Published for SISSA by Springer

Received: *February 6, 2023*
Accepted: *July 14, 2023*
Published: *July 21, 2023*

Bootstrapping string dynamics in the 6d $\mathcal{N} = (2, 0)$ theories

Carlo Meneghelli[a] **and Maxime Trépanier**[b,c]

[a] *Dipartimento SMFI, Università di Parma, Viale G.P. Usberti 7/A, 43121, Parma, PR, Italy*

[b] *Department of Mathematics, King's College London, London, WC2R 2LS, United Kingdom*

[c] *Perimeter Institute for Theoretical Physics, Waterloo, ON, N2L 2Y5, Canada*

E-mail: carlo.meneghelli@unipr.it, trepanier.maxime@gmail.com

Abstract: We present two complementary approaches to calculating the 2-point function of stress tensors in the presence of a 1/2 BPS surface defect of the 6d $\mathcal{N} = (2, 0)$ theories. First, we use analytical bootstrap techniques at large N to obtain the first nontrivial correction to this correlator, from which we extract the defect CFT (dCFT) data characterising the 2d dCFT of the 1/2 BPS plane. Along the way we derive a supersymmetric inversion formula, obtain the relevant superconformal blocks and check that crossing symmetry is satisfied. Notably our result features a holomorphic function whose appearance is related to the chiral algebra construction of Beem, Rastelli and van Rees. Second, we use that chiral algebra description to obtain exact results for the BPS sector of the dCFT, valid at any N and for any choice of surface operator. These results provide a window into the dynamics of strings of the mysterious 6d theories.

Keywords: Scale and Conformal Symmetries, Wilson, 't Hooft and Polyakov loops, Conformal and W Symmetry, 1/N Expansion

ArXiv ePrint: 2212.05020

https://doi.org/10.1007/JHEP07(2023)165

JHEP07(2023)165

Contents

1 Introduction and summary

Calculating observables in the 6d $\mathcal{N} = (2,0)$ superconformal field theories is a challenging problem. Due to their lack of a lagrangian description, the theories remain largely mysterious and outside the reach of conventional field theoretic methods.

The most established way to study these theories is through holography. At large N, the A_{N-1} and D_N $\mathcal{N} = (2,0)$ theories are dual to 11d supergravity on an $AdS_7 \times S^4$ [1] and $AdS_7 \times S^4/\mathbb{Z}_2$ [2, 3] background respectively, where the radius of AdS_7 (in Planck units) is related to N as $R_{AdS}/l_P = (8\pi N)^{1/3}$. This supergravity description is useful and leads to concrete predictions; unfortunately it is also impractical beyond the large N limit: subleading corrections probe high-energy corrections to 11d supergravity coming from M-theory, and we currently have no way to determine these systematically. It is therefore imperative to find new ways to calculate observables beyond the large N limit.

The more modern approach to calculating observables is to rely on the methods of the conformal bootstrap [4–7] and the chiral algebra subsector [8, 9]. In the context of the $\mathcal{N} = (2,0)$ theories, the chiral algebra description was used to calculate protected CFT data and obtain information about the spectrum of BPS operators [9, 10]. The bootstrap constraints on the 4-point function of stress tensor supermultiplets were studied first numerically in [11] and analytically at large N in [10, 12–15] (see also [16–20]). These works have led to remarkable progress in understanding both the (2,0) theories at large N and, through holography and the flat space limit of Mellin amplitudes [21], scattering amplitudes in M-theory.

In this paper we take a first step to generalise this approach to include correlators involving surface operators [22–25]. Surface operators are particularly interesting because they play a role analogous to the Wilson lines of gauge theories and capture interesting physical properties of these theories not accessible to local operators, such as higher-form symmetries [26–29] and measuring the string potential [30–32]. In addition to what they compute, they are useful because they provide a wealth of new observables, such as their expectation value and correlators with other operators, and thus provide a larger playground to study the 6d theories.

More precisely, we study the 2-point function of the stress tensor superprimaries in the presence of a 1/2 BPS defect V defined over a plane in $\mathbb{R}^6$. The set of such defects V is expected to be equal to the set of finite dimensional representations of the ADE group entering the classification of 6d $\mathcal{N} = (2,0)$ theories [33, 34], and in the following we keep the choice of representation arbitrary.

The bootstrap approach to this kind of correlator was first developed in the context of boundary CFTs [35] and later generalised to defect CFTs [36] and super-CFTs [37]. At large N, an effective approach to calculating correlators based on the defect version [38] of the inversion formula [39] was outlined in [40] for the Wilson line in $\mathcal{N} = 4$ SYM (see also [41–43]). Here we adapt the strategy of [40] to obtain the first nontrivial correction to our correlator at large N.

In the rest of this introduction we present a summary of our results.

JHEP07(2023)165

1.1 Summary

The superprimaries of the stress tensor multiplet are scalars transforming in the symmetric traceless representation of $\mathfrak{so}(5)$ R-symmetry, and we denote them by $\Phi^{I_1 I_2}$, with $I = 1, \dots, 5$ R-symmetry indices. Their conformal dimension is protected by supersymmetry and fixed to $\Delta = 4$. It's convenient to introduce a polarisation vector u to avoid carrying indices, and we define

$$\Phi(x, u) \equiv \Phi^{I_1 I_2}(x) u_{I_1} u_{I_2} \,, \qquad I = 1, \dots, 5 \,. \tag{1.1}$$

Note that we can enforce tracelessness by requiring u to be a null vector, $u^2 = 0$. A review of this embedding space formalism can be found in [44].

In the absence of a defect, the 2-point functions of these operators are completely fixed by conformal symmetry up to the choice of normalisation for the operators, but the presence of the defect V breaks the conformal symmetry $\mathfrak{so}(2,6) \to \mathfrak{so}(2,2) \times \mathfrak{so}(4)$ and the R-symmetry $\mathfrak{so}(5) \to \mathfrak{so}(4)$, leading to 3 independent cross-ratios $z, \bar{z}, \omega$. Writing $x^\perp$ for the coordinates perpendicular to the plane and taking n to be the unit vector specifying the embedding of $\mathfrak{so}(4) \subset \mathfrak{so}(5)$, we define these cross-ratios by

$$\frac{z + \bar{z}}{2\sqrt{z\bar{z}}} = \frac{x_1^\perp \cdot x_2^\perp}{|x_1^\perp|\,|x_2^\perp|} \,, \qquad \frac{(1-z)(1-\bar{z})}{\sqrt{z\bar{z}}} = \frac{x_{12}^2}{|x_1^\perp|\,|x_2^\perp|} \,, \qquad \frac{(1-\omega)^2}{\omega} = \frac{u_1 \cdot u_2}{(u_1 \cdot n)(u_2 \cdot n)} \,. \tag{1.2}$$

Here and below we use the short-hand notation $x_{12} \equiv x_1 - x_2$, and when restricting to coordinates along the plane we use the notation $x^\parallel$. This choice of cross-ratios is convenient and admits a geometric interpretation reviewed in section 2, see figure 3 there.

The 2-point function is then constrained by kinematics to take the form

$$\langle \Phi(x_1, u_1) \Phi(x_2, u_2) V \rangle = \frac{(u_1 \cdot n)^2 (u_2 \cdot n)^2}{|x_1^\perp|^4 |x_2^\perp|^4} \mathcal{F}(z, \bar{z}, \omega) \,. \tag{1.3}$$

As we derive in section 2 the function $\mathcal{F}$ is not arbitrary and must satisfy additional constraints from supersymmetry known as superconformal Ward identities

$$(\partial_z + \partial_\omega) \mathcal{F}(z, \bar{z}, \omega)|_{z=\omega} = 0 \,, \qquad (\partial_{\bar{z}} + \partial_\omega) \mathcal{F}(z, \bar{z}, \omega)|_{\bar{z}=\omega} = 0 \,. \tag{1.4}$$

These constraints can be understood as Cauchy-Riemann equations for the functions $\mathcal{F}(z, \bar{z}, z)$ and $\mathcal{F}(z, \bar{z}, \bar{z})$. They can be solved explicitly in terms of 2 functions $F(z, \bar{z}), \zeta(z)$

$$\begin{aligned} \mathcal{F}(z, \bar{z}, \omega) = &\frac{(z-\omega)(\bar{z}-\omega)(z-\omega^{-1})(\bar{z}-\omega^{-1})}{(z-1)^2(\bar{z}-1)^2} F(z, \bar{z}) \\ &+ \frac{z(\bar{z}-\omega)(\bar{z}-\omega^{-1})(\omega-1)^2}{\omega(\bar{z}-z)(\bar{z}-z^{-1})(z-1)^2} \bar{\zeta}(\bar{z}) + \frac{\bar{z}(z-\omega)(z-\omega^{-1})(\omega-1)^2}{\omega(z-\bar{z})(z-\bar{z}^{-1})(\bar{z}-1)^2} \zeta(z) \,. \end{aligned} \tag{1.5}$$

Our goal is then to calculate F and ζ at large N.

The structure of the correlator at large N is easy to understand from supergravity. The stress tensor superprimaries are dual to Kaluza-Klein modes on the S^4 [1, 45], and the 1/2 BPS plane in the fundamental representation of A_{N-1} is dual to an M2-brane extended

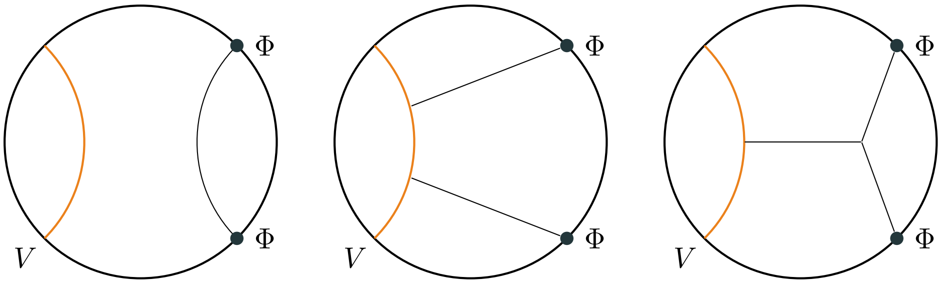

Figure 1. Leading Witten diagrams at large N. In these pictures the orange curve represents the minimal M2-brane dual to the surface operator V, and the black lines denote propagators in AdS.

along an $AdS_3 \subset AdS_7 \times S^4$ [30]; for the symmetric representation with M indices we take M coincident M2-branes [33, 34]. The leading contributions to the correlator are given by the Witten diagrams presented in figure 1, and we can evaluate their respective order in N and M by dimensional analysis. Propagators contribute as $G_{11}^{-1} \sim N^{-3}$, and conversely vertices contribute as N^3. Interactions with the M2-branes are proportional to the M2-brane tension $T_{M2} \sim N$, so for M M2-branes we expect a factor MN. Schematically we then expect

$$\langle \Phi(x_1, u_1)\Phi(x_2, u_2)V\rangle = \frac{(u_1 \cdot u_2)^2}{|x_{12}|^8} + \frac{M^2}{N}\frac{(u_1 \cdot n)^2(u_2 \cdot n)^2}{|x_1^\perp|^4|x_2^\perp|^4} + \frac{M}{N^2}(\dots) + O(N^{-3}) \tag{1.6}$$

The relative factors of M^2/N and M/N^2 are in direct correspondence with the Witten diagrams.

The first two terms correspond to disconnected correlators and are known exactly. The first is the free propagator, its coefficient is one by normalisation. The second is the square of the one-point function, which can be expressed in terms of anomaly coefficients c, d (see appendix C for a derivation)

$$\langle \Phi(x, u)V\rangle = a_2 \frac{(u \cdot n)^2}{|x^\perp|^4}\,, \qquad a_2 = -\frac{d}{\sqrt{c/2}}\,. \tag{1.7}$$

These anomaly coefficients are defined for any ADE group $\mathfrak{g}$ specifying the $\mathcal{N} = (2, 0)$ theory and any choice of representation for the surface operator V, defined by its highest weight Λ. A conjecture for their exact values was proposed for c in [9] and d in [46]; both pass many consistency checks. Writing $d_\mathfrak{g}$, $h^\vee_\mathfrak{g}$ and $r_\mathfrak{g}$ for the dimension, the dual Coxeter number and the rank of $\mathfrak{g}$, and $\rho = \frac{1}{2}\sum_{\alpha>0} \alpha$ for the sum of positive roots (the Weyl vector), they are given by

$$c = 4d_\mathfrak{g} h^\vee_\mathfrak{g} + r_\mathfrak{g}\,, \qquad d = \frac{1}{2}(\Lambda, \Lambda) + 2(\Lambda, \rho)\,. \tag{1.8}$$

In particular for $\mathfrak{su}(n)$ and for a symmetric representation of rank M, these coefficients are

$$c = 4N^3 - 3N - 1\,, \qquad d = M(N - 1)\left(1 + \frac{M}{2N}\right)\,. \tag{1.9}$$

The coefficient c agrees with the supergravity calculations of [47–50]. The coefficient d was initially obtained from holographic entanglement entropy [46], and also agrees with a calculation from the superconformal index [51] and explicit supergravity calculations for $M = 1$ at large N [52–54]. With these results, we get that in the large N limit $a_2^2 \sim M^2/N$, which matches (1.6).

The term proportional to M/N^2 in (1.6) multiplies a nontrivial function of the cross-ratios, and encodes the interaction between Φ and V via the exchange of a stress tensor multiplet. Its coefficient is also known exactly and follows from the superconformal block decomposition of $\mathcal{F}$

$$\mathcal{F}(z,\bar{z},\omega) = \frac{(z\bar{z})^2(1-\omega)^4}{(1-z)^4(1-\bar{z})^4\omega^2} \sum_{\mathcal{O}_l} \lambda_{22l} a_l \mathcal{G}_l(z,\bar{z},\omega)\,. \tag{1.10}$$

In this decomposition, the functions $\mathcal{G}$ are the superconformal blocks associated with the exchange of a given bulk supermultiplet, and the coefficients λ_{22l}, a_l are respectively the structure constants appearing in the 3-point function of local operators and 1-point functions in the presence of V. For the stress tensor, λ_{222} is known exactly and is given by

$$\lambda_{222} = \sqrt{\frac{8}{c}}\,. \tag{1.11}$$

In the large N limit we also reproduce $a_2\lambda_{222} \sim M/N^2$.

The expectations from supergravity then translate into the expansion

$$\begin{aligned} \zeta(z) &= \frac{z^2}{(1-z)^4} + \frac{2d^2}{c} - \frac{4d}{c}\zeta^{(1)}(z) + O(c^{-2}) \\ F(z,\bar{z}) &= \quad 0 \quad + \frac{2d^2}{c} - \frac{4d}{c}F^{(1)}(z,\bar{z}) + O(c^{-2})\,. \end{aligned} \tag{1.12}$$

In section 4, we adapt the strategy presented in [40] to calculate ζ, F from bootstrap techniques. The observation is as follows. The function $\mathcal{F}$ can have a branch cut at $\bar{z} = 1$ (physically this corresponds to having null separated bulk operators, refer to figure 3), and its discontinuity along that branch cut can be used to reconstruct the correlator via the dispersion relation derived in [55, 56]. From the block decomposition of $\mathcal{F}$ (1.10) one can show that the only blocks contributing to the discontinuity are either those with low enough twist $\Delta - \ell < 8$ or long multiplets with anomalous dimensions. Long multiplets arise from double-trace operators (in the large N limit they have the schematic form $\Phi_k \Box^n \partial^\ell \Phi_k$, with Φ_k a 1/2 BPS operator of dimension $2k$) and are expected to have conformal dimensions

$$\Delta = 4k + 2n + \ell + \frac{\gamma}{c} + \dots\,, \qquad k \geq 2\,, \tag{1.13}$$

so at large c their contribution to the discontinuity is subleading. We can conclude that to order c^{-1}, the only superblocks that contribute to the discontinuity are the exchange of the identity and stress tensor multiplet, which correspond to diagrams 1 and 3 of figure 1. A straightforward strategy is then to obtain their respective superblocks, calculate the discontinuity and reconstruct the correlator from its discontinuity.

Unfortunately this strategy is incomplete for two reasons. First, the dispersion relation of [55, 56] may not reconstruct the full correlator, for instance it misses the disconnected diagram in the middle of figure 1. Second, it is also not manifestly supersymmetric, and indeed to satisfy the superconformal Ward identities (1.4), the result obtained this way must be supplemented by an infinite number of conformal blocks, see for instance [40].

In section 4 we resolve both of these issues by deriving a manifestly supersymmetric inversion formula (4.3) (see [19] for a similar idea applied to 4-point functions of local operators). As we review below, in addition to the decomposition into bulk superconformal blocks (1.10), $\mathcal{F}$ admits a decomposition in defect superconformal blocks $\hat{\mathcal{G}}$

$$\mathcal{F}(z,\bar{z},\omega) = \sum_{\hat{\mathcal{O}}_l} b_{kl}^2 \hat{\mathcal{G}}_l(z,\bar{z},\omega)\,. \tag{1.14}$$

The supersymmetric inversion formula calculates the coefficients b_{kl}^2 entering this decomposition directly from the discontinuity of $\mathcal{F}$. Resumming these superblocks we are guaranteed to obtain a supersymmetric correlator, and furthermore we observe that it calculates all but two superblocks contributing to $\mathcal{F}$: the defect identity (corresponding to the middle diagram in figure 1) and the displacement supermultiplet. The contribution from the defect identity is simply the d^2/c term in (1.12), and the contribution from the displacement multiplet is fixed by kinematics [57] and proportional to d/c.

Implementing this strategy we obtain the subleading terms in (1.12)

$$\begin{aligned}
\zeta^{(1)}(z) &= \frac{z}{(1-z)^2}\,,\\
F^{(1)}(z,\bar{z}) &= \frac{z\bar{z}}{(1-z\bar{z})^6}\Big[2\left(1+z\bar{z}+(z\bar{z})^2\right)\left(1+18z\bar{z}+(z\bar{z})^2\right)\\
&\qquad\qquad -(z+\bar{z})\,(1+z\bar{z})\left(1+28z\bar{z}+(z\bar{z})^2\right)\Big]\\
&\quad+6\frac{(z\bar{z})^2\log z\bar{z}}{(1-z\bar{z})^7}\Big[(1+z\bar{z})\left(3+4z\bar{z}+3(z\bar{z})^2\right)-2\,(z+\bar{z})\,(1+3z\bar{z}+(z\bar{z})^2)\Big]\,.
\end{aligned} \tag{1.15}$$

We expect subleading corrections to $\mathcal{F}$ in d/c and $1/c$, so this result is valid for any choice of representation for V, as long as $1 \ll d \ll c$.

A nontrivial check of our result is that the correlator we obtain satisfies crossing symmetry, see figure 2. The terms corresponding to the exchange of bulk and defect identity are known to be crossing symmetric by themselves, but in addition we check that $\zeta^{(1)}, F^{(1)}$ lead to a crossing symmetric correlator.

Finally our result leads to one more surprise. A fundamental difference between the case of the Wilson line in $\mathcal{N}=4$ SYM studied in [40] and surface operators in the (2,0) theories is the lack of results from supersymmetric localization. Yet, we show in section 3 that we can obtain exact results in 6d as well. As was shown in [9], a protected subsector of the 6d theories obey the structure of a chiral algebra, and for the A_{N-1} 6d theories the corresponding chiral algebra is expected to be the $\mathcal{W}_N$-algebras. In parallel to the analogous constructions in 4d [8, 58], we show in section 3 that surface operators are identified with a module of the $\mathcal{W}_N$-algebras, and using that description we obtain exact results for the

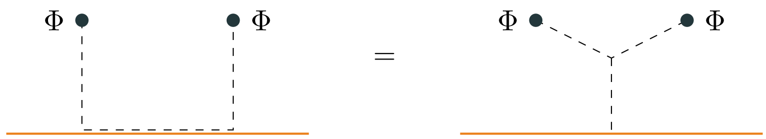

Figure 2. Crossing symmetry constraint for a 2-point function of bulk operators with a defect. On the left, the correlator is evaluated using the defect channel decomposition (1.14), on the right with the bulk channel decomposition (1.10).

protected defect CFT (dCFT) data. In particular, we are able to calculate the holomorphic function $\zeta(z)$ and find that the bootstrap result obtained above is exact

$$\zeta(z) = \frac{z^2}{(1-z)^4} + \frac{2d^2}{c} - \frac{4d}{c}\frac{z}{(1-z)^2}\,. \tag{1.16}$$

This is a nontrivial check of our bootstrap result, and unlike (1.12), this expression has no further corrections in c, d, and we conjecture that it is valid for any ADE theory and any choice of representation for V.

The rest of this paper is organised as follows. In section 2 we review the definition of the dCFT data, study the kinematics of the 2-point function, derive the superconformal Ward identities and study the two superconformal blocks decompositions of $\mathcal{F}$. In section 3 we use the chiral algebra description to obtain protected dCFT data and calculate $\zeta(z)$ (1.16). In section 4 we present a supersymmetric inversion formula and use it to obtain the dCFT data associated with the defect channel and our main result (1.15). Finally, in section 5 we check crossing symmetry and obtain dCFT data associated with the bulk channel.

We also include three appendices. Appendix A contains a review of the conformal blocks in the bulk channel, along with a derivation of the superconformal blocks relevant to this paper. Appendix B does the same for defect channel blocks. Appendix C presents the calculation of a_2 in terms of the anomaly coefficients for our choice of normalisation.

2 Kinematics

We begin by reviewing the constraints from kinematics on correlators, the definition of defect CFT data and the two superconformal blocks decomposition of the 2-point function (1.3).

2.1 Review of defect CFT data

A CFT can be defined by its spectrum of operators along with the structure constants appearing in their 3-point functions. For the A_{N-1} $\mathcal{N}=(2,0)$ theories, the spectrum is expected to contain a set of 1/2 BPS operators Φ_k ($k=2,\dots,N$) transforming in the symmetric traceless representation of $\mathfrak{so}(5)$ with k indices ($\Phi_{k=2} \equiv \Phi$ is the superprimary of the stress tensor multiplet). For these operators the 3-point functions take the form

$$\begin{aligned}&\langle \Phi_{k_1}(x_1,u_1)\Phi_{k_2}(x_2,u_2)\Phi_{k_3}(x_3,u_3)\rangle \\ &\qquad = \lambda_{k_1k_2k_3}\left(\frac{-2u_1\cdot u_2}{x_{12}^4}\right)^{\frac{k_{123}}{2}}\left(\frac{-2u_1\cdot u_3}{x_{13}^4}\right)^{\frac{k_{132}}{2}}\left(\frac{-2u_2\cdot u_3}{x_{23}^4}\right)^{\frac{k_{231}}{2}},\end{aligned} \tag{2.1}$$

JHEP07(2023)165

where we use the shorthand notation $k_{ijk} \equiv k_i + k_j - k_k$, and the factors -2 are introduced for later convenience. The constants λ are not fixed by symmetry and are pieces of CFT data defining the theory.

The value for these structure constants λ depends on the choice of normalisation of the operators, and for definiteness in this paper we take the 2-point function to be

$$\langle \Phi_k(x_1, u_2)\Phi_k(x_2, u_2)\rangle = \left(\frac{2u_1 \cdot u_2}{x_{12}^4}\right)^k . \tag{2.2}$$

In addition to this usual CFT data characterising the algebra of local operators, surface operators enrich the theory by a new set of defect CFT data that characterise correlators involving the surface operators.

The 1/2 BPS operators Φ_k can acquire an expectation value in the presence of a surface operators V and are constrained by the residual conformal symmetry to take the form

$$\langle \Phi_k(x, u)V\rangle = a_k \frac{(u \cdot n)^k}{|x^\perp|^{2k}} . \tag{2.3}$$

The coefficients a_k are independent pieces of dCFT data.

Finally, the dCFT contains defect operators $\hat{\mathcal{O}}$ that can be inserted on the defect V. These defect operators sit in multiplets of the $\mathfrak{osp}(4^*|2) \oplus \mathfrak{osp}(4^*|2)$ algebra preserved by the plane V, which includes as a bosonic subalgebra the $\mathfrak{so}(2,2)$ group of rigid 2d conformal symmetries along the plane, the $\mathfrak{so}(4)$ rotations of the space transverse to the plane and the residual $\mathfrak{so}(4)$ R-symmetry. For defect operators of conformal dimension $\hat{\Delta}$ and in representations of transverse spin s and R-symmetry spin r, we again introduce an index-free notation by contracting the operators with polarisation vectors $u_\parallel^i$ and v^m (with $i, m = 1, 2, 3, 4$ and u, v s.t. $u_\parallel^2 = v^2 = 0$)

$$\hat{\mathcal{O}}_{\hat{\Delta},s,r}(x_\parallel, u_\parallel, v) \equiv \hat{\mathcal{O}}^{i_1 i_2 \dots i_r}_{m_1 m_2 \dots m_s}(x_\parallel) u_\parallel^{i_1} u_\parallel^{i_2} \dots v^{m_1} v^{m_2} \dots \tag{2.4}$$

As above, for definiteness we assume the normalisation

$$\left\langle V[\hat{\mathcal{O}}_{\hat{\Delta},s,r}(x_1, u_1, v_1)\hat{\mathcal{O}}_{\hat{\Delta},s,r}(x_2, u_2, v_2)]\right\rangle = \frac{(2u_1 \cdot u_2)^r (2v_1 \cdot v_2)^s}{x_{12}^{2\hat{\Delta}}} . \tag{2.5}$$

Their correlators with bulk operators is fixed by kinematics up to a coefficient b

$$\left\langle \Phi_k(x_1^\parallel, x_1^\perp, u_1)V[\hat{\mathcal{O}}_{\hat{\Delta},s,r}(x_2^\parallel, u_2^\parallel, v_2)]\right\rangle = b_{k,\{\hat{\Delta},s,r\}} \frac{(u_1 \cdot n)^{k-r}(-2u_1 \cdot u_2^\parallel)^r (x_1^\perp \cdot v_2)^s}{|x_1^\perp|^{2k-\hat{\Delta}+s}\left[(x_1^\perp)^2 + (x_{12}^\parallel)^2\right]^{\hat{\Delta}}} . \tag{2.6}$$

There are two distinguished defect operators. In the case of the defect identity ($\hat{\Delta} = s = r = 0$), the correlators (2.6) reduce to a 1-point function (2.3) and $b_{k,\{0,0,0\}} \equiv a_k$. The other universal operator is the displacement supermultiplet, which arises from the broken symmetries in the presence of V. We denote it by $B[1]$ anticipating the notation for defect supermultiplets (see section 2.5), and its superprimary by $\hat{\mathcal{O}}_{B[1]}$. This supermultiplet is studied in details in [57], and the coefficient $b_{2,B[1]}$ was shown to be fixed by Ward identities to

$$\left\langle \Phi(x_1^{\parallel}, x_1^{\perp}, u_1) V[\hat{\mathcal{O}}_{B[1]}(x_2^{\parallel}, u_2^{\parallel})] \right\rangle = b_{2,B[1]} \frac{(u_1 \cdot n)(-2u_1 \cdot u_2^{\parallel})}{|x_1^{\perp}|^2 \left[(x_1^{\perp})^2 + (x_{12}^{\parallel})^2\right]^2}\,, \qquad b_{2,B[1]} = 2\sqrt{\frac{d}{c}}\,. \tag{2.7}$$

2.2 Kinematics for 2-point functions

We now turn to the correlators involving two bulk operators at points x_1 and x_2 and a defect V. The kinematics for these correlators are studied in [36] and reviewed below. Unlike the previous correlators, these correlators are not completely fixed by kinematics constraints and involve an arbitrary function of 3 cross-ratios $z, \bar{z}, \omega$.

To understand their origin, it is convenient to use the symmetries of the correlator to fix the position of the operators. Using the translations and the special conformal transformations preserved by the defect, one can show that there is a frame in which both $x_{1,2}^{\parallel} = 0$, so in that frame the only invariants under the residual symmetry are 2 linearly independent combinations built out of[1]

$$\frac{x_1^{\perp} \cdot x_2^{\perp}}{|x_1^{\perp}||x_2^{\perp}|}\,, \qquad \frac{|x_2^{\perp}|}{|x_1^{\perp}|}\,, \qquad \frac{(x_{12}^{\perp})^2}{|x_1^{\perp}||x_2^{\perp}|}\,. \tag{2.8}$$

We can recover the full conformal invariants by undoing the frame fixing, and it is convenient to take the 2 cross-ratios $z, \bar{z}$ to be (1.2) as defined in [38].

These have a nice interpretation. If we further fix $x_1^{\perp} = (1, 0, 0, 0)$ and $x_2^{\perp} = (x, y, 0, 0)$, then it's easy to show that (1.2) is solved by $z = x + iy$ and $\bar{z} = x - iy$, so in that frame $z, \bar{z}$ are interpreted as the location of one of the operators. In particular in Lorentzian kinematics both $z, \bar{z}$ are real and are interpreted as lightcone coordinates, see figure 3.

In addition to these spacetime cross-ratios, the correlator (1.3) admits many R-symmetry tensor structures, and they can be packaged as a sum over an R-symmetry cross-ratio σ

$$\sum_{j=0}^{2} F_j(z, \bar{z}) \sigma^j\,, \qquad \sigma = \frac{u_1 \cdot u_2}{(u_1 \cdot n)(u_2 \cdot n)}\,. \tag{2.9}$$

We can interpret this geometrically as follows. Polarisation vectors u take values in the projective space $\mathbb{P}^4$. The space of u subject to the tracelessness condition $u^2 = 0$ can then parametrised by coordinates y in $\mathbb{R}^3$

$$u = [y^i : \frac{1}{2}(1 - y^2) : \frac{i}{2}(1 + y^2)]\,. \tag{2.10}$$

Surface operators preserve an $\mathfrak{so}(4) \subset \mathfrak{so}(5)$, so they pick a direction n in projective space. We can choose n to be a direction in $\mathbb{R}^3$, so that V splits y into $\left\{y^{\parallel}, y^{\perp}\right\}$. As for the spacetime part we can act with preserved R-symmetry to set a frame where $y_{1,2}^{\parallel} = 0$, leaving only the invariant

$$\omega = \frac{|y_2^{\perp}|}{|y_1^{\perp}|}\,. \tag{2.11}$$

[1] Transverse rotations and dilatations preserve this frame and act on $x_1^{\perp}$, $x_2^{\perp}$.

JHEP07(2023)165

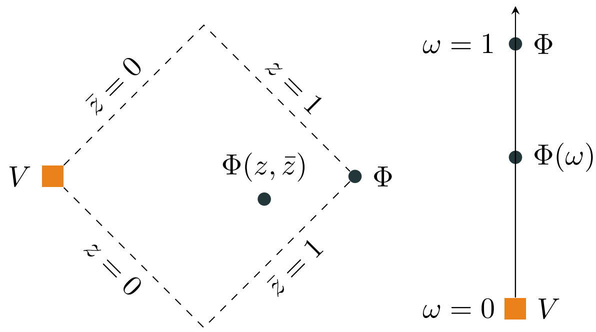

Figure 3. Interpretation for the cross-ratios $z, \bar{z}, \omega$ in lorentzian kinematics. Upon choosing an appropriate conformal frame, one can always bring the correlator (1.3) to the following kinematic configuration. On the left, we draw a plane transverse to V and containing both bulk operators. One can set V at the origin at fix the location of one Φ; $z, \bar{z}$ are then the coordinates of the remaining Φ. The dotted lines are the locations of the lightcones. On the right, we draw the transverse direction to V in R-symmetry space. We can again fix V at the origin and set the location of one Φ. The position of the other Φ is ω.

Comparing with (2.9) we can trade σ for ω using the relation

$$\sigma = \frac{(1-\omega)^2}{\omega} . \tag{2.12}$$

2.3 Superconformal Ward identities

In addition to the constraints from conformal and R-symmetry, the correlator (1.3) obeys constraints from supersymmetry. A nice way to derive them is to promote Φ to superfields taking values in superspace, so that the correlator also encodes the 2-point functions of all operators in the stress tensor multiplet.

In addition to the coordinates $x \subset \mathbb{R}^6$ and $y \subset \mathbb{R}^3$ (arising from realising geometrically the R-symmetry, see the discussion around (2.10)), we can introduce 8 Grassmann parameters $\theta_{\alpha a}$, where the spinor indices run over $\alpha = 1, \ldots, 4$ and $a = 1, 2$. Together, x, θ, y organise in the supercoordinates X of analytic superspace [17, 59]

$$X = \left(\begin{array}{c|c} x_{\alpha\beta} & \theta_{\alpha b} \\ \hline \theta_{a\beta} & y_{ab} \end{array} \right) \in GL(4|2) . \tag{2.13}$$

In this equation x is an antisymmetric matrix and y is symmetric. One can check that this supermatrix X satisfies "graded antisymmetry" with respect to the supermatrix Σ, which is the constraint (X^{st} is the supertranspose)

$$X^{st} = -X\Sigma , \qquad \Sigma = \left(\begin{array}{c|c} \mathbb{1}_4 & 0 \\ \hline 0 & -\mathbb{1}_2 \end{array} \right) . \tag{2.14}$$

With this property, X can be shown to parametrise the superspace $\mathrm{OSp}(8^*|4)/G$, with G the stabiliser of Φ (see [17] for more details).

Surface operators also have a simple description in this superspace similar to the one presented in [37] for defects in four dimensional $\mathcal{N} = 4$ SCFTs. The operator V picks a plane in $\mathbb{R}^6$ and a direction in $\mathbb{R}^3$, for definiteness we can take them along x^1, x^2 and y^1. Then V naturally defines the supermatrix

$$\Pi = \left(\begin{array}{c|c} i\gamma_{12} & 0 \\ \hline 0 & \rho_1 \end{array}\right) . \tag{2.15}$$

This decomposes X into $X_\parallel$ and $X_\perp$ according to (anti)symmetry under

$$X = X_\parallel + X_\perp\,, \qquad (\Pi X_\parallel)^{st} = -\Sigma\Pi X_\parallel\,, \qquad (\Pi X_\perp)^{st} = \Sigma\Pi X_\perp\,. \tag{2.16}$$

We are then interested in uplifting the 2-point function (1.3) to superspace. The correlator is constrained to be a function of superconformal invariants, and as for the case of 4-point functions of Φ discussed in [59], a simple counting argument shows that there are no superconformal invariants built out of fermionic coordinates θ's only. This is because our 2-point function depends on 16 fermionic coordinates (θ for both X_1 and X_2), and correspondingly the defect preserves 16 supercharges. So there are enough preserved supersymmetries to fix a frame where all the θ's vanish. In turn this implies that the 2-point function of all the superdescendants is uniquely fixed in terms of the function $\mathcal{F}$ introduced in (1.3) by finding the appropriate superspace extension to the cross-ratios $z, \bar{z}, \omega$.

To find the superspace cross-ratios, we can use the previous strategy and again fix a frame where $X_1^\parallel = X_2^\parallel = 0$. For some appropriate choice of basis for the gamma matrices, $X_\perp$ takes the form

$$X_\perp = \left(\begin{array}{cc|cc} 0 & x_\perp & 0 & \theta_\perp \\ -x_\perp^T & 0 & \eta_\perp & 0 \\ \hline 0 & \eta_\perp^T & 0 & y_\perp \\ \theta_\perp^T & 0 & y_\perp & 0 \end{array}\right) . \tag{2.17}$$

The cross-ratios are then given by the superconformal invariants built out of 2 points, which are the eigenvalues of $X_1 X_2^{-1}$. Let us call them $Z, \bar{Z}$ and Ω. If we set $\theta, \eta = 0$ it is simple to check that they match the definitions for $z, \bar{z}, \omega$ introduced in (1.2).

When $\theta, \eta \neq 0$ these eigenvalues receive corrections. To find them, we consider the 3 identities

$$\frac{1}{2}\,\mathrm{str}\left(X_1 X_2^{-1}\right)^n = Z^n + \bar{Z}^n - \Omega^n\,, \qquad n = 1, 2, 3\,. \tag{2.18}$$

For our purposes it is sufficient to take X_2 purely bosonic. Solving these equations to first order in θ, η we find (with $\pi_\pm$ some projector satisfying $\pi_+ + \pi_- = 1$; in the frame of section 2.2 they are $\pi_+ = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right)$ and $\pi_- = \left(\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}\right)$)

$$\begin{aligned} Z &= z - \frac{(\eta^T \pi_+ \theta)}{z - \omega} + \dots\,, \qquad \bar{Z} = \bar{z} - \frac{(\eta^T \pi_- \theta)}{\bar{z} - \omega} + \dots\,, \\ \Omega &= \omega - \frac{\eta^T \pi_+ \theta}{z - \omega} - \frac{\eta^T \pi_- \theta}{\bar{z} - \omega} + \dots \end{aligned} \tag{2.19}$$

Expanding $\mathcal{F}(Z, \bar{Z}, \Omega)$ in fermionic coordinates, we then get

$$F(Z, \bar{Z}, \Omega) - F(z, \bar{z}, \omega) = -\left(\partial_Z F|_z + \partial_\Omega F|_\omega\right) \frac{(\eta^T \pi_+ \theta)}{z - \omega} - \left(\partial_{\bar{Z}} F|_{\bar{z}} + \partial_\Omega F|_\omega\right) \frac{(\eta^T \pi_- \theta)}{\bar{z} - w} + \dots \tag{2.20}$$

This has unphysical poles at $z = \omega$ and $\bar{z} = \omega$. Indeed the correlator should be well-defined at these points, and requiring the absence of singularities leads to the superconformal Ward identities[2] (1.4).

2.4 Superconformal block expansion: bulk channel

Consider evaluating the correlator (1.3) using the OPE of the two bulk operators $\Phi\Phi$. The bulk operators that appear in this OPE are symmetric traceless tensors of dimension Δ, spin ℓ and R-symmetry spin R. As we review in appendix A, for each of them we can calculate a conformal block that encodes their contribution to $\mathcal{F}$. These conformal blocks consist of two parts. First, the spacetime dependence is encoded in a conformal block $g^{st}_{\Delta,\ell}(z,\bar{z})$ expressed in terms of the conformal blocks $g^{a,b}_{\Delta,\ell}$ for 4-point functions of local operators in 4d [63]

$$g^{st}_{\Delta,\ell}(z,\bar{z}) = \frac{(1-z)(1-\bar{z})}{1-z\bar{z}} g^{0,0}_{\Delta-1,\ell+1}(1-z, 1-\bar{z})\,. \tag{2.21}$$

The expression for $g^{(a,b)}_{\Delta,\ell}(z,\bar{z})$ is given in (A.14). The surprising appearance of 4d conformal blocks in the context of surface operators in 6d is part of a set of relations between conformal blocks uncovered and explained in [64].

Second, the R-symmetry dependence takes the form of a conformal block $h_R(\omega)$ and is given in terms of the Legendre polynomials $P_R(x)$

$$h_R(\omega) = \frac{R!(R+1)!}{(2R+1)!}\left(\frac{1-\omega}{1+\omega}\right) P_{R+1}\left(\frac{1+\omega}{1-\omega}\right)\,. \tag{2.22}$$

Together they contribute to $\mathcal{F}$ as

$$g_{\Delta,\ell,R}(z,\bar{z},\omega) = g^{st}_{\Delta,\ell}(z,\bar{z}) h_R(\omega)\,. \tag{2.23}$$

The derivation of these conformal blocks along with their appropriate normalisation is reviewed in appendix A.

In a SCFT, operators organise into supermultiplets, and correspondingly the contributions from conformal blocks organise into superconformal blocks. For instance, the stress tensor multiplet (denoted $\mathcal{D}[2,0]$) contains the operators listed in table 1. The only operators that can contribute to the exchange are the superprimary Φ^{IJ} and the stress tensor $T^{\mu\nu}$, so the superconformal block is a linear combination

$$\mathcal{G}_{\mathcal{D}[2,0]} = g_{4,0,2} + \alpha g_{6,2,0}\,. \tag{2.24}$$

Imposing the superconformal Ward identity (1.4) fixes the parameter $\alpha = -\frac{3}{700}$, and plugging the explicit conformal blocks we get

$$\mathcal{G}_{\mathcal{D}[2,0]}(z,\bar{z},\omega) = -12\left[1 + \frac{1(1+z)(z-\omega)(z\omega-1)(\bar{z}-1)^2}{2(1-z)(z-\bar{z})(z\bar{z}-1)(\omega-1)^2}\log(z) - (z \leftrightarrow \bar{z})\right]\,. \tag{2.25}$$

We can read the contribution to F, ζ (1.5) by taking the limits $\omega \to \bar{z}$ and $\omega \to 1$

$$\zeta_{\mathcal{D}[2,0]}(z) = -\frac{12z^2}{(1-z)^4}\left[1 + \frac{1}{2}\frac{1+z}{1-z}\log z\right]\,, \qquad F_{\mathcal{D}[2,0]} = 0\,. \tag{2.26}$$

Primary	Representation
$T^{\mu\nu}$	$[2,0,0]_6^{(0,0)}$
J^{μ}	$[1,1,0]_{11/2}^{(0,1)}$
j^{μ}	$[1,0,0]_5^{(0,2)}$
$H^{\mu\nu\rho}$	$[0,2,0]_5^{(0,1)}$
χ^I	$[0,1,0]_{9/2}^{(1,1)}$
Φ^{IJ}	$[0,0,0]_4^{(2,0)}$

Table 1. Content of the stress tensor multiplet derived in [65]. We specify the representations by their Dynkin labels as $[j_1, j_2, j_3]_\Delta^{(k_1,k_2)}$. Here $[j_1, j_2, j_3]$ denotes the Dynkin labels for $\mathfrak{so}(6)$ Lorentz symmetry; the spin l representations are $[0, l, 0]$. (k_1, k_2) denote the $\mathfrak{so}(5)$ Dynkin labels; the spin R representations are $(R, 0)$.

The superblocks that can contribute to the 2-point function of 1/2 BPS operators Φ_k are constrained by selection rules. These were derived in [16, 17, 66, 67], see also [11]. For two stress tensor superprimaries ($k = 2$) they read

$$\begin{aligned}\mathcal{D}[2,0] \times \mathcal{D}[2,0] = \mathbb{1} + \mathcal{D}[4,0] + \mathcal{D}[2,0] + \mathcal{D}[0,4] \\ + \sum_{\ell=0,2,\ldots} \mathcal{B}[2,0]_\ell + \mathcal{B}[0,2]_{\ell+1} + \sum \mathcal{L}[0,0]_{\Delta,\ell}\,.\end{aligned} \tag{2.27}$$

For the supermultiplets appearing on the right we follow the notation from [11]. The first letter indicates the shortening condition, while $[k_1, k_2]$ specify the $\mathfrak{so}(5)$ representation of the superprimary.

The representations appearing on the right are all those allowed by representation theory ($\mathcal{B}[0,0]_\ell$ multiplets are also allowed on the grounds of representation theory but they contain higher spin conserved currents). Out of these only $\mathbb{1}$, $\mathcal{D}[2,0], \mathcal{D}[4,0], \mathcal{B}[2,0]_\ell$ and $\mathcal{L}[0,0]$ have symmetric traceless tensors that can acquire an expectation value and contribute to the correlator. We derive their superblocks in appendix A and collect the results in table 2.

We note that the relation between $g^{st}_{\Delta,\ell}$ and the 4d conformal blocks $g^{(a,b)}_{\Delta,\ell}$ also extends to a relation between the superconformal blocks of table 2 and the superconformal blocks of local operators in 4d $\mathcal{N} = 2$ theories [68]. The relation to $\mathcal{N} = 2$ blocks in particular is natural because the latter depend on a single R-symmetry cross-ratio. Comparing our table 2 with their table 4, we see that the superconformal blocks share the same structure. It would be interesting to understand how the relations of [64] extend more generally to superconformal blocks for arbitrary defects and bulk operators.

2.5 Superconformal block expansion: defect channel

A second way to evaluate the 2-point function (1.3) is to use the defect operator expansion (dOE). This expresses bulk operators in terms of defect operator insertions $\hat{\mathcal{O}}$ on the

[2]This strategy has been originally applied in [59] and later in a variety on examples, see e.g. [37, 60–62].

Multiplet	Δ	R	$\mathcal{G}(z,\bar{z},\omega)$	$\zeta(z)$	$F(z,\bar{z})$
$\mathbb{1}$	0	0	1	$\frac{z^2}{(1-z)^4}$	0
$\mathcal{D}[2,0]$	4	2	(2.24)	$\frac{z^2}{(1-z)^4}k_4(1-z)$	0
$\mathcal{D}[4,0]$	8	4	(A.18)	$\frac{z^2}{(1-z)^4}k_8(1-z)$	$\frac{(z\bar{z})^2}{(1-z)^4(1-\bar{z})^4}g^{st}_{8,0}(z,\bar{z})$
$\mathcal{B}[2,0]_\ell$	$8+\ell$	2	(A.19)	$-\frac{z^2}{(1-z)^4}k_{2(\ell+6)}(1-z)$	$-\frac{(z\bar{z})^2}{(1-z)^4(1-\bar{z})^4}g^{st}_{\ell+10,\ell+2}(z,\bar{z})$
$\mathcal{L}[0,0]_{\Delta,\ell}$	$\Delta>6+\ell$	0	(A.20), (A.21)	0	$\frac{(z\bar{z})^2}{(1-z)^4(1-\bar{z})^4}g^{st}_{\Delta+4,\ell}(z,\bar{z})$

Table 2. List of supermultiplets contributing to the correlator, along with Δ, R for the superprimary. The conformal blocks entering each superblock are detailed in the appendix A.3. We also list the contributions of each superblock to ζ and F, with $k_{2h}(z)$ defined in (A.15) and $g^{st}_{\Delta,\ell}(z,\bar{z})$ defined in (2.21).

defect V. To see which defect operators can enter the dOE, recall that defect operators $\hat{\mathcal{O}}$ transform under the symmetry $\mathfrak{so}(2,2)\times\mathfrak{so}(4)\times\mathfrak{so}(4)$ preserved by the plane V. Their representations are labelled as $[s,r]_h[\bar{s},\bar{r}]_{\bar{h}}$, where $s,r,\bar{s},\bar{r}$ are $\mathfrak{su}(2)$ labels and $h,\bar{h}$ are $\mathfrak{sl}(2)$ labels. Their conformal dimension is $\hat{\Delta}=h+\bar{h}$, and their 2d spin is $h-\bar{h}$.

Consider then the correlator

$$\left\langle \Phi_k(x_1,y_1)V[\hat{\mathcal{O}}(x_2^{\parallel},y_2^{\parallel})]\right\rangle , \tag{2.28}$$

where Φ_k is the 1/2-BPS primary and $\hat{\mathcal{O}}$ is a defect operator in an arbitrary representation (we suppress the additional indices). First, notice that we can choose a frame where $x_1^{\parallel}=x_2^{\parallel}=0$, idem for $y^{\parallel}$, so there is no quantity that carries 2d spin. This means that the only defect operators $\hat{\mathcal{O}}$ that can have a nonzero correlator (2.28) are those with

$$h=\bar{h}=\frac{\hat{\Delta}}{2}\,,\qquad r=\bar{r}\,. \tag{2.29}$$

A second constraint is that the only quantity that transforms under the $\mathfrak{so}(4)$ acting on the transverse space is $x_1^{\perp}$. Therefore the only representations that can arise are the symmetric traceless, which are those for which

$$s=\bar{s}\,. \tag{2.30}$$

We label these defect operators as $\hat{\mathcal{O}}_{\hat{\Delta},s,r}$, and their correlator with bulk operators Φ_k are given in (2.6).

From these selection rules we can write a dOE for $\Phi_k V$

$$\Phi_k(x,y)V=\sum_{\{\hat{\Delta},s,r\}} b_{k,\{\hat{\Delta},s,r\}}\frac{|y^{\perp}|^{k-r}}{|x^{\perp}|^{2k-\hat{\Delta}}}C_{k,\{\hat{\Delta},s,r\}}V[\hat{\mathcal{O}}_{\hat{\Delta},s,r}(x^{\parallel},y^{\parallel},v)]\,, \tag{2.31}$$

where $C_{k,\{\hat{\Delta},s,r\}}$ are differential operators encoding the contribution from the descendants.

Acting twice with the dOE inside the correlator (1.3) gives a sum of 2-point functions of defect operators $\hat{\mathcal{O}}_{\hat{\Delta},s,r}$. As we review in appendix B, for each of them we can calculate a conformal block $\hat{g}$ that encodes their contribution to $\mathcal{F}$, we find

$$\hat{g}_{\hat{\Delta},s,r}(z,\bar{z},\omega)=(-1)^r\frac{(z\bar{z})^{\frac{\hat{\Delta}-s}{2}}}{(1-z\bar{z})}\frac{z^{s+1}-\bar{z}^{s+1}}{z-\bar{z}}\frac{\omega^{r+1}-\omega^{-(r+1)}}{\omega-\omega^{-1}}\,. \tag{2.32}$$

JHEP07(2023)165

Multiplet	$\hat{\Delta}$	r	$\hat{\mathcal{G}}$	$\zeta(z)$	$F(z,\bar{z})$
$\mathbb{1}$	0	0	1	1	1
$B[1]$	2	1	(B.12)	$-z$	$-z\bar{z}\left[1+\frac{(1-z)(1-\bar{z})}{1-z\bar{z}}\right]$
$B[2]$	4	2	(B.12)	z^2	$(z\bar{z})^2\left[1+\frac{2(1-z)(1-\bar{z})}{1-z\bar{z}}\right]$
$A[0]_s$	$2+s$	0	(B.13)	$-z^{s+2}$	$\frac{\bar{z}z^{s+2}(1-z)^2-z\bar{z}^{s+2}(1-\bar{z})^2}{(z-\bar{z})(1-z\bar{z})}$
$A[1]_s$	$4+s$	1	(B.14)	z^{s+3}	$-\frac{\bar{z}^2(\bar{z}-2)z^{s+3}(1-z)^2-z^2(z-2)\bar{z}^{s+3}(1-\bar{z})^2}{(z-\bar{z})(1-z\bar{z})}$
$L[0]_s$	$\hat{\Delta}>2+s$	0	(B.15)	0	$(z\bar{z})^{\frac{\hat{\Delta}+s}{2}}\frac{(z-1)^2(\bar{z}-1)^2}{1-z\bar{z}}\frac{z^{s+1}-\bar{z}^{s+1}}{z-\bar{z}}$

Table 3. List of defect supermultiplets contributing to the correlator. $\hat{\Delta}, r$ denote the conformal dimension and R-symmetry charge of the superprimary of the supermultiplet. For each superconformal block, its content in terms of conformal blocks is written in appendix B.3, and we write its contribution to ζ, F.

As for the bulk channels, defect operators also organise into supermultiplets, this time of the $\mathfrak{osp}(4^*|2)\oplus\mathfrak{osp}(4^*|2)$ superalgebra. The corresponding unitary supermultiplets were obtained in [57, 69, 70]. The supermultiplets that can appear in the dOE are those which contain defect operators with at most $r\leq 2$, in order to respect the decomposition (2.9) of $\mathcal{F}$. The list of superconformal blocks that correspond to these selection rules and can contribute to the correlator are given in table 3.

Note that at the unitarity bound $\hat{\Delta}\to 2+s$, the long multiplets decompose according to the recombination rules [70]

$$\begin{aligned} L[0]_{s=0}|_{\Delta\to 2} &= A[0]_0 + B[2]\,, \\ L[0]_{s\geq 1}|_{\Delta\to 2+s} &= A[0]_s + A[1]_{s-1}\,. \end{aligned} \tag{2.33}$$

Correspondingly, we find that the superconformal blocks obey the identities

$$\begin{aligned} \hat{\mathcal{G}}_{L[0]_0}|_{\Delta=2} &= \hat{\mathcal{G}}_{A[0]_0} + \hat{\mathcal{G}}_{B[2]}\,, \\ \hat{\mathcal{G}}_{L[0]_s}|_{\Delta=2+s} &= \hat{\mathcal{G}}_{A[0]_s} + \hat{\mathcal{G}}_{A[1]_{s-1}}\,. \end{aligned} \tag{2.34}$$

These recombination rules suggest that the multiplet $B[2]$ is the analytic continuation of $A[1]_s$ to $s=-1$. Indeed, we find that the superconformal blocks satisfy $\hat{\mathcal{G}}_{B[2]}=\hat{\mathcal{G}}_{A[1]_{-1}}$, and furthermore $\hat{\mathcal{G}}_{B[1]}=-\hat{\mathcal{G}}_{A[1]_{-2}}$, $\hat{\mathcal{G}}_{\mathbb{1}}=-\hat{\mathcal{G}}_{A[0]_{-2}}$.

3 Chiral algebra

We now show how to obtain exact dCFT data and calculate $\zeta(z)$ directly from the chiral algebra.

3.1 Reminder of the chiral algebra map for local operators

As shown in [9], to any 6d $\mathcal{N}=(2,0)$ superconformal field theory one can associate a chiral algebra (or VOA) by passing to the cohomology of a certain nilpotent supercharge. Under this cohomological reduction χ, representations of $\mathfrak{osp}(8^*|4)$ are mapped into representations of $\mathfrak{sl}(2)$. For the representations appearing in this work, this map gives (see table 1 in [9] for the complete list)

$$\chi \quad : \quad \mathcal{D}[k,0] \mapsto v_k\,, \qquad \mathcal{B}[k_1,k_2]_\ell \mapsto v_{k_1+k_2+4+\ell}\,, \qquad \mathcal{L}[k_1,k_2]_{\Delta,\ell} \mapsto 0\,, \tag{3.1}$$

where v_h denotes the highest weight representation of $\mathfrak{sl}(2)$ with highest weight h. This fact has a manifestation at the level of the conformal blocks in the bulk channel, see the fifth column of table 2 where one recognizes $k_{2h}(1-z)$ as the $\mathfrak{sl}(2)$ conformal blocks for the exchange of an operator of weight h. It is clear that the map (3.1) cannot be inverted as different $\mathfrak{osp}(8^*|4)$ representations map to the same representation v_h.

There is more structure to the map χ, and also the OPEs of the 6d theory reduce to OPEs for operators associated with the representations v_h, giving rise to a VOA. In the case of the $(2,0)$ theories of type A_{N-1}, the associated VOA is (conjecturally) the well-known $\mathcal{W}_N$ algebra, where the central charge takes the value c given in (1.9) (see [71] for a pedagogical introduction to $\mathcal{W}$-algebras and [72] for a review).[3]

The $\mathcal{W}_N$ algebra is generated by primary operators $W_p(z)$ $(p=2,3,\dots,N)$ of conformal weight $h=p$, where $W_2(z)=T(z)$ is the stress tensor. Under the map χ, the half-BPS operators that generate the chiral ring map to the generators of the $\mathcal{W}$-algebra as

$$\chi : \Phi_k(x,y) \mapsto W_k(z)\,, \tag{3.2}$$

up to a factor coming from the relative normalisation of Φ_k and W_k.

At large N, the operators Φ_k have a simple interpretation in holography as the "single-trace" Kaluza-Klein modes on S^4, and the spectrum of local operators is freely generated by these modes along with their derivatives. Correspondingly, at large N the Hilbert space $\mathcal{H}$ of the VOA is freely generated by $W_k(z)$ and their derivatives, as can be seen from the (unrefined) superconformal index [9, 73, 74]

$$\mathcal{I}_{A_{N-1}}(q) = \mathrm{Tr}_{\mathcal{H}}\, q^{L_0} = q^{-\frac{c}{24}} \mathrm{PE}\left(\frac{q^2+q^3+\cdots+q^N}{1-q}\right)\,, \tag{3.3}$$

with L_0 the generator of the Virasoro algebra.

The structure constants appearing in the 3-point functions of Φ_k (2.1) are captured by the VOA and can be calculated exactly at any N. Restricting the points x_i to lie on a plane (arbitrary but fixed, referred to as the *chiral algebra plane*) with coordinate $(z_i,\bar{z}_i)$ and taking the R-symmetry variables y_i to lie on a line with coordinate ω_i, one finds that upon setting $\omega_i=\bar{z}_i$ the correlator (2.1) reduces to

$$\langle \Phi_{k_1}(z_1,\bar{z}_1;\bar{z}_1)\Phi_{k_2}(z_2,\bar{z}_2;\bar{z}_2)\Phi_{k_3}(z_3,\bar{z}_3;\bar{z}_3)\rangle = \frac{\lambda_{k_1k_2k_3}}{(z_{12})^{k_{123}}(z_{13})^{k_{132}}(z_{23})^{k_{231}}}\,, \tag{3.4}$$

with the short-hand notations $z_{ij}=z_i-z_j$, $k_{ijk}=k_i+k_j-k_k$, and $\Phi_p(z,\bar{z};\bar{z})=\Phi_p(x^{\parallel}=0, x^{\perp}=\mathrm{diag}(z,\bar{z}), y^{\parallel}=0, y^{\perp}=\bar{z})$. The twisted operators $\Phi_k(z,\bar{z},\bar{z})$ sit in the cohomology

[3]This $\mathcal{W}_N$ algebra can be constructed from the corresponding current algebra using the Drinfel'd-Sokolov construction. $\mathcal{W}$-algebras also exist for the D_N and $E_{6,7,8}$ current algebras, and the proposal of [9] extends to these theories as well.

JHEP07(2023)165

used to define the VOA, and using the map χ the structure constants $\lambda_{k_1k_2k_3}$ can be identified with the corresponding structure constants of the VOA. As pointed out in [9], using results from [75] one can reproduce the supergravity results of [76] (see also [77]) at leading order at large N, which read

$$\lambda_{k_1k_2k_3} = \frac{2^{k_1+k_2+k_3-2}}{(\pi N)^{3/2}}\Gamma\left(\frac{k_1+k_2+k_3}{2}\right)\frac{\Gamma\left(\frac{k_{123}+1}{2}\right)\Gamma\left(\frac{k_{231}+1}{2}\right)\Gamma\left(\frac{k_{312}+1}{2}\right)}{\sqrt{\Gamma(2k_1-1)\Gamma(2k_2-1)\Gamma(2k_3-1)}}+\dots. \tag{3.5}$$

An example: as an example let us recall how to compute the 4-point function of the stress tensor T from the singular part of the OPE:

$$T(z)T(w) = \frac{c/2}{(z-w)^4}+\frac{2T(w)}{(z-w)^2}+\frac{\partial T(w)}{(z-w)}+\dots \tag{3.6}$$

We consider the 4-point function

$$\langle T(z_1)T(z_2)T(z_3)T(z_4)\rangle\,, \tag{3.7}$$

and think of it as a function of z_2 with the other point fixed. This is a meromorphic function, whose poles arise when z_2 approaches any of the other three points and its residues are related to lower point correlators of T by (3.6). This information is enough to completely determine the function, and following this strategy one finds that

$$\frac{\langle T(0)T(z)T(1)T(\infty)\rangle}{\langle T(0)T(z)\rangle\langle T(z)T(1)\rangle} = 1+z^4+\left(\frac{z}{z-1}\right)^4+\frac{8}{c}\left(\frac{z}{z-1}\right)^2(z^2-z+1)\,. \tag{3.8}$$

This function can be expanded in $\mathfrak{sl}(2)$ blocks as

$$\frac{\langle T(0)T(z)T(1)T(\infty)\rangle}{\langle T(0)T(z)\rangle\langle T(z)T(1)\rangle} = 1+\frac{8}{c}\,k_2(z)+\sum_{n=0}^{\infty}\lambda_n^2\,k_{2h_n}(z)\,, \tag{3.9}$$

where $k_{2h}(z) = z^h\,{}_2F_1(h,h,2h;z)$, $h_n = 2n+4$ and the structure constants are

$$\lambda_n^2 = \frac{(2n+6)!(2n+3)!(2n+1)(n+1)}{18(4n+5)!}+\frac{8}{c}\frac{(2n+3)!(2n+2)!(4n^2+14n+11)}{(4n+5)!}\,. \tag{3.10}$$

It is instructive to identify the operators that are exchanged in (3.9) and write

$$T(z_1)T(z_2) = \frac{c/2}{z_{12}^4}+\frac{2}{z_{12}^2}\mathcal{D}_{2,2;2}(z_{12},\partial_2)T(z_2)+\sum_{n=0}^{\infty}\frac{\lambda_{TT}{}^{[TT]_n}}{z_{12}^{4-h_n}}\mathcal{D}_{2,2;h_n}(z_{12},\partial_2)[TT]_n(z_2)\,, \tag{3.11}$$

where $\mathcal{D}_{2,2;h_n}$ are differential operators encoding the contributions from descendants; their explicit expression can be found in e.g. appendix A of [78] to which we also refer for further explanations. The expansion (3.11) is a completion of (3.6) to include all the regular terms where

$$[TT]_n(z) := \sum_{\ell=0}^{2n}(-1)^\ell\binom{2n}{\ell}\frac{(2n+3)!}{(\ell+3)!(2n-\ell+3)!}\mathrm{NO}[\partial^\ell T,\partial^{2n-\ell}T]-\frac{1}{4(n+1)(2n+5)}\partial^{2n+2}T\,, \tag{3.12}$$

where NO denotes normal ordering. With these conventions we can compute the norm and OPE coefficients to find

$$\langle [TT]_n(z_1)\,[TT]_m(z_2)\rangle = \frac{\delta_{n,m}\,g_n}{(z_1-z_2)^{2(2n+4)}}\,, \quad \lambda_{TT}{}^{[TT]_n} = \frac{(2n+1)(n+1)(2n+6)!}{(4n+5)!}\,, \tag{3.13}$$

and the coefficients λ_n^2 in (3.10) are recovered as

$$\lambda_n^2 = \frac{1}{(c/2)^2}\left(\lambda_{TT}{}^{[TT]_n}\right)^2 g_n\,. \tag{3.14}$$

3.2 Adding a surface defect

The VOA also captures surface operators V orthogonal to the chiral algebra plane, since the $\mathfrak{osp}(4^*|2)\oplus\mathfrak{osp}(4^*|2)$ subalgebra preserved by V contains the supercharge used to define the cohomology. This is very similar to the case of surface defects in four dimensional $\mathcal{N}=2$ SCFTs considered in [58, 79] (see also [80]): the defect V intersects the chiral algebra plane at $z=0$ and $z=\infty$, and under the map χ corresponds to inserting two vertex operators $\mathsf{V}_\Lambda(0)$ and $\bar{\mathsf{V}}_\Lambda(\infty)$ at these points, which defines a module of the associated W-algebra.

The fate of defect operators $V[\hat{\mathcal{O}}]$ under the chiral algebra map presents some important differences compared to the case of the bulk operators, which are related to the fact that these operators are bound to V and therefore cannot be translated away from the origin (or infinity). While primary bulk operators are mapped to $\mathfrak{sl}(2)$ primaries, defect primary operators can be mapped to $\mathfrak{sl}(2)$ descendants. The simplest but very important example is given by the displacement operator which, as we will see below, is mapped to $\partial\mathsf{V}_\Lambda(0)$ (or equivalently $L_{-1}\,|\mathsf{V}_\Lambda\rangle$).

The cohomological reduction carries over to representations of $\mathfrak{osp}(4^*|2)\oplus\mathfrak{osp}(4^*|2)$. In this work we consider only representations with same left/right quantum numbers (they satisfy (2.29) and (2.30)), and for these the map χ gives

$$\chi \quad : \quad B[r]\mapsto s_r\,, \qquad A[r]_s\mapsto s_{r+s+2}\,, \qquad L[r]_s\mapsto 0\,, \tag{3.15}$$

where s_d denotes a one dimensional representation of $\mathbb{C}^*$ with weight d. The weights d can be read from table 3.

$\mathcal{W}$ modules. Modules for the W-algebras introduced in section 3.1 have been extensively studied in the literature. The ones relevant to surface operators V are a special class of the so-called completely degenerate representations, see e.g. [81] (also [51, 82–84]), and are labelled by a single highest weight Λ.[4] The module is constructed by acting with the negative modes of the W-algebra generators on the highest weight vector $|\mathsf{V}_\Lambda\rangle$, which satisfies

$$L_n\,|\mathsf{V}_\Lambda\rangle = W_n^{(p)}\,|\mathsf{V}_\Lambda\rangle = 0\,, \qquad n>0\,, \qquad p=3,4,\ldots,N\,, \tag{3.16}$$

and is labelled by its weights under the W-algebra generators

$$L_0\,|\mathsf{V}_\Lambda\rangle = \Delta(\Lambda)\,|\mathsf{V}_\Lambda\rangle \quad W_0^{(p)}\,|\mathsf{V}_\Lambda\rangle = \omega_p(\Lambda)\;|\mathsf{V}_\Lambda\rangle\;, \qquad p=3,4,\ldots,N\,. \tag{3.17}$$

[4]The value of the central charge for the W-algebra in this work corresponds to setting $b=1$ in [81].

The eigenvalue of L_0 in particular is the conformal weight and is given by

$$\Delta(\Lambda) = -\frac{1}{2}(\Lambda,\Lambda) - 2\,(\rho,\Lambda) = -d(\Lambda)\,, \tag{3.18}$$

with d the anomaly coefficient introduced in (1.8). The appearance of the coefficient d supports the identification of $|\mathsf{V}_\Lambda\rangle$ as the module associated to the surface operator V with representation Λ, and remarkably indicates that the entire dependence on d is captured by the chiral algebra.

The expressions for $\omega_p(\Lambda)$ are obtained from a free field realization of the $\mathcal{W}_N$ algebra in [85].[5] For example, with their choice of normalisation for W_3, $w_3(\Lambda)$ reads

$$\begin{aligned} w_3(\Lambda) = &\sum_{1\leq i_1<i_2<i_3\leq N} (\Lambda+2\rho,\epsilon_{i_1})(\Lambda+2\rho,\epsilon_{i_2})(\Lambda+2\rho,\epsilon_{i_3}) \\ &+ 2(N-1)\sum_{1\leq i_1<i_2\leq N}(\Lambda+2\rho,\epsilon_{i_1})(\Lambda+2\rho,\epsilon_{i_2}) + 8\binom{n+2}{4}\,, \end{aligned} \tag{3.19}$$

where ϵ_i are the weights of the fundamental representation of $\mathfrak{su}(N)$. Taking for example the totally symmetric representation $\Lambda = [M,0,\dots]$, the expression above reduces to

$$w_3(M) = \frac{(M-N)(N-1)}{3N^2}\Big(M^2(N-2) + MN(N-5) - 3N^2(N+1)\Big)\,. \tag{3.20}$$

Let us look more closely at the structure of these modules. Their character is captured by the surface index computed in [74][6] (up to a factor $q^{-\frac{c}{24}}$). It can be written as a plethystic exponential

$$\mathcal{I}_\Lambda = q^{\Delta(\Lambda)-\frac{c}{24}}\,\mathrm{PE}\left(\frac{(N-1)q}{1-q} - \sum_{\alpha>0} q^{(\rho+\Lambda,\alpha)}\right), \qquad \mathrm{PE}\left(f(q)\right) \equiv \exp\left(\sum_{k=1}^{\infty}\frac{f(q^k)}{k}\right), \tag{3.21}$$

where the sum runs over the positive roots. Setting $\Lambda = 0$ we recover the superconformal index without defects given in (3.3). For the totally symmetric representations $\Lambda = [M,0,\dots]$, the index above reduces to

$$\mathcal{I}_\Lambda = q^{\Delta(\Lambda)-\frac{c}{24}}\,\mathrm{PE}\left(\frac{q^{N+M} - q^{M+1} + \sum_{k=1}^{N-1} q^k}{1-q}\right)\,. \tag{3.22}$$

The structure of these modules at large N (for any M) is encoded in this formula

$$\mathrm{PE}\left(\frac{q^{N+M} - q^{M+1} + \sum_{k=1}^{N-1} q^k}{1-q}\right) = \mathrm{PE}\left(\frac{\sum_{k=1}^{N} q^k}{1-q}\right)\prod_{\ell=1}^{M}\frac{1}{1-q^\ell} + O(q^N)\,. \tag{3.23}$$

By looking at the index for more general Λ, we notice a puzzling feature: if we count how many states in the module have the same L_0 quantum number as the state associated to

[5] This free field realisation appears naturally since the $\mathcal{W}_N$ algebra is the chiral sector of Toda field theory which in turn can be obtained by compactifying the (2,0) theories on a 4-sphere, see [86, 87].

[6] The character of fully degenerate module and the surface index can be computed independently. The fact that they are equal provides strong evidence for the conjectured identification of the corresponding modules, see [74].

the displacement operator, namely $\Delta(\Lambda)+1$, we find that this number coincides with the number of non-zero entries in the weight Λ. This feature is not new and something similar happens for surface defects in four dimensions, see [79]. We postpone the analysis of these states from the point of view of the surface defect to the future.

The module structure can be translated in the language of OPEs

$$T(z)\mathsf{V}_\Lambda(0) \sim \frac{\Delta(\Lambda)}{z^2}\mathsf{V}_\Lambda(0) + \frac{2}{z}\,\partial\mathsf{V}_\Lambda(0)\,, \tag{3.24}$$

$$W_k(z)\mathsf{V}_\Lambda(0) \sim \frac{\omega_k(\Lambda)}{z^k}\mathsf{V}_\Lambda(0) + \frac{1}{z^{k-1}}\left(\frac{k\,\omega_k(\Lambda)}{2\,\Delta(\Lambda)}\partial\mathsf{V}_\Lambda(0) + \mathsf{V}_\Lambda^{(k)}(0)\right) + \dots \tag{3.25}$$

In this way we can make direct contact with the (cohomological reduction of the) defect OPE of the Φ_k half-BPS operators. We will now use these OPEs to determine certain quantities exactly in the 6d theory with the surface defect.

A correlator of the 1/2-BPS bulk operators Φ_k (with appropriate twist $\omega = \bar{z}$) in the presence of the surface defect V is given by

$$\langle \Phi_{p_1}(z_1,\bar{z}_1;\bar{z}_1)\dots\Phi_{p_n}(z_n,\bar{z}_n;\bar{z}_n)\,V_\Lambda\rangle \propto \frac{\left\langle \bar{\mathsf{V}}_\Lambda(\infty)W_{p_1}(z_1)\dots W_{p_n}(z_n)\mathsf{V}_\Lambda(0)\right\rangle}{\left\langle \bar{\mathsf{V}}_\Lambda(\infty)\mathsf{V}_\Lambda(0)\right\rangle}\,, \tag{3.26}$$

where we have used "$\propto$" instead of "$=$" due to different normalization conventions in the two sides (recall that on the left $\langle V_\Lambda\rangle = 1$ so the normalisation is omitted). On the left we write V_Λ to emphasize the dependence on the representation Λ. On the right $\bar{\mathsf{V}}_\Lambda$ denotes the conjugate of V_Λ.

Several generalizations of the correlator (3.26) can be computed in the chiral algebra. For example, we can insert (twisted translated) defect operators at zero or infinity, which corresponds to replacing $\mathsf{V}_\Lambda(0)$ and $\bar{\mathsf{V}}_\Lambda(\infty)$ in the right hand side with the appropriate descendants.

The simplest quantity of the type (3.26) is the one point function of the half-BPS operators Φ_k starting with $k=2$ which is the stress tensor. In this case, the OPEs (3.24) immediately allow to compute

$$\frac{\left\langle \bar{\mathsf{V}}_\Lambda(\infty)T(z)\mathsf{V}_\Lambda(0)\right\rangle}{\left\langle \bar{\mathsf{V}}_\Lambda(\infty)\mathsf{V}_\Lambda(0)\right\rangle} = \frac{\Delta(\Lambda)}{z^2}\,. \tag{3.27}$$

Dividing by $\sqrt{c/2}$ to account for the normalisation of the stress tensor and recalling the identity $\Delta(\Lambda) = -d(\Lambda)$ pointed out above, we recover precisely the coefficient a_2 (1.7)!

The next quantity that can be determined using the connection to the W-algebra is the 2-point function of the stress tensor superprimary with the displacement operator given in (2.7). Setting $x^{\parallel}_{1,2} = y^{\parallel}_{1,2} = 0$, $x^\perp = (z,\bar{z})$, $y^\perp = \bar{z}$, we get a holomorphic correlator

$$\left\langle \Phi(z,\bar{z};\bar{z})V[\hat{\mathcal{O}}_{B[1]}(0,0)]\right\rangle = \frac{b_{2,B[1]}}{z^3}\,. \tag{3.28}$$

In the chiral algebra, the insertion of the displacement operator at the origin is equivalent to replace $\mathsf{V}_\Lambda(0)$ in (3.26) with its descendant $\partial\mathsf{V}_\Lambda(0)$. Using the OPEs (3.24) one easily computes

$$\frac{\left\langle \bar{\mathsf{V}}_\Lambda(\infty) T(z) \partial \mathsf{V}_\Lambda(0) \right\rangle}{\left\langle \bar{\mathsf{V}}_\Lambda(\infty) \mathsf{V}_\Lambda(0) \right\rangle} = \frac{2\Delta(\Lambda)}{z^3}\,. \tag{3.29}$$

To reproduce the coefficient $b_{2,B[1]}$ given in (2.7) we have to divide this expression by the square root of the norm of T, namely $\sqrt{c/2}$, and the square root of the norm of $\partial\mathsf{V}_\Lambda(0)$, which is $\sqrt{-2\Delta(\Lambda)}$.[7] The result is $2\sqrt{-\Delta(\Lambda)/c}$ which matches (2.7).

The next quantity we consider is the 2-point function of Φ in the presence of the defect. In this case, the protected part $\zeta(z)$ of the correlator $\langle \Phi\Phi V\rangle$ is captured by the 4-point function

$$\frac{\langle \bar{\mathsf{V}}_\Lambda(0) T(z) T(1) \mathsf{V}_\Lambda(\infty)\rangle}{\langle \bar{\mathsf{V}}_{-\Lambda}(0)\mathsf{V}_\Lambda(\infty)\rangle\langle T(z)T(1)\rangle} = 1 + \frac{\Delta(\Lambda)^2}{c} 2Z^2 + \frac{\Delta(\Lambda)}{c} 4Z\,, \qquad Z := \frac{(z-1)^2}{z}\,. \tag{3.30}$$

More precisely the quantity (3.30) is equal to $Z^2\zeta(z)$. The expression on the right is obtained using similar methods as the one used to compute (3.8), namely by reconstructing the function from its poles in z and the OPE of $T(z)$.

We can identify the protected CFT data by expanding (3.30) in conformal blocks. In the bulk channel $z \sim 1$, the function can be expanded in $\mathfrak{sl}(2)$ blocks $k_{2h}(z)$

$$Z^2\zeta(z) = 1 + \frac{4\Delta(\Lambda)}{c}\, k_4(1-z) + \sum_{n=0}^{\infty} \alpha_n\, k_{2h_n}(1-z)\,, \tag{3.31}$$

where $h_n = 4 + 2n$ and

$$\alpha_n = \frac{(2n+2)!(2n+3)!}{(2n+3)!(4n+5)} \left((2n+5)\,\frac{\Delta(\Lambda)^2}{c} + \frac{1}{n+1}\,\frac{\Delta(\Lambda)}{c} \right). \tag{3.32}$$

The operators exchanged in the TT OPEs (singular and regular) entering (3.31) are the same as the one contributing to (3.9) and (3.11). Using the explicit expressions for the bilinears in the stress tensor in (3.12) and the OPEs (3.24) one finds

$$[TT]_n(z)\mathsf{V}_\Lambda(0) \sim \frac{u_n}{z^{h_n}}\, \mathsf{V}_\Lambda(0) + \dots\,, \qquad u_n = \frac{(2n)!\,\left((n+1)(2n+5)\Delta(\Lambda)^2 + \Delta(\Lambda)\right)}{(n+2)(n+3)(2n+5)}\,. \tag{3.33}$$

Inserting the explicit expression of $\lambda_{TT}{}^{[TT]_n}$ given in (3.13) we can check that

$$\alpha_n = \frac{\lambda_{TT}{}^{[TT]_n}\, u_n}{c/2}\,, \tag{3.34}$$

as it should. By comparing to the content of table 2 we can read off the "$a\lambda$" OPE coefficients to be

$$(a\lambda)_{\mathcal{D}[2,0]} = \frac{4\Delta(\Lambda)}{c}\,, \qquad (a\lambda)_{\mathcal{D}[4,0]} = \alpha_0\,, \qquad (a\lambda)_{\mathcal{B}[2,0]_{\ell=2n-2}} = -\alpha_n\,. \tag{3.35}$$

[7] The norm is computed by recalling that $\partial\mathsf{V}_\Lambda(0) \sim L_{-1}|\mathsf{V}_\Lambda\rangle$. By inspecting the defect OPE in 6d we obtain that the conjuguate is $-\langle \mathsf{V}_\Lambda|\, L_{+1}$. We obtain the norm by using the commutation relation $[L_{+1}, L_{-1}] = 2L_0$ and the properties $|\mathsf{V}_\Lambda\rangle$: $L_{+1}|\mathsf{V}_\Lambda\rangle = 0$ and $L_0|\mathsf{V}_\Lambda\rangle = \Delta\,|\mathsf{V}_\Lambda\rangle$.

Notice that for any central charge and for any $\Delta(\Lambda)$ the operator transforming in the $\mathcal{D}[4,0]$ representation being exchanged here is a composite operator which is orthogonal to Φ_4.

To obtain OPE data associated to the defect operator expansion (dOE) we need to expand the function $\zeta(z)$ in defect channel blocks (which are simply the monomials z^k) for z close to zero

$$\zeta(z) = 2\frac{\Delta(\Lambda)^2}{c} + \sum_{n=1}^{\infty} \beta_n \, z^n \,, \qquad \beta_n = n\left(\frac{1}{6}(n^2-1) + 4\frac{\Delta(\Lambda)}{c}\right) . \tag{3.36}$$

We can identify the coefficients b^2 in 6d from table 3 and obtain

$$b^2_{\mathbb{I}} = 2\frac{\Delta(\Lambda)^2}{c} \,, \qquad b^2_{\hat{B}[1]} = -\beta_1 \,, \qquad b^2_{\hat{B}[2]} - b^2_{\hat{A}[0]_0} = \beta_2 \,, \qquad b^2_{\hat{A}[1]_s} - b^2_{\hat{A}[0]_{s+1}} = \beta_{s+3} \,, \tag{3.37}$$

where $s = 0, 1, \ldots$. Notice that there is an ambiguity here since the chiral algebra map (3.15) is not invertible and the blocks z^n may originate from multiple defect blocks in 6d. However in section 4, we show that the supermultiplets of type $A[0]_s$ do not appear in this OPE. With this additional input we can identify the coefficients b^2 in 6d uniquely from the W-algebra. It would be interesting to understand if there is a deeper reason for the absence of these $A[0]_s$ multiplets. We leave the detailed study of the W-algebra, its modules and other exact computations, such as the protected part of correlators $\langle \Phi_{k_1} \Phi_{k_2} V_\Lambda \rangle$, for future work.

4 Obtaining the correlator

The superconformal block expansion (1.14) constructs the function $\mathcal{F}$ in terms of the coefficients b^2_{kl} and the spectrum of supermultiplets contributing to the correlator. The inverse relation is known as the (lorentzian) inversion formula [38, 39] and expresses the dCFT data in terms of the discontinuity of $\mathcal{F}$ at its branch cut $\bar{z} = 1$, defined as[8]

$$\operatorname{Disc} \mathcal{F}(z, \bar{z}, \omega) = \mathcal{F}(z, \bar{z}, \omega) - \mathcal{F}(z, \bar{z}, \omega)|_{(1-\bar{z}) \to e^{2\pi i}(1-\bar{z})} \,. \tag{4.1}$$

The inversion formula of [38] applies to defects of general dimension (and codimension) and doesn't take into account R-symmetry or supersymmetry, which are specific to each setup. To address the first we can apply the formula to each R-symmetry channel $\mathcal{F}|_r(z, \bar{z})$, which is the restriction of $\mathcal{F}$ to the contributions of defect conformal blocks (2.32) of a fixed representation r (for our correlator, r can take values $0, 1, 2$).

For every r, the inversion formula defines a function $B_r(\hat{\Delta}, s)$ constructed in such a way that it has poles at isolated values of $\hat{\Delta} = \hat{\Delta}_{m,s,r}$ whenever a block with these quantum numbers contributes to $\mathcal{F}|_r$. At each pole the residue gives the OPE coefficient

$$b^2_{\hat{\Delta}_{m,s,r},s,r} = -\operatorname{Res}_{\hat{\Delta} = \hat{\Delta}_{m,s,r}} B_r(\hat{\Delta}, s) \,. \tag{4.2}$$

This gives the conformal block decomposition of $\mathcal{F}$. There is a simple observation made in [19, 88] to reorganise these conformal blocks into superconformal blocks, thus

[8] More precisely the correlator is a distribution and the discontinuity should be understood in that sense. In particular rational functions can have discontinuities.

JHEP07(2023)165

ensuring that supersymmetry is preserved. From the explicit expression for defect channel superblocks (see appendix B.3) we observe that R-symmetry blocks with $r = 2$ and given labels $\hat{\Delta}, s$ appear only once in any given superconformal block, except for the superblocks associated to the identity, $B[1]$ and $A[0]_s$ operators, for which the $r = 2$ component is absent. This means that, from the conformal block decomposition of $\mathcal{F}|_{r=2}$ we can read the superconformal block decomposition of $\mathcal{F}$, up to the OPE coefficients of identity, $B[1]$ and $A[0]_s$ operators (more on that below). Explicitly, we define the function $\mathsf{B}(\hat{\Delta}, s)$ from the inversion formula [38] applied to $\mathcal{F}|_{r=2}$ as

$$\mathsf{B}(\hat{\Delta}, s) = B_{r=2}(\hat{\Delta}+2, s) = \int_0^1 \frac{\mathrm{d}z}{2} z^{-\frac{\hat{\Delta}-s}{2}-2} \int_1^{1/z} \frac{\mathrm{d}\bar{z}}{2\pi i}(1-z\bar{z})(\bar{z}-z)\bar{z}^{-\frac{\hat{\Delta}+s}{2}-3}\,\mathrm{Disc}\,\mathcal{F}|_{r=2}(z,\bar{z}). \tag{4.3}$$

The shift in $\hat{\Delta}$ is included so that for long supermultiplets (B.15), $\mathsf{B}(\hat{\Delta}, s)$ has poles at the location of the superprimary. For short and semishort multiplets the shift is different and can be treated separately, for instance the $A_s[1]$ multiplets correspond to poles of $\mathsf{B}(\hat{\Delta}, s+1)$ at $\hat{\Delta} = s+3$, see (B.14). $\mathcal{F}|_{r=2}$ can be expressed directly in terms of ζ, F by extracting the part of (1.5) proportional to the R-symmetry block with $r = 2$ (the Chebyshev polynomial of second kind $U_2(\frac{\omega+\omega^{-1}}{2})$), which gives

$$\mathcal{F}|_{r=2} = \frac{z\bar{z}}{(1-z)^2(1-\bar{z})^2}F(z,\bar{z}) + \frac{z\bar{z}}{(z-\bar{z})(1-z\bar{z})}\left(\frac{\bar{z}}{(1-\bar{z})^2}\zeta(z) - \frac{z}{(1-z)^2}\bar{\zeta}(\bar{z})\right). \tag{4.4}$$

Following [38, 39], we can evaluate the discontinuity of $\mathcal{F}$ by relying on the bulk channel decomposition of $\mathcal{F}$, which converges in the region of integration. Consider then calculating the contribution to the discontinuity from a single bulk superblock and applying the inversion formula to it. There is a contribution to the discontinuity if the block gives rise to a branch cut for F at $\bar{z} = 1$, or as shown in [38, 40] if it gives rise to a pole in $\mathcal{F}$.

Comparing with table 2 and using the $\bar{z} = 1$ expansion of g^{st}

$$g^{st}_{\Delta,\ell}(z,\bar{z}) = (1-\bar{z})^{\frac{\Delta-\ell}{2}} k_{\Delta+l}(1-z) + \dots \tag{4.5}$$

it's easy to see that branch cuts can only arise from long operators with anomalous dimensions. At large N the exchanged operators are "double-traces" and their anomalous dimension is suppressed by a factor c^{-1}, see (1.13). Since long multiplets enter the correlator at order d^2/c, they would contribute to branch cuts at order d^2/c^2, so do not contribute to the order we consider in our calculation. This suppression of double-trace operators is a general feature of the inversion formula at large N and means the correlator is completely fixed in terms of the exchange of single-trace operators [39].

Superblocks may also lead to a pole in $\mathcal{F}$, either from a pole in $F(z, \bar{z})$ when the superprimary satisfies $\Delta - \ell < 8$, or from a pole in $\zeta(z)$. Again comparing to table 2 we find that the only superblocks which may contribute to the discontinuity by this mechanism are the (bulk) identity and the stress tensor superblock.

This observation was translated into a concrete bootstrap strategy for the 2-point function of stress tensors in the presence of a Wilson line in [40], and we adapt it in the following.

From the discontinuity of the superblocks for the bulk identity and stress tensor and the inversion formula (4.3), we calculate the dCFT data entering the superblock decomposition of $\mathcal{F}$, up to the defect identity, $B[1]$ and $A[0]_s$ blocks not captured by the inversion formula. The contributions from the defect identity and displacement supermultiplet $B[1]$ are fixed by the correlators (1.7) and (2.7) respectively which encode the choice of d and c so they are unambiguous. The contributions from the $A[0]_s$ multiplets can be inferred from the chiral algebra once the contribution from the $A[1]_s$ multiplets is known and we find that they are absent; alternatively, we can check that crossing symmetry is satisfied without adding $A[0]_s$ multiplets. Resumming all these blocks we obtain the correlator.

There is one more subtlety. The inversion formula may miss the contribution of defect multiplets with low-spins, so not reconstruct the full correlator. Using the variables $z = rw$ and $\bar{z} = r/w$, $\mathcal{F}$ may have singularities at $w = 0$. Assuming it is bounded by a power

$$|\mathcal{F}(z = rw, \bar{z} = rw^{-1})| \lesssim w^{-s_*}\,, \text{ as } w \to 0\,, \tag{4.6}$$

then the convergence of the inversion formula is guaranteed only down to spin $s > s_*$ [38]. This constraint is easy to understand from the procedure described above: poles at $w = 0$ naturally arise from an infinite sum of bulk blocks, whose discontinuity vanishes identically term by term and thus are not captured by the inversion formula.

We address this issue by checking crossing symmetry in section 5, and surprisingly we find that the inversion formula along with the input from kinematics reconstructs the *full* correlator.

In the following we use the inversion formula (4.3) to calculate the contributions from the bulk identity and stress tensor multiplet.

To simplify our calculation, it is convenient to massage the inversion formula (4.3) as follows. The contributions to B from F and ζ can be calculated separatly, with the contribution from F

$$\mathsf{B}^F(\hat{\Delta}, s) = \frac{1}{2}\int_0^1 \mathrm{d}z \frac{z^{-\frac{\hat{\Delta}-s}{2}-1}}{(1-z)^2}\int_1^{1/z}\frac{\mathrm{d}\bar{z}}{2\pi i}\frac{(1-z\bar{z})(\bar{z}-z)}{(1-\bar{z})^2}\bar{z}^{-\frac{\hat{\Delta}+s}{2}-2}\,\mathrm{Disc}\,F(z,\bar{z})\,. \tag{4.7}$$

In the present case, neither of the bulk identity and the stress tensor contribute to F, so for these $\mathsf{B}^F = 0$.

The contribution from ζ is more subtle. The function $\zeta(z)$ may only have poles at $z = 1$ by kinematics and unitarity. The integral (4.3) can be evaluated from a careful regularisation of the singularities, but a simpler approach is to express the discontinuity as a contour integral. Going back to the Euclidean inversion formula of [38], using the variables $z = rw$ and $\bar{z} = r/w$, and expressing the Chebyshev polynomials as (B.11) we get

$$\frac{1}{2\pi i}\int_0^1 \mathrm{d}r(r^{\hat{\Delta}+1} - r^{-(\hat{\Delta}+1)})\oint_{|w|=1}\mathrm{d}w(w^{s+1} - w^{-(s+1)})\left[\frac{\zeta(rw)}{(r-w)^2} - \frac{\zeta(rw^{-1})}{(rw-1)^2}\right]. \tag{4.8}$$

The contour integral picks up residues inside the disk $|w| = 1$, which can only be located at the point $w = r$.

The integral contains poles corresponding to exchanged operators and their shadow blocks. Keeping only poles corresponding to physical operators we get

$$\begin{aligned} B^{\zeta}(\hat{\Delta}, s) &= -\frac{1}{2\pi i}\int_0^1 \mathrm{d}r r^{-(\Delta+1)} \oint_{|w|=1} \mathrm{d}w w^{s+1}\left[\frac{\zeta(rw)}{(r-w)^2} - \frac{\zeta(rw^{-1})}{(rw-1)^2}\right] \\ &= -\int_0^1 \mathrm{d}r r^{-(\Delta+1)} \operatorname{Res}_{w=r}\left[w^{s+1}\left(\frac{\zeta(rw)}{(r-w)^2} - \frac{\zeta(rw^{-1})}{(rw-1)^2}\right)\right]. \end{aligned} \tag{4.9}$$

4.1 Inverting the bulk identity

As a check of the supersymmetric inversion formula (4.3) we consider the inversion of the bulk identity, which corresponds to the disconnected correlator $\langle\Phi\Phi\rangle\langle V\rangle$. From the table 2, the identity block contributes to ζ and F as

$$\zeta_{\mathbb{1}}(z) = \frac{z^2}{(1-z)^4}, \qquad F_{\mathbb{1}}(z,\bar{z}) = 0. \tag{4.10}$$

Plugging in (4.9), we calculate the residue

$$\begin{aligned} &\operatorname{Res}_{w=r}\left[w^{s+1}\left(\frac{\zeta(rw)}{(r-w)^2} - \frac{\zeta(rw^{-1})}{(rw-1)^2}\right)\right] \\ &= -\frac{(s+1)r^{s+2}}{6(1-r^2)^4}\left[s(s-1)r^4 - 2(s+3)(s-1)r^2 + (s+3)(s+2)\right]. \end{aligned} \tag{4.11}$$

Including the kernel $r^{-(\Delta+1)}$, the integral of this over r diverges near $r=1$. This divergence can be understood by expanding the integrand in powers of r using

$$(1-r^2)^{-4} = \sum_{n=0}^{\infty}\frac{(n+1)(n+2)(n+3)}{6}r^{2n}, \tag{4.12}$$

and interchanging the sum and integral. In that case the integrals converge term by term for large enough $\hat{\Delta}$ and we find

$$\mathsf{B}(\hat{\Delta}, s)|_{\text{poles}} = \sum_{m\geq 0}\frac{(m+1)(s+1)(m+s+2)(2m+s+3)}{6(2+2m+s-\hat{\Delta})}. \tag{4.13}$$

From this we read the dimension of long supermultiplets and the corresponding OPE coefficients

$$(\hat{\Delta}_{m,s})^{(0)} = 2(m+1)+s, \qquad (b^2_{m,s})^{(0)} = \frac{(m+1)(s+1)(m+s+2)(2m+s+3)}{6}. \tag{4.14}$$

We added a label $^{(0)}$ in anticipation of subleading corrections in c discussed in section 4.2.

Notice that when $m=0$, the dimension of operators sit at the unitarity bound and split into short multiplets according to the recombination rules (2.34). We can identify

JHEP07(2023)165

the resulting superblocks by shifting appropriately the labels according to where the $r = 2$ block appear (see appendix B.3) to get

$$(b^2_{0,0})^{(0)}\hat{\mathcal{G}}_{B[2]} + \sum_{s\geq 0}(b^2_{0,s+1})^{(0)}\hat{\mathcal{G}}_{A[1]_s} + \sum_{\substack{m>0\\ s\geq 0}}(b^2_{m,s})^{(0)}\hat{\mathcal{G}}_{L[0]_{\hat{\Delta}_{m,s},s}} = \left(\frac{z\bar{z}(1-\omega)^2}{(1-z)^2(1-\bar{z})^2\omega}\right)^2 . \tag{4.15}$$

In the last step we resummed the superconformal blocks. The result is indeed the contribution from the bulk identity, it decomposes into ζ and F exactly as (4.10). Since the expansion (4.15) is unambiguous, this shows that, at this order, there is no exchange of $A[0]_s$ supermultiplets.

Note that the dCFT data (4.14) doesn't depend on the choice of defect operator V, since it corresponds to a disconnected correlator. So the result simply follows from the branching rules for the decomposition of bulk operators in terms of representations of $\mathfrak{osp}(4^*|2) \oplus \mathfrak{osp}(4^*|2)$. It is clear that this decomposition in defect blocks always exists, so the bulk identity is always crossing symmetric.

Focusing on the conformal blocks with $r = 2$, this dCFT data matches the general result of [38] for a defect of dimension 2 in 6d.

4.2 Inverting the stress tensor exchange

Next we consider the contribution to the correlator arising from the exchange of a stress tensor supermultiplet. The contribution to ζ, F from the stress tensor is given in (2.26), which we reproduce here for convenience

$$\zeta_{\mathcal{D}[2,0]}(z) = -\frac{12z^2}{(1-z)^4}\left[1 + \frac{1}{2}\frac{1+z}{1-z}\log z\right] , \qquad F_{\mathcal{D}[2,0]} = 0 . \tag{4.16}$$

Calculating the residue gives

$$\begin{aligned} &\frac{r^2}{2(1-r^2)^5}\left(s + 2 + (38+9s)r^2 + r^4(20-9s) - sr^6\right) \\ &- \frac{6r^4}{(1-r^2)^6}\left(s + 3 + 6r^2 - (s-1)r^4\right)\log r . \end{aligned} \tag{4.17}$$

Again expanding $(1-r^2)$ in series around $r = 0$ we can perform the integral as above. The $\log r$ terms gives rise to double pole. They should be understood as the small c expansion of the dCFT data

$$b^2_{m,s} = (b^2_{m,s})^{(0)} + \frac{4d}{c}(b^2_{m,s})^{(1)} + \dots , \qquad \hat{\Delta}_{m,s} = 2(m+1) + s + \frac{4d}{c}\gamma^{(1)}_{m,s} + \dots \tag{4.18}$$

where the leading terms are simply the OPE data found in (4.14). This leads to an expansion

$$\frac{b^2_{m,s}}{\hat{\Delta}_{m,s} - \hat{\Delta}} = \frac{(b^2_{m,s})^{(0))}}{2(m+1)+s-\hat{\Delta}} + \frac{4d}{c}\left(\frac{(b^2_{m,s})^{(1)}}{2(m+1)+s-\hat{\Delta}} - \frac{(b^2_{m,s})^{(0)}\gamma^{(1)}_{m,s}}{(2(m+1)+s-\hat{\Delta})^2}\right) + \dots \tag{4.19}$$

Performing the integral and comparing with this expansion we can read the dCFT data

$$
\begin{aligned}
\gamma^{(1)}_{m,s} &= -\frac{6m(m+1)(m+2)}{(s+1)(2m+s+3)}\,, \\
(b^2_{m,s})^{(1)} &= -\left[(m+1)(s+1) - \frac{1}{2}(m+1)^2(5m^2+2m(2s+7)+4s+6)\right]\,,
\end{aligned}
\tag{4.20}
$$

where $m = 1, 2, \ldots$ correspond to long operators while the case $m = 0$ corresponds to $A[1]_s$ operators. Note that at this order they do not acquire an anomalous dimension.

4.3 The result

As anticipated in the beginning of this section, the supersymmetric inversion doesn't capture the contributions from the defect identity and the displacement supermultiplet; however these blocks are special and their contribution is fixed by (1.7) and (2.7) respectively.

Including these and using the dCFT data (4.14) and (4.20), the full superconformal block decomposition of the correlator is

$$
\mathcal{F}(z,\bar{z},\omega) = \frac{2d^2}{c} + \frac{4d}{c}\hat{\mathcal{G}}_{B[1]} + b^2_{0,0}\hat{\mathcal{G}}_{B[2]} + \sum_{s\geq 0} b^2_{0,s+1}\hat{\mathcal{G}}_{A[1]_s} + \sum_{\substack{m\geq 1\\ s\geq 0}} b^2_{m,s}\hat{\mathcal{G}}_{L[0]_{\hat{\Delta}_{m,s},s}} + O(c^{-2})\,,
\tag{4.21}
$$

where the OPE coefficients and spectrum are given in (4.18). Notice that all the coefficients b^2 in this expansion are positive, as expected from unitarity. For $m = 0$ the coefficients b encode the contribution of short superblocks, and we find that (4.20) agrees with the chiral algebra calculation (3.37).

Note that by comparing the coefficients $b^2_{0,s+1}$ of the $A[1]_s$ multiplets to the chiral algebra result (3.37) we can conclude that, at this order, there are no $A[0]_s$ multiplets exchanged, provided the inversion formula captures all the $A[1]_s$ multiplets. We cannot exclude the possibility that the inversion formula misses a finite number of $A[1]_s$ multiplets with $s < s_*$ for some s_*, and correspondingly the correlator could receive contributions from a finite number of $A[0]_s$ multiplets; however we find that no such multiplets are required (and are likely excluded) by crossing symmetry.

Resumming these blocks, we find our main result (1.15). The function ζ we obtain reproduces the result from the chiral algebra (1.16), which suggests that the bootstrap result captures the full protected sector of the dCFT.

The function F given in (1.15) has several interesting features. From the definition of cross-ratios (1.2) we see that $z \leftrightarrow \bar{z}$ corresponds to an equivalent kinematic configuration, so is a symmetry of F. Additionally, $(z, \bar{z}) \to (1/z, 1/\bar{z})$ is also a symmetry and corresponds to exchanging the two bulk operators.

The dependence of F on $z + \bar{z}$ is linear so that $\mathcal{F}|_{r=2}$ satisfies the expected Regge behavior (4.6) for $s_* = -1$, which is a consistency check of the inversion formula. The dependence on $z\bar{z}$ is more complicated, and we note that it has no branch cut or pole at $z\bar{z} = 1$. Note that the coefficient of the $\log z\bar{z}$ captures information about the anomalous dimensions of defect operators.

In euclidean signature, $\bar{z}$ is the complex conjugate of z, and F is manifestly real. If we analytically continue the result by taking $z, \bar{z}$ real, we get a spacelike defect in lorenztian signature (see figure 3). A different analytic continuation is to take $\sqrt{z\bar{z}} \to i\sqrt{z\bar{z}}$ with $z/\bar{z}$ fixed, which corresponds to a timelike defect [89]. In that case we have $\log z\bar{z} \to \log z\bar{z} + i\pi$, so on this sheet F develops a pole at $z\bar{z} = 1$, which is where both bulk operators become lightlike separated from the same point on the defect. The corresponding singularity can be interpreted as arising from the exchange of "double-trace" defect operators, see [89, 90].

4.4 An equivalent calculation

In the calculation of section 4.2 we apply the inversion formula to the discontinuity arising from the stress tensor block exchange (2.26). Since we know the protected sector of the dCFT from the chiral algebra in section 3 and in particular the exact $\zeta(z)$ (1.16), it is natural to write an inversion formula that already includes the contribution from these protected multiplets.

The only bulk supermultiplets that contribute to the function $\zeta(z)$ are the short bulk supermultiplets $\mathbb{1}, \mathcal{D}[2,0], \mathcal{D}[4,0]$ and $\mathcal{B}[2,0]_l$ (refer to table 2). The coefficients of that expansion are known exactly from the chiral algebra and are given in (3.35). Resumming these superblocks, we find the contribution to F which we call F_{short}[9]

$$
\begin{aligned}
&F_{\text{short}}(z,\bar{z}) \\
&\quad = \frac{2d^2}{c} - \frac{4d}{c}\left[-\frac{12z\bar{z}}{(1-z)^2(1-\bar{z})^2} + \frac{6z\bar{z}}{(z-\bar{z})(1-z\bar{z})}\left(-\frac{z(z+1)}{(1-z)^3}\log z + \frac{\bar{z}(\bar{z}+1)}{(1-\bar{z})^3}\log\bar{z} \right) \right].
\end{aligned} \tag{4.22}
$$

The full function $F = F_{\text{short}} + F_{\text{long}}$ also receives contributions from long supermultiplets, which are not protected by supersymmetry.

The inversion formula applied to the exact $\zeta(z)$ and $F_{\text{short}} + F_{\text{long}}$ is equivalent to the original one, and it only depends on the unknown F_{long} encoding the contribution of long multiplets, making it an excellent starting point for higher order computations. We can recover the results of section 4.2 by noting that to order d/c, F_{long} does not contribute. Plugging $\zeta^{(1)}$ in the inversion formula, we find that B^{ζ} vanishes identically. $F^{(1)}_{\text{short}}$ does not have a discontinuity at $\bar{z} = 1$, but it leads to poles in the inversion formula which contribute to B^F. To calculate their contributions, one needs to regularise the integrals as in [40], and one can show the result agrees with the dCFT data (4.20).

5 Crossing symmetry

A nontrivial check of our result (1.12) is that it can be decomposed both in defect and bulk channels. This suggests that, up to the missing contributions from the defect identity and displacement supermultiplet, the inversion formula (4.3) reconstructs the full correlator at this order in c. We note that crossing symmetry is satisfied independently for all three pieces appearing in (1.12), respectively with coefficients $1, \frac{d^2}{c}$ and $\frac{d}{c}$.

[9]Such a splitting was first introduced in [91] in a similar context.

JHEP07(2023)165

The leading contribution is associated with the exchange of a single bulk supermultiplet, the bulk identity, and is independent of the choice of defect. In particular it can be calculated from the trivial defect, and it is known to be universally crossing symmetric. The corresponding defect channel block decomposition is obtained in (4.15).

In this section we verify that crossing symmetry is satisfied also for the two other terms: we obtain the explicit bulk channel decomposition of $\mathcal{F}$, given by

$$\begin{aligned}\mathcal{F}(z,\bar{z},\omega) = \left(\frac{z\bar{z}(1-\omega)^2}{(1-z)^2(1-\bar{z})^2\omega}\right)^2 \Bigg[1 - \frac{4d}{c}\mathcal{G}_{\mathcal{D}[2,0]} + (a\lambda)_{0,0}\mathcal{G}_{\mathcal{D}[4,0]} \\ - \sum_{\ell\geq 0}(a\lambda)_{0,\ell+2}\mathcal{G}_{\mathcal{B}[2,0]_\ell} + \sum_{n,\ell\geq 0}(a\lambda)_{n+2,\ell}\mathcal{G}_{\mathcal{L}[0,0]_{\Delta,\ell}}\Bigg],\end{aligned} \tag{5.1}$$

where ℓ is even, the dimension of the long superblock is $\Delta = 8+2n+\ell+O(c^{-1})$ as in (1.13) and the OPE coefficients are

$$\begin{aligned}(a\lambda)_{n,\ell} = \frac{n!(n+1)!(n+\ell+2)!(n+\ell+3)!}{(2n+1)!(2n+2\ell+5)!}\times \\ \left[\frac{2d^2}{c}\delta_{n,\text{even}}(\ell+2)(2n+\ell+5)+\frac{d}{2c}(-1)^n(n+4)(n+2)(n+1)(n-1)+O(c^{-2})\right].\end{aligned} \tag{5.2}$$

The terms proportional to $\frac{d^2}{c}$ encode the exchange of the defect identity, and for $n=0$ these coefficients match the chiral algebra results (3.35) and also the leading lightcone limit obtained in [92]. The terms proportional to $\frac{d}{c}$ include the stress tensor multiplet $\mathcal{D}[2,0]$ and are the bulk channel decomposition of (1.15).

The appearance of double-trace operators with only even n suggests that the defect identity enjoys an additional $\mathbb{Z}_2$ symmetry; it would be interesting to understand it.

In the following we present a supersymmetric inversion formula, this time for the bulk channel. Applying it to the defect identity we obtain an analytic derivation of its bulk channel decomposition. The decomposition of the rest of the correlator (1.15) can be obtained in principle from the inversion formula as well, but here we obtain it directly from the result by matching an ansatz like (5.1) to high order in $z,\bar{z}\to 1$ using Mathematica.

5.1 The bulk channel inversion formula

The OPE coefficients $(a\lambda)$ entering the conformal block decomposition of $\mathcal{F}$ can be extracted from the bulk channel inversion formula of [92]. Again this formula is general and doesn't explicitly account for R-symmetry; to apply it we can decompose $\mathcal{F}$ in representations of bulk operators labelled by R. The inversion formula then defines a function $C_R(\Delta,l)$ given in terms of the double discontinuity of $\mathcal{F}|_R$

$$\begin{aligned}\text{dDisc}\,\mathcal{F}|_R(z,\bar{z}) = \cos\left(\frac{(\Delta_2-\Delta_1)\pi}{2}\right)\mathcal{F}|_R(z,\bar{z}) \\ - \frac{1}{2}e^{-i\pi\frac{\Delta_1+\Delta_2}{2}}\mathcal{F}|_R(z,e^{2\pi i}\bar{z}) - \frac{1}{2}e^{i\pi\frac{\Delta_1+\Delta_2}{2}}\mathcal{F}|_R(z,e^{-2\pi i}\bar{z})\,.\end{aligned} \tag{5.3}$$

C_R is constructed in such a way that it has poles at the location of exchanged bulk operators $\Delta=\Delta_{n,l}$ and residue containing the OPE coefficients.

To include supersymmetry we use the fact that each conformal block with $R=4$ and given Δ, ℓ appears in a single superconformal block (apart from the identity and the stress tensor supermultiplet $\mathcal{D}[2,0]$ when it does not appear), so we can read the superconformal block decomposition of $\mathcal{F}$ from the $R=4$ channel, see A.3. From the inversion formula of [92] we can define

$$\begin{aligned}\mathsf{C}^t(\Delta,\ell) &= C^t_{R=4}(\Delta+4,\ell)\\ &= -\frac{\kappa_{\Delta+\ell+4}}{2}\int_0^1 \mathrm{d}z\int_0^1 \mathrm{d}\bar{z}\frac{|z-\bar{z}|^2(1-z\bar{z})^2}{(z\bar{z})^{\frac{\Delta_\phi}{2}}[(1-z)(1-\bar{z})]^{6-\Delta_\phi}}g^{st}_{5+\ell,\Delta-1}(z,\bar{z})\,\mathrm{dDisc}\,\mathcal{F}|_{R=4}(z,\bar{z})\,.\end{aligned} \tag{5.4}$$

We included a shift $\Delta\to\Delta+4$ so that the function C has poles at the location of the superprimary operator, assuming a long supermultiplet (the corresponding superblock is given in (A.21)). In this equation, $g^{st}_{\Delta,\ell}(z,\bar{z})$ are the (spacetime part of the) bulk channel blocks (2.21) and the coefficient κ_β is given [92][10]

$$\kappa_\beta = \frac{\Gamma\left(\frac{\beta}{2}\right)^2\Gamma\left(\frac{\beta}{2}+a\right)\Gamma\left(\frac{\beta}{2}-a\right)}{2\pi^2\Gamma(\beta)\Gamma(\beta-1)}\,,\qquad a=\frac{\Delta_2-\Delta_1}{2}\,. \tag{5.5}$$

The label ^{t} in (5.4) indicates that this captures the bulk blocks appearing in the "t-channel". We also need to add the contributions from the "u-channel" obtained by exchanging the external bulk operators (here $\mathsf{C}^u=\mathsf{C}^t$)

$$\mathsf{C}(\Delta,\ell)=\mathsf{C}^t(\Delta,\ell)+(-1)^\ell\mathsf{C}^t(\Delta,\ell)\,. \tag{5.6}$$

Finally we can express $\mathcal{F}|_{R=4}$ directly in terms of ζ, F by extracting from (1.5) the contribution proportional to the R-symmetry block (2.22). We find simply

$$\mathcal{F}|_{R=4}=F(z,\bar{z})\,. \tag{5.7}$$

5.2 Inverting the defect identity

As an example of application of the inversion formula (5.4) we reproduce the defect identity contribution to the correlator, i.e. the term of order $\frac{d^2}{c}$ in (1.12). This calculation is a simple extension of the one presented in [92].

For an external operator of dimension Δ_Φ, the double discontinuity of a constant is

$$\mathrm{dDisc}\,1 = 2\sin^2\left(\frac{\pi\Delta_\Phi}{2}\right)\,. \tag{5.8}$$

In the limit $\Delta_\Phi\to 4$ the double discontinuity vanishes, but correspondingly the integral in (5.4) diverges such that C^t has a well-defined limit.

Since the discontinuity is independent of $z,\bar{z}$, the inversion formula becomes very simple to evaluate. Plugging the conformal blocks (2.21) in the inversion formula (5.4), we obtain (up to a prefactor)

$$\int_0^1 \mathrm{d}z\mathrm{d}\bar{z}\frac{[(1-z)(1-\bar{z})]^{\Delta_\Phi-4}}{(z\bar{z})^{\Delta_\Phi/2}}(z-\bar{z})(z\bar{z}-1)k_{2+\ell-\Delta}(1-z)k_{\ell+\Delta+4}(1-\bar{z})-(z\leftrightarrow\bar{z})\,. \tag{5.9}$$

[10]There is a typo there, see also the original definition in [39].

JHEP07(2023)165

Since the integral is symmetric in $z, \bar{z}$, we get a factor of 2 and the inversion formula reduces to

$$\begin{aligned}\mathsf{C}^t(\Delta,\ell) = &-\kappa_{\Delta+\ell}\sin^2\left(\frac{\pi\Delta_\Phi}{2}\right)\times \\ &\int_0^1 \mathrm{d}z\mathrm{d}\bar{z}\frac{[(1-z)(1-\bar{z})]^{\Delta_\Phi-4}}{(z\bar{z})^{\Delta_\Phi/2}}(z-\bar{z})(z\bar{z}-1)k_{2+\ell-\Delta}(1-z)k_{\ell+\Delta+4}(1-\bar{z})\,.\end{aligned} \tag{5.10}$$

Introducing the short-hand notation

$$I_{\Delta_\Phi,\beta} \equiv \int_0^1 \mathrm{d}z\frac{(1-z)^{\Delta_\Phi-2}}{z^{\Delta_\Phi/2}}k_\beta(1-z)\,, \tag{5.11}$$

we see that (5.10) factorises and the integral can be written as

$$\begin{aligned}&\int_0^1 \mathrm{d}z\mathrm{d}\bar{z}\frac{[(1-z)(1-\bar{z})]^{\Delta_\Phi-4}}{(z\bar{z})^{\Delta_\Phi/2}}(z-\bar{z})(z\bar{z}-1)k_{6+\ell-\Delta}(1-z)k_{\ell+\Delta}(1-\bar{z}) \\ &\qquad = I_{\Delta_\Phi,2+\ell-\Delta}I_{\Delta_\Phi-2,\Delta+\ell+4} - I_{\Delta_\Phi-2,2+\ell-\Delta}I_{\Delta_\Phi,\Delta+\ell+4}\,.\end{aligned} \tag{5.12}$$

The integral I can be evaluated. We expand the hypergeometric function in series, integrate and resum to get

$$\frac{\Gamma\left(1-\frac{\Delta_\Phi}{2}\right)\Gamma\left(\Delta_\Phi+\frac{\beta}{2}-1\right)}{\Gamma\left(\frac{\Delta_\Phi+\beta}{2}\right)}\,{}_3F_2\left(\begin{matrix}\frac{\beta}{2}\,,\frac{\beta}{2}\,,\frac{\beta}{2}+\Delta_\Phi-1\\ \beta\,,\frac{\beta+\Delta_\Phi}{2}\end{matrix};1\right). \tag{5.13}$$

This can be simplified using Watson's theorem (see (6) of section 4.4 of [93]), which is the identity

$${}_3F_2\left(\begin{matrix}a\,,b\,,c\\ \frac{a+b+1}{2}\,,2c\end{matrix};1\right) = \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(c+\frac{1}{2}\right)\Gamma\left(\frac{a+b+1}{2}\right)\Gamma\left(c+\frac{1-a-b}{2}\right)}{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{b+1}{2}\right)\Gamma\left(c+\frac{1-a}{2}\right)\Gamma\left(c+\frac{1-b}{2}\right)}\,. \tag{5.14}$$

With appropriate values of $a = b$ and c we find

$$I_{\Delta_\Phi,\beta} = 2^{\Delta_\Phi+\frac{\beta}{2}-2}\frac{\Gamma\left(\frac{\beta+1}{2}\right)\Gamma\left(1-\frac{\Delta_\Phi}{2}\right)^2\Gamma\left(\frac{\beta}{4}+\frac{\Delta_\Phi-1}{2}\right)}{\Gamma\left(\frac{\beta+2}{4}\right)^2\Gamma\left(\frac{\beta}{4}+\frac{2-\Delta_\Phi}{2}\right)}\,. \tag{5.15}$$

Finally, we can evaluate C^t by substituting $I_{\Delta_\Phi,\beta}$, simplifying the gamma functions and eliminating the $\sin\left(\frac{\pi\Delta_\Phi}{2}\right)$ by using the following identity for Beta functions

$$B(x,y)B(x+y,1-y) = \frac{\pi}{x\sin\pi y}\,. \tag{5.16}$$

We obtain

$$\begin{aligned}\mathsf{C}^t(\Delta,\ell)|_{\text{poles}} = &\lim_{\Delta_\Phi\to 4} 2^{2\Delta_\Phi-9-\Delta}(\ell+2)(\Delta+1)\times \\ &\left.\frac{\Gamma\left(2-\frac{\Delta_\Phi}{2}\right)^2}{\Gamma\left(\frac{4+\ell-\Delta}{4}\right)^2}\frac{\Gamma\left(\frac{\Delta+\ell+4}{4}\right)^2\Gamma\left(\frac{\Delta+\ell+2\Delta_\Phi-2}{4}\right)\Gamma\left(\frac{3+\ell-\Delta}{2}\right)\Gamma\left(\frac{2\Delta_\Phi-4+\ell-\Delta}{4}\right)}{\Gamma\left(\frac{\Delta+\ell+3}{2}\right)\Gamma\left(\frac{10+\ell-2\Delta_\Phi-\Delta}{4}\right)\Gamma\left(\frac{12+\ell+\Delta-2\Delta_\Phi}{4}\right)\Gamma\left(\frac{\Delta_\Phi}{2}\right)^2}\right|_{\text{poles}}.\end{aligned} \tag{5.17}$$

JHEP07(2023)165

This expression has poles in Δ whenever the gamma functions in the numerator diverge, which happens for[11]

$$\frac{2\Delta_\Phi - 4 + \ell - \Delta}{4} = -n \qquad \Rightarrow \qquad \Delta = 2\Delta_\Phi - 4 + \ell + 4n\,, \qquad n \in \mathbb{Z}_{\geq 0}\,. \tag{5.18}$$

Near these poles the gamma function behaves as

$$\Gamma(z)|_{z\to -n} = \frac{(-1)^n}{n!(z+n)} + \dots\,. \tag{5.19}$$

with the subleading terms regular as $z \to -n$. The residue at these poles is thus trivial to calculate. Notice also that while the limit $\Delta_\Phi \to 4$ of the gamma functions diverges above, it is finite at the poles. We obtain

$$\mathsf{C}^t(\Delta,\ell)|_{\text{poles}} = \sum_{n\geq 0} 2^{-2n-1} \frac{(\ell+2)(4n+\ell+5)(2n+1)!(2n+\ell+2)!(2n+\ell+3)!}{(4n+1)!!(4n+2\ell+5)!(4+4n+\ell-\Delta)}\,. \tag{5.20}$$

Finally, adding the contribution for the u-channel, we can read the OPE coefficients and reproduce the result (5.2) presented at the beginning of this section.

6 Conclusions and outlook

It is an interesting problem to understand how to perform calculations in a nonlagrangian theory. For such theories with a large N limit, the conformal bootstrap offers a systematic approach to calculate correlators perturbatively in $1/N$, with minimal assumptions, and thus provides a working definition of these theories.

In this paper we present a case study, the 2-point function of stress tensor superprimaries Φ in the presence of a surface operator V, and adapt and develop bootstrap techniques to calculate the first subleading contribution at large N to their correlator $\langle \Phi\Phi V\rangle$. In doing so we extract dCFT data characterising the surface operators: we find partial information about the spectrum of operators in the 2d dCFT associated to V, and their interactions with local bulk operators in the form of the coefficients a_k and b_{kl} entering respectively the 1-point functions (2.3) and bulk-defect 2-point functions (2.6).

The coefficients we calculate are the combinations $b^2_{m,s}$ in (4.14) and (4.20), and $(a\lambda)_{n,\ell}$ in (5.2). In the large N limit we expect degeneracies in the spectrum of operators, and therefore these coefficients to correspond to averages for all corresponding superblocks of the same representation. Extracting the individual coefficients a, b from this data would require lifting the degeneracies; this is not something we attempt here.

The result for the correlator and dCFT data we obtain depends on the choice of $\mathcal{N} = (2,0)$ theory and representation for V solely through the anomaly coefficients c, d (1.9), which are known exactly. Our result is valid for both the A_{N-1} and D_N series of $\mathcal{N} = (2,0)$ theories at large N, and for any representation so long as $1 \ll d \ll c$. We emphasize that

[11]There are also poles at $\Delta = 7 + \ell + 2n$ which come from poles in the conformal blocks $g_{l+5,\Delta-5}$, rather than the integral. They do not contribute to the dCFT data so we ignore them; for a proper treatment see [39].

JHEP07(2023)165

the correlator we calculate contains information about defect operators in long supermultiplets not protected by supersymmetry. For these operators we calculate their anomalous dimension to first order.

The main tool in our analysis is the supersymmetric inversion formula, presented respectively for the defect and bulk channels in (4.3) and (5.4). These formula are particularly useful at large N, since the (double) discontinuity suppresses the contribution from long operators. In the context of 4-point functions of the stress tensor, this suppression and crossing symmetry was used to obtain the 1-loop correction to the correlator by "squaring" the tree-level anomalous dimensions of all double-trace operators [88, 94]. Similarly, at order $1/c^2$, we expect contributions to the discontinuity from the tree-level anomalous dimensions of double-trace operators, leading to subleading corrections to the 2-point function of order d^2/c^2 and d/c^2.

The inversion formula is known to miss contributions to the correlator coming from defect operators with spins below a certain value s_*, so a natural question is whether the correlator we obtain should be supplemented by additional superblocks. Clearly the inversion formula misses the contributions from the defect identity and the displacement supermultiplet, but both of these supermultiplets are special and their contributions are fixed independently by kinematics, see (1.7) and (2.7). Adding these to our result, we find that the correlator we obtain also admits a bulk channel decomposition and so is a nontrivial solution to the crossing symmetry constraints. This provides substantial evidence that the inversion formula recovers completely the dynamical part of the correlator at order d/c (1.15), and so that our result is complete and unambiguous. We expect that a similar strategy based on the supersymmetric (as opposed to regular) inversion formula would also resolve the ambiguities faced in [40].

The value of s_* is not known a priori and rather enters as an assumption on the validity of the inversion formula (4.6). From the result (1.15) we can check that this assumption is verified for the R-symmetry channel $\mathcal{F}|_{r=2}$ and the inversion formula converges down to *negative* spin $s_* = -1$ (this is similar to the case of 4-point function [19]). This exceptionally low-value for s_* explains the surprisingly simple dependence on $z + \bar{z}$ in (1.15). This constrains superblocks containing a primary with $r = 2$ to sit on Regge trajectories, with $s > s_*$ for long superblocks and $s > s_* - 1$ for $A[1]_s$ (shifting s for the superprimary). For $A[1]_s$, the Regge trajectory is extended to negative spin by identifying $B[2]$ as the analytic continuation to $s = -1$ (see table 3). We note that even though the inversion formula does not converge at $s = s_*$, the coefficient of the displacement multiplet is correctly reproduced by identifying $B[1]$ as the analytic continuation of $A[1]_s$ to $s = -2$. This is surprising given that the supersymmetric inversion formula is oblivious, by construction, to the short multiplets $B[0], B[1]$ and $A[0]_s$.

Subleading corrections to the correlator are determined by long supermultiplets acquiring an anomalous dimensions, however the corresponding superblocks do not contribute to the function $\zeta(z)$, which suggests that $\zeta(z)$ is in fact *exact*. We prove that this is the case by showing that $\zeta(z)$ is captured by the chiral algebra subsector identified in [9] and can be calculated exactly using standard techniques from chiral algebras. This is an exact result for any ADE theory and any representation for V, and it encodes the OPE data of the

BPS sector of defect operators. We emphasize that this approach from chiral algebras does not assume a lagrangian description for the (2,0) theories and offers a viable alternative to supersymmetric localization.

At the technical level, the setup we study here is surprisingly simple: we could obtain all the superconformal blocks explicitly, are able to perform all the integrals and resummations exactly. We believe that this makes these surface operators an excellent playground to test and develop analytical bootstrap methods.

Beyond this technical aspect, the setup we introduce here is interesting because it makes manifest the relation between the 6d CFT of local operators and the 2d dCFT of defect operators. In this paper we use our knowledge of the 6d theory (in particular the existence of a stress tensor) to infer properties of the 2d dCFT, but it would be interesting to learn something about the 6d theories by bootstrapping directly the 2d dCFT at large N as initiated in [54].

The correlator we obtain has a natural interpretation in holography as the propagator for Kaluza-Klein modes in the graviton supermultiplet in the presence of M2-branes. It would be interesting to confirm this calculation directly from supergravity. In particular, when the number of M2-branes M is large, the M2-branes backreact on the AdS_7 geometry and give rise to the bubbling geometries [33, 34]. A hint of that change in geometry is that when $d \gg c$, the leading term in ζ is $(1-z)^{-2}$, which can be interpreted as the chiral part of the propagator of a graviton in AdS_3.

It would also be interesting to understand the structure of our result (1.15) in Mellin space, in analogy with the simplifications for the 4-point functions of local operators (see for instance [12, 21, 95, 96]). Mellin space for defects has recently been introduced in [97]. Mellin space amplitudes are also interesting for their flat space limit, and it would be interesting to study the analogous limit for defects.

Finally, this work sets the basis for further explorations of the 2d dCFT associated with V, and a natural goal for the future is to bootstrap this correlator at the next order in c. This is complicated by the degeneracies at large N which need to be resolved by considering additional correlators (a similar problem was recently studied in the context of Wilson lines [98]). We hope to report on it in the near future.

Acknowledgments

We would like to thank Lorenzo Bianchi, Gabriel Bliard, Nadav Drukker, Jean-François Fortin, Pedro Liendo, Valentina Prilepina and Pedro Vieira for enlightening discussions. MT gratefully acknowledges the support of the Institute for Theoretical and Mathematical Physics (Lomonosov State University, Moscow) where this project began, and the Simons Center for Geometry and Physics (Stony Brook University), New York University, University of Parma and the University of Turin, where part of this project was realised. Research at Perimeter Institute is supported by the Government of Canada through the Department of Innovation, Science and Economic Development and by the Province of Ontario through the Ministry of Research and Innovation. This work has been supported in part by Istituto

Nazionale di Fisica Nucleare (INFN) through the “Gauge and String Theory” (GAST) research project.

A Bulk channel blocks

In this appendix we present the superconformal blocks $\mathcal{G}$ arising in the bulk channel decomposition of the correlator (1.10). These superconformal blocks are given by combinations of conformal blocks g^{st} and h as in (2.23) and (2.24), and we begin by reviewing the derivation of the latter directly from the OPE in the limit $z, \bar{z}, \omega \to 1$. The full conformal blocks can then be recovered from the Casimir equation. Finally we present the complete list of superconformal blocks that may contribute to the correlator.

A.1 OPE and normalisation

A straightforward (if cumbersome) approach to calculating the contribution to the correlator corresponding to the exchange of a given bulk operator is to use the bulk OPE of $\Phi_k\Phi_k$. Consider a bulk operator with weights Δ, ℓ, R and nonzero 3-point function (ℓ, R are even)

$$\langle \Phi_k(x_1, y_1)\Phi_k(x_2, y_2)\mathcal{O}_{\Delta,\ell,R}(x_3, y_3, v)\rangle = \lambda_{kk,\{\Delta,\ell,R\}} \frac{|y_{12}|^{2k-R}}{|x_{12}|^{4k-\Delta}} \frac{|y_{13}|^R}{|x_{13}|^\Delta} \frac{|y_{23}|^R}{|x_{23}|^\Delta} \frac{(2x_{12} \cdot v)^\ell}{|x_{12}|^\ell} . \tag{A.1}$$

The coordinate y encodes the R-symmetry polarisation and is defined in (2.10); v is a polarisation vector for the spin. If λ is nonzero, the operator $\mathcal{O}_{\Delta,\ell,R}$ appears in the OPE of $\Phi_k\Phi_k$. Expanding near $x_2 \to x_1$ and similarly for $y_2 \to y_1$ leads to the OPE

$$\Phi_k(x_1, y_1)\Phi_k(x_2, y_2) \supset \lambda_{kk,\{\Delta,\ell,R\}} \frac{(y_{12}^2)^{\frac{2k-R}{2}}}{(x_{12}^2)^{\frac{4k-\Delta+\ell}{2}}} [1 + \dots] \, \mathcal{O}_{\Delta,\ell,R}(x_1, y_1, x_{12}) . \tag{A.2}$$

The ellipsis contains terms with derivatives in x_1 and y_1 and suppressed in the coincident limit. Since $v = x_{12}$ does not satisfy the condition $v^2 = 0$, to ensure consistency we rewrite $\mathcal{O}|_{v=x_{12}}$ using the Todorov operator D_v [99, 100] (see also [44]) defined as

$$x \cdot D_v = \left(\frac{q-2}{2} + v \cdot \frac{d}{dv}\right)\left(x \cdot \frac{d}{dv}\right) - \frac{1}{2}(x \cdot v)\frac{d^2}{dv \cdot dv} , \qquad \text{here } q = 6 . \tag{A.3}$$

This satisfies the identity (with $(n)_\ell \equiv \Gamma(n+\ell)/\Gamma(n)$ the Pochhammer symbol)

$$\mathcal{O}_{\Delta,\ell,R}(x_1, v, y_1)|_{v=x_{12}} = \frac{(2x_{12} \cdot D_v)^\ell}{(\ell!)(2)_\ell} \mathcal{O}_{\Delta,\ell,R}(x_1, v, y_1) , \tag{A.4}$$

and ensures tracelessness. The operator $\mathcal{O}_{\Delta,\ell,R}$ can have a nonzero expectation value with V whichs take the form

$$\langle \mathcal{O}_{\Delta,\ell,R}(x_1, v, y_1)V\rangle = a_{\{\Delta,\ell,R\}} \frac{|y_1^\perp|^R (x_1^\perp \cdot v)^\ell}{|x_1^\perp|^{\Delta+\ell}} . \tag{A.5}$$

Taking the expectation value of (A.2) in the presence of V and plugging (A.5) gives the contribution to the correlator for the exchange of $\mathcal{O}_{\Delta,\ell,R}$

$$\begin{aligned}&\langle\Phi_k(x_1,y_1)\Phi_k(x_2,y_2)V\rangle_{\mathcal{O}_{\Delta,\ell,R}}\\&\quad=\lambda_{kk,\{\Delta,\ell,R\}}a_{\{\Delta,\ell,R\}}\frac{(y_{12}^2)^{\frac{2k-R}{2}}}{(x_{12}^2)^{\frac{4k-\Delta+\ell}{2}}}\left[1+\dots\right]\frac{(2x_{12}\cdot D_v)^\ell}{\ell!(2)_\ell}\frac{|y_1^\perp|^R(x_1^\perp\cdot v)^\ell}{|x_1^\perp|^{\Delta+\ell}}\,.\end{aligned}\tag{A.6}$$

The action of the Todorov operator on the 1-point function can be calculated exactly using the identity [44] ($C_\ell^{(\alpha)}$ are the Gegenbauer polynomials)

$$(x\cdot D_v)^\ell(-2v\cdot w)^\ell=(\ell!)^2(x^2w^2)^{\ell/2}C_\ell^{(\frac{q}{2}-1)}\left(\frac{x\cdot w}{(x^2w^2)^{1/2}}\right)\tag{A.7}$$

In the present case this implies

$$\frac{(2x_{12}\cdot D_v)^\ell}{\ell!(2)_\ell(x_{12}^2)^{\ell/2}}\frac{(x_1^\perp\cdot v)^\ell}{|x_1^\perp|^\ell}=\frac{1}{\ell+1}C_\ell^{(2)}\left(\frac{x_{12}\cdot x_1^\perp}{|x_{12}||x_1^\perp|}\right).$$

In terms of the cross-ratios defined in (1.2) we find to leading order

$$g_{\Delta,\ell,R}(z,\bar z,\omega)=\frac{1}{\ell+1}\left|(1-z)(1-\bar z)\right|^{\frac{\Delta}{2}}|1-\omega|^{-R}C_\ell^{(2)}\left(\frac{2-z-\bar z}{2\sqrt{(1-z)(1-\bar z)}}\right)+\dots\tag{A.8}$$

with subleading terms suppressed in the limit $z,\bar z,\omega\to1$. In particular in the lightcone limit $\bar z\to1$ we recover

$$g_{\Delta,\ell,R}(z,\bar z,\omega)=|1-z|^{\frac{\Delta+\ell}{2}}|1-\bar z|^{\frac{\Delta-\ell}{2}}|1-\omega|^{-R}+\dots\,.\tag{A.9}$$

A.2 Casimir equation

A more convenient approach to compute the full conformal blocks is to use that bulk operators exchanged in the bulk channel OPE transform in representations of the 6d conformal group and $\mathfrak{so}(5)$ R-symmetry. These properties can be shown to lead respectively to two constraints satisfied by the conformal blocks $g_{\Delta,\ell,R}$ in the form of Casimir equation [36, 101]

$$\begin{aligned}&2\left[(z-1)^2z\partial_z^2+(z-1)\left(-(z+3)+\frac{2z(z-1)}{z-\bar z}+\frac{2(z-1)}{z\bar z-1}\right)\partial_z+(z\leftrightarrow\bar z)\right]g_{\Delta,\ell,R}(z,\bar z,\omega)\\&\quad=C_{2,6}(\Delta,\ell)g_{\Delta,\ell,R}(z,\bar z,\omega)\,,\end{aligned}\tag{A.10}$$

and

$$-(1-\omega)^2\left[\omega\partial_\omega^2+\partial_\omega\right]g_{\Delta,\ell,R}(z,\bar z,\omega)=C_{2,3}(-R,0)g_{\Delta,\ell,R}(z,\bar z,\omega)\,.\tag{A.11}$$

In these equations, $C_{2,d}(\Delta,\ell)$ is the quadratic Casimir of $\mathfrak{so}(d)$ for the representation of dimension Δ and spin ℓ

$$C_{2,d}(\Delta,\ell)=\Delta(\Delta-d)+\ell(\ell+d-2),\tag{A.12}$$

while the differential operators on the left are the differential representations of the same Casimir operators acting on the correlator.

JHEP07(2023)165

These two equations are separated, and accordingly their solution is given in terms of the product $g_{\Delta,\ell,R}(z,\bar{z},\omega) = g^{st}_{\Delta,\ell}(z,\bar{z})h_R(\omega)$.

The first equation (A.10) was observed in [92] to reduce to the Casimir equation of the 4d conformal blocks for 4-point functions of local operators $g^{(a,b)}_{\Delta,\ell}$, with $a = b = 0$. Following their observation it is easy to verify that a solution to the Casimir equation is given by

$$g^{st}_{\Delta,\ell} = \frac{(1-z)(1-\bar{z})}{1-z\bar{z}} g^{(0,0)}_{\Delta-1,\ell+1}(1-z, 1-\bar{z}) \,, \tag{A.13}$$

with $g^{(a,b)}_{\Delta,\ell}$ given by [63]

$$g^{(a,b)}_{\Delta,\ell}(z,\bar{z}) = \frac{z\bar{z}}{z-\bar{z}} \left(k_{\Delta+\ell}(z)k_{\Delta-\ell-2}(\bar{z}) - k_{\Delta+\ell}(\bar{z})k_{\Delta-\ell-2}(z)\right) , \tag{A.14}$$

$$k_{2h}(z) = z^h {}_2F_1\left(h - \frac{a}{2}, h + \frac{b}{2}, 2h; z\right) . \tag{A.15}$$

This solution also has the correct asymptotics required by the OPE analysis (A.8). To see this, expand the blocks first in $\bar{z} \to 1$ and then $z \to 1$.

The second equation (A.11) is closely related to the equation for Legendre polynomials, and its solution is given by

$$h_R(\omega) = \mathcal{N}_R \left(\frac{1-\omega}{1+\omega}\right) P_{R+1}\left(\frac{1+\omega}{1-\omega}\right) . \tag{A.16}$$

The normalisation factor $\mathcal{N}_R$ can be fixed by expanding this block in the limit $\omega \to 1$ and comparing to (A.8). Matching the normalisation we find

$$\mathcal{N}_R = \frac{R!(R+1)!}{(2R+1)!} . \tag{A.17}$$

The result (A.16) can be understood by realising the $\mathfrak{so}(5)$ R-symmetry as the (complexified) conformal group in 3d. From the point of view of kinematics, V specifies a plane (or boundary) in $\mathbb{R}^3$, and operators in symmetric traceless representations of rank R give scalar operators with conformal dimension $-R$. And indeed one can show that (A.16) matches the 3d boundary blocks obtained in [102].

Assembling both results we obtain the blocks given in (2.23).

A.3 Table of superconformal blocks

As reviewed in section 2.4, conformal blocks assemble in superconformal blocks describing the exchange of all the operators of a given supermultiplet appearing in the OPE. Here we tabulate the conformal blocks content of the various superconformal blocks relevant for our analysis.

The superblocks for $\mathcal{D}[2,0]$ are given in (2.24). For $\mathcal{D}[4,0]$, $\mathcal{B}[2,0]_\ell$ and $\mathcal{L}[0,0]_{\Delta,\ell}$ they are respectively

$$\mathcal{G}_{\mathcal{D}[4,0]} = g_{8,0,4} - \frac{25}{6237} g_{10,2,2} + \frac{1}{58212} g_{12,0,0} . \tag{A.18}$$

JHEP07(2023)165

$$
\begin{aligned}
\mathcal{G}_{\mathcal{B}[2,0]_\ell} = {} & g_{\ell+8,\ell,2} - \frac{3}{700} g_{\ell+10,\ell-2,0} - \frac{3}{700} g_{\ell+10,\ell+2,0} \\
& - \frac{(\ell+2)(\ell+9)}{90(2\ell+9)(2\ell+13)} g_{\ell+10,\ell+2,2} - g_{\ell+10,\ell+2,4} + \frac{3(\ell+4)(\ell+7)}{7000(2\ell+9)(2\ell+13)} g_{\ell+12,\ell,0} \\
& + \frac{3}{700} g_{\ell+12,\ell,2} + \frac{(\ell+6)^2(\ell+7)^2}{16(2\ell+11)(2\ell+13)^2(2\ell+15)} g_{\ell+12,\ell+4,2} \\
& - \frac{3(\ell+6)^2(\ell+7)^2}{11200(2\ell+11)(2\ell+13)^2(2\ell+15)} g_{\ell+14,\ell+2,0} \,.
\end{aligned} \tag{A.19}
$$

$$
\begin{aligned}
\mathcal{G}_{\mathcal{L}[0,0]_{\Delta,\ell=0}} = {} & g_{\Delta,0,0} - \frac{\Delta(\Delta+6)}{40(\Delta+1)(\Delta+5)} g_{\Delta+2,2,0} \\
& - g_{\Delta+2,2,2} + \frac{9(\Delta-4)(\Delta-2)(\Delta+4)(\Delta+6)}{8960(\Delta-3)(\Delta-1)(\Delta+3)(\Delta+5)} g_{\Delta+4,0,0} \\
& + \frac{(\Delta-4)(\Delta+6)}{36(\Delta-3)(\Delta+5)} g_{\Delta+4,0,2} \\
& + g_{\Delta+4,0,4} + \frac{(\Delta+4)^2(\Delta+6)^2}{256(\Delta+3)(\Delta+5)^2(\Delta+7)} g_{\Delta+4,4,0} \\
& - \frac{(\Delta-4)(\Delta+2)(\Delta+4)^2(\Delta+6)^2}{10240(\Delta-3)(\Delta+1)(\Delta+3)(\Delta+5)^2(\Delta+7)} g_{\Delta+6,2,0} \\
& - \frac{(\Delta+4)^2(\Delta+6)^2}{256(\Delta+3)(\Delta+5)^2(\Delta+7)} g_{\Delta+6,2,2} \\
& + \frac{\Delta^2(\Delta+2)^2(\Delta+4)^2(\Delta+6)^2}{2^{16}(\Delta-1)(\Delta+1)^2(\Delta+3)^2(\Delta+5)^2(\Delta+7)} g_{\Delta+8,0,0} \,.
\end{aligned} \tag{A.20}
$$

$$
\begin{aligned}
\mathcal{G}_{\mathcal{L}[0,0]_{\Delta,\ell}} = {} & g_{\Delta,\ell,0} - \frac{(\Delta-\ell-4)(\Delta-\ell-2)}{40(\Delta-\ell-3)(\Delta-\ell-1)} g_{\Delta+2,\ell-2,0} \\
& - g_{\Delta+2,\ell-2,2} - \frac{(\Delta+\ell)(\Delta+\ell+6)}{40(\Delta+\ell+1)(\Delta+\ell+5)} g_{\Delta+2,\ell+2,0} \\
& - g_{\Delta+2,\ell+2,2} + \frac{(\Delta-\ell)^2(\Delta-\ell+2)^2}{256(\Delta-\ell-1)(\Delta-\ell+1)^2(\Delta-\ell+3)} g_{\Delta+4,\ell+2,0} \\
& + \frac{11\Delta^4+44\Delta^3-(22\ell^2+88\ell+124)\Delta^2-4(11\ell^2+44\ell+84)\Delta+11\ell^4+88\ell^3+140\ell^2-144(\ell+1)}{2240(\Delta-\ell-3)(\Delta-\ell+1)(\Delta+\ell+1)(\Delta+\ell+5)} g_{\Delta+4,\ell,0} \\
& + \frac{\Delta^4+4\Delta^3-(2\ell^2+8\ell+19)\Delta^2-2(2\ell^2+8\ell+23)\Delta+\ell^4+8\ell^3+5\ell^2-44\ell-24}{36(\Delta-\ell-3)(\Delta-\ell+1)(\Delta+\ell+1)(\Delta+\ell+5)} g_{\Delta+4,\ell,2} \\
& + g_{\Delta+4,\ell,4} + \frac{(\Delta+\ell+4)^2(\Delta+\ell+6)^2}{256(\Delta+\ell+3)(\Delta+\ell+5)^2(\Delta+\ell+7)} g_{\Delta+4,\ell+4,0} \\
& - \frac{(\Delta-\ell)^2(\Delta-\ell+2)^2(\Delta+\ell)(\Delta+\ell+6)}{10240(\Delta-\ell-1)(\Delta-\ell+1)^2(\Delta-\ell+3)(\Delta+\ell+1)(\Delta+\ell+5)} g_{\Delta+6,\ell-2,0} \\
& - \frac{(\Delta-\ell)^2(\Delta-\ell+2)^2}{256(\Delta-\ell-1)(\Delta-\ell+1)^2(\Delta-\ell+3)} g_{\Delta+6,\ell-2,2} \\
& - \frac{(\Delta-\ell-4)(\Delta-\ell+2)(\Delta+\ell+4)^2(\Delta+\ell+6)^2}{10240(\Delta-\ell-3)(\Delta-\ell+1)(\Delta+\ell+3)(\Delta+\ell+5)^2(\Delta+\ell+7)} g_{\Delta+6,\ell+2,0} \\
& - \frac{(\Delta+\ell+4)^2(\Delta+\ell+6)^2}{256(\Delta+\ell+3)(\Delta+\ell+5)^2(\Delta+\ell+7)} g_{\Delta+6,\ell+2,2} \\
& + \frac{(\Delta-\ell)^2(\Delta-\ell+2)^2(\Delta+\ell+4+)^2(\Delta+\ell+6)^2}{2^{16}(\Delta-\ell-1)(\Delta-\ell+1)^2(\Delta-\ell+3)(\Delta+\ell+3)(\Delta+\ell+5)^2(\Delta+\ell+7)} g_{\Delta+8,\ell,0} \,.
\end{aligned} \tag{A.21}
$$

B Defect channel blocks

In this appendix we detail the derivation of the defect channel superconformal blocks. We follow the same strategy as for the bulk channel superconformal blocks presented in appendix A. We start by reviewing the calculation of the leading behavior of the conformal blocks from the OPE. This fixes the normalisation and asymptotics of the conformal blocks. We then proceed to recover the full blocks by using the Casimir equation. Finally we tabulate the relevant superconformal blocks.

B.1 OPE and normalisation

In the presence of the defect V, bulk operators admit a defect operator expansion (dOE) (2.31), which we reproduce here for convenience

$$\Phi_k(x,y)V = \sum_{\{\hat{\Delta},s,r\}} b_{k\{\hat{\Delta},s,r\}} \frac{|y^\perp|^{k-r}}{|x^\perp|^{2k-\hat{\Delta}}} C_{k,\{\hat{\Delta},s,r\}} V[\hat{\mathcal{O}}_{\hat{\Delta},s,r}(x^\parallel, y^\parallel, v)]\,. \tag{B.1}$$

The coefficients $b_{k\{\hat{\Delta},s,r\}}$ are the dCFT data entering the correlators (2.6). $C_{k,\{\hat{\Delta},s,r\}}$ are differential operators acting on $\hat{\mathcal{O}}$ and encoding the contributions of the descendants; they can be calculated by requiring the dOE to reproduce the correlators (2.6) and to leading order are given by

$$C_{k,\{\hat{\Delta},s,r\}}|_1 = (1+\dots)\frac{(x_1^\perp \cdot D_v)^s}{|x_1^\perp|^s (s!)^2}\,, \tag{B.2}$$

with terms suppressed by $|x_1^\perp|/|x_{12}^\parallel|$, and similarly for y. D_v is the Todorov operator defined in (A.3) (here with $q=4$).

From the dOE we can directly evaluate the leading contribution to $\mathcal{F}$ due to the exchange of a defect operator of weights $\hat{\Delta}, s, r$. Acting twice on (1.3) and taking the expectation value leads to an expression for the conformal block $\hat{g}_{\hat{\Delta},s,r}$ of the form

$$\hat{g}_{\hat{\Delta},s,r} = \frac{|x_1^\perp|^{\hat{\Delta}} |x_2^\perp|^{\hat{\Delta}}}{|y_1^\perp|^r |y_2^\perp|^r} C|_1 C|_2 \left\langle V[\hat{\mathcal{O}}_{\hat{\Delta},s,r}(x_1^\parallel, y_1^\parallel, v_1) \hat{\mathcal{O}}_{\hat{\Delta},s,r}(x_2^\parallel, y_2^\parallel, v_2)] \right\rangle\,, \tag{B.3}$$

where $C|_i$ are the differential operators (B.2) acting on the point i. The action of D_v on the 2-point function is easy to evaluate from the identity (A.7), and to leading order we get

$$\hat{g}_{\hat{\Delta},s,r} = \left(\frac{|x_1^\perp||x_2^\perp|}{(x_{12}^\parallel)^2}\right)^{\hat{\Delta}} \left(\frac{-(y_{12}^\parallel)^2}{|y_1^\perp||y_2^\perp|}\right)^r U_s\left(\frac{x_1^\perp \cdot x_2^\perp}{|x_1^\perp||x_2^\perp|}\right) + \dots \tag{B.4}$$

with $U_s(x)$ the Chebyshev polynomials of second kind. This is a function of the cross-ratios introduced in (1.2) and provides the normalisation for the conformal blocks.

B.2 Casimir equation

We can derive the full defect channel conformal blocks from a Casimir equation following [36, 101]. The defect operators exchanged in (1.3) transform in the 2d (global) conformal group, along with $\mathfrak{so}(4)$ rotations tranverse to the plane and $\mathfrak{so}(4)$ residual R-symmetry.

For each of these symmetries there is a corresponding Casimir equation. These take a nice form in terms of the cross-ratios

$$\begin{aligned}\chi &= \frac{(x_{12}^{\|})^2 + |x_1^{\perp}|^2 + |x_2^{\perp}|^2}{2|x_1^{\perp}||x_2^{\perp}|} = \frac{1+z\bar{z}}{2\sqrt{z\bar{z}}}\,, \qquad \cos\phi = \frac{x_1^{\perp}\cdot x_2^{\perp}}{|x_1^{\perp}||x_2^{\perp}|} = \frac{z+\bar{z}}{2\sqrt{z\bar{z}}}\,,\\ \psi &= \frac{(y_{12}^{\|})^2 + |y_1^{\perp}|^2 + |y_2^{\perp}|^2}{2|y_1^{\perp}||y_2^{\perp}|} = \frac{\omega+\omega^{-1}}{2}\,.\end{aligned} \tag{B.5}$$

They read respectively

$$\left[(1-\chi^2)\frac{\partial^2}{\partial\chi^2} - 3\chi\frac{\partial}{\partial\chi} + 4\hat{\Delta}(\hat{\Delta}-1)\right]\hat{g}_{\hat{\Delta},s,r}(\chi,\cos\phi,\psi) = 0\,, \tag{B.6}$$

$$\left[\sin^2\phi\frac{\partial^2}{\partial\cos\phi^2} - 3\cos\phi\frac{\partial}{\partial\cos\phi} + s(s+2)\right]\hat{g}_{\hat{\Delta},s,r}(\chi,\cos\phi,\psi) = 0\,, \tag{B.7}$$

$$\left[(1-\psi^2)\frac{\partial^2}{\partial\psi^2} - 3\psi\frac{\partial}{\partial\psi} + r(r+2)\right]\hat{g}_{\hat{\Delta},s,r}(\chi,\cos\phi,\psi) = 0\,. \tag{B.8}$$

The equations are separated and can be solved straightforwardly. Picking the solution with the right asymptotics, we find respectively

$$\chi^{-\hat{\Delta}}{}_2F_1\left(\frac{\hat{\Delta}+1}{2},\frac{\hat{\Delta}}{2};\hat{\Delta};\chi^{-2}\right)\,, \qquad U_s(\cos\phi)\,, \qquad U_r(\psi)\,. \tag{B.9}$$

We note that the solution for χ matches the conformal block found in [36] for a plane, and the solution for ψ matches the boundary block for $\Delta = -r$ and $d = 3$ found in [102].

The conformal block $\hat{g}_{\hat{\Delta},s,r}$ is given by the product of these 3 blocks, up to an overall normalisation factor. Taking the limit $\chi \to \infty$ and $\psi \to \infty$ and matching with the OPE result (B.4) we can fix the normalisation of the blocks to get

$$\hat{g}_{\hat{\Delta},s,r}(z,\bar{z},\omega) = (2\chi)^{-\hat{\Delta}}\,{}_2F_1\left(\frac{\hat{\Delta}+1}{2},\frac{\hat{\Delta}}{2};\hat{\Delta};\chi^{-2}\right)U_s\left(\cos(\phi)\right)U_r\left(-\psi\right)\,. \tag{B.10}$$

Expressing the cross-ratios in terms of $z,\bar{z},\omega$ and using the identity

$$U_r\left(\frac{\omega+\omega^{-1}}{2}\right) = \frac{\omega^{r+1}-\omega^{-(r+1)}}{\omega-\omega^{-1}}\,. \tag{B.11}$$

we find the conformal blocks (2.32).

B.3 Table of superconformal blocks

Finally we tabulate the various superconformal blocks appearing in the OPE. Here we list the conformal blocks content of each superconformal blocks.

The short multiplets $B[r]$ (for $r = 1,2$) have superconformal blocks

$$\hat{\mathcal{G}}_{B[r]} = \hat{g}_{2r,0,r} + \hat{g}_{2r+1,1,r-1} + \hat{g}_{2r+2,0,r-2}\,. \tag{B.12}$$

Note that for $B[0]$ is the defect identity. The special case $B[1]$ is the displacement operator supermultiplet and has a shortened superconformal block. It contains only the blocks $g_{2,0,1}$

and $g_{3,1,0}$, and correspondingly we can check that the last term in the ansatz above vanishes identically.

The semishort multiplets $A[r]_s$ (defined for $r = 0, 1$) respectively have superblocks

$$\hat{\mathcal{G}}_{A[0]_s} = \hat{g}_{2+s,s,0} + \hat{g}_{3+s,s+1,1} + \hat{g}_{4+s,s+2,0}\,, \tag{B.13}$$

and

$$\hat{\mathcal{G}}_{A[1]_s} = \hat{g}_{4+s,s,1}+\hat{g}_{5+s,s-1,0}+\hat{g}_{5+s,s+1,0}+\hat{g}_{5+s,s+1,2}+\hat{g}_{6+s,s,1}+\hat{g}_{6+s,s+2,1}+\hat{g}_{7+s,s+1,0}\,. \tag{B.14}$$

Again for the special case $s = 0$ some of these conformal blocks vanish identically, which reflects the shortening of the multiplet.

Finally the long supermultiplet have superconformal blocks

$$\begin{aligned}\hat{\mathcal{G}}_{L[s,0]_{\hat{\Delta}}} = \ &\hat{g}_{\hat{\Delta},s,0} + \hat{g}_{\hat{\Delta}+1,s+1,1} + \hat{g}_{\hat{\Delta}+1,s-1,1} + \hat{g}_{\hat{\Delta}+2,s-2,0} + \hat{g}_{\hat{\Delta}+2,s,0}\\ &+ \hat{g}_{\hat{\Delta}+2,s+2,0} + \hat{g}_{\hat{\Delta}+2,s,2} + \hat{g}_{\hat{\Delta}+3,s+1,1} + \hat{g}_{\hat{\Delta}+3,s-1,1} + \hat{g}_{\hat{\Delta}+4,s,0}\,.\end{aligned} \tag{B.15}$$

Notice that the blocks $\hat{g}$ appear, for our normalization, all with unit coefficient.

C Calculation of a_2

We can calculate the coefficient a_2 appearing in (2.3) by relating it to the anomaly coefficients c, d. The transformation rules for operators of the stress tensor multiplet are given in [57]

$$\begin{aligned}\delta T^{\mu\nu} &= \frac{1}{2}\varepsilon\gamma^{\rho(\mu}\partial_\rho J^{\nu)}\,,\\ \delta J^{\mu} &= 2\varepsilon\gamma_\nu T^{\mu\nu} + \frac{2\alpha_2}{5\alpha_3}\left(6\eta^{\rho\mu}\left(\gamma^{\nu\sigma\lambda} + 3\eta^{\sigma\nu}\gamma^{\lambda}\right) - \eta^{\mu\nu}\gamma^{\rho\sigma\lambda}\right)\check{\gamma}_I\partial_\nu H^I_{\rho\sigma\lambda}\\ &\quad + \frac{1}{10}\varepsilon\left(\gamma^{\mu\nu\rho} - 4\eta^{\mu\rho}\gamma^{\nu}\right)\check{\gamma}^{IJ}\partial_\nu j_{\rho IJ}\,,\\ \delta j^{\mu}_{IJ} &= -\frac{1}{2}\varepsilon\check{\gamma}_{IJ}J^{\mu} + \frac{1}{5\alpha_3}\varepsilon\gamma^{\mu\nu}\partial_\nu\check{\gamma}_{[I}\chi_{J]}\,,\\ \delta H^I_{\mu\nu\rho} &= \frac{\alpha_3}{8\alpha_2}\varepsilon\check{\gamma}^I\gamma_{[\mu\nu}J_{\rho]} + \frac{1}{120\alpha_2}\varepsilon\gamma_\sigma\bar{\gamma}_{\mu\nu\rho}\partial^\sigma\chi^I\,,\\ \delta\chi^I &= \alpha_2\varepsilon\gamma^{\mu\nu\rho}\left(\check{\gamma}^{IJ} + 4\delta^{IJ}\right)H^J_{\mu\nu\rho} + \alpha_3\varepsilon\gamma_\mu\left(\check{\gamma}^{IJK} + 3\delta^{IJ}\check{\gamma}^K\right)j^{\mu}_{JK}\\ &\quad + \frac{1}{\alpha_1}\varepsilon\gamma^\mu\check{\gamma}^J\partial_\mu O^{IJ}\,,\\ \delta O^{IJ} &= \alpha_1\varepsilon\check{\gamma}^{(I}\chi^{J)}\,.\end{aligned} \tag{C.1}$$

The constants $\alpha_1, \alpha_2, \alpha_3$ are arbitrary constants that can absorbed in the normalisation for the operators $\mathcal{O}, \chi, H$.

The coefficient of the 1-point function of $T^{\mu\nu}$ in the presence of V is known to be related to the anomaly coefficient d [57, 103, 104]. Acting with supersymmetry, we find that the 1-point function of the superprimary Φ has coefficient [57]

$$a_2 = \frac{5\alpha_1\alpha_3 d}{4\pi^3}\,. \tag{C.2}$$

JHEP07(2023)165

We can fix the coefficients $\alpha_1\alpha_3$ by requiring Φ to be normalised as in (2.2). A simple way to do so is to compare the 2-point function of Φ with the 2-point function of R-symmetry currents j^μ, which is fixed by conformal symmetry to take the form

$$\left\langle j^{\mu ij}(x_1) j^{\nu kl}(x_2)\right\rangle = C_j\left(\delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk}\right)\frac{I_{\mu\nu}(x_{12})}{|x_{12}|^{10}}\,, \qquad I_{\mu\nu}(x) = \delta_{\mu\nu} - 2\frac{x_\mu x_\nu}{x^2}\,. \tag{C.3}$$

The constant C_j is related to the anomaly coefficient c as [105]

$$C_j = \frac{5c}{2\pi^6}\,. \tag{C.4}$$

Acting twice with supersymmetry, a short calculation shows that

$$\langle \Phi(x_1, u_1)\Phi(x_2, u_2)\rangle = \frac{25\alpha_1^2\alpha_3^2 c}{32\pi^6}\frac{(2u_1\cdot u_2)^2}{|x_{12}|^8}\,. \tag{C.5}$$

Matching with the normalisation of Φ (2.2) fixes $(\alpha_1\alpha_3)^2$, and we should take the negative branch to match the supergravity calculation

$$\alpha_1\alpha_3 = -\frac{4\sqrt{2}\pi^3}{5\sqrt{c}}\,. \tag{C.6}$$

Plugging back into (C.2) gives the result (1.7).

References

[1] J.M. Maldacena, *The Large N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200] [INSPIRE].

[2] E. Witten, *Baryons and branes in anti-de Sitter space*, *JHEP* **07** (1998) 006 [hep-th/9805112] [INSPIRE].

[3] O. Aharony, Y. Oz and Z. Yin, *M theory on $AdS_p \times S^{(11-p)}$ and superconformal field theories*, *Phys. Lett. B* **430** (1998) 87 [hep-th/9803051] [INSPIRE].

[4] S. Ferrara, A.F. Grillo and R. Gatto, *Tensor representations of conformal algebra and conformally covariant operator product expansion*, *Annals Phys.* **76** (1973) 161 [INSPIRE].

[5] S. Ferrara, A.F. Grillo, G. Parisi and R. Gatto, *Covariant expansion of the conformal four-point function*, *Nucl. Phys. B* **49** (1972) 77 [INSPIRE].

[6] A.M. Polyakov, *Nonhamiltonian approach to conformal quantum field theory*, *Zh. Eksp. Teor. Fiz.* **66** (1974) 23 [INSPIRE].

[7] R. Rattazzi, V.S. Rychkov, E. Tonni and A. Vichi, *Bounding scalar operator dimensions in 4D CFT*, *JHEP* **12** (2008) 031 [arXiv:0807.0004] [INSPIRE].

[8] C. Beem et al., *Infinite Chiral Symmetry in Four Dimensions*, *Commun. Math. Phys.* **336** (2015) 1359 [arXiv:1312.5344] [INSPIRE].

JHEP07(2023)165

[9] C. Beem, L. Rastelli and B.C. van Rees, *$\mathcal{W}$ symmetry in six dimensions*, *JHEP* **05** (2015) 017 [arXiv:1404.1079] [INSPIRE].

[10] S.M. Chester and E. Perlmutter, *M-Theory Reconstruction from (2,0) CFT and the Chiral Algebra Conjecture*, *JHEP* **08** (2018) 116 [arXiv:1805.00892] [INSPIRE].

[11] C. Beem, M. Lemos, L. Rastelli and B.C. van Rees, *The (2, 0) superconformal bootstrap*, *Phys. Rev. D* **93** (2016) 025016 [arXiv:1507.05637] [INSPIRE].

[12] L. Rastelli and X. Zhou, *Holographic Four-Point Functions in the (2, 0) Theory*, *JHEP* **06** (2018) 087 [arXiv:1712.02788] [INSPIRE].

[13] X. Zhou, *On Superconformal Four-Point Mellin Amplitudes in Dimension $d > 2$*, *JHEP* **08** (2018) 187 [arXiv:1712.02800] [INSPIRE].

[14] P. Heslop and A.E. Lipstein, *M-theory Beyond The Supergravity Approximation*, *JHEP* **02** (2018) 004 [arXiv:1712.08570] [INSPIRE].

[15] L.F. Alday, S.M. Chester and H. Raj, *6d (2,0) and M-theory at 1-loop*, *JHEP* **01** (2021) 133 [arXiv:2005.07175] [INSPIRE].

[16] G. Arutyunov and E. Sokatchev, *Implications of superconformal symmetry for interacting (2,0) tensor multiplets*, *Nucl. Phys. B* **635** (2002) 3 [hep-th/0201145] [INSPIRE].

[17] P.J. Heslop, *Aspects of superconformal field theories in six dimensions*, *JHEP* **07** (2004) 056 [hep-th/0405245] [INSPIRE].

[18] L.F. Alday and X. Zhou, *All Tree-Level Correlators for M-theory on $AdS_7 \times S^4$*, *Phys. Rev. Lett.* **125** (2020) 131604 [arXiv:2006.06653] [INSPIRE].

[19] M. Lemos, B.C. van Rees and X. Zhao, *Regge trajectories for the (2, 0) theories*, *JHEP* **01** (2022) 022 [arXiv:2105.13361] [INSPIRE].

[20] G. Kántor, V. Niarchos, C. Papageorgakis and P. Richmond, *6D (2,0) bootstrap with the soft-actor-critic algorithm*, *Phys. Rev. D* **107** (2023) 025005 [arXiv:2209.02801] [INSPIRE].

[21] J. Penedones, *Writing CFT correlation functions as AdS scattering amplitudes*, *JHEP* **03** (2011) 025 [arXiv:1011.1485] [INSPIRE].

[22] E. Witten, *Some comments on string dynamics*, in the proceedings of the *STRINGS 95: Future Perspectives in String Theory*, (1995), p. 501–523 [hep-th/9507121] [INSPIRE].

[23] A. Strominger, *Open p-branes*, *Phys. Lett. B* **383** (1996) 44 [hep-th/9512059] [INSPIRE].

[24] O.J. Ganor, *Six-dimensional tensionless strings in the large N limit*, *Nucl. Phys. B* **489** (1997) 95 [hep-th/9605201] [INSPIRE].

[25] P.S. Howe, N.D. Lambert and P.C. West, *The Selfdual string soliton*, *Nucl. Phys. B* **515** (1998) 203 [hep-th/9709014] [INSPIRE].

[26] D. Gaiotto, A. Kapustin, N. Seiberg and B. Willett, *Generalized Global Symmetries*, *JHEP* **02** (2015) 172 [arXiv:1412.5148] [INSPIRE].

[27] M. Del Zotto, J.J. Heckman, D.S. Park and T. Rudelius, *On the Defect Group of a 6D SCFT*, *Lett. Math. Phys.* **106** (2016) 765 [arXiv:1503.04806] [INSPIRE].

[28] L. Bhardwaj and S. Schäfer-Nameki, *Higher-form symmetries of 6d and 5d theories*, *JHEP* **02** (2021) 159 [arXiv:2008.09600] [INSPIRE].

[29] F. Apruzzi, L. Bhardwaj, D.S.W. Gould and S. Schafer-Nameki, *2-Group symmetries and their classification in 6d*, *SciPost Phys.* **12** (2022) 098 [arXiv:2110.14647] [INSPIRE].

JHEP07(2023)165

[30] J.M. Maldacena, *Wilson loops in large N field theories*, Phys. Rev. Lett. **80** (1998) 4859 [hep-th/9803002] [INSPIRE].

[31] N. Drukker and M. Trepanier, *M2-doughnuts*, JHEP **02** (2022) 071 [arXiv:2111.09385] [INSPIRE].

[32] N. Drukker and M. Trépanier, *Ironing out the crease*, JHEP **08** (2022) 193 [arXiv:2204.12627] [INSPIRE].

[33] E. D'Hoker, J. Estes, M. Gutperle and D. Krym, *Exact Half-BPS Flux Solutions in M-theory II: Global solutions asymptotic to $AdS_7 \times S^4$*, JHEP **12** (2008) 044 [arXiv:0810.4647] [INSPIRE].

[34] C. Bachas, E. D'Hoker, J. Estes and D. Krym, *M-theory Solutions Invariant under $D(2,1;\gamma) \oplus D(2,1;\gamma)$*, Fortsch. Phys. **62** (2014) 207 [arXiv:1312.5477] [INSPIRE].

[35] P. Liendo, L. Rastelli and B.C. van Rees, *The Bootstrap Program for Boundary CFT_d*, JHEP **07** (2013) 113 [arXiv:1210.4258] [INSPIRE].

[36] M. Billò, V. Gonçalves, E. Lauria and M. Meineri, *Defects in conformal field theory*, JHEP **04** (2016) 091 [arXiv:1601.02883] [INSPIRE].

[37] P. Liendo and C. Meneghelli, *Bootstrap equations for $\mathcal{N} = 4$ SYM with defects*, JHEP **01** (2017) 122 [arXiv:1608.05126] [INSPIRE].

[38] M. Lemos, P. Liendo, M. Meineri and S. Sarkar, *Universality at large transverse spin in defect CFT*, JHEP **09** (2018) 091 [arXiv:1712.08185] [INSPIRE].

[39] S. Caron-Huot, *Analyticity in Spin in Conformal Theories*, JHEP **09** (2017) 078 [arXiv:1703.00278] [INSPIRE].

[40] J. Barrat, A. Gimenez-Grau and P. Liendo, *Bootstrapping holographic defect correlators in $\mathcal{N} = 4$ super Yang-Mills*, JHEP **04** (2022) 093 [arXiv:2108.13432] [INSPIRE].

[41] A. Gimenez-Grau and P. Liendo, *Bootstrapping monodromy defects in the Wess-Zumino model*, JHEP **05** (2022) 185 [arXiv:2108.05107] [INSPIRE].

[42] A. Gimenez-Grau, *Probing magnetic line defects with two-point functions*, arXiv:2212.02520 [INSPIRE].

[43] L. Bianchi, D. Bonomi and E. de Sabbata, *Analytic bootstrap for the localized magnetic field*, JHEP **04** (2023) 069 [arXiv:2212.02524] [INSPIRE].

[44] M.S. Costa, J. Penedones, D. Poland and S. Rychkov, *Spinning Conformal Correlators*, JHEP **11** (2011) 071 [arXiv:1107.3554] [INSPIRE].

[45] E. Witten, *Anti-de Sitter space and holography*, Adv. Theor. Math. Phys. **2** (1998) 253 [hep-th/9802150] [INSPIRE].

[46] K. Jensen, A. O'Bannon, B. Robinson and R. Rodgers, *From the Weyl Anomaly to Entropy of Two-Dimensional Boundaries and Defects*, Phys. Rev. Lett. **122** (2019) 241602 [arXiv:1812.08745] [INSPIRE].

[47] M. Henningson and K. Skenderis, *The Holographic Weyl anomaly*, JHEP **07** (1998) 023 [hep-th/9806087] [INSPIRE].

[48] K.A. Intriligator, *Anomaly matching and a Hopf-Wess-Zumino term in 6d, N=(2,0) field theories*, Nucl. Phys. B **581** (2000) 257 [hep-th/0001205] [INSPIRE].

JHEP07(2023)165

[49] A.A. Tseytlin, *R^4 terms in 11 dimensions and conformal anomaly of (2,0) theory*, Nucl. Phys. B **584** (2000) 233 [hep-th/0005072] [INSPIRE].

[50] M. Beccaria, G. Macorini and A.A. Tseytlin, *Supergravity one-loop corrections on AdS_7 and AdS_3, higher spins and AdS/CFT*, Nucl. Phys. B **892** (2015) 211 [arXiv:1412.0489] [INSPIRE].

[51] A. Chalabi, A. O'Bannon, B. Robinson and J. Sisti, *Central charges of 2d superconformal defects*, JHEP **05** (2020) 095 [arXiv:2003.02857] [INSPIRE].

[52] C.R. Graham and E. Witten, *Conformal anomaly of submanifold observables in AdS / CFT correspondence*, Nucl. Phys. B **546** (1999) 52 [hep-th/9901021] [INSPIRE].

[53] N. Drukker, M. Probst and M. Trépanier, *Surface operators in the 6d N = (2, 0) theory*, J. Phys. A **53** (2020) 365401 [arXiv:2003.12372] [INSPIRE].

[54] N. Drukker, S. Giombi, A.A. Tseytlin and X. Zhou, *Defect CFT in the 6d (2,0) theory from M2 brane dynamics in $AdS_7 \times S^4$*, JHEP **07** (2020) 101 [arXiv:2004.04562] [INSPIRE].

[55] J. Barrat, A. Gimenez-Grau and P. Liendo, *A dispersion relation for defect CFT*, JHEP **02** (2023) 255 [arXiv:2205.09765] [INSPIRE].

[56] L. Bianchi and D. Bonomi, *Conformal dispersion relations for defects and boundaries*, arXiv:2205.09775 [INSPIRE].

[57] N. Drukker, M. Probst and M. Trépanier, *Defect CFT techniques in the 6d $\mathcal{N} = (2,0)$ theory*, JHEP **03** (2021) 261 [arXiv:2009.10732] [INSPIRE].

[58] C. Cordova, D. Gaiotto and S.-H. Shao, *Surface Defects and Chiral Algebras*, JHEP **05** (2017) 140 [arXiv:1704.01955] [INSPIRE].

[59] F.A. Dolan, L. Gallot and E. Sokatchev, *On four-point functions of 1/2-BPS operators in general dimensions*, JHEP **09** (2004) 056 [hep-th/0405180] [INSPIRE].

[60] P. Liendo, C. Meneghelli and V. Mitev, *On Correlation Functions of BPS Operators in 3d $\mathcal{N} = 6$ Superconformal Theories*, Commun. Math. Phys. **350** (2017) 387 [arXiv:1512.06072] [INSPIRE].

[61] M. Lemos, P. Liendo, C. Meneghelli and V. Mitev, *Bootstrapping $\mathcal{N} = 3$ superconformal theories*, JHEP **04** (2017) 032 [arXiv:1612.01536] [INSPIRE].

[62] P. Liendo, C. Meneghelli and V. Mitev, *Bootstrapping the half-BPS line defect*, JHEP **10** (2018) 077 [arXiv:1806.01862] [INSPIRE].

[63] F.A. Dolan and H. Osborn, *Conformal four point functions and the operator product expansion*, Nucl. Phys. B **599** (2001) 459 [hep-th/0011040] [INSPIRE].

[64] M. Isachenkov, P. Liendo, Y. Linke and V. Schomerus, *Calogero-Sutherland Approach to Defect Blocks*, JHEP **10** (2018) 204 [arXiv:1806.09703] [INSPIRE].

[65] P.S. Howe, G. Sierra and P.K. Townsend, *Supersymmetry in Six-Dimensions*, Nucl. Phys. B **221** (1983) 331 [INSPIRE].

[66] B. Eden, S. Ferrara and E. Sokatchev, *(2,0) superconformal OPEs in $d = 6$, selection rules and nonrenormalization theorems*, JHEP **11** (2001) 020 [hep-th/0107084] [INSPIRE].

[67] S. Ferrara and E. Sokatchev, *Universal properties of superconformal OPEs for 1/2 BPS operators in $3 \leq d \leq 6$*, New J. Phys. **4** (2002) 2 [hep-th/0110174] [INSPIRE].

JHEP07(2023)165

[68] A. Gimenez-Grau and P. Liendo, *Bootstrapping Coulomb and Higgs branch operators*, *JHEP* **01** (2021) 175 [arXiv:2006.01847] [INSPIRE].

[69] M. Gunaydin and R.J. Scalise, *Unitary Lowest Weight Representations of the Noncompact Supergroup* $OSp(2m^*|2n)$, *J. Math. Phys.* **32** (1991) 599 [INSPIRE].

[70] N.B. Agmon and Y. Wang, *Classifying Superconformal Defects in Diverse Dimensions Part I: Superconformal Lines*, arXiv:2009.06650 [INSPIRE].

[71] G.M.T. Watts, *$\mathcal{W}$-algebras and their representations*, in the proceedings of the *Conformal Field Theories and Integrable Models*, Z. Horváth and L. Palla eds., Berlin, Heidelberg (1997), p. 55–84.

[72] P. Bouwknegt and K. Schoutens, *W symmetry in conformal field theory*, *Phys. Rept.* **223** (1993) 183 [hep-th/9210010] [INSPIRE].

[73] H.-C. Kim, S. Kim, S.-S. Kim and K. Lee, *The general M5-brane superconformal index*, arXiv:1307.7660 [INSPIRE].

[74] M. Bullimore and H.-C. Kim, *The Superconformal Index of the (2,0) Theory with Defects*, *JHEP* **05** (2015) 048 [arXiv:1412.3872] [INSPIRE].

[75] A. Campoleoni, S. Fredenhagen and S. Pfenninger, *Asymptotic W-symmetries in three-dimensional higher-spin gauge theories*, *JHEP* **09** (2011) 113 [arXiv:1107.0290] [INSPIRE].

[76] F. Bastianelli and R. Zucchini, *Three point functions of chiral primary operators in $d = 3, \mathcal{N} = 8$ and $d = 6, \mathcal{N} = (2,0)$ SCFT at large N*, *Phys. Lett. B* **467** (1999) 61 [hep-th/9907047] [INSPIRE].

[77] R. Corrado, B. Florea and R. McNees, *Correlation functions of operators and Wilson surfaces in the $d = 6$, (0,2) theory in the large N limit*, *Phys. Rev. D* **60** (1999) 085011 [hep-th/9902153] [INSPIRE].

[78] F. Bonetti, C. Meneghelli and L. Rastelli, *VOAs labelled by complex reflection groups and 4d SCFTs*, *JHEP* **05** (2019) 155 [arXiv:1810.03612] [INSPIRE].

[79] L. Bianchi and M. Lemos, *Superconformal surfaces in four dimensions*, *JHEP* **06** (2020) 056 [arXiv:1911.05082] [INSPIRE].

[80] P.C. Argyres, M. Lotito and M. Weaver, *Vertex algebra of extended operators in 4d N=2 superconformal field theories*, arXiv:2211.04410 [INSPIRE].

[81] N. Drukker, D. Gaiotto and J. Gomis, *The Virtue of Defects in 4D Gauge Theories and 2D CFTs*, *JHEP* **06** (2011) 025 [arXiv:1003.1112] [INSPIRE].

[82] V.A. Fateev and A.B. Zamolodchikov, *Conformal Quantum Field Theory Models in Two-Dimensions Having Z(3) Symmetry*, *Nucl. Phys. B* **280** (1987) 644 [INSPIRE].

[83] N. Afkhami-Jeddi et al., *Constraints on higher spin CFT_2*, *JHEP* **05** (2018) 092 [arXiv:1707.07717] [INSPIRE].

[84] A.R. Linshaw, *Universal two-parameter $\mathcal{W}_\infty$-algebra and vertex algebras of type $\mathcal{W}(2,3,\ldots,N)$*, *Compos. Math.* **157** (2021) 12 [arXiv:1710.02275] [INSPIRE].

[85] V.A. Fateev and S.L. Lukyanov, *The Models of Two-Dimensional Conformal Quantum Field Theory with $\mathbb{Z}_N$ Symmetry*, *Int. J. Mod. Phys. A* **3** (1988) 507 [INSPIRE].

[86] L.F. Alday, F. Benini and Y. Tachikawa, *Liouville/Toda central charges from M5-branes*, *Phys. Rev. Lett.* **105** (2010) 141601 [arXiv:0909.4776] [INSPIRE].

JHEP07(2023)165

[87] C. Cordova and D.L. Jafferis, *Toda Theory From Six Dimensions*, *JHEP* **12** (2017) 106 [arXiv:1605.03997] [INSPIRE].

[88] L.F. Alday and S. Caron-Huot, *Gravitational S-matrix from CFT dispersion relations*, *JHEP* **12** (2018) 017 [arXiv:1711.02031] [INSPIRE].

[89] E. Lauria, M. Meineri and E. Trevisani, *Radial coordinates for defect CFTs*, *JHEP* **11** (2018) 148 [arXiv:1712.07668] [INSPIRE].

[90] J. Maldacena, D. Simmons-Duffin and A. Zhiboedov, *Looking for a bulk point*, *JHEP* **01** (2017) 013 [arXiv:1509.03612] [INSPIRE].

[91] C. Beem, L. Rastelli and B.C. van Rees, *The $\mathcal{N} = 4$ Superconformal Bootstrap*, *Phys. Rev. Lett.* **111** (2013) 071601 [arXiv:1304.1803] [INSPIRE].

[92] P. Liendo, Y. Linke and V. Schomerus, *A Lorentzian inversion formula for defect CFT*, *JHEP* **08** (2020) 163 [arXiv:1903.05222] [INSPIRE].

[93] A. Erdélyi and H. Bateman, *Higher transcendental functions*, Bateman Manuscript Project, Krieger (1981), https://books.google.ca/books?id=h7UTzQEACAAJ.

[94] O. Aharony, L.F. Alday, A. Bissi and E. Perlmutter, *Loops in AdS from Conformal Field Theory*, *JHEP* **07** (2017) 036 [arXiv:1612.03891] [INSPIRE].

[95] G. Mack, *D-independent representation of Conformal Field Theories in D dimensions via transformation to auxiliary Dual Resonance Models. Scalar amplitudes*, arXiv:0907.2407 [INSPIRE].

[96] L. Rastelli and X. Zhou, *How to Succeed at Holographic Correlators Without Really Trying*, *JHEP* **04** (2018) 014 [arXiv:1710.05923] [INSPIRE].

[97] V. Goncalves and G. Itsios, *A note on defect Mellin amplitudes*, arXiv:1803.06721 [INSPIRE].

[98] P. Ferrero and C. Meneghelli, *Bootstrapping the half-BPS line defect CFT in N=4 supersymmetric Yang-Mills theory at strong coupling*, *Phys. Rev. D* **104** (2021) L081703 [arXiv:2103.10440] [INSPIRE].

[99] V.K. Dobrev, V.B. Petkova, S.G. Petrova and I.T. Todorov, *Dynamical Derivation of Vacuum Operator Product Expansion in Euclidean Conformal Quantum Field Theory*, *Phys. Rev. D* **13** (1976) 887 [INSPIRE].

[100] V.K. Dobrev et al., *Harmonic Analysis on the n-Dimensional Lorentz Group and Its Application to Conformal Quantum Field Theory*, *Lect. Notes. Phys.* **63** (1977) [INSPIRE].

[101] F.A. Dolan and H. Osborn, *Conformal partial waves and the operator product expansion*, *Nucl. Phys. B* **678** (2004) 491 [hep-th/0309180] [INSPIRE].

[102] D.M. McAvity and H. Osborn, *Conformal field theories near a boundary in general dimensions*, *Nucl. Phys. B* **455** (1995) 522 [cond-mat/9505127] [INSPIRE].

[103] A. Lewkowycz and E. Perlmutter, *Universality in the geometric dependence of Renyi entropy*, *JHEP* **01** (2015) 080 [arXiv:1407.8171] [INSPIRE].

[104] L. Bianchi, M. Meineri, R.C. Myers and M. Smolkin, *Rényi entropy and conformal defects*, *JHEP* **07** (2016) 076 [arXiv:1511.06713] [INSPIRE].

[105] R. Manvelyan and A.C. Petkou, *A Note on R currents and trace anomalies in the (2,0) tensor multiplet in $d = 6$ AdS/CFT correspondence*, *Phys. Lett. B* **483** (2000) 264 [hep-th/0003017] [INSPIRE].