, the considered solution does represent a viable means to increase, in the short term, the data-rate of the wired access network, without an intensive (and expensive) deployment of optical links.

*Keywords:* xDSL systems, crosstalk, FEXT, NEXT, energy efficiency, joint processing, time-frequency packing, spectral efficiency.

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## 1. Introduction and review of related work

Residential broadband internet access is nowadays mainly based on DSL technology [1]. Indeed, although there is a wide agreement that the ultimate technology to increase the data rates of the access network is represented by optical networks, bringing a fiber in every house and/or in its close proximity – an approach called Fiber-to-the-Curb (FTTC) – is a long and extremely expensive process. This is the main reason why optical fibers are nowadays widely used in core networks, and for backhauling in wireless networks, but their use for the access network is proceeding at a limited pace. Copper based technologies (DSL, ADSL and ADSL2+)

continue to be the dominant technologies for broadband access, and in some countries the penetration rate is still low. In [2], a detailed report commissioned by the European Union (EU), shows that, for the 27 EU countries, in 2012 DSL penetration was at 92.9% , very high-speed DSL (VDSL) penetration was at 24.9%, while fiber penetration was at 12.3%. Similar numbers hold for non-EU countries, as detailed in [3]. As a consequence, although some notable exceptions exist (e.g., Japan), copper-based solutions will be the main and dominant broadband access technology still for many years.

Accordingly, in recent years, both academia and industry have been active in finding methods to boost the performance of DSL connections, especially in terms of achievable data-rates. Initially, asymmetric DSL (ADSL) connections had a data rate of few Mbit/s [4], but over the years they have improved and nowadays they are able to offer data rates that in some cases may be around 20Mbit/s (ADSL2+ standard), or larger [5]. Indeed, (VDSL and VDSL2 have been recently standardized and provide data rates up to 52 Mbit/s and 100 Mbit/s, respectively [6, 7]. In practice, however, the actual data rates achievable on a generic copper wire may be quite lower since they usually vary in a wide range, depending on the length of the copper wire, and on how much interference it receives from other spatially adjacent lines operating DSL connections as well. Indeed, due to the fact that, to save space, copper-wires are packed together in tight ducts, they interfere each other, a phenomenon known as crosstalk. This disturbance, along with impulsive noise, is the main performance limiting factor for DSL links. In order to overcome such limitations, the seminal paper [8] suggested to use coordinated transmission at the central office by means of precoding on the downstream and multiuser detection on the upstream, showing that impressive performance improvements are attainable especially in the presence of strong crosstalk such as in VDSL systems; the ideas of [8] can be now found in the recent ITU-T standard G.vector [9], which, as reported in [10], is capable of providing data-rates even larger than 100Mbit/s for circuit lengths up to 500 meters. In [11], distributed multiuser power control for DSL systems is considered; in particular, the problem of achievable rate maximization subject to a power spectral mask constraint is examined, assuming that a non-cooperative approach is taken. Conditions for existence and uniqueness of a Nash equilibrium are provided in the simplified setting of two interfering copper wires, while the general case of a larger number of interfering wires is studied only through numerical simulations. In [12], the considered objective function is the weighted sum-rate of a bunch of lines in the same binder, again with a constraint on the transmitted power spectral mask; since the optimal solution to the said problem has a computational complexity that is exponential in the number of lines, a suboptimal algorithm, with quadratic complexity, is proposed. The dual decomposition method is instead used in [13] to optimize transmit power spectra with

an affordable computational complexity: the obtained results are shown to largely outperform distributed power control policies such as iterative waterfilling. The concept of reference line is instead introduced in [14]: the reference line plays the role of a typical interfered line based on the statistics of the network, and each line is optimized in order to limit its harmful effects on the reference line. The proposed optimization strategy has a tractable complexity and in some cases approaches the optimal rates region. Other solutions to the problem of spectrum balancing in DSL networks can be also found in [15, 16, 17].

All of the above cited papers consider the case in which the lines departing from the central office (CO) are terminated at the customer premises, and they can be thus jointly managed only at one side of the communication links, since the other sides end in different locations. On the other hand, some papers in recent years have also considered the situation in which a bunch of lines departing from the CO and arriving in the same location (i.e., the basement of a building), can be jointly managed at both ends of the communication links [18, 19, 20], showing the advantages of such approach. In particular, the paper [18] designs joint transmit-receive linear processing schemes to minimize the transmit power subject to a quality-of-service constraint. It turns out that at the reception side linear minimum mean square error detection is performed, while waterfilling-like precoding is carried out at the transmitter side. The study [19], using methodologies from circuit theory, shows that for a copper DSL binder of 200 line connections the ultimate available shared bandwidth is on the order of 100Gbit/s, while the paper [20] shows that data-rates up to 1Gbit/s can be achieved through proper joint processing of four twisted pairs (category 3) over short distances (up to 300m).

This paper is focused on a scenario similar to that studied in [18, 19, 20]. As shown in Fig. 1, the lines departing from a CO and arriving in the same location are terminated in a common device, wherein the signals received in the downstream can be jointly processed, and resources (such as transmit power and line/carrier selection) for the upstream can be jointly allocated; this common device is finally connected to the end user's equipments through some technology, which might be for instance either a Gigabit Ethernet local area network, or single (and short) DSL connections. We believe that the potential of this architecture has not yet been fully understood by telecom operators. Indeed, considering that it will take several years, if not decades, to extensively deploy fibers in close proximity of the end users, and that this will be an extremely expensive task, the proposal to jointly manage bunches of copper wires arriving in the same location can guarantee to end users data-rates that are considerably larger than those achieved with current ADSL connections and with no expensive digging for fiber deployment. In particular, the structure of Fig. 1 has the following advantages:

- The customers whose lines are jointly managed might benefit from the statistical multiplexing gain, and could thus achieve data-rates much larger than that achieved with an ordinary DSL connection.
- Joint transmit/receive processing and smart resource allocation strategies might be employed, aimed at crosstalk mitigation and at improving the attainable data-rates.
- In low traffic conditions the transceivers of some copper lines might be switched off, thus permitting to achieve remarkable energy savings, an issue that these days has been receiving more and more attention (see for instance the recent papers [21, 22, 23] and references therein).
- In densely populated urban areas and in business districts not yet reached by optical fibers, implementing the considered scheme would permit having in each building data-pipes at rates around 1Gbit/s, and with no digging costs.

In this paper, thus, the following contributions are provided.

- A resource allocation algorithm, aimed at selecting the pairs (line, subcarrier) to be used for transmission, and the relative transmit power, will be proposed in order to increase the system energy efficiency.
- Motivated by the fact that the DSL access multiplexer (DSLAM) installed at the CO is no longer connected to the remote end users DSL modems, compliance to the standard DSL modulation formats is no longer required; as a consequence, the use of alternative modulation schemes, based on time-frequency packing, will be investigated, with the aim of achieving larger spectral efficiency as compared to classical modulation formats employing orthogonal carriers [24].

This paper is organized as follows. Next Section contains the system model, while Section 3 considers the problem of resource allocation aimed at maximizing the energy efficiency. Section 4 contains the design of a time-frequency packed linear modulation scheme for improved spectral efficiency. Numerical results are given in Section 5 while, finally, Section 6 wraps up the paper.

## 2. System Model

Consider the scenario of Fig. 1 and focus on a group of  $N$  lines departing from the DSLAM and arriving in the same physical location. According to the DSL standard, frequency-division duplexing (FDD) is used to separate uplink and

downlink transmission, so that the transmissions in the two opposite directions do not interfere each other. As a consequence, near-end crosstalk (NEXT) disappears and far-end crosstalk (FEXT) is the only source of disturbance.

We assume the use of a linear modulation with base pulse  $p(t)$  of duration  $T_p$  which is shifted of multiples of  $T$  in the time domain and of multiple of  $F$  in the frequency domain. Note that in the ADSL2 standard we have  $T = T_p$  and  $F = 1/T$ , so that signals transmitted on different carriers are orthogonal. For now, we instead use a more general signal model, that will turn out useful in the sequel of the paper (Section 4), when we will consider the use of non-standard modulation formats for increased spectral efficiency. The baseband equivalent of the transmitted signal on the  $N$  wires can be written as the following ( $N \times 1$ ) vector-valued function<sup>1</sup>:

$$\mathbf{x}(t) = \sqrt{E_s T F} \sum_m \sum_l \mathbf{x}_l(m) p(t - mT) e^{j2\pi l F t}, \quad (1)$$

where  $x_l^i(m)$  is the data-symbol transmitted on the  $i$ -th wire in the  $m$ -th symbol interval and on the  $l$ -th subcarrier, with  $E[|x_l^i(m)|^2] = p_l^i(m)$  ( $E[\cdot]$  denotes statistical expectation, and  $p_l^i(m)$  is the transmitted power associated to the  $i$ -th wire in the  $m$ -th symbol interval and on the  $l$ -th subcarrier).

The received signal can be modeled as the following vector-valued signal:

$$\mathbf{y}(t) = \sqrt{E_s T F} \sum_m \sum_l \mathbf{H}_l(m) \mathbf{x}_l(m) p(t - mT) e^{j2\pi l F t} + \mathbf{n}(t),$$

where  $\mathbf{H}_l(m)$  is the DSL channel matrix in the  $m$ -th time interval and on subcarrier  $l$ . The matrix  $\mathbf{H}_l(m) \in \mathbb{C}^{N \times N}$  contains the channel gains on tone  $l$ . Its  $(i, j)$ -th entry, say  $h_l^{(i,j)}(m)$ , is the complex gain of the channel from transmitter  $j$  to receiver  $i$ ; note that the diagonal elements of  $\mathbf{H}_l(m)$  contain the direct channels whilst the off-diagonal elements contain the crosstalk channels. This matrix is usually column-wise diagonally dominant. Due to the stationary nature of the copper wires, the channel is assumed to be constant in time. In the frequency domain, instead, it is considered flat over each subcarrier of bandwidth  $F$ , while it changes from subcarrier to subcarrier. Note that, due to FEXT, the signal observed on a given line contains contributions from the symbols transmitted on all the lines. Finally,  $\mathbf{n}(t)$  represents the thermal noise; its  $i$ -th entry is the additive noise received on the  $i$ -th wire, and is modeled as a circularly symmetric zero-mean white Gaussian noise with power spectral density (PSD)  $\sigma^2$ .

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<sup>1</sup>A similar model holds for the upstream signal too.

In order to convert the received signals to discrete-time, filters matched to the time-frequency shifted replicas of the base pulse are employed at the receiver side. We thus obtain, for the case of filters matched to the  $(n, k)$ -th time-frequency pair, the following test statistics:

$$\begin{aligned} \mathbf{y}_k(n) &= \int \mathbf{y}(t)p^*(t - nT)e^{-j2\pi kFt} dt = \\ &= \sqrt{E_sTF} \left[ \mathbf{H}_k(n)\mathbf{x}_k(n) + \sum_{m \neq n} \sum_{l \neq k} A_{m,l}(nT, kF)\mathbf{H}_l(m)\mathbf{x}_l(m) \right] + \mathbf{z}_k(n), \end{aligned} \quad (2)$$

where  $A_{m,l}(n, k) = \int p(t - mT)p^*(t - nT)e^{-j2\pi(k-l)Ft} dt$  is called *ambiguity function* and  $\mathbf{z}_{n,k} = \int p^*(t - nT)e^{-j2\pi kFt} \mathbf{n}(t) dt$ .

In order to detect the data vector  $\mathbf{x}_k(n)$ , a square processing window of length  $(2P+1)$  and  $(2L+1)$  in the time and frequency domain, respectively, is considered. Otherwise stated, the data vectors  $\mathbf{y}_{k'}(n')$ ,  $k' = k - L, k - L + 1, \dots, k + L$ ,  $n' = n - P, n - P + 1, \dots, n + P$  are jointly processed to detect the vector  $\mathbf{x}_k(n)$ . Upon straightforward mathematical manipulations, it can be shown that the above data vectors can be grouped in the following  $[(2L + 1)(2P + 1)N]$ -dimensional data vector:

$$\tilde{\mathbf{y}}_k(n) = \tilde{\mathbf{H}}_k(n)\mathbf{x}_k(n) + \sum_{m \neq n} \sum_{l \neq k} \tilde{\mathbf{H}}_l(m)\mathbf{x}_l(m) + \tilde{\mathbf{z}}_k(n), \quad (3)$$

where  $\tilde{\mathbf{H}}_k(n)$  is the matrix *signature* of the symbols contained in the vector  $\mathbf{x}_k(n)$ , and  $\tilde{\mathbf{z}}_k(n)$  collects the noise contribution.

For the case in which  $T = T_p = 1/F$ , intercarrier and intersymbol interference disappear. We thus obtain a classical orthogonal multicarrier modulation format, and the signal model in (2) simplifies to:

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{z}_k. \quad (4)$$

Note that in (4) we have assumed  $E_sTF = 1$ , and the time-index has disappeared too. Indeed, due to the cited stationarity assumptions, the statistical properties of the received signal are independent of time, and we can focus on a generic symbol interval with no loss of generality. Note that, in writing (4), we have explicitly neglected (a) the FEXT contribution from other lines departing from the same DSLAM and arriving in different locations; and, (b) the FEXT from possible additional lines arriving in the same physical location as the ones jointly processed. For the sake of simplicity, we assume that their contribution is included in the additive Gaussian noise term.

### 2.1. The full-duplex case

In this section we extend the signal model in (4) to the case in which a full-duplex transmission is used. Indeed, given the fact that the lines departing from the CO are no longer directly linked to the DSL modems at the customers' premises, it is possible to use modulation formats and frequency allocation schemes that are not compliant with the DSL standard. In the following, we will also consider a full duplex transmission wherein the entire available bandwidth is used both for the upstream and the downstream. In this scenario, the transmissions in the two opposite directions interfere each other and the NEXT term has to be taken into account as well. The discrete-time baseband equivalent of the downstream signal received on the  $N$  lines is, in this case, expressed as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{H}_k^{\text{NEXT}} \mathbf{x}_k^{\text{NEXT}} + \mathbf{n}_k. \quad (5)$$

In (5), the terms  $\mathbf{H}_k$ ,  $\mathbf{x}_k$ , and  $\mathbf{n}_k$  have the same meaning as in (4), the  $N$ -dimensional vector  $\mathbf{x}_k^{\text{NEXT}}$  represents the upstream data symbols on the  $k$ -th carrier, while, finally, the  $(N \times N)$ -dimensional matrix  $\mathbf{H}_k^{\text{NEXT}}$  contains the NEXT complex channel gains, its  $(i, j)$ -th entry describing the NEXT gain from the  $j$ -th line to the  $i$ -th line.

In the next Section, we will consider both (4) and (5), while in Section 4 we will revert to the more general signal model of Eq. (3).

## 3. Energy-efficient resource allocation

In this section, we propose a resource allocation algorithm aimed at minimizing the transmit power subject to a QoS constraint. Since we are jointly considering  $N$  copper wires, and assuming that on each wire there are  $C$  available carriers, there are a total of  $NC$  physical channels, wherein by physical channel we mean the pair (line, carrier). For each assigned data-rate that is to be transmitted through the  $N$  jointly processed lines, and for a target BER, we have thus to choose the physical channels to be used, and, also, the transmit power and the modulation cardinality on each chosen channel, so that we are able to support the required data-rate  $\tilde{R}$  with the required BER  $\tilde{\beta}$ .

Before giving a formal definition of the described optimization problem, we need to add further details on the modulation scheme and reception algorithm. We assume that, on each subcarrier, a QAM modulation scheme can be employed whose cardinality belongs to the set  $\mathcal{M} = \{4, 16, 64, 256, 1024, 4096, 16384\}$  (note that these numbers are compliant with the ADSL standard, wherein each carrier can be loaded with up to 15 bits in each signaling interval.). At the receiver side, a soft estimate of the transmitted data symbols is obtained through an



LMMSE receiver;<sup>2</sup> otherwise stated, assuming, with no loss of generality, that we are interested in  $x_k^j$ , namely the data symbol transmitted on the  $(j, k)$ -th physical channel ( $j$ -th line and  $k$ -th carrier), its soft-estimate, say  $\hat{x}_k^j$ , is formed as

$$\hat{x}_k^j = \left( \mathbf{d}_k^j \right)^H \mathbf{y}_k, \quad (6)$$

wherein  $(\cdot)^H$  denotes conjugate transpose, and

$$\mathbf{d}_k^j = p_k^j \left( \mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^H + \sigma^2 \mathbf{I}_N \right)^{-1} \mathbf{h}_k^j \quad (7)$$

is the LMMSE receive filter, where  $\mathbf{h}_k^j$  is the  $j$ -th column of the matrix  $\mathbf{H}_k$ , and  $\mathbf{P}_k$  is a  $(N \times N)$ -dimensional diagonal matrix whose  $(j, j)$ -th entry contains the transmitted power  $p_k^j$  on the  $k$ -th carrier of the  $j$ -th line. Using standard linear algebra techniques, it is easy to show that the corresponding Signal-to-Interference-plus-Noise Ratio (SINR),  $\gamma_k^j$  say, is written as

$$\gamma_k^j = p_k^j \left( \mathbf{H}_k^{-j} \right)^H \left( \mathbf{H}_k^{-j} \mathbf{P}_k^{-j} \left( \mathbf{H}_k^{-j} \right)^H + \sigma^2 \mathbf{I}_N \right)^{-1} \mathbf{H}_k^j, \quad (8)$$

wherein  $\mathbf{H}_k^{-j}$  and  $\mathbf{P}_k^{-j}$  can be obtained by eliminating the  $j$ -th column in the matrix  $\mathbf{H}_k$  and  $\mathbf{P}_k$ , respectively.

Let now  $p_{\max}$  be the maximum allowed transmit power on each physical channel, and define the system sum-power as the overall transmitted power, i.e. the sum of the powers transmitted on all the active physical channels. We aim at minimizing the system sum-power, subject to the following constraints:

$$\begin{cases} \text{supported rate} = \tilde{R}, \\ \text{BER} \leq \tilde{\beta}. \end{cases} \quad (9)$$

We assume here that the target data-rate  $\tilde{R}$  is an integer multiple of  $2/T$ , with  $T$  the inverse of the bandwidth of each physical channel<sup>3</sup>. The minimization of the sum-power is made with respect to the choice of the physical channels and of the transmit power (to be taken not larger than  $p_{\max}$ ), and cardinality of the modulation

<sup>2</sup>We are considering a linear receiver for the sake of simplicity, but more elaborate receivers might also be conceived. For instance, the use of soft-input-soft-output detection algorithms is a straightforward option that brings also remarkable performance improvements [25].

<sup>3</sup>Such a bandwidth equals the carrier spacing and in the ADSL standard is in turn equal to 4.3125 kHz, so we are assuming that  $\tilde{R}$  can be increased in steps of about 8 kbit/s.

(to be taken in the set  $\mathcal{M}$ ) on each chosen active channel. Following [26], the following bound can be used for the BER of an  $M$ -QAM modulation system:

$$BER \leq 2 \exp\left(\frac{-1.5\eta\gamma}{M-1}\right), \quad M \geq 4, \quad (10)$$

with  $\gamma$  the Signal-to-Noise Ratio (SNR) and  $\eta$  the coding gain<sup>4</sup>. Although expression (10) holds for the AWGN channel, it is a reasonable choice also for the case in which there is co-channel interference and an LMMSE receiver is adopted. We will thus use (10) with  $\gamma$  replaced by the SINR defined in (8). Accordingly, the BER constraint in (9) can be replaced by the following constraint on the received SINR:

$$\text{SINR} \geq \frac{M-1}{1.5\eta} \ln\left(\frac{1}{\beta}\right). \quad (11)$$

Now, solution of the considered problem would require in principle implementation of the following steps:

1. Given the target data-rate  $\tilde{R}$ , let  $\tilde{q} = \tilde{R}T$  be the number of bits that are to be transmitted in the signaling interval  $T$  (recall that  $\tilde{q}$  is an integer);
2. For each possible allocation of the  $\tilde{q}$  bits on the physical channels, compute the transmitted powers needed to achieve the target SINR<sup>5</sup>;
3. Choose, among all the possible allocations, the one corresponding to the minimum system sum-power.

Regarding step 2), it might also happen that, for any possible allocation of the  $\tilde{q}$  bits, it is not possible to find a set of transmit powers in the range  $[0, p_{\max}]$  such that the target SINR is achieved; in this case, the considered problem is unfeasible and we have to either reduce the desired rate (i.e.,  $\tilde{q}$ ), or accept that the BER target is not met.

Clearly, the computational complexity of this optimal procedure is prohibitive, given the fact that an exhaustive search over all possible allocations of the  $\tilde{q}$  bits among the available physical channels, along with the choice of the relative modulation cardinality is a combinatorial problem. In what follows we thus propose a simple suboptimal algorithm, which, despite its simplicity, will be shown to achieve good performance results. The algorithm is sequential, in the sense that

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<sup>4</sup>Of course more appropriate BER approximations could be used to model the effect of coding; however, the main goal here is to show that a joint resource management of the lines yields better performance than their separate use, independently from the type of channel code used.

<sup>5</sup>Such a task can be easily performed through an iterative procedure as described in [27], or a batch procedure as in [28].

data bits are sequentially allocated to the physical channels, in groups of two bits each. Given  $\tilde{q} = 2$ , we start by choosing the best physical channel, i.e., we choose the pair (line, carrier) with the largest channel gain, and allocate two bits on this channel; otherwise stated, we start by choosing line  $i$  and carrier  $k$  if:

$$\|\mathbf{h}_k^i\| \geq \max_{i' \neq i, k' \neq k} \|\mathbf{h}_{k'}^{i'}\|. \quad (12)$$

Then, we have to allocate the next group of two bits. We have the following choices:

- a) We can use the already active  $(i, k)$ -th physical channel by switching from a QPSK to a 16-QAM modulation;
- b) We can use one of the remaining  $NC - 1$  physical channels by using a QPSK modulation;

We evaluate the sum-power needed to reach the target SINR for configuration a) and for the  $NC - 1$  configurations of choice b), and choose the configuration with the smallest sum-power. Note that at this step we have thus a complexity linear in  $NC$ , the number of available physical channels. The computational complexity can be made even smaller if, in performing this step, we neglect interference, i.e., we choose the most convenient configuration nulling the crosstalk contribution in the received SINR.<sup>6</sup> Now that we have allocated the first 4 bits we can proceed in a similar way to allocate the next group of 2 bits. In general, at the generic step of the interval we will have a certain number, say  $\Gamma$ , of active physical channels (and each active channel will be using a modulation with a certain cardinality), and  $NC - \Gamma$  empty physical channels. To proceed, we have thus to allocate additional two bits and, again, this can be done either by multiplying by 4 the cardinality of the modulation on one of the channels already in use, or turning on a new physical channel with a QPSK modulation. Of course, the solution corresponding to the minimum sum-power will be taken. In the simplified form of the algorithm, we have just to compare the best (i.e., with the largest channel coefficient) unused channel with the best active channels for each modulation cardinality in the set  $\mathcal{M}$ . The following remarks can be now done. First of all, since at each step we have to choose among  $NC$  different configurations, and since the number of steps is  $\tilde{q}/2$ , we have that the overall complexity of the proposed algorithm scales linearly with  $N$ ,  $C$  and  $\tilde{q}$ . The computational complexity savings are based on the sequential nature of the algorithm: bits are allocated in groups of two, and at each allocation

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<sup>6</sup>Indeed in this case we have to compare configuration a) with only one configuration b), i.e. the one with the largest channel coefficient.

the channels already in use cannot be dismissed, they can only be upgraded to a modulation with larger cardinality. Finally, it is worth noting that, of course, it might also happen that, for too large target data-rates, the system is not able to meet the required target SINR. This occurrence is detected by the fact that, at a given step, in any of the possible allocations of the two additional bits, the target SINR cannot be reached at least for one active physical channel.<sup>7</sup>

Although alternative suboptimal algorithm may be obtained, in the following numerical results will show that the proposed resource allocation algorithm exhibits excellent performance.

### 3.1. The full-duplex case

As stated previously, many DSL standards employ FDD in order to avoid near-end crosstalk (NEXT). In the scenario of Fig. 1, instead, NEXT cancellation strategies may be employed and the entire available bandwidth can be used both for upstream and downstream communications. Referring to the signal model in (5), since the downstream receiver has obviously knowledge of the data vector  $\mathbf{x}_k^{\text{NEXT}}$ , NEXT can be cancelled by considering the modified observables:

$$\begin{aligned}\tilde{\mathbf{y}}_k &= \mathbf{y}_k - \hat{\mathbf{H}}_k^{\text{NEXT}} \mathbf{x}_k^{\text{NEXT}} \\ &= \mathbf{H}_k \mathbf{x}_k + (\mathbf{H}_k^{\text{NEXT}} - \hat{\mathbf{H}}_k^{\text{NEXT}}) \mathbf{x}_k^{\text{NEXT}} + \mathbf{n}_k.\end{aligned}\tag{13}$$

In the above equation,  $\hat{\mathbf{H}}_k^{\text{NEXT}}$  is an estimate of the NEXT channel matrix, that can be formed by using pilot symbols transmitted by the downstream device when the DSLAM located in the CO is silent. As an example, using a least-squares approach [29], such an estimate is obtained as

$$\hat{\mathbf{H}}_k^{\text{NEXT}} = \mathbf{Y}_k \mathbf{E}^\dagger,\tag{14}$$

with  $\mathbf{E}$  the  $N \times n_t$  matrix of the pilot symbols,  $(\cdot)^\dagger$  denoting matrix pseudo-inversion,  $n_t \geq N$  the number of symbol intervals devoted to training, and  $\mathbf{Y}_k$  is an  $(N \times n_t)$ -dimensional matrix containing the downstream received data when the downstream transmitter sends the pilot symbols upstream. Based now on the modified observables (13), an LMMSE receiver may be used as in (6), a modified expression for the SINR can be obtained, and, finally, the proposed resource allocation algorithm may be now run using the entire available bandwidth of the copper wires. For the sake of brevity we do not give further analytical details on

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<sup>7</sup>In this case we are dealing with an unfeasible power control problem; details on this can be found in the papers [27, 28].

this approach, and we just will show some performance results showing what happens when NEXT is cancelled and full-duplex transmission is considered. As a final remark, we note that in a standard ADSL system wherein lines are individually managed, NEXT cancellation may happen only in the CO DSLAM, and not in the proximity of the end users.

#### 4. Time-frequency Packing for Improved Spectral Efficiency

In the previous section we have shown that a joint processing of copper wires at both ends of the link enables the adoption of smart resource allocation procedures aimed at energy efficiency maximization; such a joint processing also permits, alternatively to energy efficiency maximization, rate maximization, an approach however that we have not been considering in this paper for the sake of brevity. It is worth noting that although the resource allocation algorithm of the previous section has been obtained by using a signal model representative of the ADSL2 standard, there is no longer any need to stick to the ADSL2 standard signal model. Otherwise stated, since the lines departing from the CO do not terminate in the ADSL modems of the remote customers, but they are jointly managed by an intermediate device in the close proximity of the end users, alternative modulation schemes might be used in order to improve the system performance. In this section, we thus refer to the signal model in Eq. (3), and, inspired by [24], we investigate on time-frequency packed modulations aiming at systems with increased spectral efficiency.

Indeed, although orthogonal signals with Gaussian inputs achieve capacity on the additive white Gaussian channel, in [24] it is shown that, when finite order modulations are considered, the efficiency of the communication system can be improved by giving up the orthogonality condition, even when a simple symbol by symbol receiver is used. Notably, even though the spacing among subcarriers is smaller than the reciprocal of the signaling interval, the modulator and the demodulator can be still implemented by using Fast Fourier Transforms (FFTs) [30]. While [24] considered a single-user transmission link (i.e., a system impaired by the thermal noise only), in the following we extend the concepts developed in [24] to the considered multiuser scenario, wherein FEXT disturbance exists, including the adoption of an extended window LMMSE receiver.

For the sake of simplicity, in the following we will restrict our attention to the case in which QPSK modulation is used on all the carriers; of course extension to the case of adaptive cardinality of the modulation can be done with ordinary efforts. We also assume that the data-symbols have unit-energy, so that the transmitted signal power is ruled by the parameter  $E_s$ . The signal in (3) is passed through an LMMSE filter. Denoting by  $\mathbf{v}_k^i(n) \in \mathbb{C}^{(2L+1)(2P+1)N}$  the LMMSE filter used to detect the symbol  $x_k^i(n)$ , and by  $\tilde{\mathbf{h}}_k^i(n)$  the  $i$ -th column of the matrix  $\tilde{\mathbf{H}}_k(n)$ , the

estimated symbol, say  $\hat{x}_k^i(n)$ , is written as:

$$\begin{aligned}\hat{x}_k^i(n) &= \mathbf{v}_k^i(n)^H \tilde{\mathbf{y}}_k(n) = \\ &= \sqrt{E_s T F} \mathbf{v}_k^i(n)^H \left[ \tilde{\mathbf{h}}_k^i(n) x_k^i(n) + \left( \sum_{j \neq i} \tilde{\mathbf{h}}_k^j(n) x_k^j(n) + \right. \right. \\ &\quad \left. \left. + \sum_{m \neq n} \sum_{l \neq k} \tilde{\mathbf{H}}_l(m) \mathbf{x}_l(m) + \tilde{\mathbf{z}}_k(n) \right) \right],\end{aligned}\quad (15)$$

where

$$\mathbf{v}_k^i(n) = \frac{\mathbf{B}_k^i(n)^{-1} \tilde{\mathbf{h}}_k^i(n)}{(E_s T F)^{-1} + \tilde{\mathbf{h}}_k^i(n)^H \mathbf{B}_k^i(n)^{-1} \tilde{\mathbf{h}}_k^i(n)}, \quad (16)$$

and

$$\mathbf{B}_k^i(n) = \mathbf{C}_{\tilde{\mathbf{z}}_k} + E_s T F \sum_{j \neq i} \tilde{\mathbf{h}}_k^j(n) \tilde{\mathbf{h}}_k^j(n)^H + E_s T F \sum_{m \neq n} \sum_{l \neq k} \tilde{\mathbf{H}}_l(m) \tilde{\mathbf{H}}_l(m)^H. \quad (17)$$

is the noise plus interference covariance matrix, with  $\mathbf{C}_{\tilde{\mathbf{z}}_k}$  the covariance matrix of the vector  $\tilde{\mathbf{z}}_k(n)$ .

Now, we are willing to have a reliable approximation of the spectral efficiency of a communication system whose output is given by (15). We will now use an information-theoretic technique that permits obtaining the achievable spectral efficiency (ASE), which represents an upper bound that can be approached through the use of a channel code performing close to the Shannon limit. Although, thus, ASE cannot be really attained by any practical system, strategies are available to approach it quite closely, so it represents a valid performance metric.

In keeping with [24] and, in turn, [31], instead of simply neglecting the interference terms in (15) due to adjacent terms in time and frequency, we model such interference as an additional zero-mean Gaussian disturbance. Then, the PSD of the interference plus the noise can be expressed as follows:

$$N_I = E_s T F \left( \frac{1}{(E_s T F)^{-1} + \tilde{\mathbf{h}}_k^i(n)^H \mathbf{B}_k^i(n)^{-1} \tilde{\mathbf{h}}_k^i(n)} \right)^2 \tilde{\mathbf{h}}_k^i(n)^H \mathbf{B}_k^i(n)^{-1} \tilde{\mathbf{h}}_k^i(n). \quad (18)$$

Otherwise stated, we introduce the following auxiliary channel model

$$\hat{x}_k^i(n) = \sqrt{E_s T F} \mathbf{v}_k^i(n)^H \tilde{\mathbf{h}}_k^i(n) x_k^i(n) + w_k^i(n), \quad (19)$$

wherein  $w_k^i(n)$  is assumed to be a zero-mean Gaussian random variate with variance  $N_I$ . We are now interested in evaluating the ultimate performance limits (in

terms of spectral efficiency) when using a symbol-by-symbol receiver designed for the auxiliary channel model (19) when the actual channel model is the one in (15). This is an instance of mismatched detection and the achievable information rate in bits per channel use can be shown to be expressed as [31]:

$$I(x_k^i(n); \hat{x}_k^i(n)) = \mathbb{E}_{x_k^i(n); \hat{x}_k^i(n)} \left[ \log_2 \left( \frac{M p_{\hat{X}_k^i(n)|X_k^i(n)}(\hat{x}_k^i(n)|x_k^i(n))}{\sum_x p_{\hat{X}_k^i(n)|X_k^i(n)}(\hat{x}_k^i(n)|a)} \right) \right], \quad (20)$$

where  $p_{\hat{X}_k^i(n)|X_k^i(n)}(\hat{x}_k^i(n)|x_k^i(n))$  is a Gaussian probability density function (pdf) with mean  $\sqrt{E_s T F} \mathbf{v}_k^i(n)^H \tilde{\mathbf{h}}_k^i(n) x_k^i(n)$ , and variance  $N_I$ , while the outer statistical average, with respect to  $x_k^i(n)$  and  $\hat{x}_k^i(n)$ , is carried out according to the real channel model of (15), [24, 31]. Note that (20), which can be computed via Monte-carlo simulations, represents the information rate on the  $k$ -th subcarrier of the  $i$ -th line in the  $n$ -th signaling interval; due to the frequency selectivity of the channel, the average information rate of the  $i$ -th line is obtained as

$$\text{AIR} = \frac{1}{C} \sum_k I(x_k^i(n); \hat{x}_k^i(n)), \quad (21)$$

where, we recall,  $C$  is the number of subcarriers. Given (21), the ASE, measured in bit/s/Hz, is finally expressed as

$$\text{ASE} = \frac{\text{AIR}}{FT}, \quad (22)$$

which is to be now maximized with respect to  $T$  and  $F$ .

Similar reasoning can be used to evaluate the spectral efficiency in the case in which full-duplex transmission over the entire line bandwidth is adopted. Mathematical details are however omitted for the sake of brevity.

## 5. Simulation results

In this section, we discuss some numerical results giving evidence of the merits of the architecture illustrated in Fig. 1, as well as showing the performance improvements that can be obtained with respect to standard ADSL connections wherein lines are treated separately. We will refer to papers [32], [33], and [34] to model the channel matrices  $\mathbf{H}_k$  and  $\mathbf{H}_k^{\text{NEXT}}$ .

We begin with the resource allocation algorithm proposed in Section 3. We will refer here to the simplified version of the proposed resource allocation algorithm. In Fig. 2 we report the curve rate versus sum-power for the proposed solution, and for comparison purposes, we also show a curve corresponding to the case in

which ADSL lines are treated separately, and the rate is randomly split among them. It has been assumed that  $N = 20$  lines are jointly processed, the number of subcarriers per line is  $C = 480$  (this number is compliant with the ADSL2 standard), the target error probability is  $\tilde{\beta} = 10^{-6}$ , and a coding gain  $\eta = 3\text{dB}$  has been assumed. The simulated loop length is 200m. While in an underloaded scenario the two solutions exhibit the same performance, it is clearly seen that as the requested data-rate increases, our solution is capable of delivering, for a fixed amount of transmit power, larger data-rates, or, equivalently, for a given delivered data-rate, our solution requires smaller values of transmit power. Results also show that our solution extends the maximum deliverable data-rate with the required QoS constraint, whereas the curve corresponding to the plain ADSL2 standard stops at about 440Mbit/s.

Consider now the use of time-frequency packed modulations. Fig. 3 depicts the ASE versus the time spacing  $T$  and the frequency spacing  $F$  with  $10 \log_{10} (E_s/\sigma^2) = 70 \text{ dB}$ . There are several pairs  $(T, F)$  allowing to obtain an ASE larger than that achieved by the orthogonal signaling ( $F = 4000$  and  $T = T_p$ ), and it is seen that gains about 20% may be achieved. The figure was obtained for a DSL link of 800m length, and with a unit-length processing window, both in time and frequency (i.e.,  $L = P = 0$ ). Fig. 4 represents the same data as Fig. 3 for the case in which  $10 \log_{10} (E_s/\sigma^2) = 30 \text{ dB}$ . Both figures assume a QPSK modulation scheme and the number of lines is  $N = 25$ . The time and frequency spacing that optimize the ASE are  $(T = T_p, F \simeq 3100\text{Hz})$  and  $(T \simeq 2.375 \cdot 10^{-4}\text{s}, F \simeq 3550\text{Hz})$  Hz for Fig. 3 and 4, respectively. It is thus clear that the optimal values of spacing depend on the value of the received SNR. Note also that larger values of the ASE can be obtained by considering a larger time-frequency processing window, i.e., by choosing  $L > 0$  and  $P > 0$ . The results so far shown have not considered the use of full-duplex. Let us assume now to cancel the NEXT contribution and let us assess its impact on the ASE. In Table 1, we report, for some loops lengths, the optimal values of the time-frequency spacings, the ASE when FDD and orthogonal signaling is used (referred to as ASE1), the ASE when FDD and time-frequency packing is used (referred to as ASE2), and, finally, the ASE when full-duplex transmission, NEXT cancellation and time-frequency packing is used (referred to as ASE3). The results clearly show the benefit of NEXT cancellation, the overall ASE is almost doubled with respect to the case in which FDD is employed. Fig. 5, additionally, shows the ASE versus the SNR for fixed values of  $T$  and  $F$ . In particular, we have considered the values of  $T$  and  $F$  which are optimal for the case in which SNR=70dB, and have plotted the ASE versus the SNR. Results show that time-frequency packing continues to exhibit an ASE larger than that of OFDM for a wide range of SNR values. It is finally worth recalling that the above results have been obtained under the assumption that an



LMMSE receiver is used at the receiver; as shown in [35], even larger values of ASE can be however obtained by using non-linear receivers based, e.g., on trellis processing.

## 6. Conclusions

This paper has been focused on the investigation of the advantages that a joint processing of copper wires at both ends of the communication link may grant. It has been shown that the architecture depicted in Fig. 1 enables the use of resource allocation procedures taking advantage from the structure of the crosstalk, and, also, paves the way to the use of modulation schemes better than OFDM in terms of spectral efficiency (which is more than doubled in the full-duplex case with echo cancellation). Although, as already discussed, copper lines cannot compete with fiber-based optical channels, the solutions proposed in this paper may be useful in the short-to-medium term, while we wait for a thorough deployment of the fiber, as well as in developing countries wherein optical fibers are not yet at the horizon.

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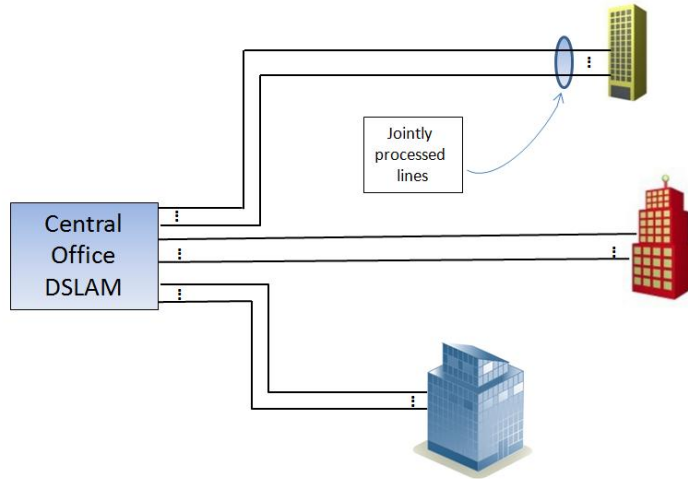


Figure 1: Joint processing of copper wires at both sides of the last mile.

<b>Loop length [m]</b>	800	1600	2400	3200
<b>optimal T [s]</b>	$2.500 \cdot 10^{-4}$	$2.500 \cdot 10^{-4}$	$2.500 \cdot 10^{-4}$	$2.375 \cdot 10^{-4}$
<b>optimal F [Hz]</b>	3100	3100	3100	3100
<b>ASE 1 [bit/s/Hz]</b>	2.0000	1.9997	1.1716	0.6039
<b>ASE 2 [bit/s/Hz]</b>	2.3968	2.3364	1.3041	0.6786
<b>ASE 3 [bit/s/Hz]</b>	4.7445	4.5523	2.3813	1.2427

Table 1: ASE 1: ASE when FDD and OFDM are used. ASE 2: ASE when FDD and the optimal  $T$  and  $F$  values are considered. ASE 3: ASE when there is no FDD, the optimal  $T$  and  $F$  values are used and the NEXT cancellation is performed.

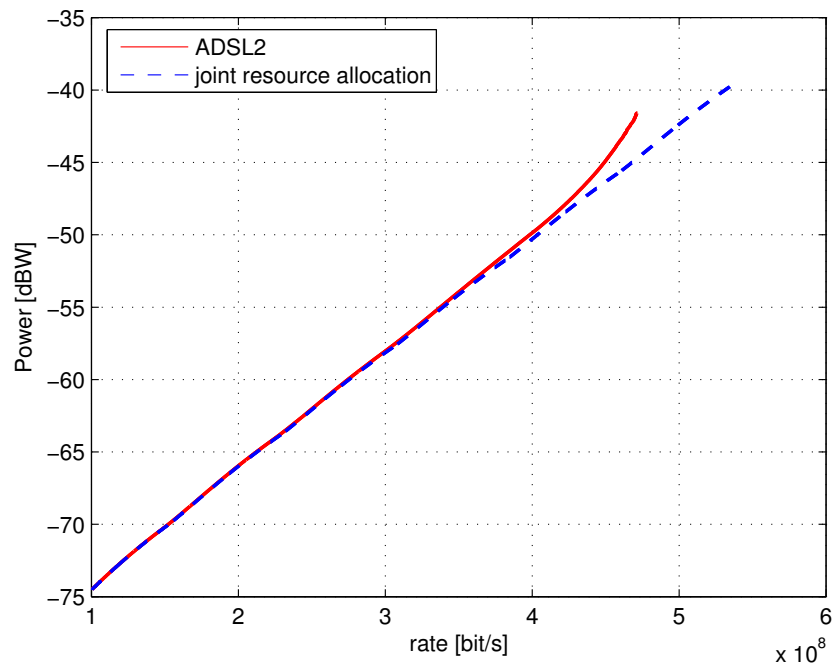


Figure 2: Total power consumed by the system versus the rate. The total number of possible active users is 20.

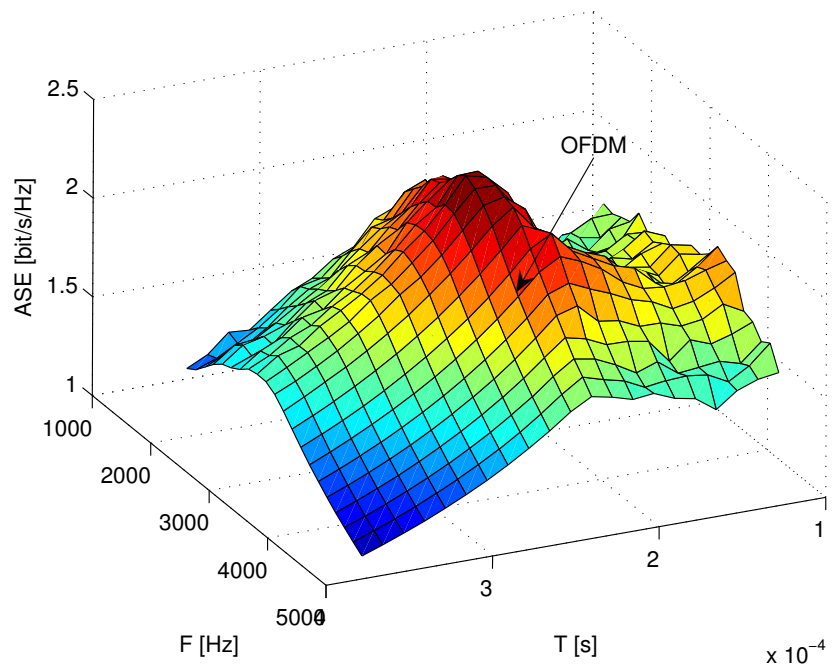


Figure 3: ASE versus  $T$  and  $F$ .  $10 \log_{10}(E_s/\sigma^2) = 70$  dB. Loop lengths: 800m.  $N = 25$ .

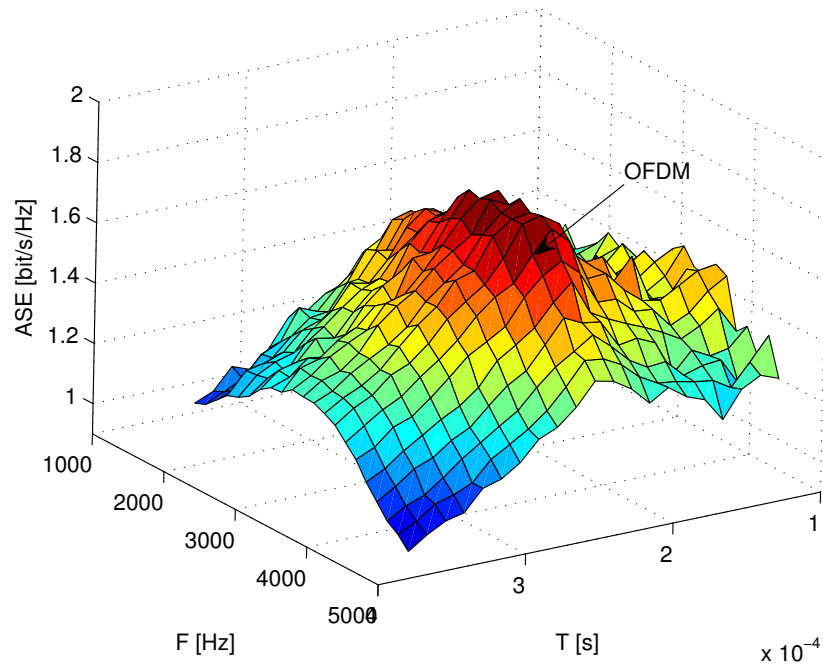


Figure 4: ASE versus  $T$  and  $F$ .  $10 \log_{10}(E_s/\sigma^2) = 30$  dB. Loop lengths: 800m.  $N = 25$ .



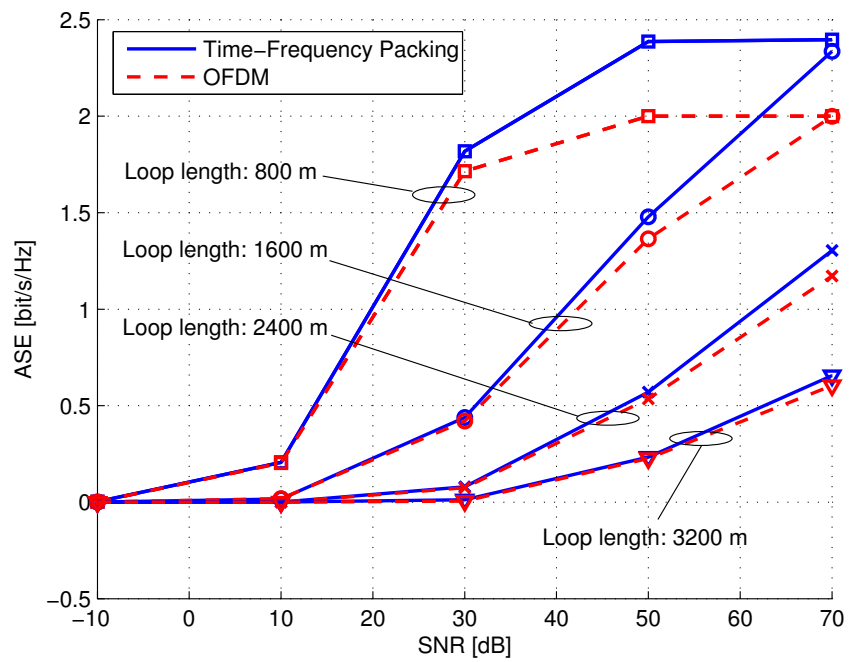


Figure 5: ASE versus SNR with  $T = 2.5$  ms and  $F = 3100$  Hz and for several loop lengths.  $N = 25$ .