Received: 9 February 2022 | Revised: 3 March 2022 | Accepted: 10 May 2022

DOI: 10.1111/meca.12400

METROECONOMICA
INTERNATIONAL REVIEW OF ECONOMICS

WILEY

ORIGINAL RESEARCH

# When is the long run?—Historical time and adjustment periods in demand-led growth models

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**Funding information**
Open Access Funding provided by Universita degli Studi di Roma La Sapienza within the CRUI-CARE Agreement.

**Abstract**

In recent years, Post-Keynesian analysis has been characterized by a renewed interest in long-run theories of growth and distribution. While many authors have focused on the convergence of demand-led growth models to a fully adjusted equilibrium, relatively little attention has been given to the time required to reach this long-run position. In order to fill the gap, this paper seeks to answer the question of *when is the long run* in demand-led growth models. By making use of numerical integration, it analyses the time of adjustment from one steady-state to the other in two well-known demand-led growth models: the Sraffian Supermultiplier and the fully adjusted version of the neo-Kaleckian model. The results show that the adjustment period is generally beyond an economically meaningful time span, suggesting that researchers and policy makers ought to pay more attention to the models' predictions during the traverse rather than focusing on steady-state positions.

**KEYWORDS**

adjustment period, effective demand, growth, neo-Kaleckian model, Sraffian Supermultiplier, time, traverse

**JEL CLASSIFICATION**

B41, B51, E11, E12

# 1 | INTRODUCTION

This paper takes Joan Robinson seriously.[1] In her famous 1980 article, Robinson claimed that "to construct models that cannot be applied is merely an idle amusement" (p. 223–224). Yet, the construction of any supposedly realistic model cannot abstain from the consideration that historical time – rather than logical time – rules reality. Accordingly, it is "a common error to confuse a comparison of static positions with a movement between them" (ibid., p. 228). This contribution is chiefly interested in the duration of the movement between steady-state positions in demand-led growth models.

Post-Keynesian analysis has been characterized by a renewed interest in long-run theories of growth and distribution. In recent years, academic debates about Post-Keynesian theories of growth and distribution have been characterized by a renewed interest in long-run modeling compared to short-run analysis – or "chain*(s)* of short-period situations" (Kalecki, 1971, p. 165). While Post-Keynesian growth theory benefited from this shift, gaining more rigor and coherency, more fundamental questions were often overlooked; in particular, few or no academic discussions can be found as regards the essential question of *when is the long run* (Robinson, 1980, p. 226) and how we can evaluate growth models in historical time. In other terms, inquiries about the nature and duration of the traverse effectively fell by the wayside. Bringing these issues to the front of the debate is thus of key importance to avoid committing Post-Keynesian growth theory to what we might call the 'Marshallian leap', making "the step from a model to reality by an act of faith" (ibid.).

Along these lines, the present contribution attempts to shed light on a dormant debate on traverse analysis and the persistence of out-of-equilibrium dynamics, thus recovering and deepening Joan Robinson's insights on the differences between logical and historical time in economic analysis. Accordingly, the paper seeks to answer Joan Robinson's (1980) question *'when is the long run?'*, evaluating in historical time the adjustment periods to the long-run position in two prominent demand-led models focused on the role of autonomous demand in driving economic growth, namely the Sraffian Supermultiplier model and the long-run version of the neo-Kaleckian model presented by Allain (2015) and Lavoie (2016).[2] In other terms, the article describes the temporal sequence required to establish a new long-run position following an initial increase in the growth rate of autonomous expenditures. More specifically, in accordance with the line of research pioneered by Sato (1963, 1980), Sato (1966) and Atkinson (1969), the paper adopts the method of numerical integration to solve the systems of differential equations regulating the out-of-equilibrium dynamics of the two models. In order to do that, we calibrate both models in line with the existing theoretical and empirical literature.

The paper is organized as follows. Section 2 presents the two models under scrutiny, that is, the Sraffian Supermultiplier model and the long-run version of the neo-Kaleckian model presented by

[1] The *incipit* of the paper draws upon the opening line of one of most influential articles in the field of neoclassical growth models, that is, Mankiw et al. (1992).

[2] Some words on the rationale behind the choice of the two models are in order. First, both models rely on the role of autonomous components of demand in driving economic growth. Given that "the literature on autonomous growth has itself been cast in terms that are intrinsically long run" (Skott, 2019, p. 238), the comparison of the two models allow to coherently answer the question that inspires the paper. Second, both models reach a fully adjusted position – equaling the actual and normal rate of capacity utilization in the long run – thus preventing the emergence of the second Harrod problem. Third, both models are demand-led, allowing to summarize the analytical compatibilities and divergences of Kaleckian and Sraffian insights on growth in a relatively simple way. More specifically, the structure of the economy is the same in both models, with the notable exception of investment – as discussed in Appendix A. On one hand, the Allain-Lavoie (2015) model relies on a neo-Kaleckian investment function whereby the accumulation rate depends on discrepancies between the actual and the normal utilization rates; on the other, investment is treated as fully induced in the Sraffian Supermultiplier model. As shown later in the article, the different investment theories adopted by the two models produce varied out-of-equilibrium trajectories.

Allain (2015) and Lavoie (2016). Section 3 discusses the adopted parameter calibration, then presenting the numerical solution of the two models and our main findings. Section 4 discusses the sensitivity of the models' time of adjustment following a change in the parameter space. Last, Section 5 concludes, discussing the interpretation and implications of the results.

## 2 | SRAFFIAN AND KALECKIAN LONG-RUN GROWTH MODELS

This section provides a synthetic review of the models under scrutiny. A more in-depth discussion of the Sraffian Supermultiplier model (Subsection 2.1) can be found in Serrano (1995 b); Serrano and Freitas (2017); Girardi and Pariboni (2016) and Gallo (2019). As regards the long-run version of the neo-Kaleckian model (Subsection 2.2) with autonomous demand and Harridan dynamics, see Allain (2015, 2018, 2021) and Lavoie (2016).

In order to make the Supermultiplier and neo-Kaleckian frameworks fully comparable, the two models are presented for an open economy with government activity. Moreover, we include a linear depreciation rate of physical capital.[3]

### 2.1 | The Sraffian Supermultiplier model

Following Serrano and Freitas (2017), this Subsection presents the Sraffian Supermultiplier model assuming an open economy with government activity. The model can be represented as a 3-equation in 3 variables – autonomous demand growth $\left(g_t^Z\right)$, the investment share $(h_t)$ and the rate of capacity utilization $(u_t = Y_t / Y^p)$

$$g_t^Y = g_t^Z + \frac{h_t \gamma \left(u_t - u_n\right)}{s + m - h_t} \tag{1}$$

$$g_t^K = \frac{h_t u_t}{v} - \delta \tag{2}$$

$$g_t^Z = \overline{g^Z} \tag{3}$$

Equation (1) describes the evolution of economic activity as depending on autonomous demand growth $\left(g_t^Z\right)$ plus an additional proportional rate of growth of output resulting from the supermultiplier when capacity utilization is not at its normal degree $(u_n)$, that is, the second term of the equation. Moreover, $s$ indicates the “tax-adjusted marginal propensity to save” (Girardi & Pariboni, 2015, p. 526), $m$ is the propensity to import, and $\gamma$ represents “a parameter that measures the reaction of the growth rate of the marginal propensity to invest to the deviation of the actual degree of capacity utilization” (Serrano & Freitas, 2017, p. 74). Assuming a constant capital-capacity ratio $(v = K_t/Y^p)$, the evolution of capital accumulation is given by the rate of growth of capacity output minus the depreciation rate $(\delta)$, as in equation (2).[4] Lastly, equation (3) constitutes the closure of the model for an exogenously given rate of growth of autonomous demand $\left(\overline{g^Z}\right)$.

[3] For the derivation of variables from levels to growth rates, see Appendix A. The list of variables used in the paper is reported in Appendix B, while a list and description of parameters can be found in Table 1 below.

[4] Under the assumption of fully induced investment, it ought to be noted that this is a mere accounting identity, as showed in Appendix A.

TABLE 1 Parameter values

| Par. | Description | Value | Source |
|---|---|---|---|
| $\delta$ | Depreciation rate (annual) | 0.084 | Fazzari et al. (2020) |
| $u_n$ | Normal rate of capacity utilization | 0.8242 | Setterfield and Budd (2011) |
| $v$ | Capital-capacity ratio (annual) | 0.9890 | Author's calculation, based on Fazzari et al. (2020) |
| $s$ | Propensity to save | 0.5 | Fazzari et al. (2020) |
| $m$ | Propensity to import | 0.17 | Gallo (2021), Girardi and Pariboni (2016) |
| $\gamma$ | Sensitivity of the investment share to $u_t - u_n$ | 0.15 | Nomaler et al. (2021) |
| $\beta$ | Sensitivity of the investment rate to $u_t - u_n$ | 0.25 | Allain (2021) |
| $\mu$ | Sensitivity of animal spirits to $u_t - u_n$ | 0.18 | Author's calculation, based on Allain (2021) |

*Source*: author's calculation, various sources.

The model settles in its long-run steady state when the fully adjusted position (Vianello, 1985) is reached, that is, $u_t = u_n$ and actual output and capital grow at the same pace, that is, $g_t^Y = g_t^K$. Therefore, the long-run equilibrium position of the model is characterized by:

$$h^* = \frac{v}{u_n}\left(\overline{g^Z} + \delta\right) \tag{4}$$

$$u^* = u_n \tag{5}$$

$$g^{Z*} = \overline{g^Z} \tag{6}$$

Accordingly, in the long run all growth rates ought to equal the exogenous expansion of autonomous components of demand, that is, $g^* = g^{K*} = g^{Y*} = \overline{g^Z}$.

Let us now analyze more-in-depth the process of economic growth and out-of-equilibrium dynamics. The adjustment to the long-run equilibrium is carried out by the two endogenous variables of the system, that is, the rate of capacity utilization $u_t$ and the investment share $h_t$. In line with Serrano and Freitas (2017), the two adjustment mechanisms[5] are modeled as follows:

$$\dot{u} = u_t\left(g_t^Y - g_t^K\right) \tag{7}$$

$$\dot{h} = h_t\gamma\left(u_t - u_n\right) \tag{8}$$

Substituting equation (1 and 2) into equation (7), we obtain the system of two first-order non-linear differential equations that will be solved numerically in Section (3):

$$\begin{cases} \dot{u} = & u_t\left[g_t^Z + \dfrac{h_t\gamma\left(u_t - u_n\right)}{s + m - h_t} - \dfrac{h_t}{v}u_t + \delta\right] \\ \dot{h} = & h_t\gamma\left(u_t - u_n\right) \end{cases} \tag{9}$$

Summarizing, discrepancies between actual and normal degrees of capacity utilization can only be of transient nature, producing growth effects in the short but not in the long run, in

[5] Henceforth, changes of a variable over time will be denoted with the dot symbol, for example, $\dot{u} = du/dt$.

which the fully adjusted position is reached.[6] More specifically, during the adjustment process when $u_t \gtreqless u_n$, it follows that $\dot{h} \gtreqless 0$, and whenever $g_t^Y \gtreqless g_t^K$, then $\dot{u} \gtreqless 0$. The rest of the paper will focus on evaluating in historical time the transiency of these effects.

### 2.2 | The long-run neo-Kaleckian model with autonomous expenditures and a Harrodian mechanism

The Allain-Lavoie long-run version of the neo-Kaleckian model can be presented as the following 3-equations system in 3 variables – autonomous demand growth $\left(g_t^Z\right)$, animal spirits $(\alpha_t)$ and the autonomous demand-capital ratio $(z_t)$:

$$g_t^I = \alpha_t + \beta\left(u_t - u_n\right) \tag{10}$$

$$g_t^S = \frac{(s+m)u_t}{v} - z_t \tag{11}$$

$$g_t^Z = \overline{g^Z} \tag{12}$$

Equation (10) constitutes the conventional version of the neo-Kaleckian investment function with a normal rate of capacity utilization.[7] The term $\alpha_t$ captures animal spirits, which along with $z_t$, vary in the long run to prevent the emergence of the second Harrod problem, as we will see later.[8] Equation (11) represents the saving function proposed by Lavoie (2016) in line with Serrano (1995 a,b); it incorporates in the neo-Kaleckian model a "non-proportional saving function with a constant term that in the long run grows at an exogenously given rate" (Lavoie, 2016, p. 173).

In the short run, animal spirits and the autonomous demand-capital ratio are assumed to be constant. Accordingly the *ex-post* equality of the growth rates of investment and saving yields the following short-run equilibrium rate of growth of investment and saving:

$$g_{sr}^{I*} = g_{sr}^{S*} = \alpha + \beta\left(u_{sr}^* - u_n\right) \tag{13}$$

Solving for the short-run goods market equilibrium of $g_t^S = g_t^I$, it follows that the short-run rate of capacity utilization is equal to:

$$u_{sr}^* = \frac{\left(\alpha + z - \beta u_n\right)v}{s + m - \beta v} \tag{14}$$

If not by a fluke, the short-run rate of capacity utilization $u_{sr}^*$ – that brings about the goods market equilibrium – will diverge from its long-run value $u_n$. More specifically, short-run discrepancies between the actual and normal rates of capacity utilization are given by:

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[6] For a discussion of the stability of the system, see Appendix A.

[7] It is worth stressing that $g_t^I$ denotes the accumulation rate, that is, $I_t/K_t$, not the growth rate of investment. For further discussion, see equation (A17) in Appendix A.

[8] Amadeo (1986) was the first author to associate the constant term in the investment function with animal spirits, and the paper maintains his terminology. However, as acknowledged by Lavoie (2016), $\alpha_t$ could be interpreted as capturing all determinants of investment unexplained by the model, "such as technological change, the profit rate or the profit share, credit or monetary conditions, the leverage ratio of firms, radical uncertainty and so on" (*ibid.*, p. 177).

$$u_{sr}^{*} - u_n = \frac{(\alpha + z)v - (s+m)u_n}{s+m-\beta v} \tag{15}$$

Consequently, the equilibrium accumulation rate in the short run is given by:

$$g_{sr}^{K*} = g_{sr}^{I*} - \delta = \alpha + \beta\left(u_{sr}^{*} - u_n\right) - \delta \tag{16}$$

where $g_{sr}^{K*}$ is the growth rate of the capital stock corresponding to the goods market equilibrium.[9]

However, during the traverse towards the long-run steady state, animal spirits $\alpha$ and the $z$ ratio will vary, ensuring the long-run convergence of economic growth to autonomous demand growth $\left(g^{*} = g^{K*} = g^{Y*} = \overline{g^{Z}}\right)$ and of the actual rate of capacity utilization towards its normal degree $(u = u_n)$. Therefore, the long-run equilibrium position of the model is characterized by:

$$z^{*} = \frac{(s+m)u_n}{v} - \overline{g^{Z}} - \delta \tag{17}$$

$$\alpha^{*} = \overline{g^{Z}} + \delta \tag{18}$$

$$u^{*} = u_n \tag{19}$$

$$g^{Z*} = \overline{g^{Z}} \tag{20}$$

As mentioned above, the long-run adjustment process is carried out through changes in animal spirits and in the autonomous demand-capital ratio. More specifically, animal spirits react to discrepancies between the short-run equilibrium of the capacity utilization rate – that is, the one that ensures the *ex-post* adjustment of saving to investment – and the normal degree. Furthermore, the $z$ ratio adjusts to discrepancies between the exogenous growth rate of autonomous demand and the short-run equilibrium rate of economic growth.[10]

$$\dot{\alpha} = \mu\left(g_{sr}^{I*} - \alpha_t\right) = \beta\mu\left(u_{sr}^{*} - u_n\right) \tag{21}$$

$$\dot{z} = z_t\left(\overline{g^{Z}} - g_{sr}^{K*}\right) \tag{22}$$

[9] In order to make the Allain-Lavoie model fully comparable with the Supermultiplier, a small amendment is introduced, including a linear depreciation rate of the capital stock. The novelty does not alter significantly the long-run equilibrium results, as showed in Appendix A.

[10] For the discussion of the derivation and the economic rationale of the two adjustments, see Allain (2015, 2018), Lavoie (2016) and Skott (2017). Regarding the animal spirits adjustment, the paper adopts the specification suggested by Allain (2015, 2018) rather than the one put forward by Lavoie (2016), who expresses the adjustment in terms of the growth rate of $\alpha$. However, as noted by Skott (2017, p. 188) "There is no reason […] to assume that the rate of change should be proportional to the level of $\gamma$ [*$\alpha$ in the notation of this paper*] for any given discrepancy", as would result from Lavoie's specification $\left(\hat{\alpha} = \mu\left(g_{sr}^{I*} - \alpha_t\right) \Rightarrow \dot{\alpha} = \alpha_t\mu\left(g_{sr}^{I*} - \alpha_t\right)\right)$.

Substituting equations (15) and (16) into the above equations, we obtain the system of two first-order non-linear differential equations describing out-of-equilibrium dynamics in the long-run neo-Kaleckian model:

$$\begin{cases} \dot{\alpha} = & \beta\mu\left[\dfrac{(\alpha_t + z)\,v - (s+m)u_n}{s+m-\beta v}\right] \\ \dot{z} = & z_t\left[\overline{g^Z} - \alpha_t - \beta\left(\dfrac{(\alpha_t + z)\,v - (s+m)u_n}{s+m-\beta v}\right) + \delta\right] \end{cases} \tag{23}$$

As discussed by Lavoie (2016, p. 185–186), the system is dynamically stable "when there is short-run Keynesian stability as long as the effect of Harridan instability is not overly strong". As showed in Appendix A, this implies that the system converges towards its long-run equilibrium when $s + m - \beta v > 0$ and $\mu < \frac{(s+m)u_n}{v} - \overline{g^Z} - \delta$. Moreover, it is worth stressing that – similarly to the Supermultiplier model - "the growth rate of autonomous expenditures cannot be too large, for otherwise the share of autonomous consumption expenditures would need to be negative" (Lavoie, 2016, p. 193). In our framework:

$$z^* > 0 \quad \Rightarrow \quad \overline{g^Z} < \frac{(s+m)u_n}{v} - \delta \tag{24}$$

## 3 | NUMERICAL SOLUTION

Since an analytical solution in terms of the time of adjustment for both systems of differential equations cannot be found, the method of numerical integration is adopted.

Before moving to parameter calibration in Subsection (3.1), an important consideration is in order. When adopting numerical integration methods, one must be careful about the interpretation of the time dimension, which stems from the time frequency of the calibration. Since the analysis focuses on long-run growth models, an annual calibration appears to be the most sensible. Therefore, by setting parameter values and initial conditions so as to ensure that all relevant variables and growth rates are compatible with yearly processes (e.g. autonomous demand growth is around 2.5% per year), the time unit could be consistently taken as equal to one calendar year. More specifically, under the assumption that the long-run adjustment of capacity to demand does not occur faster than the unit period considered (Gandolfo, 2012), the yearly calibration allows to coherently interpret the out-of-equilibrium trajectories in calendar time with $dt = 1$ year, as more extensively discussed by Gallo (2021).

### 3.1 | Parameter calibration and initial values

Parameter values are set in accordance with the empirical evidences for the US economy in the post-war period, as well as in line with previous model calibrations.

The values assigned to the parameters are summarized in Table 1.

The value of the annual depreciation rate ($\delta$) is taken from Fazzari et al. (2020). As discussed by the authors in their Supplementary Appendix, the value is consistent with the empirical evidences for the US economy. The normal rate of capacity utilization ($u_n = 82.42\%$) is set in accordance to Setterfield and Budd (2011). As regards $u_n$, it is worth mentioning that the value matches the empirical evidences for other advanced capitalist economies, for example, it is relatively close to the value

(0.8104) calculated by Gallo (2019).[11] The capital-capacity ratio ($v$) is also obtained from Fazzari et al. (2020). By using BEA data both on non-residential investment and non-residential capital stock, the authors estimate a long-run capital-output ratio in the US equal to 1.2. Therefore, given $u_n = 82.42\%$, the capital-capacity ratio will be equal to $v = \frac{K}{Y_p} = \frac{K}{Y_n}\frac{Y_n}{Y_p} = 1.2 \times 0.8242 = 0.9890$. The benchmark values of the propensities to save and to import are set in accordance to the empirical evidence in the US economy as discussed by Girardi and Pariboni (2016), Fazzari et al. (2020) and Gallo (2021).[12]

Last, the supermultiplier-specific parameter $\gamma$ – which measures the reaction of the investment share to changes in the utilization rate – is taken from Nomaler et al. (2021). The remaining parameters $\beta$ and $\mu$ (specific to the neo-Kaleckian model) are both taken from Allain (2021).[13] Since these sensitivities greatly influence the numerical solution of the two systems of differential equations under scrutiny, more attention should be given to them. Therefore, the next section will assess how changing the value of these parameters affects the time of adjustment in the two models.

A discussion of the choice of the initial conditions is now in order (Table 2). First, we ought to recall that the main goal of the exercise conducted in this Section is to show the persistence of out-of-equilibrium dynamics following an increase in autonomous-demand growth. Accordingly, let us suppose that prior to the shock the economy was in its fully adjusted position $u_0 = u_n = 82.42\%$, growing at an exogenously given annual growth rate of autonomous demand of 2.5%.[14] Accordingly, from equation (4) and (5) and on the basis of the parameter calibration discussed above, it follows that the initial value of the investment share ($h_0$) in the Supermultiplier model is equal to 13.08% – in line with the empirical evidences for the US economy (Fazzari et al., 2020; Gallo, 2021; Girardi & Pariboni, 2016). Similarly, equations (17) and (18) imply that $z_0 = 43.93\%$ and $\alpha_0 = 10.9\%$ in the amended neo-Kaleckian model.

At time $t = 0$, the annual growth rate of autonomous demand permanently increases from 2.5% to 3.5% (e.g. as a consequence of an increase in government spending), thus affecting the long-run growth path of both models and giving rise to the long-run traverse discussed in the next subsection.

[11] Consistent with the models presented in Section 2, treating the normal degree of capacity as parametric implies that it is not affected by temporary changes in demand. For a more detailed critical discussion of the notion of normal capacity, the interested reader may refer to Ciccone (1986); Kurz (1986). For an empirical support of the idea that normal utilization is exogenous to the level of demand, see Haluska, Summa, and Serrano (2021) and Haluska, Braga, and Summa (2021).

[12] The value of $s$ might seem greater than expected compared to estimates and calibration that use personal consumption expenditure to derive the propensity to save. However, as discussed by Cynamon and Fazzari (2017) and Fazzari et al. (2020, Supplementary Appendix), the most appropriate way to calculate $s$ in models with autonomous expenditures is by using the adjusted household demand approach, which yields a point estimate of about 0.5 for the US (Fazzari et al., 2020, Supplementary Appendix, p. 2).

[13] It ought to be noted that the value of $\mu$ in the present calibration exercise is slightly above the one in Allain (2021), who sets its value equal to $\mu = 0.4z^*$. Accordingly, since in this paper the derived equilibrium autonomous demand-capital ratio ($z^*$) is higher, $\mu$ will be higher as well.

[14] The value of the year-on-year growth rate of autonomous demand is taken from Fazzari et al. (2020). It is worth noting that this is consistent with the empirical evidences for the US economy; according to the definition of autonomous demand used by Girardi and Pariboni (2016), the average annual growth rate of the variable in the US for the period 1979–2013 is just slightly higher (2.54%).

**TABLE 2** Initial conditions

| Variable | Description | Value |
|---|---|---|
| $g_0^Z$ | Autonomous demand (annual) growth rate | 0.035 |
| $u_0$ | Capacity utilization rate | 0.8242 |
| $\alpha_0$ | Animal spirits | 0.109 |
| $z_0$ | Autonomous demand-capital ratio | 0.4493 |
| $h_0$ | Investment share | 0.1308 |

*Source*: Author's calculation.

## 3.2 | How long is the long run?

This subsection shows by means of numerical integration the behavior of the two models following a permanent increase in the growth rate of autonomous demand, relying on the calibration summarized in Tables 1 and 2.

### 3.2.1 | The long-run convergence in the supermultiplier model

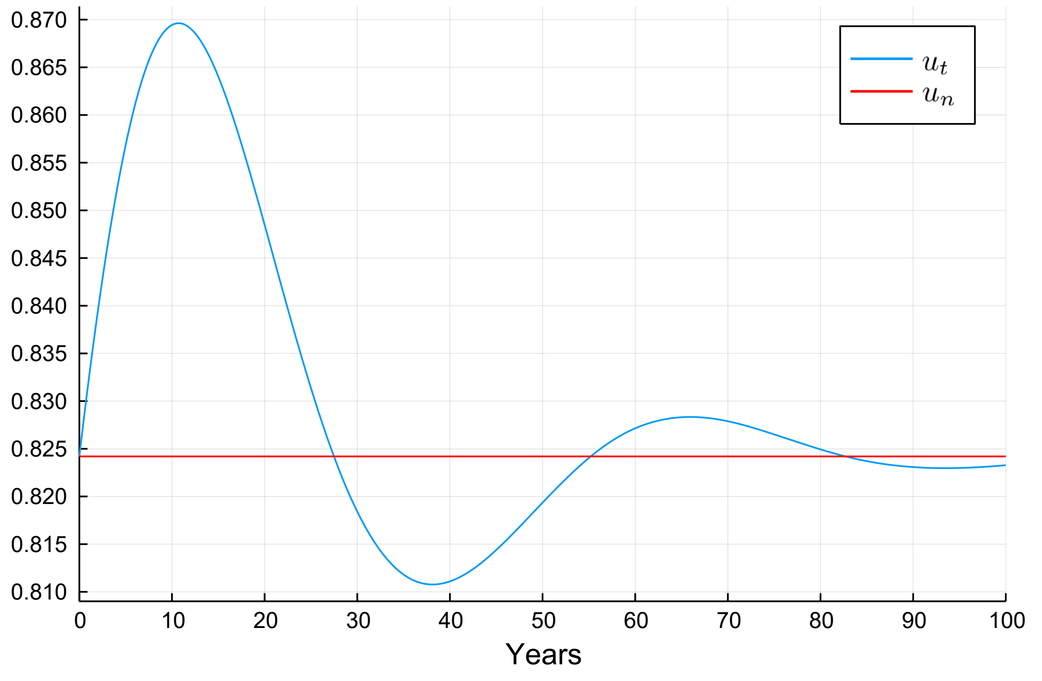
As discussed in Subsection (2.1), the two adjusting variables of the Sraffian Supermultiplier model are the rate of capacity utilization and the investment share. In the long-run steady-state, they should come back, respectively, to the normal rate of capacity utilization ($u_n$) and to the equilibrium investment share ($h^*$) given by equation (4).

Figure 1 shows the behavior of the two adjusting variables in the long run, following an increase in the growth rate of autonomous demand at time 0, from 2.5% to 3.5%.[15]

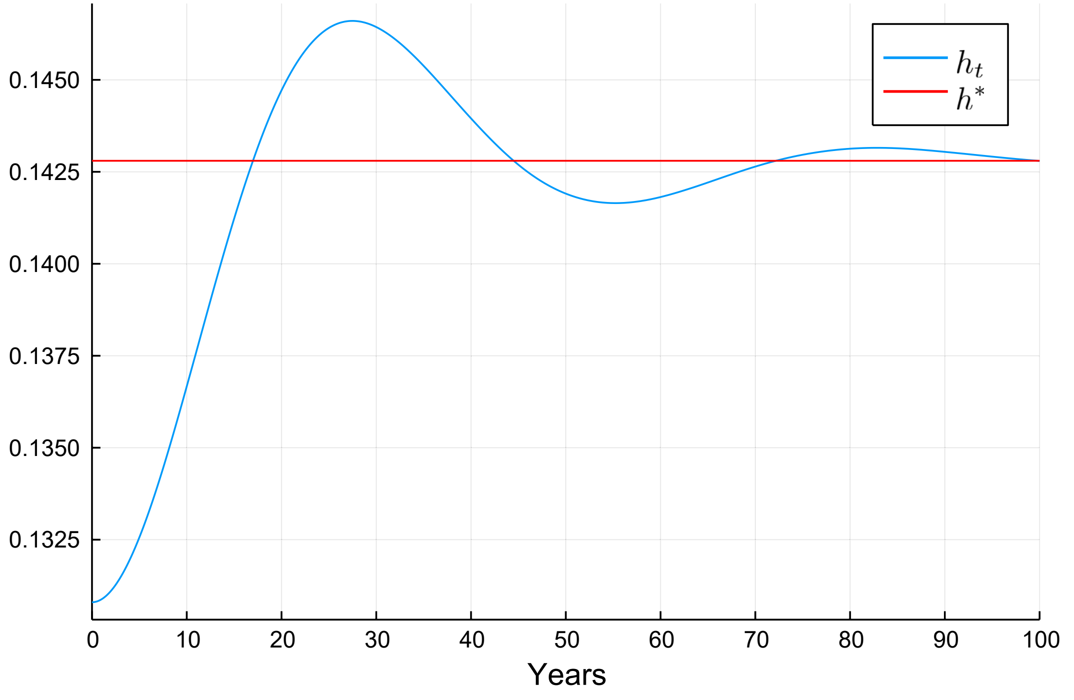
Following the permanent autonomous demand shock at time 0, output will increase as well, and hence entrepreneurs will push more on the utilization of productive capacity. More specifically, in the first phase of the long-run traverse, the output growth rate will be greater than the accumulation rate, that is, $g_t^Y > g_t^K$. Following the demand shock, entrepreneurs will thus increase their utilization of productive capacity ($\dot{u} > 0$) through equation (7). In this first phase, the economy will be characterized by a situation of above-normal utilization ($u_t > u_n$), triggering the investment share adjustment $\left(\dot{h} > 0\right)$, as per equation (8). The gap between the accumulation rate and output growth is closed only after a period of about 10 years, after which the actual rate of capacity utilization starts to decrease again towards the normal rate ($\dot{u} < 0$). However, as long as the gap between $u_t$ and $u_n$ remains positive, the investment share will keep rising. The investment share peak is reached only after more than 25 years, corresponding to a temporary situation of normal utilization. However, as long as the actual rate of utilization keeps decreasing ($\dot{u} < 0$), the economy will enter a period of under-utilization of productive capacity, which leads in turn to an investment share adjustment of reverse sign.

The economy proceeds through damped oscillations following the pattern described above, converging towards its long-run equilibrium position. Only after about 50 years do the dynamics of the rate of capacity utilization and the investment share begin to stabilize around their steady-state

[15] It is worth noting that Freitas and Serrano (2013, p. 41) report a graph that is very similar to the ones below. However, they express time as logical indexes ($t_0$, $t_1$, …) instead of historical time (months, quarters, years, etc.).




**(A)** The long-run dynamic of the rate of capacity utilization




**(B)** The long-run dynamic of the investment share

FIGURE 1 The long run in the supermultiplier model values.[16] Generally speaking, the simulation postulates that it takes a very long period of time for the model to settle down in the fully adjusted equilibrium.

### 3.2.2 | The long-run convergence in the amended neo-Kaleckian model

In the model presented in Subsection (2.2), the two adjusting variables are the autonomous demand-capital ratio ($z_t$) and the animal spirits proxy variable ($\alpha_t$). In the steady state, their values are given by equations (17 and 18). Figure 2 shows the behavior of the two adjusting variables in the long run, following the same increase in the growth rate of autonomous demand described above.

An increase in the growth rate of autonomous expenditures above the accumulation rate will generate an increase in the value of the autonomous demand-capital ratio, that is, $\dot{z} > 0$ via equation (22). Entrepreneurs will hence absorb the demand boom by pressing additional capital resources into productive use, resulting in an increase in the short-run rate of capacity utilization $\left(u^*_{sr}\right)$. As $u^*_{sr}$ rises above the normal rate of capacity utilization, the Harrodian mechanism (equation 21) will be activated, resulting in an increase in animal spirits $\dot{\alpha} > 0$. At the same time, the increase in $u^*_{sr}$ will compensate the effect of the higher autonomous demand growth rate, gradually closing the gap between $\overline{g^Z}$ and $g^K_{sr}$. When the latter exceeds the former after about 5 years, the $z$ ratio will begin its descent towards its long-run position. In this time span, $\alpha$ will keep rising until the discrepancy between the short-run utilization rate and the normal rate remains positive; however, as $\dot{z}$ is now negative, the gap between $u^*_{sr}$ and $u_n$ is shrinking. Under the parameter constellation discussed above, it takes about 20 years for this gap to be closed after an initial 1% increase in $g^Z_t$. After the actual rate of capacity utilization has fallen short of the normal rate, the Harridan mechanism will work in the opposite direction, that is, $\dot{\alpha} < 0$. The process will go on until both $\alpha_t$ and $z_t$ stabilize around their long-run steady-state values at which point the traverse will end.

Even though the long-run traverse is somewhat shorter than that of the supermultiplier model, vicinity of the new equilibrium is reached after a period of more than 30 years.[17] In other terms, when evaluated in historical time, both the supermultiplier and the amended neo-Kaleckian model share a very slow pace of adjustment. The asymptotic convergence to the fully adjusted equilibrium is a sluggish one.

[16] It is worth noting that the simulation results are partly consistent with the empirical evidences presented by Girardi and Pariboni (2020). Using panel data on 20 OECD countries, the authors show that a permanent one percent increase in autonomous-demand growth has long-lasting effects (more than 40 years) on the business investment share. While the timing of the adjustment is consistent with the simulation results presented above, its precise dynamics is not: in analytical terms, the calibration of the model gives rise to a spiral sink, implying that the investment share converges to its new long-run position through damped oscillations. Conversely, the instrumental-variables estimation of Girardi and Pariboni (2020) provides empirical support to the idea of stable convergence (real sink).

[17] While the exercise has relied on numerical methods only, it is possible to infer on the analytical level that differences in the times of adjustment largely depend on the structure of the investment theories of the two models as well as on their varied dynamic adjustments.

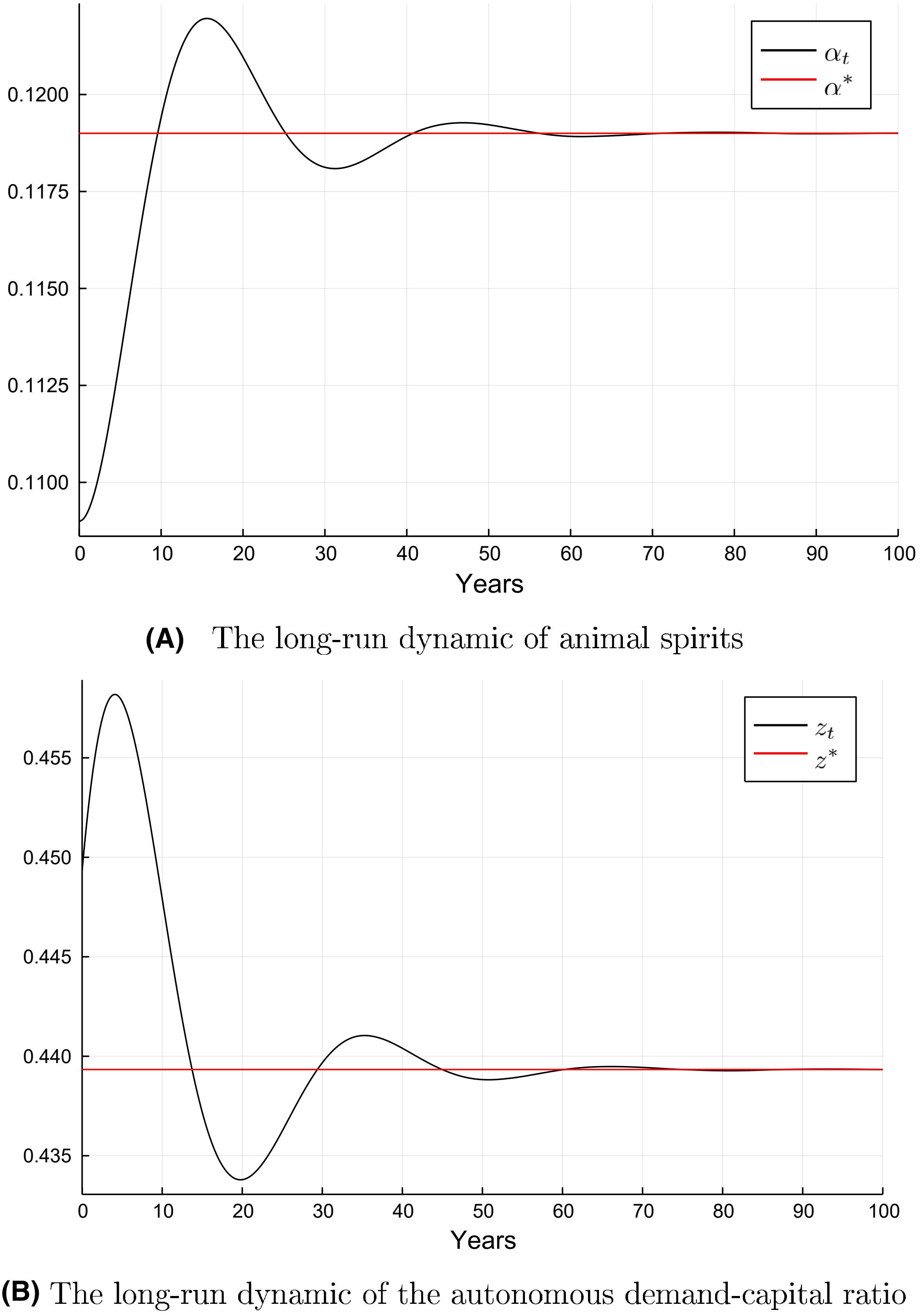



**FIGURE 2** The long run in the amended neo-Kaleckian model

## 4 | SENSITIVITY ANALYSIS

As noted earlier in the paper, the existing literature has already extensively discussed issues related to the stability of the long-run equilibria of the two models.[18] Therefore, this section will confine itself to assess the sensitivity of the models' speed of adjustment when the parameter space is modified.

In order to assess the speed of convergence of a system of differential equation, there exist known analytical methods based on eigenvalue computation. For instance, Gabaix et al. (2016) use the dominant eigenvalue, that is, the largest in absolute value, to provide a convenient description of the speed of the dynamic adjustment. However, this method is not available for the models under scrutiny. As showed in Appendix A, the eigenvalues for both models are complex with nonzero imaginary parts and hence cannot be ordered.

Therefore, the sensitivity analysis would need to rely on numerical methods only. In order to do that, a convenient visualization tool is provided in the Online Appendix C of this paper. With the aid of a web app, the interested reader could easily perform a re-parametrization of the two models, within the broad ranges reported in Table 3.

While all parameters influence – to different degrees – the *magnitude* of the dynamic adjustments and the stability of the long-run equilibria, one could easily verify that the two reaction coefficients $\gamma$ and $\beta$ are the only ones that sensibly influence the *speed* of adjustment of the Supermultiplier and of the amended neo-Kaleckian model, respectively. Unfortunately, these two parameters are exactly the ones for which we do not have sufficient empirical support. Whilst the existing literature provides a sufficiently solid ground to justify the baseline values for most parameters, these foundations become more shaky when it comes to the reaction coefficients, as also noted by Nomaler et al. (2021) for the Supermultiplier model.

Solving numerically the system for bigger and smaller values of $\gamma$ and $\beta$ would allow to assess how the two parameters affect the speed of the dynamic adjustment in the Supermultiplier and the amended neo-Kaleckian model, respectively.

Let us start with the Supermultiplier (Figure 3). A reduction of $\gamma$ from a baseline value of 0.15 to 0.05 stabilizes the system, making the adjustment slower but less persistent.[19] In the first phase of the long-run traverse, the increase of the rate of capacity utilization is bigger with $\gamma = 0.05$;

**TABLE 3** Parameters and exogenous variables - minimum and maximum values

| Par. | Description | Min.Value | Max.Value |
|---|---|---|---|
| $g^Z$ | Autonomous demand growth | 0.01 | 0.12 |
| $\delta$ | Depreciation rate (annual) | 0.01 | 0.2 |
| $u_n$ | Normal rate of capacity utilization | 0.5 | 1 |
| $v$ | Capital-capacity ratio (annual) | 0.7 | 3 |
| $s$ | Propensity to save | 0.2 | 0.6 |
| $m$ | Propensity to import | 0 | 0.3 |
| $\gamma$ | Sensitivity of the investment share to $u_t - u_n$ | 0 | 1 |
| $\beta$ | Sensitivity of capital formation to $u_t - u_n$ | 0 | 1 |

*Source*: Author's calculation, see Appendix C.

[18] See Appendix A for the derivation of the stability conditions of both models.

[19] For sufficiently small values of $\gamma$, the eigenvalues becomes real and distinct, and the system could converge monotonically towards the long-run equilibrium. See Appendix A for further discussion.

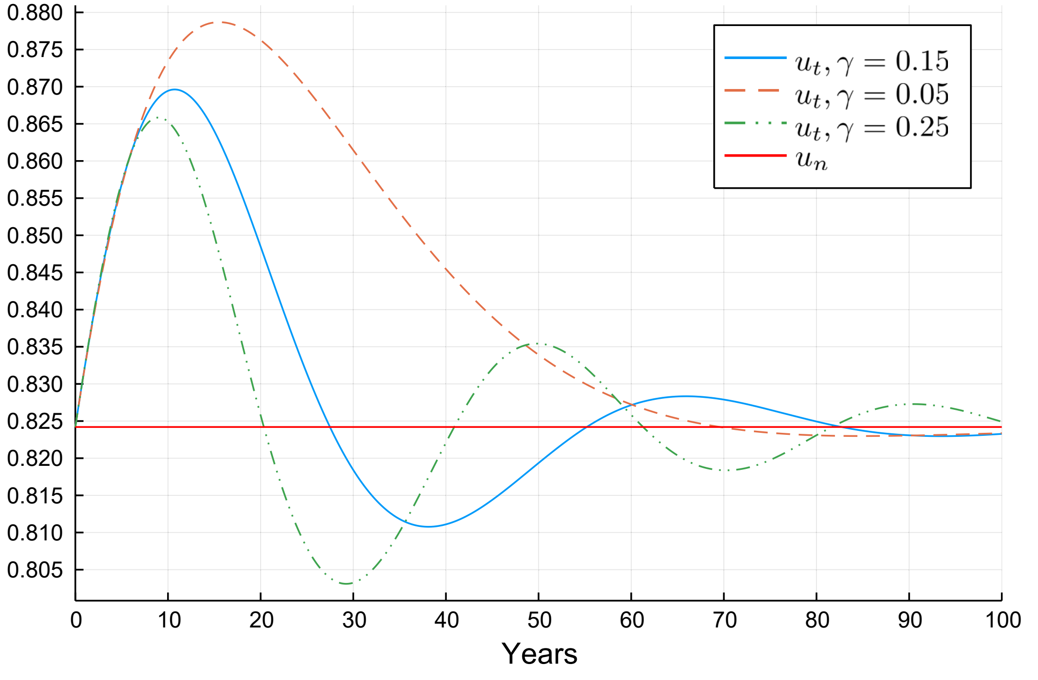
after a 1% increase in the growth rate of autonomous demand, $u_t$ peaks only after a period of about 15 years. Following that, the model slowly converges towards the fully adjusted position. Moreover, this case provides a good occasion to stress another important point in traverse analysis of demand-led growth models (and hence valid both for the Supermultiplier and for the amended neo-Kaleckian model). As it can be seen, a reduction of $\gamma$ (or similarly of $s$ and $m$ or an increase of $u_n$, $u_0$, $\overline{g^Z}$, $v$ or $\delta$) has the effect of increasing the maximum value reached by the rate of capacity utilization during the adjustment. At and near the peak, $u_t$ may become bigger than one, which would be logically inconsistent (an economy cannot *at any time* achieve an income level higher than the maximum one determined by its potential). In this sense, as noted by Lavoie and Ramírez-Gastón (1997, p. 162): “to look at the requirements of the steady state is insufficient to assess whether or not the new steady state is possible; the rate of capacity utilization must also remain below unity at all times during the traverse”. Let us now assess what happens in the opposite case, looking at the traverse trajectory corresponding to an increase in $\gamma$ to a value of 0.25. In this case, the time required to approach the long-run equilibrium position after the initial shock decreases, while making the model less stable, with the dumped fluctuations being not completely absorbed after a period of more than 70 years.

Let us now discuss what happens in the Allain-Lavoie model if the sensitivity of accumulation to the discrepancy between the actual and the normal utilization rates ($\beta$) changes (Figure 4). In particular, if $\beta$ increases from the baseline value of 0.25–0.35, the time of adjustment is reduced and the model stabilizes faster, approaching the long-run equilibrium in less than 30 years. Conversely, with a lower $\beta$ of 0.15, the model becomes less stable and takes more time to converge. Similar to what has been discussed in the previous paragraph, with a lower $\beta$ one must be careful about whether the traverse path ensures that the rate of capacity utilization remains below unity during the entire process. As all time steps over the long-run traverse correspond to a situation of short-run equilibrium in the goods market, this condition could be easily verified by computing the value of the rate of capacity utilization via equation (14). Besides $\beta$, the other parameters that determine the value of $u^*_{sr}$ also ensure whether or not the rate stays below unity at all times. In particular, $u^*_{sr}$ may rise above 1 with a lower $s$, $m$ and $u_n$ or with a higher $\alpha_0$, $z_0$, $v$ and $\delta$.

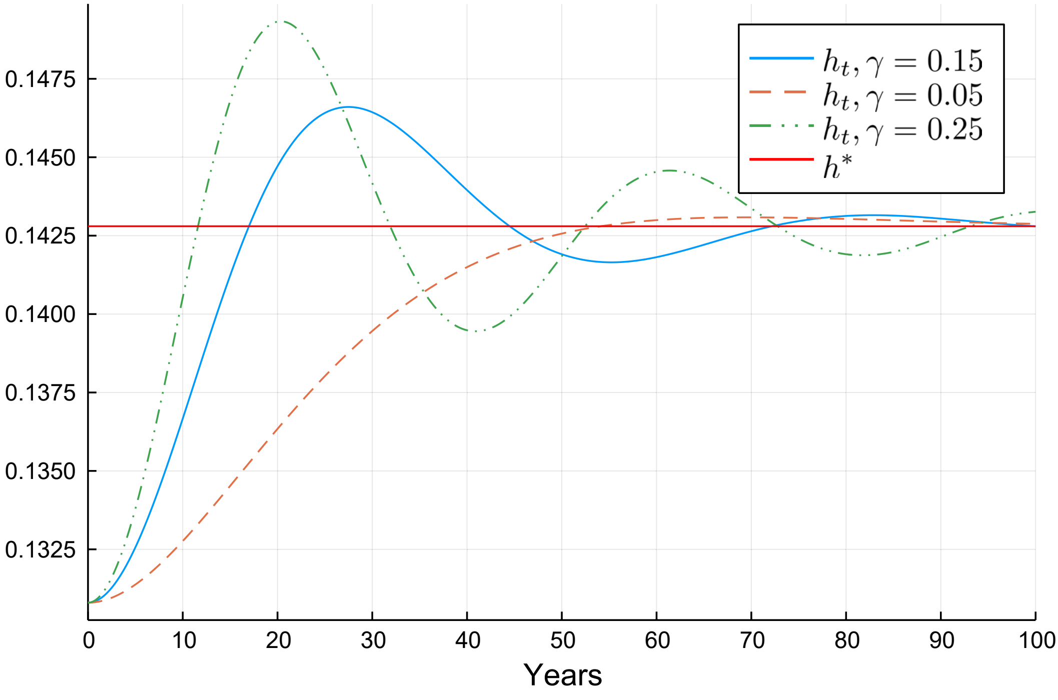
In both cases, a change of $\pm 0.1$ in both $\gamma$ and $\beta$ does not alter the general conclusion in Section 3 regarding the relative speeds of adjustment of the two models. Regardless, further econometric analysis would be needed to assess the size of these reaction parameters (provided the empirical soundness of the assumed adjustments), allowing to determine both whether the two models predict stability of the long-run equilibrium and to have reliable point estimates of the predicted adjustment to the fully adjusted position.

## 5 | CONCLUDING REMARKS

The paper has attempted to answer Joan Robinson's (1980) question *‘when is the long run?’*, evaluating in historical time the long-run traverse in two prominent demand-led growth models, namely the Supermultiplier model and the long-run neo-Kaleckian model. In doing so, it provided a description of the temporal sequence needed to achieve a new long-run position after an initial increase in the growth rate of autonomous demand. After presenting the two models, the paper discussed a reasonable calibration in line with the existing theoretical and empirical literature. The calibration allowed to provide a numerical solution of the systems of differential equations that regulate out-of-equilibrium dynamics in both models. The simulation exercise showed that the convergence to the fully adjusted equilibrium is sluggish, with adjustment periods of about 50 and 30 years for the Supermultiplier and

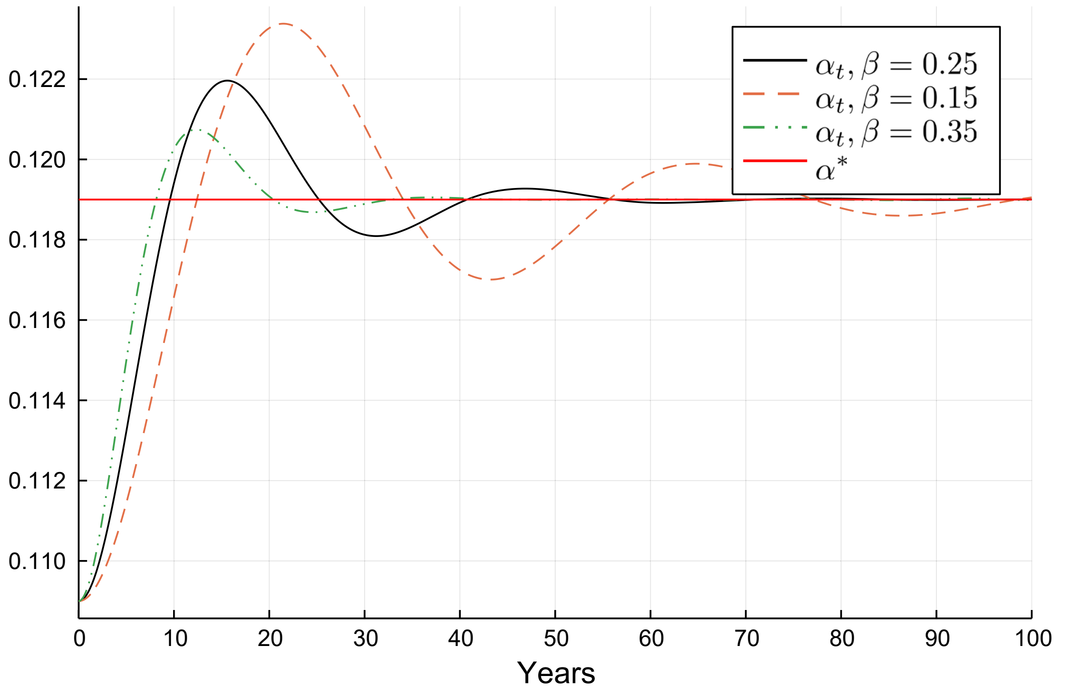



**(A)** The long-run dynamic of the rate of capacity utilization

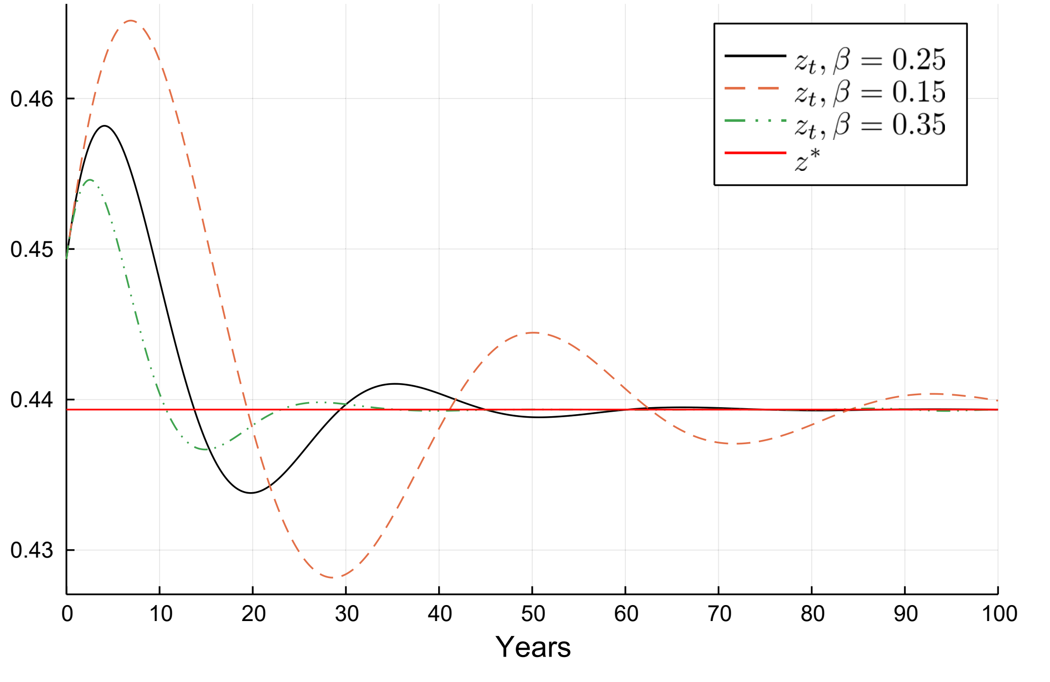



**(B)** The long-run dynamic of the investment share

FIGURE 3 The long-run traverse in the supermultiplier model with changing $\gamma$




**(A)** The long-run dynamic of animal spirits




**(B)** The long-run dynamic of the autonomous demand-capital ratio

**FIGURE 4** The long-run traverse in the amended neo-Kaleckian model with changing $\beta$

the neo-Kaleckian model, respectively. Furthermore, the analysis assessed how changes in the parameter space affect the adjustment periods in both models. More specifically, it showed that the reaction coefficients of the investment share (for the Supermultiplier) and of the investment rate (for the neo-Kaleckian model) affect sensibly the duration of the long-run traverse, leaving scope for further empirical research to derive point estimates of these coefficients.

A conclusion as tempting as naive that could be drew from the exercise is that – if interpreted as a length of time – *the long run may be longer than expected*. As the simulations presented in the previous sections have shown, the two models under scrutiny share a very slow pace of adjustment. In other terms, in historical time the adjustment period to a new steady-state position may be long enough to be economically meaningless.

The long run, however, is not a length of time, but a process. Accordingly, as Robinson (1965, p. 17) notes, "it is absurd, though unfortunately common, to talk as though 'in the long run' we shall reach a date at which the equilibrium corresponding to today's conditions will have been realized". In the length of time spanning from a change in today's conditions to the realization of a new equilibrium, further changes are likely to affect the growth process: history has a pervasive influence on the determination of economic outcomes and growth processes (Setterfield, 1995, 1997). Rather than focusing, as the existing literature sometimes does, on the mere comparison between two equilibrium positions, researchers should pay more attention to the properties that characterize the models' trajectories during the traverse, for example, by discussing out-of-equilibrium growth effects, path dependency and so on (Morlin et al., 2021). The examination of the models' timescale and adjustment period is a fundamental piece of information and a key factor for understanding the relation between the theoretical framework and the real world. Very rarely this information is exploited for economic analysis and policy recommendations, with researchers and policy makers finding themselves more at ease with thinking in logical rather than historical time.

Lastly, it is worth stressing that the goal of this exercise has not been to quantify the actual duration of the traverse, but first and foremost to shed light on issues and methods that have not received the deserved attention by growth theorists. On the methodological side, the results presented in the paper have been derived by making use of numerical methods of analysis to solve two systems of differential equations that cannot be solved analytically. The mathematical tool is well known by economists and growth theorists, but neglected for the analysis of the traverse and out-of-equilibrium dynamics. Using more thoroughly these methods may result in a significant gain of explanatory power of the models used for the analysis of real-world economies.

## ACKNOWLEDGMENTS

I am most grateful to Mark Setterfield, Olivier Allain, Marc Lavoie, Willi Semmler, Luca Zamparelli, Paulo Dos Santos, Maria Cristina Barbieri Góes, the editor and two anonymous referees for their helpful comments and suggestions on earlier drafts of this article. I would also like to thank participants in the 2021 FMM Conference, the 2021 EAEPE Conference and the 2020 STOREP Conference. All remaining errors are, of course, my own.

Open Access Funding provided by Università degli Studi di Roma La Sapienza within the CRUI-CARE Agreement.

## CONFLICT OF INTEREST

No potential conflict of interest was reported by the author.

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**How to cite this article:** Gallo, E. (2022). When is the long run?—Historical time and adjustment periods in demand-led growth models. *Metroeconomica*, 1–24. https://doi.org/10.1111/meca.12400

## APPENDIX A: DERIVATION OF THE MODELS, STABILITY AND EQUILIBRIUM

### Open Economy with Government Activity

Let us start from the output equation of an open economy with government activity:

$$Y_t = C_t + I_t + G_t + (X_t - M_t) \tag{A1}$$

where the current level of aggregate output ($Y_t$) is defined as the sum of aggregate consumption ($C_t$), private investment ($I_t$), public expenditures ($G_t$) and net exports ($X_t - M_t$). Consumption, government spending, exports and imports can be modeled as follows:

$$C_t = C_{Yt} + \overline{C_{0t}} = c(1-t)Y_t + \overline{C_{0t}} \tag{A2}$$

$$G_t = \overline{G_t} \tag{A3}$$

$$X_t = \overline{X_t} \tag{A4}$$

$$M_t = mY_t \tag{A5}$$

Equation (A2) assumes that aggregate consumption is partly induced - via the tax-adjusted propensity to consume $c\ (1-t)$ - and partly autonomous from the current level of income $(\overline{C_{0t}})$. Autonomous consumption can be understood as 'that part of aggregate consumption financed by credit and, therefore, unrelated to the current level of output resulting from firms' production decisions' (Freitas & Serrano, 2015, p. 4). Government spending (equation A3) and exports (equation A4) are both treated as autonomous, the first because public consumption and investment depend on the arbitrary decisions of the general government, the second because exports does not depend on the level of national income, but on that of the rest on the world. For the sake of simplicity, imports of goods and services are assumed to be linearly dependent of the level of income, via the propensity to import $m$ (equation A5).

The modeling choice regarding aggregate investment is what effectively constitutes the main difference between the Sraffian Supermultiplier and the Neo-Kaleckian model, as showed below.

**Sraffian Supermultiplier Model**

According to the baseline Supermultiplier model, private investment is treated as fully induced (equation A6), reflecting the simple idea that at the aggregate level firms will invest only as long as there is demand for their products. Therefore, $I_t$ can be model sic *et simpliciter* as the product of the investment share ($h_t$) times national income.

$$I_t = h_t Y_t \tag{A6}$$

Since $\dot{K}_t = I_t - \delta K_t$, the accumulation rate can be derived as follows:

$$g_t^K = \frac{\dot{K}_t}{K_t} - \delta = \frac{I_t}{K_t} - \delta = \frac{h_t Y_t}{K_t} - \delta = h_t \frac{Y_t}{Y^p}\frac{Y^p}{K_t} - \delta = \frac{h_t u_t}{v} - \delta \tag{A7}$$

where $Y^p$ is full-capacity output. Let us now solve for the level of output, substituting equations (A2-A6) in equation (A5):

$$Y_t = \left(\frac{1}{s+m-h_t}\right)\left(\overline{C_{0t}} + \overline{G_t} + \overline{X_t}\right) = \left(\frac{1}{s+m-h_t}\right) Z_t = SM_t Z_t \tag{A8}$$

where $s$ denotes the tax-adjusted propensity to save, that is, $s = 1 - c\ (1-t)$. The term $SM_t$ denotes the supermultiplier, that is, $1/(s+m-h_t)$.

Differentiating equation (A8), we obtain the growth rate of output as the sum of the growth rate of autonomous demand and of the supermultiplier, under the assumption that the investment share behaves in line with equation (8):

$$g_t^Y = g_t^Z + \frac{h_t \gamma\ (u_t - u_n)}{s+m-h_t} \tag{A9}$$

Lastly, the model closure is given by the assumption of an exogenously given growth rate of autonomous demand:

$$g_t^Z = \overline{g^Z} \tag{A10}$$

Let us now analyze the stability of the fully adjusted equilibrium, whose necessary and sufficient condition is that the determinant of the Jacobian's matrix evaluated at the equilibrium point with $u^* = u_n$ and $h^* = \frac{v}{u_n}\left(\overline{g^Z} + \delta\right)$ is positive and its trace is negative:

$$J^* = \begin{bmatrix} \left[\frac{\partial \dot{h}}{\partial h}\right]_{h^*,u^*} & \left[\frac{\partial \dot{h}}{\partial u}\right]_{h^*,u^*} \\ \left[\frac{\partial \dot{u}}{\partial h}\right]_{h^*,u^*} & \left[\frac{\partial \dot{u}}{\partial u}\right]_{h^*,u^*} \end{bmatrix} = \begin{bmatrix} 0 & \frac{\gamma v\left(\overline{g^Z}+\delta\right)}{u_n} \\ -\frac{u_n^2}{v} & \left(\overline{g^Z} + \delta\right)\left(\frac{\gamma v}{s+m-\frac{v}{u_n}\left(\overline{g^Z}+\delta\right)} - 1\right) \end{bmatrix} \tag{A11}$$

$$\det J^* = \gamma u_n \left(\overline{g^Z} + \delta\right) \tag{A12}$$

$$tr\, J^* = \left(\overline{g^Z} + \delta\right)\left(\frac{\gamma v}{s + m - \frac{v}{u_n}\left(\overline{g^Z} + \delta\right)} - 1\right) \tag{A13}$$

Since $\gamma$, $u_n$ and $\overline{g^Z}$ are assumed to be positive, the determinant is necessarily positive. Similarly to Freitas and Serrano (2015), the stability condition boils down to the sign of the Tr $J^*$, which is ensured by the following condition:

$$1 - s + m + \gamma v + \frac{v}{u_n}\left(\overline{g^Z} + \delta\right) < 1 \tag{A14}$$

where $1 - s + m$ may also be interpreted as the tax and imports-adjusted propensity to spend. Equation (A14) implies three conditions:

1. The value of the reaction parameter $\gamma$ should be sufficiently low, implying that induced investment ought not to adjust capacity to demand too fast outside the fully adjusted position (Freitas & Serrano, 2015). In other terms, the effect of Harridan instability needs not to be overly strong;
2. The growth rate of autonomous demand $\overline{g^Z}$ cannot be too large;
3. The tax and imports-adjusted propensity to spend $(1 - s + m)$ needs not to be too large and it must be smaller than unity in the entire adjustment process.

If the condition in equation (A14) is fulfilled, then the system converges to its long-run equilibrium. The trajectory of the system depends on the discriminant of its eigenvalues:

$$\lambda_{1,2} = \frac{tr\, J^* - \sqrt{\Delta}}{2} \text{ with } \Delta = \left(tr\, J^*\right)^2 - 4\det J^* \tag{A15}$$

Therefore, if $trJ^*$, $\Delta < 0$ and det $J^* > 0$, the eigenvalues will be complex with nonzero imaginary part; $u_t$ and $h_t$ will converge via damped oscillation (spiral sink). Conversely, if $trJ^* < 0$ and det $J^*$, $\Delta > 0$, both eigenvalues will be real and distinct; the system will converge monotonically towards the fully adjusted position (real sink). Relying on the parameter calibration discussed in Section 3, the first case applies. Figure A1 shows the resulting phase plane.

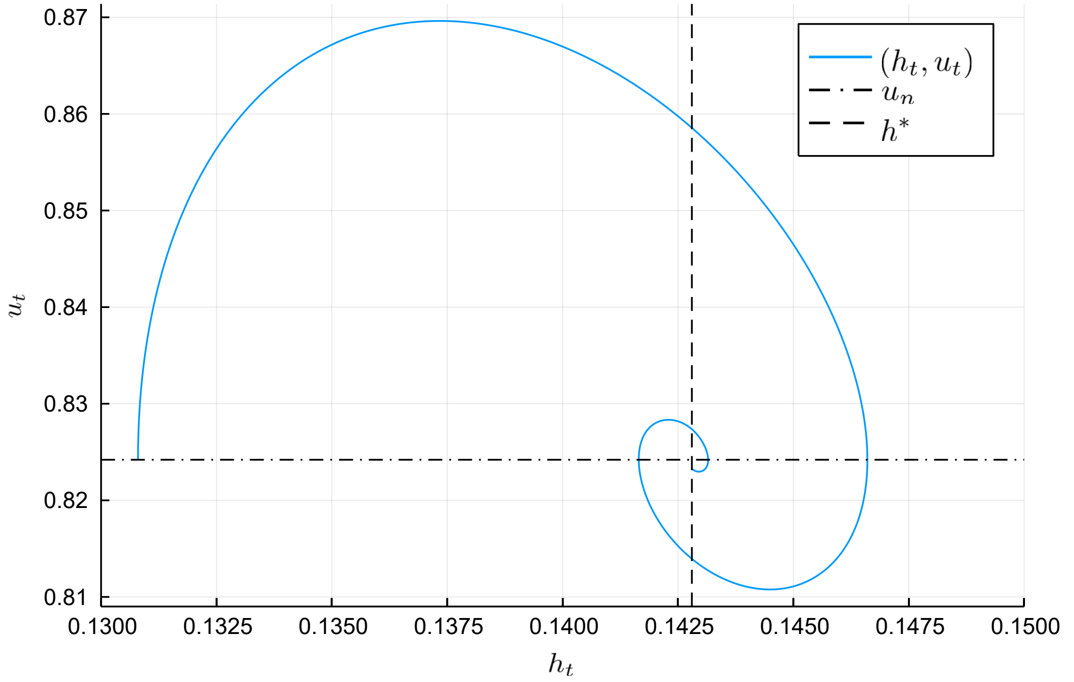


**FIGURE A1** Phase plane of the amended Supermultiplier model. *Source*: authors' representation

**Long-run Neo-Kaleckian Model**

Neo-Kaleckian models treat capital formation as dependent on the rate of capacity utilization. More specifically, adopting the formulation proposed for the first time by Amadeo (1986), the investment rate will depend on the secular growth rate of sales ($\alpha_t$) plus discrepancies between the actual and the normal or 'planned' (Steindl, 1952) utilization rates, via the parameter $\beta$:

$$I_t = [\alpha_t + \beta (u_t - u_n)] K_t \tag{A16}$$

which - under the assumption of a linear depreciation coefficient - implies that the accumulation rate will be equal to:

$$g_t^K = g_t^I - \delta = \frac{I_t}{K_t} - \delta = \alpha_t + \beta (u_t - u_n) - \delta \tag{A17}$$

The saving equation in levels is then given by:

$$S_t = Y_t - C_t - G_t - (X_t - M_t) = [1 - c(1 - t) + m]Y_t - \left(\overline{C_{0t}} + \overline{G_t} + \overline{X_t}\right) = (s + m)Y_t - Z_t \tag{A18}$$

Dividing everything by the capital stock and multiplying/dividing the first term on the right-hand side by full-capacity output, it follows that:

$$g_t^S = \frac{S_t}{K_t} = (s + m)\frac{Y_t}{Y^p}\frac{Y^p}{K_t} - \frac{Z_t}{K_t} = \frac{(s + m)u_t}{v} - z_t \tag{A19}$$

Same as for the Supermultiplier model, the model is closed by the assumption of an exogenously given growth rate of autonomous demand - Equation (A10) above.

Let us now evaluate the Jacobian matrix in the long-run fully adjusted equilibrium $\alpha^* = \overline{g_t^Z} + \delta$ and $z^* = \frac{(s+m)u_n}{v} - \overline{g^Z} - \delta$:

$$
\begin{aligned}
\boldsymbol{J}^* &= \begin{bmatrix} \left[\frac{\partial \dot{\alpha}}{\partial \alpha}\right]_{\alpha^*,z^*} & \left[\frac{\partial \dot{\alpha}}{\partial z}\right]_{\alpha^*,z^*} \\ \left[\frac{\partial \dot{z}}{\partial \alpha}\right]_{\alpha^*,z^*} & \left[\frac{\partial \dot{z}}{\partial z}\right]_{\alpha^*,z^*} \end{bmatrix} \\
&= \begin{bmatrix} \frac{\beta v \mu}{s+m-\beta v} & \frac{\beta v \mu}{s+m-\beta v} \\ -\left[\frac{(s+m)u_n}{v} - \overline{g^Z} - \delta\right]\left(1 + \frac{\beta v}{s+m-\beta v}\right) & \frac{-\beta v}{s+m-\beta v}\left[\frac{(s+m)u_n}{v} - \overline{g^Z} - \delta\right] \end{bmatrix}
\end{aligned}
\tag{A20}
$$

$$
\det J^* = \frac{\beta v \mu}{s+m-\beta v}\left(\frac{(s+m)u_n}{v} - \overline{g^Z} - \delta\right) \tag{A21}
$$

$$
tr\, J^* = \frac{\beta v}{s+m-\beta v}\left[\mu - \frac{(s+m)u_n}{v} + \overline{g^Z} + \delta\right] \tag{A22}
$$

Given that $\beta$ and $v$ are assumed to be positive, the determinant is positive whenever the Keynesian stability condition holds ($\beta < (s+m)/v$) and the equilibrium autonomous demand-capital ratio $z^*$ is positive, that is, whenever $\overline{g^Z} < (s+m)u_n/v - \delta$. If the Keynesian stability condition holds, then it can be shown that the trace is negative whenever:

$$
\mu < \frac{(s+m)u_n}{v} - \overline{g^Z} - \delta \Rightarrow \mu < z^* \tag{A23}
$$

Taken together, the stability conditions of the long-run neo-Kaleckian model imply that:

1. The value of the reaction parameter $\beta$ should be sufficiently low, implying that the reaction of capital formation to discrepancies in utilization rates is not too strong;
2. The growth rate of autonomous demand $\overline{g^Z}$ cannot be too large;
3. Similarly to the Supermultiplier model, capacity ought to adjust fairly slowly to demand, that is, the Harridan mechanism need not to be overly strong (Equation A23);

As discussed above, the discriminant of the system's eigenvalues will determine its trajectory. Similar to the Supermultiplier model, the calibration of the model suggests that – at least in the baseline parametrization – the eigenvalues are complex ($\Delta < 0$). Therefore, the systems converges through damped oscillations towards the fully adjusted position (spiral sink).

Figure A2 shows the 2D phase space plot of the Allain-Lavoie system.

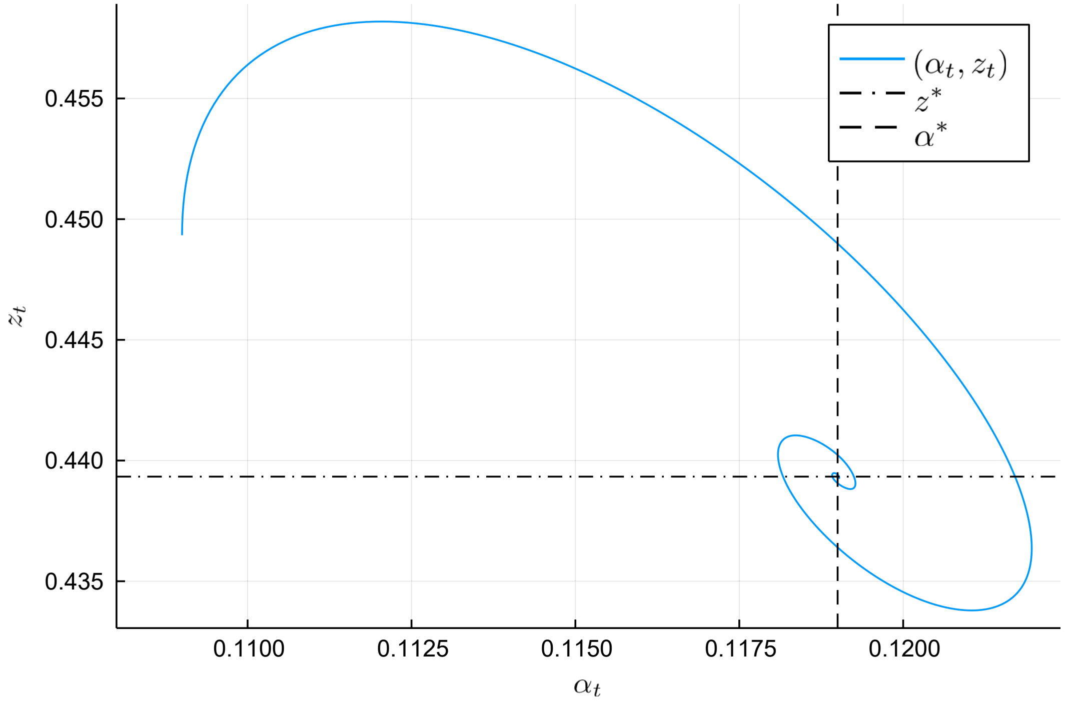


**FIGURE A2** Phase plane of the amended neo-Kaleckian model. *Source*: authors' representation.

## APPENDIX B: VARIABLES

| | |
|---|---|
| $\alpha_t$ | Animal spirits (also, expected growth rate of sales) |
| $h_t$ | Investment share (also, marginal propensity to invest) |
| $g_t^I$ | Investment rate |
| $g_t^K$ | Growth rate of the capital stock |
| $g_t^S$ | Saving rate |
| $g_t^Y$ | Growth rate of output |
| $g_t^Z$ | Growth rate of autonomous demand |
| $u_t$ | Capacity utilization rate |
| $z_t$ | Autonomous demand-capital ratio |

## APPENDIX C: SENSITIVITY ANALYSIS

The interested reader could easily perform a re-parametrization of the two models through the following interactive Web App – created with Shiny R: http://ettoregallo.shinyapps.io/When_is_the_long_run.