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On the Use of Multiple Satellites to Improve the Spectral Efficiency of Broadcast Transmissions

Alessandro Ugolini, Amina Piemontese, Andrea Modenini, Giulio Colavolpe

DII, University of Parma, Italy, email: {alessandro.ugolini,amina.piemontese,andrea.modenini,giulio}@unipr.it

Abstract—We consider the use of multiple co-located satellites to improve the spectral efficiency of broadcast transmissions. In particular, we assume that two satellites transmit on overlapping geographical coverage areas, with overlapping frequencies. We first describe the theoretical framework based on network information theory and, in particular, on the theory for multiple access channels. The application to different scenarios will be then considered, including the bandlimited additive white Gaussian noise channel with average power constraint and different models for the nonlinear satellite channel. The comparison with the adoption of frequency division multiplexing (FDM) is also provided. The main conclusion is that a strategy based on overlapped signals is convenient with respect to FDM, although it requires the adoption of a multiuser detection strategy at the receiver.

I. INTRODUCTION

In today's satellite communication systems, the scarcity of frequency spectrum and the ever growing demand for data throughput has increased the need for resource sharing. In recent years, users of professional broadcast applications such as content contribution, distribution, and professional data services have demanded more spectrally efficient solutions.

Satellite service providers often have the availability of *co-located* satellites: two (or more) satellites are said to be co-located when, from a receiver on Earth, they appear to occupy the same orbital position. Co-location of satellites is typically used to cover the fully available spectrum by activating transponders on different satellites that cover non-overlapping frequencies or as a stand-by in-orbit redundancy, when the backup satellite is activated in case of failure of the main satellite. However, the second satellite can also be exploited to try to increase the capacity of the communications link.

In this paper, we address a scenario in which the backup satellite is activated in addition to the main one, to improve the spectral efficiency (SE) of the overall communication system. Time alignment between the signals transmitted by the two satellites is not possible and this prevents the possibility to use a few techniques available for a scenario where, for each user, the signals from the two transmitters are perfectly aligned in time [1]. Moreover, channel state information is not available at the transmitter. Here, we study the information rate achievable by a system where the two satellites transmit on overlapping geographical coverage areas with overlapping frequencies, and compare our results with that achievable by the frequency division multiplexing (FDM) strategy.

The two-satellites scenario has been studied in [2], [3], where the satellite channel is approximated as a linear additive white Gaussian noise (AWGN) channel, and the information theoretic analysis has been carried out under the limiting

assumption of Gaussian inputs. We instead examine three different models for the system: the linear AWGN channel, the peak-power-limited AWGN channel [4]–[6], and the DVB-S2 satellite channel [7]. The studied system is an instance of broadcast channel [8]–[10] with multiple transmitters. However, we are interested in a scenario in which the same information must be sent to every receiver. This situation corresponds, for example, to the delivery of a TV broadcast channel. We show that all these scenarios can be analyzed by means of network information theory and we will resort to multiple access channels (MACs) [8], [11].

Our analysis reveals that, if we allow multiuser detection, the strategy based on overlapping signals achieves higher SEs with respect to that achievable by using FDM. Interestingly, we show that there are cases in which a single satellite can outperform both multiple satellites strategies.

The remainder of this paper is structured as follows: in Section II we present a general system model valid for all cases, and in Section III we briefly review the theory of MACs. In Section IV we discuss the achievable rates by FDM. In Sections V, VI, and VII we analyze the three different scenarios and Section VIII concludes the paper.

II. SYSTEM MODEL

Figure 1 depicts a schematic view of the baseband model we are considering. A single operator properly sends two separate data streams to the two satellites. In such a way, the impact of the feeder uplink interference is negligible in this scenario. Data streams are linearly modulated signals, expressed as

$$x_i(t) = \sum_k x_k^{(i)} p(t - kT) \quad i = 1, 2, \quad (1)$$

where $x_k^{(i)}$ is the k -th symbol transmitted on data stream i , $p(t)$ is the shaping pulse, and T is the symbol time.

Each satellite then relays the signal, denoted as $s_i(t)$, to several users scattered in its coverage area. For each user, then, the received signal is the sum of the two signals coming from the satellites, with a possible power unbalance γ^2 due to different path attenuations (we assume $1/2 \leq \gamma \leq 1$). Without loss of generality, we assume that the attenuated signal is $s_2(t)$, otherwise we can exchange the role of the two satellites. The received signal is also affected by an AWGN process $w(t)$ with power spectral density N_0 . Hence, the received signal has the following expression

$$y(t) = s_1(t) + \gamma e^{j\phi(t)} s_2(t) + w(t), \quad (2)$$

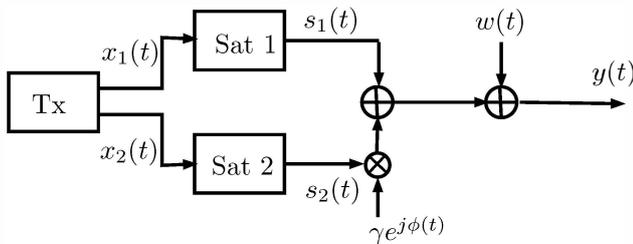


Figure 1. Block diagram of the analyzed system.

where $s_1(t)$ and $s_2(t)$ are the signals at the output of the two satellites, and $\phi(t)$ is a possible phase noise process, assumed to be perfectly known at the receiver. Signals $s_1(t)$ and $s_2(t)$ are transmitted with overlapping frequencies, and the overall signal has bandwidth W . Since we are analyzing a broadcast scenario in which different receivers experience different (and unknown) levels of power unbalance, we impose that the two satellites transmit with the same rate. This constraint will be better clarified in the next sections.

A simple alternative strategy to overlapping frequencies, that allows to avoid interference between the two transmitted signals, is FDM. The bandwidth W is divided into two equal subbands assigned to the different satellites.¹ In this case, the received signal has expression

$$y(t) = s_1(t)e^{j\pi f_c t} + \gamma e^{-j\pi f_c t + j\phi(t)} s_2(t) + w(t), \quad (3)$$

where f_c is the frequency separation between the two signals.

Channel state information is not available at the transmitter and no cooperation among the users is allowed. This is because, as mentioned, the target is on broadcasting applications.

The transmission from the two satellites can be coordinated, but through simple geometrical considerations it can be easily shown that even with a coverage area of a few tens of kilometers and two co-located signals separated in angle by a fraction of degree, time alignment is not possible. In other words, if the signals from the two satellites come perfectly aligned at a given receiver in the area, there will be other receivers for which a misalignment of a few symbols (usually more than ten) is observed. On the other hand, our information-theoretic analysis does not depend on the time alignment of the two signals, and hence we will assume synchronous users to simplify the study.

We are interested in maximizing the overall SE of the system, defined as

$$SE = \frac{I}{TW} \quad [\text{bit/s/Hz}],$$

where I is the maximum mutual information of the channel. However, since in the scenario of interest the values of T and W are fixed, without loss of generality we will assume that $TW = 1$ and we will refer to the terms information rate and SE interchangeably.

¹An unequal subband allocation does not make sense since the power unbalance is different for different receivers in the coverage area and, in any case, unknown to the transmitter.

III. MULTIPLE ACCESS CHANNELS

In this section, we review some results on classical MACs [8]. We consider the transmission of independent signals from the two satellites. We denote by R_1 the SE of the first satellite and by R_2 that of the second satellite. At this point, we make no assumptions on the channel inputs, since a better characterization of the input distributions is presented in the next sections. However, independently of the form assumed by the input distribution, the boundaries of the SE region can be expressed, for each fixed signal-to-noise power ratio, as [8]

$$R_1 \leq I(x_1; y|x_2) \triangleq I_1$$

$$R_2 \leq I(x_2; y|x_1) \triangleq I_2$$

$$R_1 + R_2 \leq I(x_1, x_2; y) \triangleq I_J,$$

where $I(x_1; y|x_2)$, $I(x_2; y|x_1)$ and $I(x_1, x_2; y)$ represent, respectively, the mutual information between x_1 and y conditioned to x_2 , that between x_2 and y conditioned to x_1 and that between the couple (x_1, x_2) and y ; we omit the dependence on t and we adopt definitions I_1 , I_2 , and I_J to simplify the notation.

Figure 2 is useful to gain a better understanding of the behavior of SE regions. Point D corresponds to the maximum achievable SE from satellite 1 to the receiver when satellite 2 is not sending any information. Point C corresponds to the maximum rate at which satellite 2 can transmit as long as satellite 1 transmits at its maximum rate.² The maximum of the sum of the SEs, however, is obtained on points of the segment B-C; these points can be achieved by joint decoding for both users. It does not make sense to adopt different rates for the two satellites, since each satellite ignores whether its signal will be attenuated or not and this attenuation will vary for different receivers. As a consequence, the only boundary point of the SE region we can achieve is point E which lies on the line $R_1 = R_2$. We define $I_{J,p}$ the pragmatic sum of rates corresponding to point E: it is easy to see, through graphical considerations, that

$$I_{J,p} = \min(I_J, 2I_2).$$

Point F is the intersection between the capacity region and line $R_2 = -R_1 + I_1$ and it corresponds to a sum-rate equal to I_1 . The position of point E depends on the power and on the power unbalance. Depending on these two values, E can be found in different positions: in particular, if E lies on the left of F, we can notice that $I_{J,p} < I_1$, hence it is convenient to use a single satellite with rate I_1 rather than activating the second satellite.

IV. ACHIEVABLE RATES BY FREQUENCY DIVISION MULTIPLEXING

Since the two signals transmitted by the FDM model (3) operate on disjoint bandwidths, they are independent and the information rate achievable by this system is equal, in case $\gamma = 1$, to that of a single transmitter with double signal-to-noise power ratio. We define by I_{FDM} the achievable rate by FDM, and by $I_{\text{FDM},p}$ that achievable by FDM under the equal rate constraint. The latter is clearly equal to twice the

²If we exchange the role of the two satellites, the same considerations hold for points A and B instead of D and C.

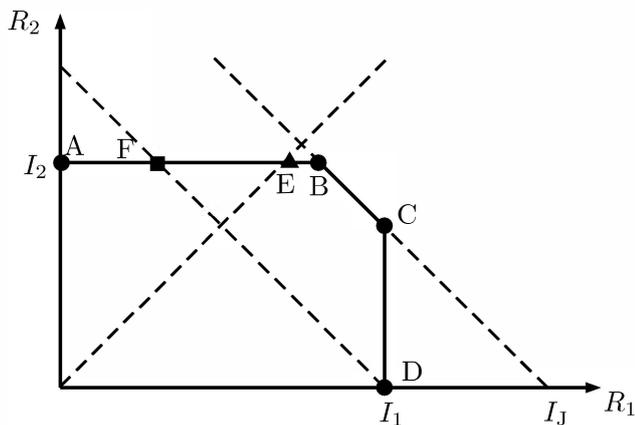


Figure 2. Achievable rate region in the case $I_2 < I_1$.

minimum SE of the two subchannels. We demonstrate that the rate achievable by FDM is always lower or equal than that achievable with two signals with overlapping frequencies in the absence of nonlinear distortions; the same result holds for the pragmatic rates and can be stated in the following theorem, whose proof can be found in the Appendix.

Theorem. *Let us consider the ideal multiple access channel*

$$y(t) = x_1(t) + \gamma x_2(t) + w(t). \quad (4)$$

The following inequalities hold

$$I_J \geq I_{\text{FDM}} \quad (5)$$

$$I_{J,p} \geq I_{\text{FDM},p} \quad (6)$$

with equality if and only if $x_1(t)$ and $x_2(t)$ are Gaussian random processes and $\gamma^2 = 0$ dB.

The strategy based on FDM is perfectly equivalent, in terms of SE, to a strategy based on time-division multiplexing (TDM), in which time is divided into slots of equal length, and each satellite is allotted a slot during which only that satellite transmits and the other remains silent. During its slot, each satellite is allowed to use twice the power. However, on satellites, due to peak power constraints, it is not possible to double the power and the satellite amplifiers are not conceived for power bursts. Hence TDM strategy will not be considered.

V. ADDITIVE WHITE GAUSSIAN NOISE CHANNEL WITH AVERAGE POWER CONSTRAINT

A first case study, useful to draw some preliminary considerations about the theoretical limits for the system we are considering, is that of the classical AWGN channel with average power constraint. For this case, we have that the two satellites of Figure 1 have no effect on the signal, hence the received signal reads

$$y(t) = x_1(t) + \gamma e^{j\phi(t)} x_2(t) + w(t).$$

We express the average power constraint as

$$\mathbb{E}[|x_i(t)|^2] \leq P \quad i = 1, 2,$$

where P is the maximum allowed average power.

For this channel, assuming independent Gaussian inputs, the SE I_J is given by the classical Shannon capacity, taking into account the total transmitted power, and reads

$$I_J = \log_2 \left(1 + \frac{(1 + \gamma^2)P}{N} \right),$$

where $N = N_0W$ is the noise power in the considered bandwidth. If, instead, we adopt the FDM model (3), the SE can be computed as the average of the SEs of two subchannels, each transmitting on half the bandwidth:

$$I_{\text{FDM}} = \frac{1}{2} \log_2 \left(1 + 2\frac{P}{N} \right) + \frac{1}{2} \log_2 \left(1 + 2\gamma^2\frac{P}{N} \right).$$

These SEs are independent of the phase noise $\phi(t)$. When we introduce the equal rate constraint, it is straightforward to show that we have the following pragmatic SEs

$$I_{J,p} = \min \left(I_J, 2 \log_2 \left(1 + \gamma^2\frac{P}{N} \right) \right)$$

$$I_{\text{FDM},p} = \log_2 \left(1 + 2\gamma^2\frac{P}{N} \right).$$

In Figure 3 we show the SE I_J as a function of P/N , for different values of power unbalance γ , together with the SE that can be obtained when a single satellite is available ($\gamma \rightarrow 0$). The figure also shows I_{FDM} for the same values of γ . We see that FDM is capacity-achieving when $\gamma = 1$ (i.e., $I_J = I_{\text{FDM}}$ when $\gamma = 1$, as also clear from the equations and as foreseen by the Theorem) but it is suboptimal in the case of power unbalance.

In Figure 4 we report the pragmatic SEs for the cases of Figure 3. For signals with overlapping frequencies, $I_{J,p}$ is lower than I_J only in the range of low P/N values, corresponding to the case $2I_2 < I_J$. The transitions are indicated by the change of slope in the curves. We also see that, for high power unbalance, a portion of $I_{J,p}$ lies below single-satellite SE.

In case of FDM, we clearly see how the user with the lower SE limits $I_{\text{FDM},p}$. The curves coincide for $\gamma = 1$, while they suffer from a significant performance loss w.r.t. I_{FDM} for high values of power unbalance. If the power unbalance is very high, FDM performs even worse than a single satellite.

VI. ADDITIVE WHITE GAUSSIAN NOISE CHANNEL WITH PEAK POWER CONSTRAINT

A more realistic scenario to theoretically address the problem of signals transmitted from satellites is to consider a peak-power-limited signal rather than an average-power-limited one. The adoption of a peak power constraint comes naturally from the use of a saturated nonlinear high-power amplifier (HPA) at the satellite. In this section, we repeat the analysis of Section V in a peak-power-limited scenario. We impose the constraint on the transmitted symbols $x_k^{(i)}$ of (1). An alternative statement of the problem could consist in imposing the constraint on the modulated signals; however, no expression for the SE exists (to the best of our knowledge), except for the bounds in [5].

Many considerations still hold in this case, however there are some significant differences that will be pointed out. We first review the results in [6] for the case of a single transmitter, then we extend the reasoning to the case of two transmitters.

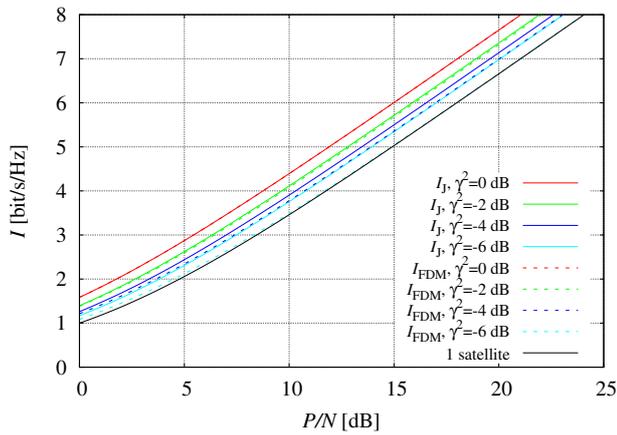


Figure 3. Joint spectral efficiency for different values of γ (AWGN channel with average power constraint).

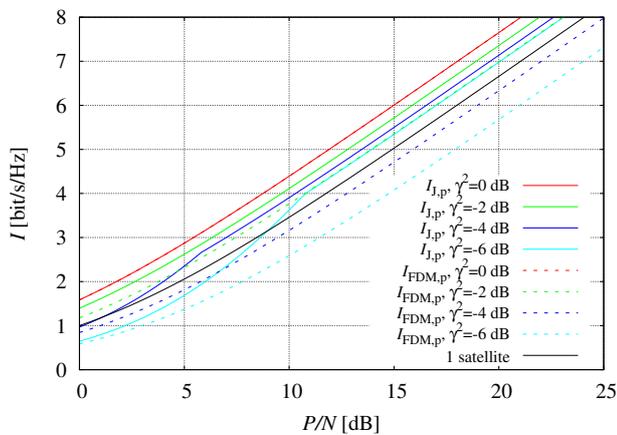


Figure 4. Pragmatic spectral efficiency for different values of γ (AWGN channel with average power constraint).

A. Analysis for Single Transmitter

If we assume that pulses $p(t) = \text{sinc}(t/T)$ in (1), the model (2) for $\gamma \rightarrow 0$ simplifies to the following discrete-time memoryless channel model

$$y_k = x_k + w_k, \quad (7)$$

where y_k is the observable, $x_k = x_k^{(1)}$ is the k -th symbol transmitted by satellite 1, and w_k is AWGN with variance $N = N_0W$. The input symbols x_k must be subject to a peak-power constraint, that can be expressed in the form

$$|x_k|^2 \leq P. \quad (8)$$

Channel (7) under constraint (8) was completely studied in [6]: the capacity-achieving distribution is discrete in amplitude and uniform in phase, and has the following expression

$$p(r, \theta) = \frac{1}{2\pi} \sum_{\ell=1}^m q_\ell \delta(r - p_\ell), \quad (9)$$

with $x_k = r e^{j\theta}$. The distribution is formed of m concentric circles, each having weight q_ℓ and radius p_ℓ . The constraints

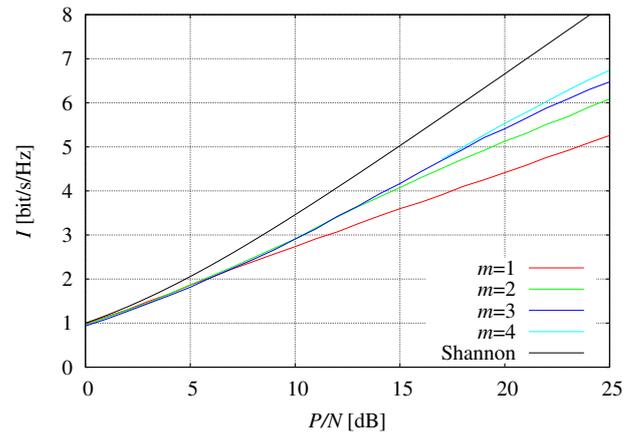


Figure 5. Spectral efficiency for single transmitter (AWGN channel with peak power constraint).

of the problem, in polar coordinates, become

$$0 \leq p_\ell \leq \sqrt{P} \quad (10)$$

$$p_{\ell+1} > p_\ell \quad (11)$$

$$0 \leq q_\ell \leq 1 \quad (12)$$

$$\sum_{\ell=1}^m q_\ell = 1. \quad (13)$$

For the distribution (9), we can express the information rate $I(x_k; y_k)$ as

$$\begin{aligned} I(x_k; y_k) &= \mathbb{E} \left[\log_2 \frac{p(y_k|x_k)}{p(y_k)} \right] \quad (14) \\ &= \mathbb{E} \left[\log_2 \frac{e^{-\frac{|y_k - r e^{j\theta}|^2}{N}}}{\sum_{\ell=1}^m q_\ell e^{-\frac{|y_k|^2 + p_\ell^2}{N}} I_0 \left(\frac{2|y_k| p_\ell}{N} \right)} \right], \end{aligned}$$

where the expectation is taken with respect to r , θ , and w_k , and $I_0(\cdot)$ is the modified Bessel function of the first kind and order zero. The optimal values of m , q_ℓ and p_ℓ cannot be found in closed form, but they are subject to optimization [6]. For this reason, we evaluated (14) for increasing values of m and, for each value, we optimized the m radii to achieve the highest information rate. Optimization results for $1 \leq m \leq 4$ are plotted in Figure 5. We see that, as expected, as P/N increases, the optimal distribution is formed of a higher number of circles. For comparison, the same figure also reports the classical Shannon capacity for average-power-constrained channels.

B. Analysis for Two Transmitters

Aim of this section is to extend the results of Section VI-A to the case of two transmitters. For this scenario, we make the assumption that the optimal distributions of the two inputs are still in the form (9). This result has been demonstrated for real inputs [12], but not for complex inputs, as the case of interest here. For this reason, the computed information rate is a lower bound to the actual channel capacity, whose expression is unknown. Under this assumption, the input

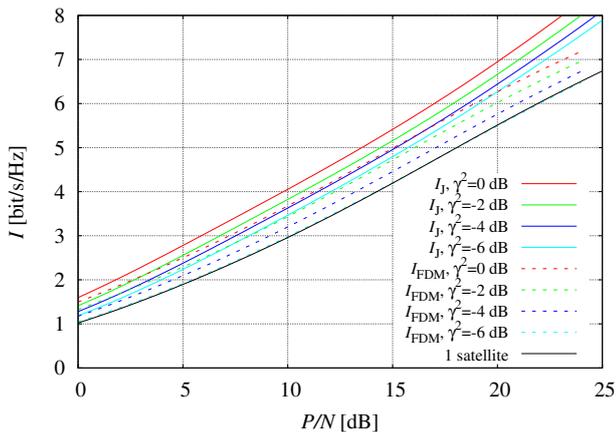


Figure 6. Joint spectral efficiency for different values of γ (AWGN channel with peak power constraint).

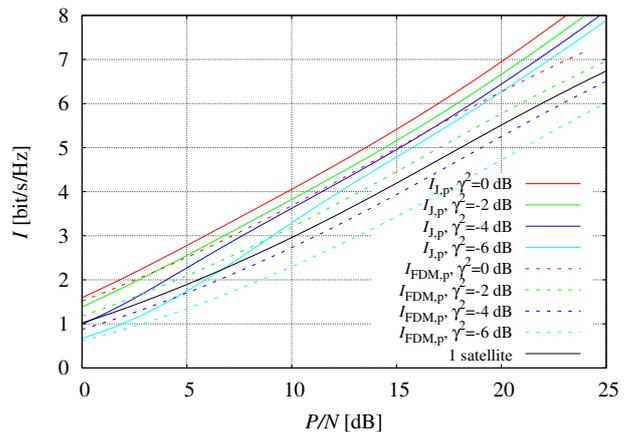


Figure 7. Pragmatic spectral efficiency for different values of γ (AWGN channel with peak power constraint).

amplitude distributions are

$$p(r_i) = \sum_{\ell=1}^{m_i} q_{\ell}^{(i)} \delta(r_i - p_{\ell}^{(i)}), \quad i = 1, 2$$

and the received signal is an extension of (7):³

$$\begin{aligned} y_k &= x_k^{(1)} + \gamma x_k^{(2)} + w_k \\ &= r_1 e^{j\theta_1} + \gamma r_2 e^{j\theta_2} + w_k, \end{aligned}$$

with each of the two inputs satisfying constraints (10)–(13). In this scenario, we can express the joint information rate $I_J = I(x_1, x_2; y)$ as an extension of (14):

$$I_J = \mathbb{E} \left[\log_2 \frac{(2\pi)^2 e^{-\frac{|y_k - r_1 e^{j\theta_1} - \gamma r_2 e^{j\theta_2}|^2}{N}}}{\sum_{\ell=1}^{m_1} \sum_{i=1}^{m_2} q_{\ell}^{(1)} q_i^{(2)} \Lambda_{\ell,i}} \right], \quad (15)$$

where

$$\Lambda_{\ell,i} = \int_0^{2\pi} \int_0^{2\pi} e^{-\frac{|y_k - p_{\ell}^{(1)} e^{j\theta_1} - \gamma p_i^{(2)} e^{j\theta_2}|^2}{N}} d\theta_1 d\theta_2.$$

We have computed (15) for different levels of power unbalance between the two received signals, assuming that the transmitters use the distributions optimized for the single transmitter case. The joint information rate is shown in Figure 6, where the curve labeled *1 satellite* is obtained as the envelope of the four curves of Figure 5. We report here, for comparison, the information rate computed when FDM is used, assigning half of the bandwidth to each of the satellites. We see that, unlike the case of average power constraint, FDM is not the optimal choice, not even in the absence of power unbalance. This result comes from a straightforward application of the Theorem in Section IV. In effect, since the two input distributions are not Gaussian, the inequality (5) is strict.

As already mentioned, in a broadcast scenario we have the further constraint that the two transmitters must use the

³We point out that a phase noise term should be considered in the second signal. However, since this shift is assumed to be perfectly known at the receiver and the input distributions are invariant w.r.t. a phase rotation, we do not add it to our model.

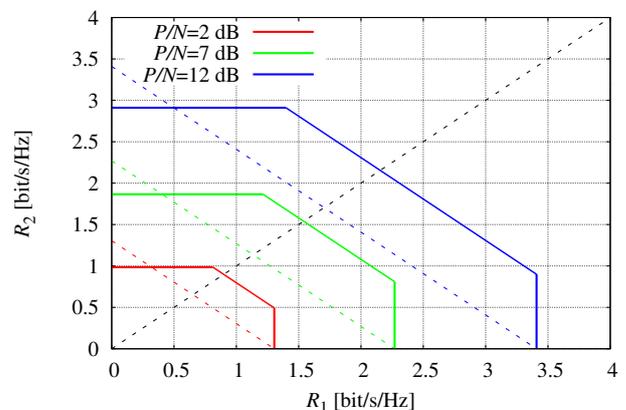


Figure 8. Spectral efficiency regions for $\gamma^2 = -2$ dB.

same rate. When we impose this constraint to the rates shown in Figure 6, we obtain the pragmatic rates in Figure 7. We see again that all curves, except those with absence of power unbalance, have suffered a degradation, and we also notice that, for low values of P/N and a high power unbalance, the use of a single satellite may be convenient over the use of two transmitters.

This fact can be better understood studying the SE regions of the channel for different values of P/N and power unbalance, reported in Figures 8 and 9. From the analysis of these figures, we can conclude that the maximum sum-rate cannot always be achieved and we can have a numerical insight of the values of P/N and γ^2 that allow to improve the rates with respect to a case with only a single transmitter. In particular, when the power unbalance is low ($\gamma^2 = -2$ dB) the maximum sum-rate is achieved with all three P/N values shown. On the other hand, with high power unbalance ($\gamma^2 = -6$ dB) it is clear that maximum sum-rate can be achieved only at high P/N , whereas when the power is low the performance of two satellites is worse than that of a single satellite.

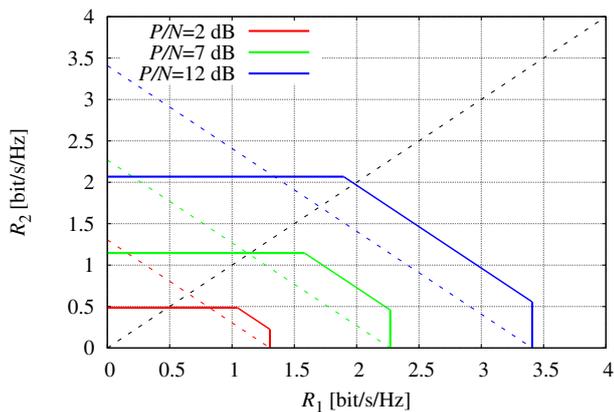
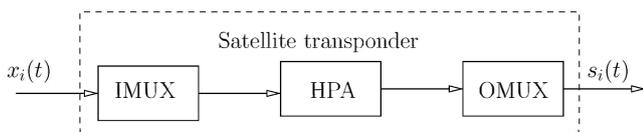

 Figure 9. Spectral efficiency regions for $\gamma^2 = -6$ dB.


Figure 10. Block diagram of the considered satellite channel.

VII. SATELLITE CHANNEL

This section studies the performance for the system in Figure 1 when the satellites are composed by the block diagram depicted in Figure 10. The block diagram shows an input multiplexing (IMUX) filter which removes the adjacent channels, a HPA, and an output multiplexing (OMUX) filter aimed at reducing the spectral broadening caused by the nonlinear amplifier. The HPA AM/AM and AM/PM characteristics and the IMUX/OMUX impulse responses are described in [7], and the OMUX filter has -3 dB bandwidth equal to 38 MHz. Although the HPA is a nonlinear memoryless device, the overall system has memory due to the presence of IMUX and OMUX filters.

The transmitted signals at the input of the two satellites are linearly modulated as in (1) where the information symbols $x_k^{(i)}$ are drawn from a discrete constellation and $p(t)$ is a properly normalized pulse. The received signal reads as in (2). We employ the adaptive receiver proposed in [13], [14]: a sufficient statistics for detection is extracted by using oversampling [15], and a fractionally-spaced minimum mean square error (FS-MMSE) equalizer, working at twice the symbol rate, acts as adaptive filter followed by a multiuser detector. The multiuser detector computes the a posteriori probabilities of the symbols as

$$p(y_k | x_k^{(1)}, x_k^{(2)}) \propto \exp \left\{ -\frac{|y_k - \beta(x_k^{(1)} + \gamma x_k^{(2)})|^2}{N_0} \right\}$$

where y_k is the sample at the output of the FS-MMSE equalizer and β is a possible (complex-valued) bias.

Similarly to previous sections, we also consider a FDM scenario: the transponder bandwidth is equally divided into two subchannels as schematically depicted in Figure 11. Then,

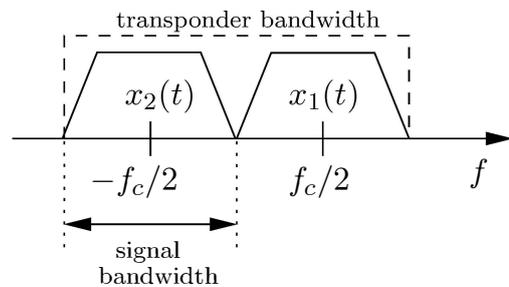


Figure 11. Transponder bandwidth allocation for FDM.

the FDM receiver performs detection separately with two FS-MMSE equalizers and symbol-by-symbol receivers.

The complexity of the channel model does not allow (to the best of our knowledge) to obtain results in a closed form as in previous sections. Hence, the achievable SEs $I_{J,P}$ and $I_{FDM,P}$ for these two scenarios are computed through the Monte Carlo method proposed in [16]. We point out that these values are a lower bound to the actual SE and they are achievable with the specific adopted receiver. The ensuing SE curves describe the possible achievable gains in a scenario that is more realistic than those described in previous sections. All results will be reported as function of P_{sat}/N , where P_{sat} is the power at saturation of the HPA.

A. Numerical results

We study a satellite channel whose filters and HPA have the characteristics provided by DVB-S2 standard (the HPA is that called “conventional” in [7]). We consider transmitted signals with baudrate 37 Mbaud, adopting the classical constellations of satellite communications, i.e., QPSK, 8PSK, 16APSK and 32APSK. The adopted shaping pulse $p(t)$ has root raised cosine spectrum with roll-off $\alpha = 0.1$. The input back-off is set to 0 dB for PSK modulations, and 3 dB for APSK modulations.⁴

Figure 12 shows the envelope of the pragmatic SE $I_{J,P}$ for the four considered modulations, without power unbalance. The figure also shows SE for FDM and for a single satellite. In case of FDM, each signal has baudrate $1/T = 18.5$ Mbaud, and the frequency spacing is equal to $f_c = (1 + \alpha)/T = 20.35$ MHz.⁵ We can see from the figure that two overlapped signals can achieve a higher SE than both FDM and the single satellite. It is interesting to notice that, although the channel model is affected by nonlinear effects, inequality (6) still holds true even in this case. We also notice that, at high P_{sat}/N , FDM performs worse even than a single satellite. This loss is due to the interchannel interference (ICI) from the second FDM signal, which lies in the same OMUX bandwidth. In fact, due to the spectral regrowth after the HPA, the two FDM signals are no more orthogonal. This effect is proved in Figure 13, that compares the FDM curve with two SE curves: ideal FDM in the absence of ICI, and a single satellite with twice the power

⁴We found these values to be optimal from other activities beyond this paper. We also point out that the impact of interchannel interference due to transponders transmitting on adjacent frequencies is negligible for all the presented scenarios, and hence it will not be considered [17].

⁵Other values of frequency spacing have been tested, but 20.35 MHz has been found to be practically optimal for this scenario.

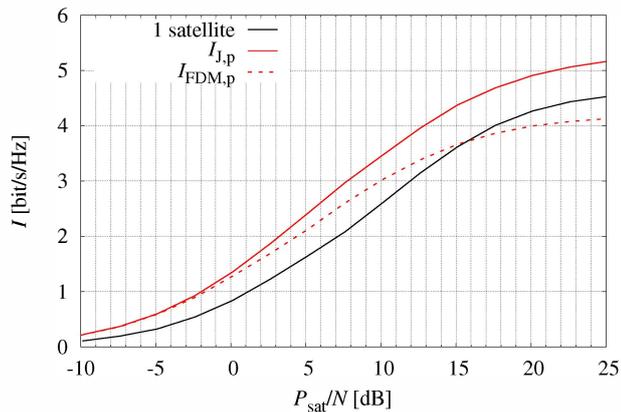


Figure 12. Spectral efficiency achievable by DVB-S2 constellations for two satellites and $\gamma^2=0$ dB.

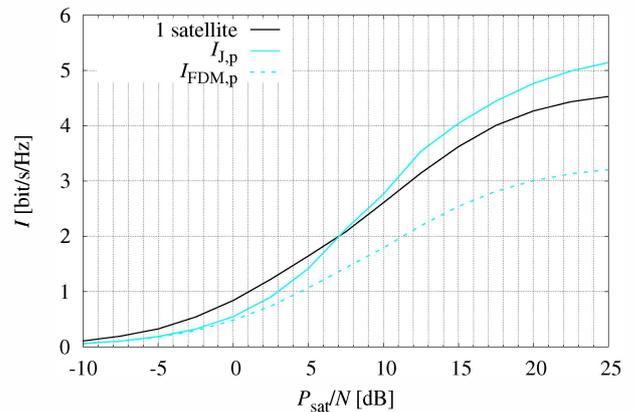


Figure 14. Spectral efficiency achievable by DVB-S2 constellations for two satellites and $\gamma^2=-6$ dB.

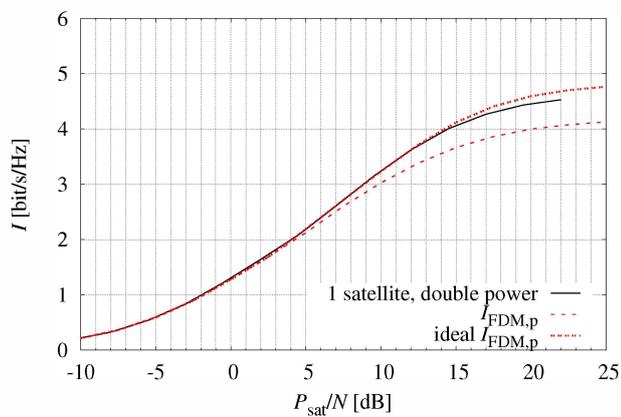


Figure 13. Spectral efficiency achievable by DVB-S2 constellations for two satellites using FDM and $\gamma^2=0$ dB.

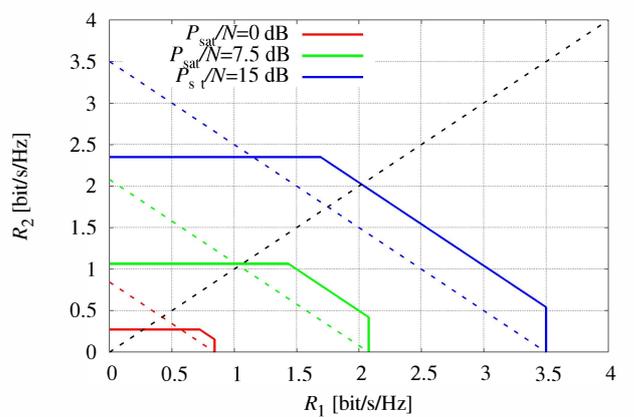


Figure 15. Spectral efficiency regions for DVB-S2 constellations for two satellite transmitting with P_{sat}/N equal to 0, 7.5 and 15 dB and $\gamma^2=-6$ dB.

P_{sat} . Similarly to the linear channel, ideal FDM can achieve the same SE as the single satellite with double power, but in the actual case ICI has an impact on performance.

Figure 14 shows SE curves for the same scenario as in Figure 12, but with power unbalance equal to 6 dB. Overlapped signals again outperform FDM for every P_{sat}/N value but, since the equal rate constraint limits the performance to that of the lower power signal, a single satellite has higher SE at low P_{sat}/N . This behavior can be seen from the SE regions in Figure 15, and it must be noticed that it is perfectly in line with results found for the peak limited AWGN channel, despite a huge difference between the two models.

VIII. CONCLUSIONS

We studied achievable rates by a system using two co-located satellites. We exploited the second satellite to improve the spectral efficiency. We studied three models: AWGN with average power constraint, AWGN with peak power constraint, and the DVB-S2 satellite channel. For all cases we considered signals with overlapping frequencies or FDM. Overlapped signals resulted to be convenient in all cases w.r.t. FDM, but we showed that there are cases in which a single satellite can

outperform both: these cases depend on the power unbalance and on the received signal powers.

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APPENDIX

We now prove the theorem stated in Section IV; we first prove a preliminary result concerning the differential entropies of two continuous random variables.

Lemma. *Let x and y be two independent continuous complex random variables, with probability density functions $p(x)$ and $p(y)$ and differential entropies $h(x)$ and $h(y)$. Then*

$$h(x + y) \geq 1 + \frac{h(x) + h(y)}{2} \quad (16)$$

with equality if and only if x and y are independent Gaussian random variables with the same variance.

Proof: For the entropy power inequality

$$2^{h(x+y)} \geq 2^{h(x)} + 2^{h(y)} \quad (17)$$

$$= 2^{1+\frac{h(x)+h(y)}{2}} \cosh\left(\frac{h(x)-h(y)}{2} \ln 2\right)$$

$$\geq 2^{1+\frac{h(x)+h(y)}{2}} \quad (18)$$

where equalities in (17) and (18) hold if and only if x and y are Gaussian and have same variance. Eq. (16) is finally derived by taking \log_2 of (18). ■

We then consider the rates achievable by FDM. Under the assumption of ideal FDM transmission, a sufficient statistics is obtained by sampling the continuous waveforms. The observables for the two subchannels are

$$y_1 = x_1 + w_1$$

$$y_2 = \gamma x_2 + w_2$$

where x_1 and x_2 are the signal samples, w_1 and w_2 are white Gaussian noise processes with power $N/2$ instead of N , since FDM works with half the bandwidth w.r.t. the case of a single transmitter. The mutual information of FDM is the average of the mutual information for the two channels, i.e.

$$I_{\text{FDM}} = \frac{h(y_1) + h(y_2)}{2} - \log_2\left(\pi e \frac{N}{2}\right),$$

and the pragmatic rate is

$$I_{\text{FDM,p}} = h(y_2) - \log_2\left(\pi e \frac{N}{2}\right).$$

Since the mutual information is a non decreasing function of the signal-to-noise ratio [18], clearly it is $I_{\text{FDM,p}} \leq I_{\text{FDM}}$.

We finally prove the theorem stated in Section IV.

Proof: We first prove inequalities (5) and

$$2I_2 \geq I_{\text{FDM,p}}. \quad (19)$$

The samples at the output of channel (4) can be equivalently expressed as $y = y_1 + y_2$ and the mutual information of this equivalent expression reads $I_J = h(y_1 + y_2) - \log_2(\pi e N)$. Hence,

$$I_J - I_{\text{FDM}} = h(y_1 + y_2) - \frac{h(y_1) + h(y_2)}{2} - 1$$

from which, by an application of the Lemma, we derive inequality (5). The mutual information I_2 , instead, reads

$$I_2 = h(y|x_1) - \log_2(\pi e N)$$

$$= h(y_2 + w_1) - \log_2(\pi e N)$$

and

$$2I_2 - I_{\text{FDM,p}} = 2h(y_2 + w_1) - h(y_2) - \log_2(2\pi e N)$$

which becomes (19) from Lemma.

Since $I_{J,p} = \min(I_J, 2I_2)$ and $I_{\text{FDM}} \geq I_{\text{FDM,p}}$, clearly (6) follows with equality if and only if x_1 and x_2 are Gaussian with $\gamma^2=0$ dB. ■

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