

## Appendix A1

In this first appendix is copied the Matlab code '*MarkovInfer\_MobInd.m*' described in the Chapters 3, 4, 5 and employed to process Excel datasheets from the archive AIDA dealing with Italian firms (see Chapter 6). Some outputs inside the workspaces, created by means of such tool, have been analysed in Chapters 7 and 8.

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1 %
2 % The author of this script is Danilo Aringhieri.
3 %
4 clear all
5 License=char({'','This software is released by Danilo Aringhieri, who is its sole author. ',...
6 ' Everyone is warned against making use of the specific content of this script, ',...
7 ' wholly or partially, for any scientific work ',...
8 ' without including Danilo Aringhieri as a coauthor. '});
9 disp(License)
10 disp(' Press the "Enter" button on the keyboard to START...')
11 response=input(' ','s');
12 while isempty(response)==0
13 disp(' Please, press the "Enter" button to START!')
14 response=input(' ','s');
15 end
16 Intro1=char({'',' A spreadsheet, able to read and save xls-type or xlsx-type files, ',...
17 ' must be installed on your computer and the variable values, to be imported, ',...
18 ' must be tabled by columns and rows. ',...
19 ' Opening columns on the left may be filled with some lists of strings ',...
20 ' (firms" names in alphabetic characters, alphanumeric identity codes or addresses ) ',...
21 ' as well as opening rows on the top may be filled with the dates of observations ',...
22 ' or any other details written in string characters, without compromising the import. ',...
23 ' DO YOU WANT TO CONTINUE ? ',...
24 ' (Answer "Y" for "Yes" or "N" for "Not", then press [Enter] )});
25 disp(Intro1)
26 response=input(' ','s');
27 while strcmp(response,{'Y','y','N','n'})==[0,0,0,0]
28 disp(' (You must answer exclusively "Y" or "N" ! ) ')
29 response=input(' ','s');
30 end
31 if sum(strcmp(response,{'N','n'}))==1
32 disp(char({' ',' OK: you are going to return directly ',...
33 ' to the prompt state in the Command Window! ',' '}))
34 clear Intro1 response
35 return
36 elseif sum(strcmp(response,{'Y','y'}))==1
37 clear Intro1
38 end
39 Input0=char({'',' (You can STOP at any time the program ',...
40 ' typing "Ctrl" + "C" on the keyboard ! )});
41 disp(Input0)
42 Input1=char({'',' To import automatically your data, ',...
43 ' WRITE THE NAME OF YOUR FILE INCLUDING, ',...
44 ' its extension (".xls" or ".xlsx"), ',...
45 ' then press [Enter]. ',...
46 ' (any other command will provide an error message as a result; ',...
47 ' if it should appear a warning message, please wait few seconds. '});
48 disp(Input1)
49 filename=input(' ','s');
50 [AuxData1,Titles]=xlsread(filename);
51 dim_AuxData1=size(AuxData1);
52 for k=1:dim_AuxData1(2)
53 nan_col(k)=(sum(isnan(AuxData1(:,k))))/dim_AuxData1(1);
54 end

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55 % If the k-th column in AuxData1 is made only by 'NaN' elements,
56 % it is nan_col(k)=1 otherwise, if there exist some finite numbers
57 % in the same column, it holds nan_col(k)<1.
58 not_nan_col=find(nan_col-1);
59 % The elements of the row vector 'not_nan_col' identify the positions,
60 % in the matrix AuxData1, of all the columns not completely filled
61 % with 'NaN-type' elements.
62 dim_notnan_col=size(not_nan_col);
63 AuxData2=AuxData1(:,[not_nan_col]);
64 % All the columns in the original datasheet (and thus in the matrix AuxData1, too),
65 % wholly filled with 'NaN-type' elements has been now cancelled from AuxData1:
66 % the result is the matrix Auxdata2.
67 NumNotData1=char({' This tool tolerates that, in the original datasheet, ',...
68 ' some columns are filled with numbers which ARE NOT NUMERICAL DATA, ',...
69 ' like when the statistical units are indentified (also) by ordinals. ',...
70 ' ALL these kind of numerical non-data columns ',...
71 ' HAVE TO STAY AT THE LEFT of the variable values !! ',...
72 ' Do some numerical non-data columns exist in the original datasheet? ',...
73 ' ( Answer "Y" if they exist or "N" otherwise, then press [Enter] ) '});
74 disp(NumNotData1)
75 response=input(' ');
76 while strcmp(response,{'Y','y','N','n'})=[0,0,0,0]
77 disp(' You must answer exclusively "Y" or "N" ! ')
78 response=input(' ');
79 end
80 if sum(strcmp(response,{'N','n'}))==1
81 All_Data=AuxData2;
82 elseif sum(strcmp(response,{'Y','y'}))==1
83 NumNotData2=char({' How many are the numerical non-data columns in the original sheet ? '});
84 disp(NumNotData2)
85 nondata_col=input(' ');
86 if (isscalar(nondata_col)==0 || isreal(nondata_col)==0 || isnan(nondata_col)==1 || isinf(nondata_col)==1 ||...
87 nondata_col<0 || floor(nondata_col)~=nondata_col)
88 while (isscalar(nondata_col)==0 || isreal(nondata_col)==0 || isnan(nondata_col)==1 || isinf(nondata_col)==1 ||...
89 nondata_col<0 || floor(nondata_col)~=nondata_col)
90 if isscalar(nondata_col)==0
91 disp(' ( You pressed the [ENTER] button or inserted a vector or an array ! ) ')
92 disp(' BUT you must insert exclusively a positive integer ! ')
93 nondata_col=input(' ');
94 elseif (isscalar(nondata_col)==1 && (isnan(nondata_col)==1 || isinf(nondata_col)==1 || isreal(nondata_col)==0))
95 disp(' Your input is a NaN value or infinite or a complex number: WHY? ')
96 nondata_col=input(' ');
97 elseif nondata_col<0
98 disp(' ( You must insert exclusively a positive number ! ) ')
99 nondata_col=input(' ');
100 elseif (nondata_col>0 && floor(nondata_col)~=nondata_col)
101 disp(' ( You must insert exclusively a positive integer ! ) ')
102 nondata_col=input(' ');
103 end
104 end
105 elseif (isscalar(nondata_col)==1 && isreal(nondata_col)==1 && isnan(nondata_col)==0 && isinf(nondata_col)==0 ...
106 && nondata_col>0 && floor(nondata_col)~=nondata_col)
107 end
108 All_Data=AuxData2(:,nondata_col+1:dim_notnan_col(2));

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109 end
110 % Now all the columns wholly filled with numerical values, which are not values
111 % for the variable, are cancelled from Auxdata2: the result is the matrix
112 % All_Data, only containing either the available variable's values
113 % for every statistical unit or a 'NaN' element, where and where the variable is unknown.
114 % The number of columns in All_Data is the number of all observation periods
115 % recorded in the sheet of the variable.
116 clear NumNotData1 NumNotData2
117 dim_All_Data=size(All_Data);
118 % Now the tool is going to extract, from the matrix 'All_Data', the subsamples
119 % of the incumbents, that is all the statistical units in the datasheet expressing
120 % a value for everyone of the observation periods registered, for the variable of interest.
121 for k=1:dim_All_Data(1)
122 incumbent_row(k)=prod(isfinite(All_Data(k,:)));
123 end
124 % The k-th individual in the spreadsheet is an incumbent if and only if
125 % isfinite(All_Data(k,:)) is a vector whose elements are all equal to 1
126 % so the component 'incumbent_row(k)= prod(isfinite(All_Data(k,:)))'
127 % is 1 for an incumbent but it is zero for any agent lacking of a datum for some periods.
128 incumbent_posit=find(incumbent_row);
129 % The elements of the row vector 'incumbent_posit' identify the positions,
130 % in the matrix All_Data, of all agents (one agent corresponds to one and only one row)
131 % which are incumbents. The element inside All_Data which are not incumbent
132 % are going to be excluded from the workspace and the final result
133 % is 'incumbData', the cleaned matrix of the only incumbent agents.
134 incumbData=All_Data([incumbent_posit,:]);
135 dim_incumb_pos=size(incumbent_posit);
136 num_of_incumb=dim_incumb_pos(2);
137 share_of_incumb=num_of_incumb/dim_All_Data(1);
138 dim_incumb=size(incumbData);
139 disp(char({' ' The number of all individuals ',...
140 ' in the original spreadsheet '}))
141 fprintf(' is %-1.0d \n',dim_All_Data(1))
142 disp(' but number of the extracted incumbents ')
143 fprintf(' is %-1.0d statistical units. \n',num_of_incumb)
144 MaxValues=max(incumbData);
145 MinValues=min(incumbData);
146 disp(char({' ' What is the dimensional unit of your data? ',...
147 ' (Use the expression you like and press [Enter] '}))
148 units=input(' 's');
149 disp(char({' ' Insert the time unit in which your observations are marked, ',...
150 ' then press [Enter] '}))
151 time_scan=input(' 's');
152 Input2_1=char({' ' It is assumed observation dates are in chronological order: ',...
153 ' so they will be implicitly marked, according to their relative position ',...
154 ' in the datasheet, by progressive positive integers assigned to the auxiliary variable ',...
155 ' "t_start" of the time when a transition starts. ',...
156 ' ' All the numerical data from your file ',...
157 ' will be recorded into the matrix "incumbData" in the workspace. ',...
158 ' The opening rows and columns in a string format will be copied ',...
159 ' into the array "Titles" which will be cancelled soon after. ',...
160 ' The "NaN" elements inside "incumbData" correspond, inside the spreadsheet, ',...
161 ' to empty cells or to cells where no values for the variable are available. ',...
162 ' (press [Enter] to go on...) '});

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163 disp(Input2_1)
164 response=input('','s');
165 while isempty(response)==0
166 disp('(press [Enter] to go on...')
167 response=input('','s');
168 end
169 Input2_2=char({' Three vectors will be created: ',...
170 ' "dim_incumb", "MaxValues" and "MinValues" . ',...
171 ','; "dim_incumb" consists of the dimensions of incumbData:',...
172 ' dim_incumb(1) is the number of incumbents enumerated inside the datasheet. ',...
173 ' dim_incumb(2) is the number of periods when the variable"s values have been measured. ',...
174 ','; " MaxValues " and " MinValues " are row vectors: ',...
175 ' " MaxValues(m) " and " MinValues(m) " are respectively ',...
176 ' the biggest datum and the smallest one registered in the m-th column of "incumbData" ',...
177 ' referring to the m-th observation period. ',...
178 ' (press [Enter] to see) ',';});
179 disp(Input2_2)
180 clear resp
181 response=input('','s');
182 while isempty(response)==0
183 disp('(press [Enter] to see)')
184 response=input('','s');
185 end
186 disp(' According to your sample, the vector " MinValues " is: ')
187 if dim_incumb(2)==2
188 fprintf(' [%-1.3f, ',MinValues(1));...
189 fprintf('%-1.3f] \n',MinValues(2))
190 else
191 fprintf(' [%-1.3f, ',MinValues(1));...
192 fprintf('%-1.3f, ',MinValues(2:dim_incumb(2)-1));...
193 fprintf('%-1.3f] \n',MinValues(dim_incumb(2)))
194 end
195 disp(' and the vector " MaxValues " is: ')
196 if dim_incumb(2)==2
197 fprintf(' [%-1.3f, ',MaxValues(1));...
198 fprintf('%-1.3f] \n',MaxValues(2))
199 else
200 fprintf(' [%-1.3f, ',MaxValues(1));...
201 fprintf('%-1.3f, ',MaxValues(2:dim_incumb(2)-1));...
202 fprintf('%-1.3f] \n',MaxValues(dim_incumb(2)))
203 end
204 clear Titles
205 lowest_datum=min(MinValues');
206 highest_datum=max(MaxValues');
207 fprintf(' (all the values are "%s") \n',units)
208 disp(' ')
209 disp(' In your whole spreadsheet, absolute minimum and maximum values are: ')
210 fprintf(' Minimal datum = %-1.3f \n',lowest_datum)
211 fprintf(' Maximal datum = %-1.3f \n',highest_datum)
212 fprintf(' (%s) \n',units)
213 disp(char({' Knowing such extremes is helpful to decide the partition ',...
214 ' on which the states, available for the transitions of agents, ',...
215 ' will be set. ',...
216 ' (Press [Enter] to continue...) '}))

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217 response=input('','s');
218 while isempty(response)==0
219 disp('(press [Enter] to continue...')
220 response=input('','s');
221 end
222 clear AuxData1 AuxData2 dim_AuxData1 dim_incumb_pos
223 disp(char({' Are you interested in knowing also ','...
224 ' the set of quantiles of a certain degree, ','...
225 ' along with the median and the mean, of your imported data? ','...
226 ' You might adopt the quantiles to create the partition ','...
227 ' on which the states for the transition matrix are based. ','...
228 ' ("Y" for "Yes" or "N" for "Not", then [Enter]) '}))
229 response=input('','s');
230 while strcmp(response,{'Y','y','N','n'})==[0,0,0,0]
231 disp(' (You must answer exclusively "Y" or "N" ! ) ')
232 response=input('','s');
233 end
234 if sum(strcmp(response,{'Y','y'}))==1
235 disp(char({' Insert the degree of quantiles you are interested in ','...
236 ' (e.g. "4" for the quartiles, "5" for the quintiles, ','...
237 ' "10" for the deciles, "100" for centiles, and so on. '})));
238 qntl_degree=input(' ');
239 if (isscalar(qntl_degree)==0 || isreal(qntl_degree)==0 || isnan(qntl_degree)==1 || isinf(qntl_degree)==1 || ...
240 qntl_degree<=0 || floor(qntl_degree)~=qntl_degree)
241 while (isscalar(qntl_degree)==0 || isreal(qntl_degree)==0 || isnan(qntl_degree)=1 || isinf(qntl_degree)=1 || ...
242 qntl_degree<=0 || floor(qntl_degree)~=qntl_degree)
243 if isscalar(qntl_degree)==0
244 disp(' ( You pressed the [ENTER] button or inserted a vector or an array ! ) ')
245 disp(' BUT you must insert exclusively a positive integer ! ) ')
246 qntl_degree=input(' ');
247 elseif (isscalar(qntl_degree)==1 && (isnan(qntl_degree)==1 || isinf(qntl_degree)==1 || isreal(qntl_degree)==0))
248 disp(' Your input is a NaN value or infinite or a complex number: WHY? ')
249 qntl_degree=input(' ');
250 elseif qntl_degree<=0
251 disp(' ( You must insert exclusively a positive number ! ) ')
252 qntl_degree=input(' ');
253 elseif (qntl_degree>0 && floor(qntl_degree)~=qntl_degree)
254 disp(' ( You must insert exclusively a positive integer ! ) ')
255 qntl_degree=input(' ');
256 end
257 end
258 elseif (isscalar(qntl_degree)==1 && isreal(qntl_degree)==1 && isnan(qntl_degree)==0 && isinf(qntl_degree)==0 ...
259 && qntl_degree>0 && floor(qntl_degree)==qntl_degree)
260 end
261 q=zeros(1,qntl_degree);
262 for k=1:qntl_degree
263 q(k)=k/qntl_degree;
264 end
265 Quantiles=(quantile(incumbData,q,1));
266 Means=mean(incumbData);
267 Medians=median(incumbData);
268 Modes=mode(incumbData);
269 disp(char({'',' The matrix of quantiles you have chosen is named " Quantiles " ','...
270 ' and it is shown below: every observation period in the datasheet corresponds orderly ','...

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271 ' to a row in "Quantiles" and its columns from the second to the last one ',...
272 ' corresponds to the quantile ordering for such a date. ',' Quantiles = '))
273 disp(Quantiles)
274 disp(char({' The means, medians and modes of the data for each period, ',...
275 ' instead, are ordered in three row vectors, ',...
276 ' "Means", "Medians" and "Modes" respectively; ',...
277 ',' Means = '}))
278 disp(Means)
279 disp(' Medians = ')
280 disp(Medians)
281 disp(' Modes = ')
282 disp(Modes)
283 disp(' Press [Enter] to continue... ')
284 response=input(' ','s');
285 while isempty(response)==0
286 disp('(press [Enter] to continue...))
287 response=input(' ','s');
288 end
289 elseif sum(strcmp(response,{'N','n'}))=1
290 end
291 Input3=char({'',' ALL AGENTS are going to be GROUPED ',...
292 ' IN "n" CLASSES OF NOT-PREDETERMINED WIDTH, ',...
293 ' splitting the whole spectrum of the variable in the following manner: ',...
294 ','[0;sup(1)], [sup(1);sup(2)], [sup(2);sup(3)],..., [sup(n-1);infinity] ',...
295 ' where sup(1) is bigger than the minimal datum in the sheet',...
296 ' and sup(n-1) is smallest than the maximal datum. ',...
297 ',' Each class represents one of the states among which a transition happens ',...
298 ' and will be identified by a progressive positive integer. ',...
299 ',' INSERT THE NUMBER OF CLASSES ',...
300 ' and press [Enter]: ');
301 disp(Input3)
302 classes_num=input(' ');
303 while (isscalar(classes_num)==0 || isreal(classes_num)==0 || isnan(classes_num)==1 || isinf(classes_num)==1 ...
304 || classes_num<=0 || floor(classes_num)~=classes_num)
305 if isscalar(classes_num)==0
306 disp(char({' ( You pressed the [ENTER] button or inserted a vector or an array ! ) ',...
307 ' BUT you must insert exclusively a positive integer ! ) '}))
308 classes_num=input(' ');
309 elseif (isscalar(classes_num)==1 && (isnan(classes_num)==1 || isinf(classes_num)==1 || isreal(classes_num)==0))
310 disp(' Your input is a NaN value or a not finite one or it is not a real number: WHY? ')
311 classes_num=input(' ');
312 elseif classes_num<=0
313 disp(' ( You must insert exclusively a positive number ! ) ')
314 classes_num=input(' ');
315 elseif (classes_num>0 && floor(classes_num)~=classes_num)
316 disp(' ( You must insert exclusively a positive integer ! ) ')
317 classes_num=input(' ');
318 end
319 end
320 disp(' ')
321 fprintf(' Now write the sup for everyone of the first %-1.0d classes, \n',classes_num-1)
322 disp(char({' which are the inner points of your partition ',...
323 ' of the variable spectrum [ Minimal datum, Maximal datum ]. '}))
324 for k=1:classes_num-1

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325 fprintf('Insert the top (expressed as number of "%s") of the %-.10d° class:\n',units,k)
326 sup_k=input( ' ');
327 if k==1
328 if not(isscalar(sup_k)==1 && isreal(sup_k)==1 && isnan(sup_k)==0 && isinf(sup_k)==0 &&...
329 sup_k>0 && sup_k>lowest_datum && sup_k<highest_datum)
330 while not(isscalar(sup_k)==1 && isreal(sup_k)==1 && isnan(sup_k)==0 && isinf(sup_k)==0 &&...
331 sup_k>0 && sup_k>lowest_datum && sup_k<highest_datum)
332 if ( isscalar(sup_k)==0)
333 disp(' ( You pressed the [ENTER] button or inserted a vector or an array ! ) ')
334 disp(' BUT you must insert exclusively a positive integer ! ) ')
335 sup_k=input( ' ');
336 elseif (isscalar(sup_k)==1 && (isnan(sup_k)==1 || isinf(sup_k)==1 || isreal(sup_k)==0))
337 disp(' Your input is a NaN value or infinite or a complex number: WHY? ')
338 sup_k=input( ' ');
339 elseif sup_k<=0
340 disp(' ( You cannot insert a negative number or zero ! ) ')
341 sup_k=input( ' ');
342 elseif sup_k<=lowest_datum
343 disp(char({' Insert a value bigger than Minimal datum (but smaller than Maximal datum): '}))
344 sup_k=input( ' ');
345 elseif sup_k>=highest_datum
346 disp(char({' Insert a value smaller than Maximal datum (and bigger than Minimal datum): '}))
347 sup_k=input( ' ');
348 end
349 end
350 else
351 end
352 sup(1)=sup_k;
353 elseif k>1
354 if not(isscalar(sup_k)==1 && isreal(sup_k)==1 && isnan(sup_k)==0 && isinf(sup_k)==0 &&...
355 sup_k>0 && sup_k>lowest_datum && sup_k<highest_datum && sup(k-1)<sup_k)
356 while not(isscalar(sup_k)==1 && isreal(sup_k)==1 && isnan(sup_k)==0 && isinf(sup_k)==0 &&...
357 sup_k>0 && sup_k>lowest_datum && sup_k<highest_datum && sup(k-1)<sup_k)
358 if ( isscalar(sup_k)==0)
359 disp(' ( You pressed the [ENTER] button or inserted a vector or an array ! ) ')
360 disp(' BUT you must insert exclusively a positive integer ! ) ')
361 sup_k=input( ' ');
362 elseif (isscalar(sup_k)==1 && (isnan(sup_k)==1 || isinf(sup_k)==1 || isreal(sup_k)==0))
363 disp(' Your input is a NaN value or infinite or a complex number: WHY? ')
364 sup_k=input( ' ');
365 elseif sup_k<=0
366 disp(' ( You cannot insert a negative number or zero ! ) ')
367 sup_k=input( ' ');
368 elseif sup_k<=lowest_datum
369 disp(char({' Insert a value bigger than Minimal datum (but smaller than Maximal datum): '}))
370 sup_k=input( ' ');
371 elseif sup_k>=highest_datum
372 disp(char({' Insert a value smaller than Maximal datum (and bigger than Minimal datum): '}))
373 sup_k=input( ' ');
374 elseif sup_k<=sup(k-1)
375 disp(char({' Insert a value greater than the previous one '}))
376 sup_k=input( ' ');
377 end
378 end

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379 else
380 end
381 sup(k)=sup_k;
382 end
383 end
384 clear sup_k
385 partition=[0,sup,Inf];
386 Input4=char({' The extremes of the partition you have chosen ',...
387 ' are arranged orderly as components inside the vector "partition". '});
388 disp(Input4)
389 clear Input1 Input2_1 Input2_2 Input3 Input4
390 disp(' ( Press [Enter] to continue... ) ')
391 response=input(' ');
392 while isempty(response)==0
393 disp('(press [Enter] to continue...)')
394 response=input(' ');
395 end
396 %
397 % From this point, forward to the creation of the array "mov_k_tstart",
398 % we are referring to the statistical inference about first order
399 % Markov chains presented in Sections 2.1 and 2.2 from the 1956 paper
400 % by Anderson & Goodman, "Statistical inference about Markov Chains".
401 %
402 % n_k_tstart is the number of agents belonging to the k-th class
403 % at the starting period (t_start), ready to make a one-period transition.
404 %
405 % sh_k_tstart is the percentage of units belonging to the k-th class at t_start.
406 % For such array, as well as for "n_k_tstart", t_start is an integer
407 % varying from 1 to dim_incumb(2), differently from the other arrays
408 % which count agents or transitions
409 %
410 % num_k_h_tstart is the number of agents making a transition
411 % from the state 'k' occupied at time t_start
412 % to the state 'h' occupied at time t_start+1
413 %
414 % mov_k_tstart is the percentage of moving agents, which were inside the k-th class
415 % at the starting period t_start, observed inside any other class,
416 % different from the k-th one, at the successive period t_start+1
417 %
418 % nstar_k_tult is the cumulated number of all one-period transitions
419 % starting from the k-th class and occurring between t_start=1
420 % and t_start=t_ult where (t_ult >= 1 and t_ult <= dim_incumb(2)-1
421 %
422 % n_k_h_tult is the cumulated number of one-period transitions occurring
423 % from state 'k' at time t_start=1 to state 'h'
424 % at every subsequent period t_start=t_ult <= dim_incumb(2)-1
425 %
426 % p_stat_k_h is the maximum likelihood estimation of the first order
427 % stationary transition probability from the k-th class
428 % to the h-th one, computed between t_start=1 and t_start=dim_incumb(2)-1
429 %
430 % p_nonstat_k_h_tstart is the maximum likelihood estimation of the first order,
431 % not-stationary transition probability from the k-th state
432 % to the h-th one, computed between t_start and t_start+1

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433 %
434 % PAY ATTENTION: differently from the other counting arrays,
435 % "n_k_tstart" is defined for everyone of the observation dates
436 % in the spreadsheet, from t_start=1 to t_start=dim_incumb(2),
437 % instead of till to dim_incumb(2)-1 only.
438 %
439 n_k_tstart=zeros(classes_num,dim_incumb(2));
440 num_k_h_tstart=zeros(classes_num,classes_num,dim_incumb(2)-1);
441 for t_start=1:dim_incumb(2)-1
442 for k=1:classes_num
443 for h=1:classes_num
444 num_k_h_tstart(k,h,t_start)=sum((incumbData(:,t_start)>=partition(k) & incumbData(:,t_start)<partition(k+1))&...
445 (incumbData(:,t_start+1)>=partition(h) & incumbData(:,t_start+1)<partition(h+1)));
446 end
447 end
448 end
449 for t_start=1:dim_incumb(2)
450 for k=1:classes_num
451 n_k_tstart(k,t_start)=sum(incumbData(:,t_start)>=partition(k) & incumbData(:,t_start)<partition(k+1));
452 sh_k_tstart(k,t_start)=n_k_tstart(k,t_start)./dim_incumb(1);
453 end
454 end
455 p_nonstat_k_h_tstart=zeros(classes_num,classes_num,dim_incumb(2)-1);
456 for t_start=1:dim_incumb(2)-1
457 for k=1:classes_num
458 for h=1:classes_num
459 if n_k_tstart(k,t_start)==0
460 continue
461 end
462 p_nonstat_k_h_tstart(k,h,t_start)=num_k_h_tstart(k,h,t_start)./n_k_tstart(k,t_start);
463 end
464 end
465 end
466 n_k_h_tult=zeros(classes_num,classes_num,dim_incumb(2)-1);
467 nstar_k_tult=zeros(classes_num,dim_incumb(2)-1);
468 for k=1:classes_num
469 for t_ult=1:dim_incumb(2)-1
470 for h=1:classes_num
471 n_k_h_tult(k,h,t_ult)=sum(num_k_h_tstart(k,h,1:t_ult));
472 end
473 end
474 end
475 for k=1:classes_num
476 for t_ult=1:dim_incumb(2)-1
477 nstar_k_tult(k,t_ult)=sum(n_k_h_tult(k,,:t_ult));
478 end
479 end
480 p_stat_k_h=zeros(classes_num,classes_num);
481 for k=1:classes_num
482 for h=1:classes_num
483 if nstar_k_tult(k,dim_incumb(2)-1)==0
484 continue
485 end
486 p_stat_k_h(k,h)=n_k_h_tult(k,h,dim_incumb(2)-1)./nstar_k_tult(k,dim_incumb(2)-1);

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487 end
488 end
489 p_up_nonstat_k_h_tstart=zeros(classes_num,classes_num,dim_incumb(2)-1);
490 p_dn_nonstat_k_h_tstart=zeros(classes_num,classes_num,dim_incumb(2)-1);
491 for t_start=1:dim_incumb(2)-1
492 p_up_nonstat_k_h_tstart(:,:,t_start)=triu(p_nonstat_k_h_tstart(:,:,t_start),1);
493 p_dn_nonstat_k_h_tstart(:,:,t_start)=tril(p_nonstat_k_h_tstart(:,:,t_start),-1);
494 end
495 p_up_stat_k_h=triu(p_stat_k_h,1);
496 p_dn_stat_k_h=tril(p_stat_k_h,-1);
497 %
498 % Now we are going to reckon the products of both the stationary transition matrix
499 % and the not-stationary ones between the first period of observations in the datasheet
500 % and everyone of the subsequent periods until the last one, in order to calculate
501 % also the aggregate values of the mobility indices of our interest.
502 %
503 % power_p_stat_k_h_tult is the compound stationary transition matrix
504 % from the first period t_start=1 to each of the following
505 % periods t_ult=1,2,..., dim_incumb(2)-1
506 %
507 % aggr_p_nonstat_k_h_tult is the compound not-stationary transition matrix
508 % from the first period t_start=1 to everyone
509 % of the following t_ult=1,2,...,dim_incumb(2)-1.
510 %
511 for t_ult=1:dim_incumb(2)-1
512 power_p_stat_k_h_tult(:,:,t_ult)=(p_stat_k_h)^(t_ult);
513 end
514 for t_start=1:dim_incumb(2)-1
515 prdct(:,:,t_start)=eye(classes_num,classes_num);
516 end
517 aggr_p_nonstat_k_h_tult=zeros(classes_num,classes_num,dim_incumb(2)-1);
518 for t_ult=1:dim_incumb(2)-1
519 if t_ult==1
520 aggr_p_nonstat_k_h_tult(:,:,t_ult)=p_nonstat_k_h_tstart(:,:,1);
521 continue
522 end
523 for t_start=1:t_ult
524 prdct(:,:,t_start+1)=prdct(:,:,t_start)*p_nonstat_k_h_tstart(:,:,t_start);
525 end
526 aggr_p_nonstat_k_h_tult(:,:,t_ult)=prdct(:,:,t_start+1);
527 end
528 clear prdct
529 %
530 % Chi-Square Test and Likelihood Ratio Criterion to test the null hypothesis
531 % that the one-period transition probabilities from a first order Markov chain
532 % are constant respect to the time (Anderson & Goodman 1956, Sect. 3.2).
533 %
534 disp(char({' ' In this part we are going to check up the 1st null hypothesis ' ,...
535 ' that the one-period transition probabilities from a first order Markov chain, ' ,...
536 ' based on the imported data, are constant respect to the time, ' ,...
537 ' applying the Chi-Square Test and the Maximum Likelihood Ratio Criterion (MLRC) ' ,...
538 ' as proposed in Section 3.2 of the 1956 paper by Anderson & Goodman. ' ,''}))
539 r_X_timedepend=zeros(classes_num,classes_num,dim_incumb(2)-1);
540 r_MLR_timedepend=zeros(classes_num,classes_num,dim_incumb(2)-1);

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541 for k=1:classes_num
542 for h=1:classes_num
543 for t_start=1:dim_incumb(2)-1
544 if p_stat_k_h(k,h)==0
545 continue
546 end
547 r_X_timedepend(k,h,t_start)=((p_nonstat_k_h_tstart(k,h,t_start)-p_stat_k_h(k,h)).^2)./p_stat_k_h(k,h);
548 end
549 end
550 end
551 for k=1:classes_num
552 for h=1:classes_num
553 for t_start=1:dim_incumb(2)-1
554 if p_stat_k_h(k,h)==0
555 r_MLR_timedepend(k,h,t_start)=1;
556 else
557 r_MLR_timedepend(k,h,t_start)=(p_stat_k_h(k,h)/p_nonstat_k_h_tstart(k,h,t_start)).^num_k_h_tstart(k,h,t_start);
558 end
559 end
560 end
561 end
562 % "df" is the number of degree of freedom in both the Chi-Square Test
563 % and the Likelihood Ratio Criterion.
564 a_k=zeros(1,classes_num);
565 b_k=zeros(1,classes_num);
566 for k=1:classes_num
567 a_k(k)=length(find(p_stat_k_h(k,:)));
568 % By construction, "n_k_tstart" is the number of individuals staying in 'k'
569 % for each observation date; the number of observation dates is "dim_incumb(2)"
570 % BUT we are interested in the number of transition periods
571 % with a positive number of individuals concluding a transition.
572 % From the last observation date in the database, no transitions start !!
573 % So, for the degrees of freedom, we are not interested in "n_k_tstart(:,dim_incumb(2))."
574 % We have to count all the positive values of "n_k_tstart"
575 % only from the first date to the penultimate one, marked as "dim_incumb(2)-1",
576 % because the number of periods in the datasheet is just "dim_incumb(2)-1".
577 b_k(k)=length(find(n_k_tstart(k,1:dim_incumb(2)-1)));
578 end
579 df_timedepend_k(:)=(a_k(:)-1).*(b_k(:)-1);
580 df_timedepend_tot=(a_k-1)*(b_k-1);
581 %
582 % "chisq_timedepend_oss_k" is a column vector, relative to the specific imported data matrix,
583 % collecting, for each starting state 'k', the observed value of the Chi-Square statistics
584 % representing the null hypothesis respect to the alternative.
585 % "chisq_timedepend_oss" is the scalar of the total observed statistics
586 % for the null hypothesis against the alternative, without distinguishing
587 % the different starting states.
588 % Both of them admit as asymptotic distributon the Chi-Square one.
589 %
590 chisq_timedepend_oss_k=sum(n_k_tstart(:,1:dim_incumb(2)-1).*squeeze(sum(r_X_timedepend,2)),2);
591 chisq_timedepend_oss=sum(chisq_timedepend_oss_k);
592 %
593 % "p_X2_timedepend_k" and "p_X2_timedepend_tot" are the probabilities that the value of a chi-square variable
594 % may be greater than the observed chi-square statistics, for the hypothesis of 1st order Markovian

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595 % stationary transition probabilities, respectively in the cases of distinguished starting states 'k'
596 % and without any distinction.
597 %
598 for k=1:classes_num
599 p_X2_timedepend_k(k)=chi2cdf(chisq_timedepend_oss_k(k),df_timedepend_k(k),'upper');
600 end
601 disp(char({' According to the Chi Square Test, the probability that, ',...
602 ' depending on every starting state "k", Markovian first-order ',...
603 ' transition probabilities, based on the imported data, are constant in time, ',...
604 ' is the k-th component in the vector : ',';',' p_X2_timedepend_k(k) = ')))
605 disp(p_X2_timedepend_k)
606 p_X2_timedepend_tot=chi2cdf(chisq_timedepend_oss,df_timedepend_tot,'upper');
607 disp(char({' and the probability that the whole 1st order Markov chain, ',...
608 ' based on the data, is stationary, is: ',';',' p_X2_timedepend_tot = ')))
609 disp(p_X2_timedepend_tot)
610 if chisq_timedepend_oss<=df_timedepend_tot
611 disp(char({' The value of the Chi^2 statistics computed onto the data for the tested sample ',...
612 ' isn"t greater than number of degree of freedom "df_timedepend_tot", so we can conclude ',...
613 ' that the 1st null hypothesis is confirmed. })))
614 elseif chisq_timedepend_oss>df_timedepend_tot
615 if p_X2_timedepend_tot>=5/100
616 disp(char({' The value of the Chi^2 statistics computed onto the data for the tested sample ',...
617 ' is greater than number of degree of freedom "df_timedepend_tot", but the probability ',...
618 ' "p_X2_timedepend_tot" for such an event isn"t lower than 5%, ',...
619 ' so we can accept the 1st null hypothesis at the 5% significance level. })))
620 elseif p_X2_timedepend_tot<5/100
621 disp(char({' The value of the Chi^2 statistics computed onto the data for the tested sample ',...
622 ' is greater than number of degree of freedom "df_timedepend_tot" and the probability ',...
623 ' "p_X2_timedepend_tot" for such an event is lower than 5%, ',...
624 ' so we must reject the 1st null hypothesis at the 5% significance level. })))
625 end
626 end
627 disp(' ( Press [Enter] to continue... ) ')
628 response=input(' ','s');
629 while isempty(response)==0
630 disp('(press [Enter] to continue...')
631 response=input(' ','s');
632 end
633 %
634 % "MLRcrit_timedepend_k" is a column vector, relative to the imported data, collecting,
635 % for each starting state 'k', the opposite of the double logarithm
636 % of the observed value of the Maximum Likelihood Ratio
637 % for the null hypothesis against its alternative.
638 % "MLRcrit_timedepend_tot" is the scalar of the total observed statistics
639 % of the Maximum Likelihood Ratio Criterion (MLRC)
640 % for the null hypothesis against the alternative, without distinguishing
641 % the different starting states.
642 % Both of them admit, as an asymptotic distributon, the Chi-Square one.
643 %
644 lambda_timedepend_oss_k=prod(squeeze(prod(r_MLR_timedepend,2)),2);
645 MLRcrit_timedepend_k=-2*log(lambda_timedepend_oss_k);
646 MLRcrit_timedepend_tot=sum(MLRcrit_timedepend_k);
647 disp(char({' According to the Maximum Likelihood Ratio Criterion, ',...
648 ' the probability, depending on each starting state "k", that Markovian first-order ',...

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649 ' transition probabilities, from the imported data, are time-independent, ',...
650 ' is the k-th component of the vector : ',' p_MLR_timedepend_k(k) = '))
651 %
652 % The degrees of freedom for the Maximum Likelihood Ratio Criterion
653 % are the same, "nu_X_k" and "nu_X_tot", as for the previous Chi-Square Test.
654 %
655 % "p_MLR_timedepend_k" and "p_MLR_timedepend_tot" are the probabilities that the value of a chi-square variable
656 % may be greater than the observed MLRC statistics, for the hypothesis of 1st order Markovian
657 % stationary transition probabilities, respectively by distinguishing, or not,
658 % the starting states 'k'.
659 %
660 for k=1:classes_num
661 p_MLR_timedepend_k(k)=chi2cdf(MLRcrit_timedepend_k(k),df_timedepend_k(k),'upper');
662 end
663 disp(p_MLR_timedepend_k)
664 p_MLR_timedepend_tot=chi2cdf(MLRcrit_timedepend_tot,df_timedepend_tot,'upper');
665 disp(char({' and the probability that the whole 1st order Markov chain, ',...
666 ' built up on the data, is stationary, is: ',' p_MLR_timedepend_tot = ')))
667 disp(p_MLR_timedepend_tot)
668 if MLRcrit_timedepend_tot<=df_timedepend_tot
669 disp(char({' The value of the MLRC statistics computed onto the data for the tested sample ',...
670 ' isn't greater than number of degree of freedom "df_timedepend_tot", so we can conclude ',...
671 ' that the 1st null hypothesis is confirmed by the MLR Criterion. ','})))
672 elseif MLRcrit_timedepend_tot>df_timedepend_tot
673 if p_MLR_timedepend_tot>=5/100
674 disp(char({' The value of the MLRC statistics computed onto the data for the tested sample ',...
675 ' is greater than number of degree of freedom "df_timedepend_tot", but the probability ',...
676 ' "p_MLR_timedepend_tot" for such an event isn't lower than 5%, ',...
677 ' so the 1st null hypothesis is accepted by the MLR Criterion ',...
678 ' at the 5% significance level. ','})))
679 elseif p_MLR_timedepend_tot<5/100
680 disp(char({' The value of the MLRC statistics computed onto the data for the tested sample ',...
681 ' is greater than number of degree of freedom "df_timedepend_tot" and the probability ',...
682 ' "p_MLR_timedepend_tot" for such an event is lower than 5%, ',...
683 ' so the 1st null hypothesis is rejected by the MLR Criterion ',...
684 ' at the 5% significance level. ','})))
685 end
686 end
687 disp(' ( Press [Enter] to continue... ) ')
688 response=input(' ','s');
689 while isempty(response)==0
690 disp('(press [Enter] to continue...')
691 response=input(' ','s');
692 end
693 %
694 % Chi-Square Test and Likelihood Ratio Criterion to test the null hypothesis
695 % that the Markov chain from the data is a stationary first-order chain
696 % respect to the alternative that it is a stationary second-order one
697 % (Anderson & Goodman 1956, Sect. 3.3).
698 %
699 disp(char({' We want also to verify the 2nd null hypothesis ',...
700 ' that the Markov chain from the data is a stationary 1st-order chain, ',...
701 ' against the alternative that it is a stationary 2nd-order one, ',...
702 ' by the relative Chi-Square Test and the Maximum Likelihood Ratio Criterion (MLRC) ',...

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703 ' as proposed in Section 3.3 of the 1956 paper by Anderson & Goodman. ');'))
704 num_k_h_l_tstart=zeros(classes_num,classes_num,classes_num,dim_incumb(2)-2);
705 %
706 % num_k_h_l_tstart is the number of individuals making a transition
707 % from the state 'k' occupied at time t_start
708 % to the intermediate state 'h' occupied at time t_start+1
709 % and finally to the state 'l' occupied at time t_start+2,
710 % varying t_start between 1 and dim_incumb(2)-1.
711 %
712 for t_start=1:dim_incumb(2)-2
713 for k=1:classes_num
714 for h=1:classes_num
715 for l=1:classes_num
716 num_k_h_l_tstart(k,h,l,t_start)=sum((incumbData(:,t_start)>=partition(k) & incumbData(:,t_start)<partition(k+1))&...
717 (incumbData(:,t_start+1)>=partition(h) & incumbData(:,t_start+1)<partition(h+1))...
718 &(incumbData(:,t_start+2)>=partition(l) & incumbData(:,t_start+2)<partition(l+1)));
719 end
720 end
721 end
722 end
723 %
724 % p_stat_k_h_l is the maximum likelihood estimation, for a Markovian 2nd order
725 % stationary chain, of the probability to find an agent in the k-th class
726 % at time t_start, in the intermediate h-th one at the date t_start+1
727 % and finally in the l-th class at t_start+2, varying t_start
728 % between t_start=1 and t_start=dim_incumb(2)-2.
729 %
730 p_stat_k_h_l=zeros(classes_num,classes_num,classes_num);
731 num_k_h_l=squeeze(sum(num_k_h_l_tstart,4));
732 %
733 % num_k_h_l is the number of one-period transitions from the state 'k' at the date t_start
734 % to the intermediate class 'h' at t_start+1 and to the final one 'l'
735 % at time t_start+2, varying t_start between 1 and dim_incumb(2)-2.
736 %
737 % nstarM2ord_k_h is the number of agents making a transition from the k-th class
738 % to the h-th one, between t_start and t_start+1, whatever the final state
739 % occupied at t_start+2, varying t_start between 1 and dim_incumb(2)-2.
740 %
741 nstarM2ord_k_h=squeeze(sum(num_k_h_tstart(:,1:(dim_incumb(2)-2)),3));
742 % A first alternative definition for "nstarM2ord_k_h" is:
743 % nstarM2ord_k_h(:,:)=(n_k_h_tult(:,dim_incumb(2)-1)-num_k_h_tstart(:,dim_incumb(2)-1));
744 for k=1:classes_num
745 for h=1:classes_num
746 for l=1:classes_num
747 if nstarM2ord_k_h(k,h)==0
748 continue
749 end
750 p_stat_k_h_l(k,h,l)=num_k_h_l(k,h,l)/nstarM2ord_k_h(k,h);
751 end
752 end
753 end
754 for k=1:classes_num
755 for h=1:classes_num
756 % A second alternative definition for "nstarM2ord_k_h" is:

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757 % nstarM2ord_k_h(k,h)=n_k_h_tult(k,h,dim_incumb(2)-1)-num_k_h_tstart(k,h,dim_incumb(2)-1);
758 sum2_M2ord_k_h(k,h)=n_k_h_tult(k,h,dim_incumb(2)-1)-num_k_h_tstart(k,h,1);
759 end
760 end
761 % An alternative definition for "sum2_M2ord_k_h" is:
762 % sum2_M2ord_k_h(:,:)=sum(num_k_h_tstart(:,2:dim_incumb(2)-1),3);
763 %
764 % p_statM2ord_k_h is the maximum likelihood estimation, for a Markovian 2nd order
765 % stationary chain, of the probability to find an agent in the k-th class
766 % at time t_start+1 and in the h-th class at t_start+2,
767 % whatever the starting state at the date t_start,
768 % varying t_start between 1 and dim_incumb(2)-2.
769 %
770 p_statM2ord_k_h=zeros(classes_num,classes_num);
771 for k=1:classes_num
772 for h=1:classes_num
773 if nstar_k_tult(k,dim_incumb(2)-1)-nstar_k_tult(k,1)==0
774 continue
775 end
776 p_statM2ord_k_h(k,h)=sum2_M2ord_k_h(k,h)/(nstar_k_tult(k,dim_incumb(2)-1)-nstar_k_tult(k,1));
777 % An alternative definition for "p_statM2ord_k_h" is:
778 % p_statM2ord_k_h(k,h)=sum2_M2ord_k_h(k,h)/(nstar_k_tult(k,dim_incumb(2)-1)-n_k_tstart(k,1));
779 end
780 end
781 r_X_1stVS2nd_k_h_l=zeros(classes_num,classes_num,classes_num);
782 r_MLR_1stVS2nd_k_h_l=zeros(classes_num,classes_num,classes_num);
783 for k=1:classes_num
784 for h=1:classes_num
785 for l=1:classes_num
786 if p_statM2ord_k_h(h,l)==0
787 continue
788 end
789 r_X_1stVS2nd_k_h_l(k,h,l)=(p_stat_k_h_l(k,h,l)-p_statM2ord_k_h(h,l)).^2./p_statM2ord_k_h(h,l);
790 end
791 end
792 end
793 for k=1:classes_num
794 for h=1:classes_num
795 for l=1:classes_num
796 if p_statM2ord_k_h(h,l)==0
797 r_MLR_1stVS2nd_k_h_l(k,h,l)=1;
798 else
799 r_MLR_1stVS2nd_k_h_l(k,h,l)=(p_statM2ord_k_h(h,l)/p_stat_k_h_l(k,h,l)).^num_k_h_l(k,h,l);
800 end
801 end
802 end
803 end
804 % "df" is the number of degree of freedom in both the Chi-Square Test
805 % and the Likelihood Ratio Criterion.
806 c_k=zeros(1,classes_num);
807 d_h=zeros(1,classes_num);
808 for k=1:classes_num
809 c_k(k)=length(find(p_statM2ord_k_h(k,:)));
810 end

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811 for h=1:classes_num
812 d_h(h)=length(find(nstarM2ord_k_h(:,h)));
813 end
814 df_1stVS2nd_k=(c_k-1).*(d_h-1);
815 df_1stVS2nd_tot=(c_k-1)*(d_h-1);
816 %
817 % "chisq_1stVS2nd_oss_h" is a column vector, relative to the imported data matrix,
818 % collecting, for each intermediate state 'k', the observed value of the Chi-Square statistics
819 % representing the null hypothesis respect to the alternative.
820 % "chisq_1stVS2nd_oss" is the scalar of the total observed statistics
821 % for the null hypothesis against the alternative, without distinguishing
822 % the different intermediate states.
823 % Both of them admit as asymptotic distributon the Chi-Square one.
824 %
825 chisq_1stVS2nd_oss_h=squeeze(sum(nstarM2ord_k_h.*sum(r_X_1stVS2nd_k_h_l,3),1));
826 chisq_1stVS2nd_oss=sum(chisq_1stVS2nd_oss_h);
827 %
828 % "p_X2_1stVS2nd_h" and "p_X2_1stVS2nd_tot" are the probabilities that the value of a chi-square variable
829 % may be greater than the observed chi-square statistics, for the hypothesis of 1st order
830 % Markovian stationary chain, respectively in the cases of distinguished intermediate classes
831 % 'h' and without any distinction.
832 %
833 for h=1:classes_num
834 p_X2_1stVS2nd_h(h)=chi2cdf(chisq_1stVS2nd_oss_h(h),df_1stVS2nd_k(h),'upper');
835 end
836 p_X2_1stVS2nd_tot=chi2cdf(chisq_1stVS2nd_oss,df_1stVS2nd_tot,'upper');
837 disp(char({' According to the Chi Square Test, the probability that, ',...
838 ' depending on every intermediate state "h", the 2nd-order stationary Markov chain, ',...
839 ' based on the imported data, reduces itself to a stationary 1st order one starting from "h", ',...
840 ' is the h-th component in the vector : ',' p_X2_1stVS2nd_h(h) = '}))
841 disp(p_X2_1stVS2nd_h)
842 disp(char({' and the probability that the whole stationary Markov chain, ',...
843 ' based on the data, is a 1st order chain but not a 2nd order one, is: ',',...
844 ' p_X2_1stVS2nd_tot = '}))
845 disp(p_X2_1stVS2nd_tot)
846 if chisq_1stVS2nd_oss<=df_1stVS2nd_tot
847 disp(char({' The value of the Chi^2 statistics computed onto the data for the tested sample ',...
848 ' isn't greater than number of degree of freedom "df_1stVS2nd_tot", so we can conclude ',...
849 ' that the 2nd null hypothesis is confirmed. '}))
850 elseif chisq_1stVS2nd_oss>df_1stVS2nd_tot
851 if p_X2_1stVS2nd_tot>=5/100
852 disp(char({' The value of the Chi^2 statistics computed onto the data for the tested sample ',...
853 ' is greater than number of degree of freedom "df_1stVS2nd_tot", but the probability ',...
854 ' "p_X2_1stVS2nd_tot" for such an event isn't lower than 5%, ',...
855 ' so the 2nd null hypothesis is accepted at the 5% significance level. '}))
856 elseif p_X2_1stVS2nd_tot<5/100
857 disp(char({' The value of the Chi^2 statistics computed onto the data for the tested sample ',...
858 ' is greater than number of degree of freedom "df_1stVS2nd_tot" and the probability ',...
859 ' "p_X2_1stVS2nd_tot" for such an event is lower than 5%, ',...
860 ' so the 2nd null hypothesis is rejected at the 5% significance level. '}))
861 end
862 end
863 disp(' ( Press [Enter] to continue... ) ')
864 response=input(' ','s');

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865 while isempty(response)==0
866 disp('(press [Enter] to continue...')
867 response=input(' ');
868 end
869 %
870 % "MLRcrit_1stVS2nd_h" is a column vector, based on the imported data, collecting,
871 % for each intermediate state 'h', the opposite of the double logarithm
872 % of the observed value of the Maximum Likelihood Ratio
873 % for the null hypothesis against its alternative.
874 % "MLRcrit_1stVS2nd_tot" is the scalar of the total observed statistics
875 % of the Maximum Likelihood Ratio Criterion (MLRC)
876 % for the null hypothesis against the alternative, without distinguishing
877 % the different intermediate classes.
878 % Both of them admit, as an asymptotic distributon, the Chi-Square one.
879 %
880 lambda_1stVS2nd_oss_h=prod(squeeze(prod(r_MLR_1stVS2nd_k_h_l,3)),1);
881 MLRcrit_1stVS2nd_h=-2*log(lambda_1stVS2nd_oss_h);
882 MLRcrit_1stVS2nd_tot=sum(MLRcrit_1stVS2nd_h);
883 %
884 % The degrees of freedom for the Maximum Likelihood Ratio Criterion
885 % are the same, "df_1stVS2nd_k" and "df_1stVS2nd_tot", as for the previous Chi-Square Test.
886 %
887 % "p_MLR_1stVS2nd_h" and "p_MLR_1stVS2nd_tot" are the probabilities that the value of a chi-square variable
888 % may be greater than the observed MLRC statistics, for the hypothesis of a 1st order Markovian
889 % stationary chain, respectively by distinguishing, or not, the intermediate states 'h'.
890 %
891 for h=1:classes_num
892 p_MLR_1stVS2nd_h(h)=chi2cdf(MLRcrit_1stVS2nd_h(h),df_1stVS2nd_k(h),'upper');
893 end
894 p_MLR_1stVS2nd_tot=chi2cdf(MLRcrit_1stVS2nd_tot,df_1stVS2nd_tot,'upper');
895 disp(char({' According to the Maximum Likelihood Ratio Criterion, ',...
896 ' the probability that, depending on every intermediate state "h" ',...
897 ' the 2nd-order stationary Markov chain, based on the imported data, ',...
898 ' reduces itself to a stationary 1st order one starting from "h", ',...
899 ' is the h-th component in the vector : ',' p_MLR_1stVS2nd_h(h) = ')))
900 disp(p_MLR_1stVS2nd_h)
901 disp(char({' and the probability that the whole stationary Markov chain, ',...
902 ' based on the data, is a 1st order chain but not a 2nd order one, is: ',',...
903 ' p_MLR_1stVS2nd_tot = ')))
904 disp(p_MLR_1stVS2nd_tot)
905 if MLRcrit_1stVS2nd_tot<=df_1stVS2nd_tot
906 disp(char({' The value of the MLRC statistics computed onto the data for the tested sample ',...
907 ' isn"t greater than number of degree of freedom "df_1stVS2nd_tot", so we can conclude ',...
908 ' that the 2nd null hypothesis is confirmed by the MLR Criterion. ')))
909 elseif MLRcrit_1stVS2nd_tot>df_1stVS2nd_tot
910 if p_MLR_1stVS2nd_tot>=5/100
911 disp(char({' The value of the MLRC statistics computed onto the data for the tested sample ',...
912 ' is greater than number of degree of freedom "df_1stVS2nd_tot", but the probability ',...
913 ' "p_MLR_1stVS2nd_tot" for such an event isn"t lower than 5%, ',...
914 ' so the 2nd null hypothesis is accepted by the MLR Criterion ',...
915 ' at the 5% significance level. ')))
916 elseif p_MLR_1stVS2nd_tot<5/100
917 disp(char({' The value of the MLRC statistics computed onto the data for the tested sample ',...
918 ' is greater than number of degree of freedom "df_1stVS2nd_tot" and the probability ',...

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919 ' "p_MLR_1stVS2nd_tot" for such an event is lower than 5%, ',...
920 ' so the 2nd null hypothesis is rejected by the MLR Criterion ',...
921 ' at the 5% significance level. ' ');))
922 end
923 end
924 disp(' Press [Enter] to continue... ')
925 response=input(' ');
926 while isempty(response)==0
927 disp('(press [Enter] to go on...')
928 response=input(' ');
929 end
930 % Computation of the mean permanence times, along with their theoretical standard deviations,
931 % for the variable in the datasheet estimated for a 1st ord., stationary Markov chain;
932 % of our two suggestions to estimate the mean permanence times for a 1st ord. not-stationary chain
933 % with their related theoretical standard deviations.
934 % Finally, computation of the empirical mean permanence times mechanically counted in the datasheet,
935 % together with their corrected sample standard deviations.
936 for k=1:classes_num
937 mean_permtime_M1stat_k(k)=1./(1-p_stat_k_h(k,k));
938 sigma_mean_permtime_M1stat_k(k)=sqrt(p_stat_k_h(k,k))./(1-p_stat_k_h(k,k));
939 mean_permtime_M1nonstat_k(k)=1+sum(aggr_p_nonstat_k_h_tult(k,k,:),3);
940 for t_ult=1:dim_incumb(2)-1
941 coeffc(t_ult)=2*t_ult+1;
942 aggr_p_nonstat_k_k(k,t_ult)=aggr_p_nonstat_k_h_tult(k,k,t_ult);
943 end
944 end
945 sigma_mean_permtime_M1nonstat_k=((1+aggr_p_nonstat_k_k*(coeffc))'-(mean_permtime_M1nonstat_k.^2)).^(1/2);
946 disp(char({' "mean_permtime_M1stat_k" is the vector of average times ',...
947 ' spent by an individual in everyone of the k-th classes ',...
948 ' (defined as a geometric series whose common ration is the one-period ',...
949 ' transition probability from "k" to "k", "p_stat_k_h(k,k)", ',...
950 ' and is computed along with the vector of the associated standard deviations, ',...
951 ' "sigma_mean_permtime_M1stat_k", for a stationary first-order Markov chain ',...
952 ' (see, for instance, the explanation in Appendix B of the 1955 Prais's paper). ' ');))
953 fprintf(' If the state index "k" varies from 1 to %d: \n',classes_num)
954 disp(char({' ; mean_permtime_M1stat_k(k) = '}))
955 fprintf(' (all these values are expressed in "%s") \n',time_scan)
956 disp(mean_permtime_M1stat_k)
957 disp(' sigma_mean_permtime_M1stat_k(k) = ')
958 fprintf(' (in "%s") \n',time_scan)
959 disp(sigma_mean_permtime_M1stat_k)
960 disp(' Press [Enter] to continue... ')
961 response=input(' ');
962 while isempty(response)==0
963 disp('(press [Enter] to go on...')
964 response=input(' ');
965 end
966 disp(char({' "mean_permtime_M1nonstat_k" is instead our proposal for the average time ',...
967 ' spent inside the k-th class, according to a 1st order not stationary Markov chain. ',...
968 ' We have defined it by analogy with the sum of the first "dim_incumb(2)" ',...
969 ' terms of the geometric series for the stationary case. ' ');))
970 "mean_permtime_M1nonstat_k" is the sum of 1 and p_nonstat_k_h_tstart(k,k,1) ',...
971 ' with the products of the first two, three, four (and so on....) ',...
972 ' not-stationary transition matrices from "k" to "k" for a 1st order Markov chain. ',...

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973 ' It is computed together with the vector of the corresponding standard deviations, ',...
974 ' "sigma_mean_permtime_M1nonstat_k", for a not stationary first-order Markov chain ',...));
975 fprintf(' For "k" varying from 1 to %d:\n',classes_num)
976 disp(char({' ', mean_permtime_M1nonstat_k(k) = '}))
977 fprintf(' (in "%s") \n',time_scan)
978 disp(mean_permtime_M1nonstat_k)
979 disp(' sigma_mean_permtime_M1nonstat_k(k) = ')
980 fprintf(' (in "%s") \n',time_scan)
981 disp(sigma_mean_permtime_M1nonstat_k)
982 disp(' Press [Enter] to continue... ')
983 response=input(' ','s');
984 while isempty(response)==0
985 disp('(press [Enter] to go on...')
986 response=input(' ','s');
987 end
988 disp(char({' "mean_permtime_observed_k" is the vector of the mean numbers ',...
989 ' of observation periods spent by any statistical unit ',...
990 ' to remain in the k-th classes: they are automatically counted ',...
991 ' scanning each row of the incumbents inside the spreadsheet. ',...
992 ' The vector of the corresponding corrected sample standard deviations ',...
993 ' is also computed: it is "sigma_mean_permtime_observed_k". ',...
994 ' Such third approach is not based on a predetermined underlying model: ',...
995 ' the mean permanence times and the associated sample standard deviations ',...
996 ' are counted row by row inside the incumbents" matrix, ',...
997 ' from any date to the successive one. ',...}));
998 % for k=1:classes_num
999 % tot_permtime_obs_k(k)=sum(num_k_h_tstart(k,k,:),3);
1000 % end
1001 % mean_permtime_obs_k=tot_permtime_obs_k./dim_incumb(1);
1002 % % "mean_permtime_obs_k" and "mean_permtime_observed_k" must coincide
1003 % % if they are the same quantity computed in two different manners.
1004 %
1005 Agent_k_h_tstart=zeros(dim_incumb(1),classes_num,classes_num,dim_incumb(2)-1);
1006 for Ag_id=1:dim_incumb(1)
1007 for t_start=1:dim_incumb(2)-1
1008 for k=1:classes_num
1009 for h=1:classes_num
1010 Agent_k_h_tstart(Ag_id,k,h,t_start)=(incumbData(Ag_id,t_start)>=partition(k) &
incumbData(Ag_id,t_start)<partition(k+1))&...
1011 (incumbData(Ag_id,t_start+1)>=partition(h) & incumbData(Ag_id,t_start+1)<partition(h+1));
1012 %
1013 % "Agent_k_h_tstart(Ag_id,k,h,t_start)" is an array whose elements are logical scalar outputs:
1014 % if the individual identified by the integer 'Ag_id' is in state 'k' at 't_start'
1015 % and it will be in state 'h' at 't_start+1', such array "Agent_k_h_tstart" is worth '1',
1016 % otherwise it is worth zero.
1017 end
1018 end
1019 end
1020 end
1021 for Ag_id=1:dim_incumb(1)
1022 for k=1:classes_num
1023 Agent_permtime_observed_k(Ag_id,k)=sum(Agent_k_h_tstart(Ag_id,k,k,:));
1024 % "Agent_permtime_observed_k(Ag_id,k)" is the total number of observation periods
1025 % measured for the individual, identified by the integer 'Ag_id', during which it remains

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1026 % in state 'k' from the time 't_start' to 't_start+1', whatever 't_start'.
1027 mean_permtime_observed_k(k)=sum(Agent_permtime_observed_k(:,k))./dim_incumb(1);
1028 sigma_mean_permtime_observed_k(k)=((sum((Agent_permtime_observed_k(:,k)-mean_permtime_observed_k(k)).^2)).
1029 /(dim_incumb(1)-1)).^(1/2);
1030 end
1031 end
1032 fprintf(' For "k" varying from 1 to %d: \n',classes_num)
1033 disp(char({' ' mean_permtime_observed_k(k) = '}))
1034 fprintf(' (in "%s") \n',time_scan)
1035 disp(mean_permtime_observed_k)
1036 disp(' sigma_mean_permtime_observed_k(k) = ')
1037 fprintf(' (in "%s") \n',time_scan)
1038 disp(sigma_mean_permtime_observed_k)
1039 disp(' Press [Enter] to continue... ')
1040 response=input(' ','s');
1041 while isempty(response)==0
1042 disp('(press [Enter] to go on...')
1043 response=input(' ','s');
1044 end
1045 %
1046 % Now the setting of the weights {omega_i} of the starting states
1047 % and the function v(|i-j|) of the jumps begins, both included in the definitions
1048 % of the family of mobility indices by Alcalde-Unzu and of the Directional Index.
1049 disp(char({' Now it begins the setting of the weights {omega_i} of the starting states ',...
1050 ' and the function v(|i-j|) of the jumps, both included in the definitions ',...
1051 ' of the family of mobility indices by Alcalde-Unzu and of the Directional Index. ','})))
1052 default=char({' By default the weights {omega_i}, are the rates of individuals ',...
1053 ' in each state "i" at the first period in the datasheet, t_start=1. ',...
1054 ' If you want to change, it is possible to assign a number, ',...
1055 ' different from the starting probability, ',...
1056 ' weighting every single initial state "i" inside such Indices; ',...
1057 ', ' Do you want to modify the default weights? ',...
1058 ' ( "Y" for "Yes" or "N" for "Not", then [Enter] )});
1059 Input_5ter=char({' ',' ' Write, in the command line, the handle " @(x) " ',...
1060 ' followed by the mathematical shape of your v(|i-j|), ',...
1061 ' for example @(x)exp(x^(1/3)) or @(x)log(2*x^3) . ',...
1062 ' You can express it respect to the auxiliary variable "x", ',...
1063 ' so that x = |i-j| : anyway, to avoid conflicts, ',...
1064 ' you must refer the handle "@" and the function to the same variable. ',...
1065 ' In any case, the Alcalde-Unzu Index and the Directional one will be computed ',...
1066 ' simultaneously for the same function v(|i-j|). ','});
1067 disp(default)
1068 resp_w=input(' ','s');
1069 while strcmp(resp_w,{'Y','y','N','n'})==[0,0,0,0]
1070 disp(' (You must answer exclusively "Y" or "N" ! ) ')
1071 resp_w=input(' ','s');
1072 end
1073 if sum(strcmp(resp_w,{'N','n'}))==1
1074 for t_start=1:dim_incumb(2)-1
1075 immob_num(t_start)=trace(num_k_h_tstart(:,t_start));
1076 for k=1:classes_num
1077 mov_k_tstart(k,t_start)=(n_k_tstart(k,t_start)-num_k_h_tstart(k,k,t_start))./...
1078 (dim_incumb(1)-immob_num(t_start));
1079 end

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1080 end
1081 w_k=(sum(mov_k_tstart,2)./(dim_incumb(2)-1));
1082 elseif sum(strcmp(resp_w,{ 'Y','y' }))==1
1083 change=char({' If you choose a scalar, as a weight for each state, ',...
1084 ' BEWARE that THE SUM OVER ALL THE STATES MUST BE EQUAL TO 1. ',...
1085 ', ' If you dislike this option about {omega_i} ',...
1086 ' you have to modify directly the part of this script ',...
1087 ' concerning the definitions of "w_k(k)". ',...
1088 ', BE also CAREFUL NOT TO PINCH the ENTER button if you don"t want to quit !!});
1089 disp(change)
1090 for k=1:classes_num
1091 fprintf('Insert the numerical value of the %-1.0d° weight w(%-1.0d):\n',k,k)
1092 w_k(k)=input(' ');
1093 while w_k(k)<0 || w_k(k)>1 || isnan(w_k(k))==1 || isreal(w_k(k))==0 || isscalar(w_k(k))==0
1094 if w_k(k)<0
1095 disp(' This input is a probability: you have to insert a positive number !! ')
1096 w_k(k)=input(' ');
1097 elseif w_k(k)>1
1098 disp(char({' This input is a probability of a single event: ',...
1099 ' it cannot be a number greater than 1 !! '}))
1100 w_k(k)=input(' ');
1101 elseif (isnan(w_k(k))==1 || isscalar(w_k(k))==0)
1102 disp(' This input is a probability: you have to insert a scalar !! ')
1103 w_k(k)=input(' ');
1104 elseif isreal(w_k(k))==0
1105 disp(' This input is a probability: you have to insert a real scalar !! ')
1106 w_k(k)=input(' ');
1107 end
1108 end
1109 end
1110 sum_w_k=single(sum(w_k));
1111 if (sum_w_k>1 || sum_w_k<1)
1112 while (sum_w_k>1 || sum_w_k<1)
1113 warn=char({' Pay attention: ',...
1114 ' THE SUM OF ALL THE PREVIOUS INPUTS IS NOT EQUAL TO 1 !! ',...
1115 ' YOU NEED TO INSERT ALL THE WEIGHTS AGAIN. ',',',...
1116 ' Be careful not to pinch the enter button !});
1117 disp(warn)
1118 for k=1:classes_num
1119 fprintf('Insert the numerical value of the %-1.0d° weight w(%-1.0d):\n',k,k)
1120 w_k(k)=input(' ');
1121 while w_k(k)<0 || w_k(k)>1 || isnan(w_k(k))==1 || isreal(w_k(k))==0 || isscalar(w_k(k))==0
1122 if w_k(k)<0
1123 disp(' This input is a probability: you have to insert a positive number !! ')
1124 w_k(k)=input(' ');
1125 elseif w_k(k)>1
1126 disp(char({' This input is a probability of a single event: ',...
1127 ' it cannot be a number greater than 1 !! '}))
1128 w_k(k)=input(' ');
1129 elseif (isnan(w_k(k))==1 || isscalar(w_k(k))==0)
1130 disp(' This input is a probability: you have to insert a scalar !! ')
1131 w_k(k)=input(' ');
1132 elseif isreal(w_k(k))==0
1133 disp(' This input is a probability: you have to insert a real scalar !! ')

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1134 w_k(k)=input(' ');
1135 end
1136 end
1137 end
1138 sum_w_k=single(sum(w_k));
1139 end
1140 else
1141 end
1142 end
1143 disp(Input_5ter)
1144 clear resp_w change
1145 %
1146 % We are going to introduce
1147 %
1148 % V_k_h(h,k) the suitable function of the difference (h-k), weighting
1149 % the significance of a jump from the k-th class to the h-th one
1150 % V_k_h(h,k) = v(|h-k|)
1151 %
1152 % Below an auxiliary variable, named "x", will be used.
1153 %
1154 v=input(' v(|i-j|) = v(x)= ');
1155 %
1156 % The function handle " v ", inserted by you, corresponds to the weight function v(|i-j|)
1157 % namely the handle in x=0 is the weight v(|i-j|) computed in |i-j|=0,
1158 % the handle in x=1 is the weight v(|i-j|) computed in |i-j|=1, and so on.
1159 %
1160 while isa(v,'function_handle')==0
1161 disp(' Your input is not a function handle: ')
1162 disp(' please follow the instructon above and retype your function. ')
1163 v=input(' v(|i-j|) = v(x)= ');
1164 end
1165 for h=1:classes_num
1166 for k=1:classes_num
1167 V_k_h(h,k)=feval(v,abs(h-k));
1168 Vs_k_h(h,k)=sign(h-k)*feval(v,abs(h-k));
1169 %
1170 % for the matrix V_k_h, 'h' is a row index and 'k' is a column index
1171 % and they appear, as parameters, also in the scalar 'abs(h-k)'
1172 % which is a non negative integer.
1173 %
1174 end
1175 end
1176 V_nan=isnan(V_k_h);
1177 V_infinite=isinf(V_k_h);
1178 V_real=isreal(V_k_h);
1179 while (isequal(V_nan,zeros(classes_num,classes_num))==0 | isequal(V_infinite,zeros(classes_num,classes_num))==0 |...
1180 V_real==0)
1181 disp(char({' ', ' Note that the shape " v(|i-j|) " you have chosen ',...
1182 ' is not defined over the whole domain of the couple (i,j): ',...
1183 ' it could invalidate the computation of the Index. ',...
1184 ' You can check it by observing the position of all the possible ',...
1185 ' NaN values, infinite values or non-real values ',...
1186 ' in the following matrix " V_k_h(h,k) " : ','}'))
1187 disp(V_k_h)

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1188 disp(char({' ' So insert a more convenient functional " v(|i-j) " ',...
1189 ' defined for each possible value of the couple (i,j). ',...
1190 ' The name "x" for the auxiliary variable is only a suggestion: ', ''}));
1191 v=input(' v(|i-j) = v(x)= ');
1192 while isa(v,'function_handle')==0
1193 disp(' Your input is not a function handle: ')
1194 disp(' please follow the instructon above and retype your function. ')
1195 v=input(' v(|i-j) = v(x)= ');
1196 end
1197 for h=1:classes_num
1198 for k=1:classes_num
1199 V_k_h(h,k)=feval(v,abs(h-k));
1200 end
1201 end
1202 V_nan=isnan(V_k_h);
1203 V_infinite=isinf(V_k_h);
1204 V_real=isreal(V_k_h);
1205 end
1206 InputAlfa=char({' ',' ' Lastly, it is necessary to fix the value of the power "alfa" ',...
1207 ' appearing, in the family of Mobility Indices by Alcalde-Unzu, ',...
1208 ' over the difference (P-Identity): ',...
1209 ' it is a real number, not smaller than 1, ',...
1210 ' allowing for different distance functions. '});
1211 disp(InputAlfa)
1212 alfa=input(' Power order "alfa" = ');
1213 while (isscalar(alfa)==0 || isnan(alfa)==1 || isinf(alfa)==1 || alfa <= 1 || isreal(alfa)==0)
1214 if isscalar(alfa)==0
1215 disp(' ( You pressed the [ENTER] button or inserted a vector or an array ! ) ')
1216 disp(' BUT you must insert exclusively a positive number ! ) ')
1217 alfa=input(' ');
1218 elseif (isscalar(alfa)==1 && (isnan(alfa)==1 || isinf(alfa)==1 || isreal(alfa)==0))
1219 disp(' Your input is a NaN value or a not finite one or it is not a real number: WHY? ')
1220 alfa=input(' ');
1221 elseif alfa <= 1
1222 disp(' ( You must insert exclusively a positive number, not smaller than 1 ! ) ')
1223 alfa=input(' ');
1224 end
1225 end
1226 % Here it begins the computation of the Alcalde-Unzu Index based
1227 % on a first-order, not-stationary Markov process
1228 Id=eye(classes_num);
1229 v_zero=feval(v,0);
1230 for t_start=1:dim_incumb(2)-1
1231 for k=1:classes_num
1232 Max(k)=max([V_k_h(1,k),V_k_h(classes_num,k)]);
1233 aux_Z(k)=(Max(k)+v_zero).^(1/alfa);
1234 end
1235 Z=aux_Z*(w_k)';
1236 end
1237 for t_start=1:dim_incumb(2)-1
1238 for k=1:classes_num
1239 Y1_nonstat_V_alfa_k_tstart(k,t_start)=(((abs(p_nonstat_k_h_tstart(k,;t_start)-Id(k,:)).^alfa)*V_k_h(:,k)).^(1/alfa);
1240 Y1up_nonstat_V_alfa_k_tstart(k,t_start)=(((abs(p_up_nonstat_k_h_tstart(k,;t_start)-Id(k,:)).^alfa)*V_k_h(:,k)).^(1/alfa);
1241 Y1dn_nonstat_V_alfa_k_tstart(k,t_start)=(((abs(p_dn_nonstat_k_h_tstart(k,;t_start)-Id(k,:)).^alfa)*V_k_h(:,k)).^(1/alfa);

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1242 end
1243 Imov_w_V_alfa_tstart(t_start)=(w_k(:))*Y1_nonstat_V_alfa_k_tstart(:,t_start);
1244 Imovup_w_V_alfa_tstart(t_start)=(w_k(:))*Y1up_nonstat_V_alfa_k_tstart(:,t_start);
1245 Imovdn_w_V_alfa_tstart(t_start)=(w_k(:))*Y1dn_nonstat_V_alfa_k_tstart(:,t_start);
1246 end
1247 for k=1:classes_num
1248 Y1_stat_V_alfa_k(k)=(((abs(p_stat_k_h(k,:)-ld(k,:)).^alfa)*V_k_h(:,k)).^(1/alfa);
1249 Y1up_stat_V_alfa_k(k)=(((abs(p_up_stat_k_h(k,:)-ld(k,:)).^alfa)*V_k_h(:,k)).^(1/alfa);
1250 Y1dn_stat_V_alfa_k(k)=(((abs(p_dn_stat_k_h(k,:)-ld(k,:)).^alfa)*V_k_h(:,k)).^(1/alfa);
1251 end
1252 Imov_stat_w_V_a=(w_k(:))*Y1_stat_V_alfa_k(:);
1253 Imovup_stat_w_V_a=(w_k(:))*Y1up_stat_V_alfa_k(:);
1254 Imovdn_stat_w_V_a=(w_k(:))*Y1dn_stat_V_alfa_k(:);
1255 %
1256 % Imov_w_V_alfa_tstart(t_start) and Imov_stat_w_V_a
1257 % are the not-normalized versions of the Mobility-as-a-Movement Index,
1258 % respectively for a not-stationary 1st-order Markov chain and for the stationary one.
1259 % The values of the Normalized Indices
1260 % Imovnorm_w_V_alfa_tstart(t_start) and Imovnorm_stat_w_V_a
1261 % are:
1262 %
1263 Imovnorm_w_V_alfa_tstart=Imov_w_V_alfa_tstart./Z;
1264 Imovnorm_up_w_V_alfa_tstart=Imovup_w_V_alfa_tstart./Z;
1265 Imovnorm_dn_w_V_alfa_tstart=Imovdn_w_V_alfa_tstart./Z;
1266 Imovnorm_stat_w_V_a=Imov_stat_w_V_a./Z;
1267 Imovnorm_up_stat_w_V_a=Imovup_stat_w_V_a./Z;
1268 Imovnorm_dn_stat_w_V_a=Imovdn_stat_w_V_a./Z;
1269 StayShares_k_tstart=zeros(classes_num,dim_incumb(2)-1);
1270 Ratemov_k_tstart=zeros(classes_num,dim_incumb(2)-1);
1271 for k=1:classes_num
1272 for t_start=1:dim_incumb(2)-1
1273 StayShares_k_tstart(k,t_start)=num_k_h_tstart(k,k,t_start)./dim_incumb(1);
1274 if (w_k(k))*Y1_nonstat_V_alfa_k_tstart(k,t_start)==0 && Imov_w_V_alfa_tstart(t_start)==0)
1275 Ratemov_k_tstart(k,t_start)=0;
1276 elseif (w_k(k))*Y1_nonstat_V_alfa_k_tstart(k,t_start)~=0 && Imov_w_V_alfa_tstart(t_start)==0)
1277 Ratemov_k_tstart(k,t_start)=Inf;
1278 else
1279 Ratemov_k_tstart(k,t_start)=(w_k(k).*Y1_nonstat_V_alfa_k_tstart(k,t_start))./Imov_w_V_alfa_tstart(t_start);
1280 end
1281 end
1282 end
1283 Ratemov_stat_k=zeros(1,classes_num);
1284 for k=1:classes_num
1285 if (w_k(k))*Y1_stat_V_alfa_k(k)==0 && Imov_stat_w_V_a==0)
1286 Ratemov_stat_k(k)=0;
1287 elseif (w_k(k))*Y1_stat_V_alfa_k(k)~=0 && Imov_stat_w_V_a==0)
1288 Ratemov_stat_k(k)=Inf;
1289 else
1290 Ratemov_stat_k(k)=(w_k(k).*Y1_stat_V_alfa_k(k))./Imov_stat_w_V_a;
1291 end
1292 end
1293 % Here the computation of the Alcalde-Unzu Index ends;
1294 % the calculus of the Directional Mobility Index, based
1295 % on a first-order, not-stationary Markov process, begins.

```

```

1296 clear default Input0 Input_5ter InputAlfa
1297 m1=Vs_k_h(1,:)*w_k(:);
1298 m2=Vs_k_h(classes_num,:)*w_k(:);
1299 %
1300 % m1 and m2 are, respectively, the global minimum and the global maximum
1301 % theoretically possible for the not-normalized index.
1302 % They are necessary to carry out the normalization of the Directional Mobility Index
1303 % to the target [-1;+1] in the following manner:
1304 % if the not normalized index is negative then
1305 % normalized index = -1*(not normalized index) / m1;
1306 % else if the not normalized index is non negative then
1307 % normalized index = (not normalized index) / m2;
1308 %
1309 for t_start=1:dim_incumb(2)-1
1310 for k=1:classes_num
1311 Y2_nonstat_V_k_tstart(k,t_start)=p_nonstat_k_h_tstart(k,;,t_start)*Vs_k_h(:,k);
1312 Y2up_nonstat_V_k_tstart(k,t_start)=p_up_nonstat_k_h_tstart(k,;,t_start)*Vs_k_h(:,k);
1313 Y2dn_nonstat_V_k_tstart(k,t_start)=p_dn_nonstat_k_h_tstart(k,;,t_start)*Vs_k_h(:,k);
1314 end
1315 ldir_w_V_tstart(t_start)=w_k(:)*Y2_nonstat_V_k_tstart(:,t_start);
1316 ldirup_w_V_tstart(t_start)=w_k(:)*Y2up_nonstat_V_k_tstart(:,t_start);
1317 ldirdn_w_V_tstart(t_start)=w_k(:)*Y2dn_nonstat_V_k_tstart(:,t_start);
1318 end
1319 %
1320 % ldir_w_V_tstart
1321 % is the not-normalized version of the Directional Mobility Index.
1322 % ldirup_w_V_tstart
1323 % and ldirdn_w_V_tstart
1324 % are respectively its upward and downward components.
1325 % The new value of the Normalized Directional Mobility Index
1326 % ldirnorm_w_V_tstart
1327 % along with its respective upward and downward components
1328 % are:
1329 %
1330 for t_start=1:dim_incumb(2)-1
1331 if ldir_w_V_tstart(t_start)>=0
1332 ldirnorm_w_V_tstart(t_start)=ldir_w_V_tstart(t_start)./m2;
1333 ldirnorm_up_w_V_tstart(t_start)=ldirup_w_V_tstart(t_start)./m2;
1334 ldirnorm_dn_w_V_tstart(t_start)=ldirdn_w_V_tstart(t_start)./m2;
1335 elseif ldir_w_V_tstart(t_start)<0
1336 ldirnorm_w_V_tstart(t_start)=-ldir_w_V_tstart(t_start)./m1;
1337 ldirnorm_up_w_V_tstart(t_start)=-ldirup_w_V_tstart(t_start)./m1;
1338 ldirnorm_dn_w_V_tstart(t_start)=-ldirdn_w_V_tstart(t_start)./m1;
1339 end
1340 end
1341 Rate_dirup_w_V_tstart=ones(1,dim_incumb(2)-1);
1342 for t_start=1:dim_incumb(2)-1
1343 if (ldirnorm_up_w_V_tstart(t_start)==0 && ldirnorm_w_V_tstart(t_start)==0)
1344 Rate_dirup_w_V_tstart(t_start)=0;
1345 elseif (ldirnorm_up_w_V_tstart(t_start)~=0 && ldirnorm_w_V_tstart(t_start)==0)
1346 Rate_dirup_w_V_tstart(t_start)=Inf;
1347 else Rate_dirup_w_V_tstart(t_start)=ldirnorm_up_w_V_tstart(t_start)./ldirnorm_w_V_tstart(t_start);
1348 end
1349 end

```

```

1350 Rate_dirdn_w_V_tstart=ones(1,dim_incumb(2)-1);
1351 for t_start=1:dim_incumb(2)-1
1352 if (ldirnorm_dn_w_V_tstart(t_start)==0 && ldirnorm_w_V_tstart(t_start)==0)
1353 Rate_dirdn_w_V_tstart(t_start)=0;
1354 elseif (ldirnorm_dn_w_V_tstart(t_start)~=0 && ldirnorm_w_V_tstart(t_start)==0)
1355 Rate_dirdn_w_V_tstart(t_start)=Inf;
1356 else
1357 Rate_dirdn_w_V_tstart(t_start)=ldirnorm_dn_w_V_tstart(t_start)/ldirnorm_w_V_tstart(t_start);
1358 end
1359 end
1360 Ratedir_k_tstart=zeros(classes_num,dim_incumb(2)-1);
1361 for k=1:classes_num
1362 for t_start=1:dim_incumb(2)-1
1363 if (w_k(k)*Y2_nonstat_V_k_tstart(k,t_start)==0 && ldir_w_V_tstart(t_start)==0)
1364 Ratedir_k_tstart(k,t_start)=0;
1365 elseif (w_k(k)*Y2_nonstat_V_k_tstart(k,t_start)~=0 && ldir_w_V_tstart(t_start)==0)
1366 Ratedir_k_tstart(k,t_start)=Inf;
1367 else
1368 Ratedir_k_tstart(k,t_start)=(w_k(k).*Y2_nonstat_V_k_tstart(k,t_start))/ldir_w_V_tstart(t_start);
1369 end
1370 end
1371 end
1372 % Computation of the Directional Mobility Index based
1373 % on a stationary 1st-order Markov chain:
1374 % The not normalized version of the Index is:
1375 for k=1:classes_num
1376 Y2_stat_V_k(k)=p_stat_k_h(k,:)*Vs_k_h(:,k);
1377 Y2up_stat_V_k(k)=p_up_stat_k_h(k,:)*Vs_k_h(:,k);
1378 Y2dn_stat_V_k(k)=p_dn_stat_k_h(k,:)*Vs_k_h(:,k);
1379 end
1380 ldir_stat_w_V=w_k(:)*Y2_stat_V_k(:);
1381 ldirup_stat_w_V=w_k(:)*Y2up_stat_V_k(:);
1382 ldirdn_stat_w_V=w_k(:)*Y2dn_stat_V_k(:);
1383 % The normalized version of the Directional Index is:
1384 if ldir_stat_w_V>=0
1385 ldirnorm_stat_w_V=ldir_stat_w_V./m2;
1386 ldirnorm_up_stat_w_V=ldirup_stat_w_V./m2;
1387 ldirnorm_dn_stat_w_V=ldirdn_stat_w_V./m2;
1388 elseif ldir_stat_w_V<0
1389 ldirnorm_stat_w_V=-ldir_stat_w_V./m1;
1390 ldirnorm_up_stat_w_V=-ldirup_stat_w_V./m1;
1391 ldirnorm_dn_stat_w_V=-ldirdn_stat_w_V./m1;
1392 end
1393 %
1394 % Here the calculus of the Directional Mobility Index ends.
1395 % It begins the computation of some other mobility measures
1396 % based on a first-order, not-stationary Markov process.
1397 InputBet=char({'', ' Also one of the indices, proposed by Shorrocks in 1978 ', ...
1398 ' and based on the determinant of the transition matrix ', ...
1399 ' as a period invariant measure, depends on a parameter "b", ', ...
1400 ' the power over the absolute value of the determinant: ', ...
1401 ' "b" is a positive real number ', ...
1402 ' which must be fixed by the user, like the previous power order "alfa". '});
1403 disp(InputBet)

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1404 bet=input(' Power order "b" = ');
1405 while (isscalar(bet)==0 || isnan(bet)==1 || isinf(bet)==1 || bet<=0 || isreal(bet)==0)
1406 if isscalar(bet)==0
1407 disp(' ( You pressed the [ENTER] button or inserted a vector or an array ! ) ')
1408 disp(' BUT you must insert exclusively a positive number ! ) ')
1409 bet=input(' ');
1410 elseif (isscalar(bet)==1 && (isnan(bet)==1 || isinf(bet)==1 || isreal(bet)==0))
1411 disp(' Your input is a NaN value or a not finite one or it is not a real number: WHY? ')
1412 bet=input(' ');
1413 elseif bet<=0
1414 disp(' ( You must insert exclusively a positive number ! ) ')
1415 bet=input(' ');
1416 end
1417 end
1418 for k=1:classes_num
1419 for h=1:classes_num
1420 Intens_k_h(k,h)=abs(h-k);
1421 end
1422 end
1423 % Here it begins the calculation of remaining seven indices
1424 % based on a not-stationary, 1st-order, Markov Chain:
1425 for t_start=1:dim_incumb(2)-1
1426 ltrace_tstart(t_start)=(classes_num-trace(p_nonstat_k_h_tstart(:,t_start)))/(classes_num-1);
1427 % Trace index of mobility
1428 ldet_b_tstart(t_start)=1-(abs(det(p_nonstat_k_h_tstart(:,t_start))))^bet;
1429 % Determinant index of mobility
1430 % The weakness of this index when it is applied to actual raw data:
1431 % it is sufficient that the estimated transition matrix has a null column
1432 % or a null row to make the index be worth 1 !
1433 lpredict_tstart(t_start)=classes_num*(sum(sum((p_nonstat_k_h_tstart(:,t_start)).^2,2))-1)/(classes_num-1);
1434 % Index of predictability
1435 l2ndBarthol_tstart(t_start)=(classes_num/(classes_num-1))*(sum(sum(p_nonstat_k_h_tstart(:,t_start).*Intens_k_h)));
1436 % Second Index of Bartholomew
1437 eigvals_tstart(:,t_start)=eig(p_nonstat_k_h_tstart(:,t_start));
1438 abs_eigvals_tstart=abs(eigvals_tstart);
1439 sorted_abs_eigvals_tstart=sort(abs_eigvals_tstart,'descend');
1440 l2ndeigval_tstart(t_start)=1-sorted_abs_eigvals_tstart(2,t_start); % Mobility index of the 2nd biggest eigenvalue
1441 lalleigvals_tstart(t_start)=(classes_num-sum(abs_eigvals_tstart(:,t_start)))/(classes_num-1);
1442 % All eigenvalues mobility index
1443 h_tstart(t_start)=-log(2)/log(sorted_abs_eigvals_tstart(2,t_start)); % Asymptotic half life
1444 lhalflife_tstart(t_start)=exp(-h_tstart(t_start)); % Asymptotic-half-life mobility Index
1445 end
1446 % The alternative versions of the remaining seven indices,
1447 % based on a stationary 1st-order Markov Chain, are:
1448 ltrace_stat=(classes_num-trace(p_stat_k_h(:,:)))/(classes_num-1); % Trace index
1449 ldet_b_stat=1-(abs(det(p_stat_k_h(:,:))))^bet; % Determinant index
1450 lpredict_stat=classes_num*(sum(sum((p_stat_k_h(:,:)).^2,2))-1)/(classes_num-1);
1451 % Index of predictability
1452 l2ndBarthol_stat=(classes_num/(classes_num-1))*(sum(sum(p_stat_k_h(:,:).*Intens_k_h)));
1453 % Second Index of Bartholomew
1454 eigvals_stat(:)=eig(p_stat_k_h(:,:));
1455 abs_eigvals_stat=abs(eigvals_stat);
1456 sorted_abs_eigvals_stat=sort(abs_eigvals_stat,'descend');
1457 l2ndeigval_stat=1-sorted_abs_eigvals_stat(2); % Index of the 2nd biggest eigenvalue

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1458 lalleigvals_stat=(classes_num-sum(abs_eigvals_stat(:)))/(classes_num-1);
1459 % All eigenvalues index
1460 h_stat=-log(2)/log(sorted_abs_eigvals_stat(2)); % Asymptotic half life
1461 lhalflife_stat=exp(-h_stat); % Asymptotic-half-life Index
1462 clear InputBet
1463 % The following section makes the calculus of the aggregate mobility indices,
1464 % that is all the mobility indices of interest defined on a multiperiod time interval,
1465 % both in the case of a not-stationary first-order Markov chain and for a stationary one.
1466 %
1467 % The Alcalde-Unzu family of aggregated mobility measures:
1468 % when the compound (multistep) transition matrix is not-stationary
1469 for t_ult=1:dim_incumb(2)-1
1470 for k=1:classes_num
1471 Y1nonstat_V_alfa_k_tult(k,t_ult)=(((abs(aggr_p_nonstat_k_h_tult(k,;t_ult)-ld(k,;)))^alfa)*V_k_h(,;k)).^(1/alfa);
1472 end
1473 Aggr_nonstat_lmov_w_V_alfa_tult(t_ult)=(w_k(:))*Y1nonstat_V_alfa_k_tult(:,t_ult);
1474 end
1475 Aggr_nonstat_lmovnorm_w_V_alfa_tult=Aggr_nonstat_lmov_w_V_alfa_tult./Z(1);
1476 % when the compound transition matrix is stationary
1477 for t_ult=1:dim_incumb(2)-1
1478 for k=1:classes_num
1479 Y1_stat_V_alfa_k_tult(k,t_ult)=(((abs(power_p_stat_k_h_tult(k,;t_ult)-ld(k,;)))^alfa)*V_k_h(,;k)).^(1/alfa);
1480 end
1481 Aggr_stat_lmov_w_V_alfa_tult(t_ult)=(w_k(:))*Y1_stat_V_alfa_k_tult(:,t_ult);
1482 end
1483 Aggr_stat_lmovnorm_w_V_alfa_tult=Aggr_stat_lmov_w_V_alfa_tult./Z(1);
1484 %
1485 % The aggregated Directional Mobility Index:
1486 % when the compound (multistep) transition matrix is not-stationary
1487 %
1488 aggr_pup_nonstat_k_h_tult=zeros(classes_num,classes_num,dim_incumb(2)-1);
1489 aggr_pdn_nonstat_k_h_tult=zeros(classes_num,classes_num,dim_incumb(2)-1);
1490 for t_ult=1:dim_incumb(2)-1
1491 aggr_pup_nonstat_k_h_tult(:,;t_ult)=triu(aggr_p_nonstat_k_h_tult(:,;t_ult),1);
1492 aggr_pdn_nonstat_k_h_tult(:,;t_ult)=tril(aggr_p_nonstat_k_h_tult(:,;t_ult),-1);
1493 end
1494 for t_ult=1:dim_incumb(2)-1
1495 for k=1:classes_num
1496 Y2_nonstat_V_k_tult(k,t_ult)=aggr_p_nonstat_k_h_tult(k,;t_ult)*Vs_k_h(,;k);
1497 Y2up_nonstat_V_k_tult(k,t_ult)=aggr_pup_nonstat_k_h_tult(k,;t_ult)*Vs_k_h(,;k);
1498 Y2dn_nonstat_V_k_tult(k,t_ult)=aggr_pdn_nonstat_k_h_tult(k,;t_ult)*Vs_k_h(,;k);
1499 end
1500 Aggr_nonstat_ldir_w_V_tult(t_ult)=w_k(:)*Y2_nonstat_V_k_tult(:,t_ult);
1501 Aggr_nonstat_ldirup_w_V_tult(t_ult)=w_k(:)*Y2up_nonstat_V_k_tult(:,t_ult);
1502 Aggr_nonstat_ldirdn_w_V_tult(t_ult)=w_k(:)*Y2dn_nonstat_V_k_tult(:,t_ult);
1503 end
1504 for t_ult=1:dim_incumb(2)-1
1505 if Aggr_nonstat_ldir_w_V_tult(t_ult)>=0
1506 Aggr_nonstat_ldirnorm_w_V_tult(t_ult)=Aggr_nonstat_ldir_w_V_tult(t_ult)./m2;
1507 Aggr_nonstat_ldirnorm_up_w_V_tult(t_ult)=Aggr_nonstat_ldirup_w_V_tult(t_ult)./m2;
1508 Aggr_nonstat_ldirnorm_dn_w_V_tult(t_ult)=Aggr_nonstat_ldirdn_w_V_tult(t_ult)./m2;
1509 elseif Aggr_nonstat_ldir_w_V_tult(t_ult)<0
1510 Aggr_nonstat_ldirnorm_w_V_tult(t_ult)=-Aggr_nonstat_ldir_w_V_tult(t_ult)./m1;
1511 Aggr_nonstat_ldirnorm_up_w_V_tult(t_ult)=-Aggr_nonstat_ldirup_w_V_tult(t_ult)./m1;

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1512 Aggr_nonstat_lDIRnorm_dn_w_V_tult(t_ult)=-Aggr_nonstat_lDIRdn_w_V_tult(t_ult)./m1;
1513 end
1514 end
1515 %
1516 % when the compound (multistep) transition matrix is stationary
1517 aggr_pup_stat_k_h_tult=zeros(classes_num,classes_num,dim_incumb(2)-1);
1518 aggr_pdn_stat_k_h_tult=zeros(classes_num,classes_num,dim_incumb(2)-1);
1519 for t_ult=1:dim_incumb(2)-1
1520 aggr_pup_stat_k_h_tult(:,t_ult)=triu(power_p_stat_k_h_tult(:,t_ult),1);
1521 aggr_pdn_stat_k_h_tult(:,t_ult)=tril(power_p_stat_k_h_tult(:,t_ult),-1);
1522 end
1523 for t_ult=1:dim_incumb(2)-1
1524 for k=1:classes_num
1525 Y2_stat_V_k_tult(k,t_ult)=power_p_stat_k_h_tult(k,t_ult)*Vs_k_h(:,k);
1526 Y2up_stat_V_k_tult(k,t_ult)=aggr_pup_stat_k_h_tult(k,t_ult)*Vs_k_h(:,k);
1527 Y2dn_stat_V_k_tult(k,t_ult)=aggr_pdn_stat_k_h_tult(k,t_ult)*Vs_k_h(:,k);
1528 end
1529 Aggr_stat_lDIR_w_V_tult(t_ult)=w_k(:)*Y2_stat_V_k_tult(:,t_ult);
1530 Aggr_stat_lDIRup_w_V_tult(t_ult)=w_k(:)*Y2up_stat_V_k_tult(:,t_ult);
1531 Aggr_stat_lDIRdn_w_V_tult(t_ult)=w_k(:)*Y2dn_stat_V_k_tult(:,t_ult);
1532 end
1533 for t_ult=1:dim_incumb(2)-1
1534 if Aggr_stat_lDIR_w_V_tult(t_ult)>=0
1535 Aggr_stat_lDIRnorm_w_V_tult(t_ult)=Aggr_stat_lDIR_w_V_tult(t_ult)./m2;
1536 Aggr_stat_lDIRnorm_up_w_V_tult(t_ult)=Aggr_stat_lDIRup_w_V_tult(t_ult)./m2;
1537 Aggr_stat_lDIRnorm_dn_w_V_tult(t_ult)=Aggr_stat_lDIRdn_w_V_tult(t_ult)./m2;
1538 elseif Aggr_stat_lDIR_w_V_tult(t_ult)<0
1539 Aggr_stat_lDIRnorm_w_V_tult(t_ult)=-Aggr_stat_lDIR_w_V_tult(t_ult)./m1;
1540 Aggr_stat_lDIRnorm_up_w_V_tult(t_ult)=-Aggr_stat_lDIRup_w_V_tult(t_ult)./m1;
1541 Aggr_stat_lDIRnorm_dn_w_V_tult(t_ult)=-Aggr_stat_lDIRdn_w_V_tult(t_ult)./m1;
1542 end
1543 end
1544 % The other mobility measures, based on the transition matrix
1545 % of a first order Markov chain:
1546 % when the multistep transition matrix is not-stationary
1547 for t_ult=1:dim_incumb(2)-1
1548 Aggr_nonstat_ltrace_tult(t_ult)=(classes_num-trace(aggr_p_nonstat_k_h_tult(:,t_ult)))/(classes_num-1);
1549 % Aggregated Trace index
1550 Aggr_nonstat_lDET_b_tult(t_ult)=1-(abs(det(aggr_p_nonstat_k_h_tult(:,t_ult))))^(bet/t_ult);
1551 % Aggregated Determinant Index
1552 Aggr_nonstat_lpredict_tult(t_ult)=classes_num*(sum(sum((aggr_p_nonstat_k_h_tult(:,t_ult)).^2,2))-1)/(classes_num-1);
1553 % Aggregated Index of predictability
1554 Aggr_nonstat_l2ndBarthol_tult(t_ult)=(classes_num/(classes_num-
1555 1))*(sum(sum(aggr_p_nonstat_k_h_tult(:,t_ult).*Intens_k_h)));
1556 % Aggregated Second Index of Bartholomew
1557 eigvals_aggrnonstat_tult(:,t_ult)=eig(aggr_p_nonstat_k_h_tult(:,t_ult));
1558 abs_eigvals_aggrnonstat_tult=abs(eigvals_aggrnonstat_tult);
1559 sorted_abs_eigvals_aggrnonstat_tult=sort(abs_eigvals_aggrnonstat_tult,'descend');
1560 Aggr_nonstat_l2ndeigval_tult(t_ult)=1-sorted_abs_eigvals_aggrnonstat_tult(2,t_ult); % Aggregated index of the 2nd biggest
1561 eigenvalue
1562 Aggr_nonstat_lalleigvals_tult(t_ult)=(classes_num-sum(abs_eigvals_aggrnonstat_tult(:,t_ult)))/(classes_num-1);
1563 % Aggregated All eigenvalues mobility index
1564 h_aggrnonstat_tult(t_ult)=-log(2)/log(sorted_abs_eigvals_aggrnonstat_tult(2,t_ult)); % Asymptotic half life
1565 Aggr_nonstat_lhalflife_tult(t_ult)=exp(-h_aggrnonstat_tult(t_ult)*t_ult); % Aggregated Asymptotic-half-life mobility Index

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1564 end
1565 % when the multistep transition matrix is stationary
1566 for t_ult=1:dim_incumb(2)-1
1567 Aggr_stat_ltrace_tult(t_ult)=(classes_num-trace(power_p_stat_k_h_tult(:,t_ult)))/(classes_num-1);
1568 % Aggregated Trace index
1569 Aggr_stat_ldet_b_tult(t_ult)=1-(abs(det(power_p_stat_k_h_tult(:,t_ult)))^(bet/t_ult);
1570 % Aggregated Determinant Index
1571 Aggr_stat_lpredict_tult(t_ult)=classes_num*(sum(sum((power_p_stat_k_h_tult(:,t_ult)).^2,2))-1)/(classes_num-1);
1572 % Aggregated Index of predictability
1573 Aggr_stat_l2ndBarthol_tult(t_ult)=(classes_num/(classes_num-1))*(sum(sum(power_p_stat_k_h_tult(:,t_ult).*Intens_k_h)));
1574 % Aggregated Second Index of Bartholomew
1575 eigvals_aggrstat_tult(:,t_ult)=eig(power_p_stat_k_h_tult(:,t_ult));
1576 abs_eigvals_aggrstat_tult=abs(eigvals_aggrstat_tult);
1577 sorted_abs_eigvals_aggrstat_tult=sort(abs_eigvals_aggrstat_tult,'descend');
1578 Aggr_stat_l2ndeigval_tult(t_ult)=1-sorted_abs_eigvals_aggrstat_tult(2,t_ult); % Aggregated index of the 2nd biggest
eigenvalue
1579 Aggr_stat_lalleigvals_tult(t_ult)=(classes_num-sum(abs_eigvals_aggrstat_tult(:,t_ult)))/(classes_num-1);
1580 % Aggregated All eigenvalues mobility index
1581 h_aggrstat_tult(t_ult)=-log(2)/log(sorted_abs_eigvals_aggrstat_tult(2,t_ult)); % Asymptotic half life
1582 Aggr_stat_lhalflife_tult(t_ult)=exp(-h_aggrstat_tult(t_ult)*t_ult); % Aggregated Asymptotic-half-life mobility Index
1583 end
1584 %
1585 % In the following lines are defined the bidimensional arrays
1586 % where the values of all the mobilities indices are gathered
1587 % (one particular index for each row) for every observation period
1588 % (one column for each date).
1589 nonstat_Mobilities=[ldirnorm_w_V_tstart;Imovnorm_w_V_alfa_tstart;ltrace_tstart;ldet_b_tstart;...
1590 lpredict_tstart;l2ndBarthol_tstart;l2ndeigval_tstart;lalleigvals_tstart;lhalflife_tstart];
1591 stat_Mobilities=[Aggr_stat_ldirnorm_w_V_tult(1);Aggr_stat_lmovnorm_w_V_alfa_tult(1);...
1592 Aggr_stat_ltrace_tult(1);Aggr_stat_ldet_b_tult(1);Aggr_stat_lpredict_tult(1);...
1593 Aggr_stat_l2ndBarthol_tult(1);Aggr_stat_l2ndeigval_tult(1);Aggr_stat_lalleigvals_tult(1);...
1594 Aggr_stat_lhalflife_tult(1)];
1595 Aggr_nonstat_Mobilities=[Aggr_nonstat_ldirnorm_w_V_tult;Aggr_nonstat_lmovnorm_w_V_alfa_tult;...
1596 Aggr_nonstat_ltrace_tult;Aggr_nonstat_ldet_b_tult;...
1597 Aggr_nonstat_lpredict_tult;Aggr_nonstat_l2ndBarthol_tult;...
1598 Aggr_nonstat_l2ndeigval_tult;Aggr_nonstat_lalleigvals_tult;Aggr_nonstat_lhalflife_tult];
1599 Aggr_stat_Mobilities=[Aggr_stat_ldirnorm_w_V_tult;Aggr_stat_lmovnorm_w_V_alfa_tult;...
1600 Aggr_stat_ltrace_tult;Aggr_stat_ldet_b_tult;Aggr_stat_lpredict_tult;...
1601 Aggr_stat_l2ndBarthol_tult;Aggr_stat_l2ndeigval_tult;Aggr_stat_lalleigvals_tult;...
1602 Aggr_stat_lhalflife_tult];
1603 clear ld coeffc aggr_p_nonstat_k_k
1604 %
1605 % GRAPHICS
1606 for t_start=1:dim_incumb(2)-1
1607 t(t_start)=t_start+1;
1608 end
1609 % One-period mobility indices vs the final time
1610 % of any possible one-step, not-stationary, 1st ord. Markovian transition
1611 % (from t_start to t_start+1) from the datasheet.
1612 nonstat_dir_mov_trace=figure('Name',...
1613 '1st ord., not-stationary Markov chain: Directional Mobility, Mobility as a movement, Trace Index',...
1614 'NumberTitle','off');
1615 scatter(t,ldirnorm_w_V_tstart,40,'d','filled')
1616 hold on

```

```

1617 scatter(t,Imovnorm_w_V_alfa_tstart,40,'o','filled')
1618 scatter(t,ltrace_tstart,50,'g','s','filled')
1619 line(t,ldirnorm_w_V_tstart)
1620 line(t,Imovnorm_w_V_alfa_tstart)
1621 line(t,ltrace_tstart)
1622 hold off
1623 legend('Directional Mobility','Mobility as a movement','Trace Mobility Index')
1624 nonstat_halfife_predic_Barthol=figure('Name',...
1625 '1st ord., not-stationary Markov chain: Asymptotic-half-life Mobility, Index of predictability, 2nd Bartholomew Ind.',...
1626 'NumberTitle','off');
1627 scatter(t,lhalfife_tstart,50,'g','s','filled')
1628 hold on
1629 scatter(t,lpredict_tstart,40,'o','filled')
1630 scatter(t,l2ndBarthol_tstart,40,'b','d','filled')
1631 line(t,lhalfife_tstart)
1632 line(t,lpredict_tstart)
1633 line(t,l2ndBarthol_tstart)
1634 hold off
1635 legend('Asymptotic-half-life Ind.','Ind. of predictability','2nd Bartholomew Ind.')
1636 nonstat_2ndEig_allEig_det=figure('Name',...
1637 '1st ord., not-stationary Markov chain: 2nd Eigenvalue Mobility Index, All-eigenvalue Index, Determinant Index',...
1638 'NumberTitle','off');
1639 scatter(t,l2ndeigval_tstart,40,'d','filled')
1640 hold on
1641 scatter(t,lalleigvals_tstart,40,'o','filled')
1642 scatter(t,ldet_b_tstart,50,'g','s','filled')
1643 line(t,l2ndeigval_tstart)
1644 line(t,lalleigvals_tstart)
1645 line(t,ldet_b_tstart)
1646 hold off
1647 legend('2nd-Eigenvalue Mobil. Ind.','All-eigenvalue Ind.','Determinant Mobility Ind.')
1648 %
1649 % Aggregated-in-time mobility indices vs the final time t_ult
1650 % (from the first period to each of the successive ones)
1651 % of any possible multistep, not-stationary transition
1652 % based on a 1st ord. Markov chain from the datasheet.
1653 aggnonstat_dir_mov_trace=figure('Name',...
1654 '1st ord., not-stationary Markov ch.: aggregation in time-Directional Mobility, Mob. as a movement, Trace Ind.',...
1655 'NumberTitle','off');
1656 scatter(t,Aggr_nonstat_ldirnorm_w_V_tult,40,'d','filled')
1657 hold on
1658 scatter(t,Aggr_nonstat_Imovnorm_w_V_alfa_tult,40,'o','filled')
1659 scatter(t,Aggr_nonstat_ltrace_tult,50,'g','s','filled')
1660 line(t,Aggr_nonstat_ldirnorm_w_V_tult)
1661 line(t,Aggr_nonstat_Imovnorm_w_V_alfa_tult)
1662 line(t,Aggr_nonstat_ltrace_tult)
1663 hold off
1664 legend('Aggregated Directional Ind.','Aggregat. Mobility as a movement','Aggregated Trace Ind.')
1665 aggnonstat_halfife_predic_Barthol=figure('Name',...
1666 '1st ord., not-stationary Markov ch.: aggregation in time-Asymptotic-half-life Ind., Ind. of predictability, 2nd Bartholomew
Ind.',...
1667 'NumberTitle','off');
1668 scatter(t,Aggr_nonstat_lhalfife_tult,50,'g','s','filled')
1669 hold on

```

```

1670 scatter(t,Aggr_nonstat_lpredict_tult,40,'o','filled')
1671 scatter(t,Aggr_nonstat_l2ndBarthol_tult,40,'d','filled')
1672 line(t,Aggr_nonstat_lhalflife_tult)
1673 line(t,Aggr_nonstat_lpredict_tult)
1674 line(t,Aggr_nonstat_l2ndBarthol_tult)
1675 hold off
1676 legend('Aggregated Asymptotic-half-life Ind. ','Aggreg. Ind. of predictability','Aggreg. 2nd Bartholomew Ind. ')
1677 aggnonstat_2ndEig_allEig_det=figure('Name',...
1678 '1st ord., not-stationary Markov ch.: aggregation in time-2nd Eigenvalue Mobil. Ind., All-eigenvalue Ind., Determinant
Ind.',...
1679 'NumberTitle','off');
1680 scatter(t,Aggr_nonstat_l2ndeigval_tult,40,'b','d','filled')
1681 hold on
1682 scatter(t,Aggr_nonstat_lalleigvals_tult,40,'o','filled')
1683 scatter(t,Aggr_nonstat_lidet_b_tult,40,'g','s','filled')
1684 line(t,Aggr_nonstat_l2ndeigval_tult)
1685 line(t,Aggr_nonstat_lalleigvals_tult)
1686 line(t,Aggr_nonstat_lidet_b_tult)
1687 hold off
1688 legend('Aggregated 2nd-Eigenvalue Ind. ','Aggreg. All-eigenvalue Ind. ','Aggreg. Determinant Ind. ')
1689 %
1690 % Aggregated-in-time mobility indices vs the final time t_ult
1691 % (from the first period to each of the successive ones)
1692 % of any possible multistep, stationary transition
1693 % based on a 1st ord. Markov chain from the datasheet.
1694 aggststat_dir_mov_trace=figure('Name',...
1695 '1st ord., stationary Markov ch.: aggregation in time-Directional Mobility, Mob. as a movement, Trace Ind.',...
1696 'NumberTitle','off');
1697 scatter(t,Aggr_stat_lDIRnorm_w_V_tult,40,'d','filled')
1698 hold on
1699 scatter(t,Aggr_stat_lmovnorm_w_V_alfa_tult,40,'o','filled')
1700 scatter(t,Aggr_stat_ltrace_tult,50,'g','s','filled')
1701 line(t,Aggr_stat_lDIRnorm_w_V_tult)
1702 line(t,Aggr_stat_lmovnorm_w_V_alfa_tult)
1703 line(t,Aggr_stat_ltrace_tult)
1704 hold off
1705 legend('Aggregated Directional Ind. ','Aggregat. Mobility as a movement','Aggregated Trace Ind. ')
1706 aggststat_halfife_predic_Barthol=figure('Name',...
1707 '1st ord., stationary Markov ch.: aggregation in time-Asymptotic-half-life Ind., Ind. of predictability, 2nd Bartholomew
Ind.',...
1708 'NumberTitle','off');
1709 scatter(t,Aggr_stat_lhalflife_tult,50,'g','s','filled')
1710 hold on
1711 scatter(t,Aggr_stat_lpredict_tult,40,'o','filled')
1712 scatter(t,Aggr_stat_l2ndBarthol_tult,40,'d','filled')
1713 line(t,Aggr_stat_lhalflife_tult)
1714 line(t,Aggr_stat_lpredict_tult)
1715 line(t,Aggr_stat_l2ndBarthol_tult)
1716 hold off
1717 legend('Aggregated Asymptotic-half-life Ind. ','Aggreg. Ind. of predictability','Aggreg. 2nd Bartholomew Ind. ')
1718 aggststat_2ndEig_allEig_det=figure('Name',...
1719 '1st ord., stationary Markov ch.: aggregation in time-2nd Eigenvalue Mobil. Ind., All-eigenvalue Ind., Determinant Ind.',...
1720 'NumberTitle','off');
1721 scatter(t,Aggr_stat_l2ndeigval_tult,40,'b','d','filled')

```

```
1722 hold on
1723 scatter(t,Aggr_stat_lalleigvals_tult,40,'o','filled')
1724 scatter(t,Aggr_stat_Idet_b_tult,40,'g','s','filled')
1725 line(t,Aggr_stat_l2ndeigval_tult)
1726 line(t,Aggr_stat_lalleigvals_tult)
1727 line(t,Aggr_stat_Idet_b_tult)
1728 hold off
1729 legend('Aggregated 2nd-Eigenvalue Ind.','Aggreg. All-eigenvalue Ind.','Aggreg. Determinant Ind.')
1730 clear response k h l t_start t_ult Ag_id
```

## Appendix A2

In this second appendix there are sixteen extensive tables filled with the detailed outputs elaborated by '*MarkovInfer\_MobInd.m*' regarding the chi-squared test and the maximum likelihood ratio criterium, applied to national Italian data from AIDA, about reliability assessments between the null hypothesis that data follow a 1<sup>st</sup>-order time-homogeneous Markov Chain, on one side, and the alternatives of a 1<sup>st</sup>-order not-stationary Chain and a 2<sup>nd</sup>-order stationary one on the other side. Only two sectors are reported: Foods & Beverages Manufactures and Innovative Small & Medium Enterprises. A general analysis on the overall dependability results, for the 28 combinations between an economic proxy and an industrial sector, can be found in the second part of Chapter 7.

These tables are an example of the type and the amount of information which it is possible to draw, about hypothesis tests, from the Matlab workspaces produced through our device processing Italian firm data.

**A2.1 Null hypothesis of a time-homogeneous 1<sup>st</sup>-order chain against the alternative of a 1<sup>st</sup>-order not-stationary one for Food Industry (ATECO 2007 C10)**

Food Industry, ATECO cod. C10 (N=4486 residings firms)										
Total Sales (Revenues from Sales and Services)										
Chi Squared test										
k-th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for k-th class	72	64	64	64	72	64	64	72	72	48
Observed $\chi^2$ value for k-th class	182,414	109,201	139,699	130,057	161,696	127,082	168,699	175,767	147,724	63,377
$P(k)$	1,520e-11	0,00037	1,463e-07	2,0912e-06	7,620e-09	4,608e-06	2,267e-11	1,1774e-10	3,736e-07	0,06759
Total num. of Degrees of freedom	656									
$\chi^2$ observed total value	1405,71721265542									
Total null-hyp. Probability	1,12141421435526e-56									
Maximum Likelihood Ratio Criterion (MLRC)										
k-th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for k-th class	72	64	64	64	72	64	64	72	72	48
Observed MLR value for k-th class	137,224	105,258	131,410	121,016	157,517	134,172	172,691	174,508	138,437	55,196
$P(k)$	5,753e-06	0,0008894	1,453e-06	2,194e-05	2,510e-08	6,845e-07	6,282e-12	1,725e-10	4,233e-06	0,2212527
Total num. of Degrees of freedom	656									
MLR observed total value	1327,43005361992									
Total null-hyp. Probability	8,59833997571484e-48									

Food Industry, ATECO cod. C10 (N=4497 residings firms)										
Total Assets										
Chi Squared test										
k-th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for k-th class	64	56	56	56	56	72	72	64	40	32
Observed $\chi^2$ value for k-th class	105,644	105,608	110,818	159,676	180,540	203,398	191,168	157,280	109,359	48,681
$P(k)$	0,0008178	6,891e-05	1,791e-05	6,680e-12	5,133e-15	1,796e-14	9,587e-13	8,112e-10	2,323e-08	0,0297433
Total num. of Degrees of freedom	568									
$\chi^2$ observed total value	1372,17524595382									
Total null-hyp. Probability	2,43360688671702e-68									
Maximum Likelihood Ratio Criterion (MLRC)										
k-th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for k-th class	64	56	56	56	56	72	72	64	40	32
Observed MLR value for k-th class	81,670	99,301	105,966	161,924	172,189	169,960	185,620	137,478	103,549	39,041
$P(k)$	0,0674153	0,0003249	6,293e-05	3,145e-12	9,490e-14	6,767e-10	5,572e-12	2,736e-07	1,546e-07	0,1827724
Total num. of Degrees of freedom	568									
MLR observed total value	1256,69951943128									
Total null-hyp. Probability	4,85028945105562e-54									

Food Industry, ATECO cod. C10 (N=4497 residings firms)										
Total Debts (Total Payables)										
Chi Squared test										
<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	72	48	72	56	72	64	64	64	56	40
Observed $\chi^2$ value for <i>k</i> -th class	111,136	74,301	108,751	129,506	98,247	142,181	129,353	132,655	142,390	66,011
<i>P</i> ( <i>k</i> )	0,0021125	0,0088023	0,0033603	9,373e-08	0,0216506	7,211e-08	2,525e-06	1,037e-06	1,824e-09	0,0059464
Total num. of Degrees of freedom	608									
$\chi^2$ observed total value	1134,53254227022									
Total null-hyp. Probability	2,76002829568050e-34									
Maximum Likelihood Ratio Criterion (MLRC)										
<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	72	48	72	56	72	64	64	64	56	40
Observed MLR value for <i>k</i> -th class	91,894	73,419	100,109	135,610	97,029	142,920	126,350	125,173	133,347	50,668
<i>P</i> ( <i>k</i> )	0,0570249	0,0105553	0,0159208	1,492e-08	0,0263244	5,831e-08	5,582e-06	7,584e-06	2,968e-08	0,1202537
Total num. of Degrees of freedom	608									
MLR observed total value	1076,51735032308									
Total null-hyp. Probability	1,44148963098415e-28									

Food Industry, ATECO cod. C10 (N=4493 residings firms)

Total Production Value

Chi Squared test

<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	64	64	72	72	72	64	64	72	64	56
Observed $\chi^2$ value for <i>k</i> -th class	143,895	114,300	151,772	118,068	167,098	110,879	143,654	179,561	158,316	79,745
$P(k)$	4,400e-08	0,0001136	1,244e-07	0,0005066	1,580e-09	0,0002530	4,717e-08	3,679e-11	5,899e-10	0,0202533
Total num. of Degrees of freedom	664									
$\chi^2$ observed total value	1367,28869651755									
Total null-hyp. Probability	5,51420573045303e-51									

Maximum Likelihood Ratio Criterion (MLRC)

<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	64	64	72	72	72	64	64	72	64	56
Observed MLR value for <i>k</i> -th class	100,179	99,022	131,315	103,676	173,319	117,828	151,206	179,419	153,685	66,053
$P(k)$	0,0025901	0,0032729	2,470e-05	0,0085898	2,471e-10	4,843e-05	5,112e-09	3,844e-11	2,425e-09	0,1683955
Total num. of Degrees of freedom	664									
MLR observed total value	1275,70192203943									
Total null-hyp. Probability	4,92594220921146e-41									

## A2.2 Null hypothesis of a time-homogeneous 1<sup>st</sup>-order chain against the alternative of a time homogeneous 2<sup>nd</sup>-order one for Food Industry (ATECO 2007 C10)

Food Industry, ATECO cod. C10 (N=4486 residings firms)										
Total Sales (Revenues from Sales and Services)										
Chi Squared test										
k-th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for k-th class	81	64	72	64	81	64	72	81	72	30
Observed $\chi^2$ value for k-th class	1826,578	908,769	1944,657	1041,458	1987,612	1352,568	1529,795	1131,816	519,940	222,494
$P(k)$	0	1,442e-149	0	1,502e-176	0	1,358e-240	5,512e-272	7,669e-185	4,646e-69	2,827e-31
Total num. of Degrees of freedom	681									
$\chi^2$ observed total value	12465,6892409152									
Total null-hyp. Probability	0									
Maximum Likelihood Ratio Criterion (MLRC)										
k-th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for k-th class	81	64	72	64	81	64	72	81	72	30
Observed MLR value for k-th class	214,637	347,448	483,932	349,724	419,317	359,240	320,337	247,292	247,843	120,094
$P(k)$	4,901e-14	1,438e-40	2,513e-62	5,636e-41	4,236e-47	1,106e-42	4,918e-33	9,951e-19	3,713e-21	9,839e-13
Total num. of Degrees of freedom	681									
MLR observed total value	3109,86530536454									
Total null-hyp. Probability	8,96495832546461e-306									

Food Industry, ATECO cod. C10 (N=4497 residings firms)										
Total Assets										
Chi Squared test										
<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	56	49	42	56	49	72	81	64	20	24
Observed $\chi^2$ value for <i>k</i> -th class	7474,688	310,351	619,574	419,424	528,404	250,673	439,999	188,205	216,632	146,078
$P(k)$	0	1,156e-39	8,417e-104	4,261e-57	1,300e-81	1,336e-21	9,061e-51	3,704e-14	5,607e-35	1,767e-19
Total num. of Degrees of freedom	513									
$\chi^2$ observed total value	10594,0300877899									
Total null-hyp. Probability	0									
Maximum Likelihood Ratio Criterion (MLRC)										
<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	56	49	42	56	49	72	81	64	20	24
Observed MLR value for <i>k</i> -th class	169,661	217,652	210,683	186,300	187,469	119,564	240,388	146,693	172,012	88,591
$P(k)$	2,268e-13	4,023e-23	2,552e-24	6,646e-16	4,510e-18	0,0003668	1,040e-17	1,947e-08	3,519e-26	2,463e-09
Total num. of Degrees of freedom	513									
MLR observed total value	1739,01336239156									
Total null-hyp. Probability	6,08166169741971e-133									

Food Industry, ATECO cod. C10 (N=4497 residings firms)										
Total Debts (Total Payables)										
Chi Squared test										
<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	64	42	72	56	72	64	64	64	56	25
Observed $\chi^2$ value for <i>k</i> -th class	6048,461	506,601	545,682	550,259	759,384	514,400	523,139	525,622	348,374	793,522
$P(k)$	0	5,189e-81	6,435e-74	2,443e-82	2,567e-115	1,440e-71	3,066e-73	1,025e-73	7,845e-44	2,808e-151
Total num. of Degrees of freedom	579									
$\chi^2$ observed total value	11115,4450248406									
Total null-hyp. Probability	0									
Maximum Likelihood Ratio Criterion (MLRC)										
<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	64	42	72	56	72	64	64	64	56	25
Observed MLR value for <i>k</i> -th class	187,102	217,058	286,990	243,613	261,975	208,603	275,746	193,503	264,685	196,523
$P(k)$	5,374e-14	1,899e-25	1,889e-27	3,046e-25	2,163e-23	3,200e-17	4,374e-28	6,117e-15	7,435e-29	1,429e-28
Total num. of Degrees of freedom	579									
MLR observed total value	2335,79776743187									
Total null-hyp. Probability	5,88886111187446e-209									

Food Industry, ATECO cod. C10 (N=4493 residings firms)

Total Production Value

Chi Squared test

<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	72	72	72	81	63	72	72	72	72	42
Observed $\chi^2$ value for <i>k</i> -th class	1917,760	1980,835	1054,113	2221,693	894,438	1948,053	1394,967	984,802	614,157	249,266
<i>P</i> ( <i>k</i> )	0	0	2,415e-174	0	3,013e-147	0	4,154e-244	2,522e-160	5,361e-87	2,980e-31
Total num. of Degrees of freedom	690									
$\chi^2$ observed total value	13260,0851755110									
Total null-hyp. Probability	0									

Maximum Likelihood Ratio Criterion (MLRC)

<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	72	72	72	81	63	72	72	72	72	42
Observed MLR value for <i>k</i> -th class	178,100	437,578	443,577	385,212	370,662	393,504	320,561	263,383	313,814	137,305
<i>P</i> ( <i>k</i> )	5,766e-11	8,770e-54	7,018e-55	3,859e-41	3,927e-45	8,111e-46	4,505e-33	1,288e-23	6,281e-32	4,760e-12
Total num. of Degrees of freedom	690									
MLR observed total value	3243,69786676381									
Total null-hyp. Probability	0									

### A2.3 Null hypothesis of a time-homogeneous 1<sup>st</sup>-order chain against the alternative of a 1<sup>st</sup>-order not-stationary one for Innovative Small & Medium Enterprises (InnoSME)

Innovative Small & Medium Enterprises (InnoSME), (N=115 residings firms)										
Total Sales (Revenues from Sales and Services)										
Chi Squared test										
k-th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for k-th class	32	32	40	40	32	40	48	24	40	24
Observed $\chi^2$ value for k-th class	32,862	27,590	32,438	48,178	40,959	31,512	33,805	28,790	51,217	16,238
$P(k)$	0,4246	0,6895	0,7966	0,1756	0,1332	0,8289	0,9398	0,2281	0,1101	0,8793
Total num. of Degrees of freedom	352									
$\chi^2$ observed total value	343,588521899431									
Total null-hyp. Probability	0,615774151051117									
Maximum Likelihood Ratio Criterion (MLRC)										
k-th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for k-th class	32	32	40	40	32	40	48	24	40	24
Observed MLRC value for k-th class	19,055	31,598	28,032	46,999	45,738	30,629	34,715	26,825	42,503	14,889
$P(k)$	0,9658	0,4868	0,9228	0,2076	0,0548	0,8569	0,9245	0,3127	0,3637	0,9239
Total num. of Degrees of freedom	352									
MLRC observed total value	320,983091862259									
Total null-hyp. Probability	0,880978268094164									

Innovative Small & Medium Enterprises (InnoSME), (N=115 residings firms)										
Total Assets										
Chi Squared test										
<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	16	32	40	32	32	16	40	32	16	16
Observed $\chi^2$ value for <i>k</i> -th class	13,489	37,275	45,753	39,349	41,102	17,174	38,268	25,670	16,348	11,841
<i>P</i> ( <i>k</i> )	0,6367	0,2393	0,2456	0,1740	0,1300	0,3744	0,5484	0,7780	0,4290	0,7548
Total num. of Degrees of freedom	272									
$\chi^2$ observed total value	286,268876573403									
Total null-hyp. Probability	0,264394237272064									
Maximum Likelihood Ratio Criterion (MLRC)										
<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	16	32	40	32	32	16	40	32	16	16
Observed MLRC value for <i>k</i> -th class	9,3634	31,779	39,718	38,595	40,285	18,716	34,190	26,078	20,485	9,461
<i>P</i> ( <i>k</i> )	0,8977	0,4777	0,4828	0,1960	0,1493	0,2836	0,7285	0,7601	0,1991	0,8932
Total num. of Degrees of freedom	272									
MLRC observed total value	268,672395505715									
Total null-hyp. Probability	0,545634163366739									

Innovative Small & Medium Enterprises (InnoSME), (N=115 residings firms)										
Total Debts (Total Payables)										
Chi Squared test										
k-th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for k-th class	16	24	48	40	40	32	40	32	24	8
Observed $\chi^2$ value for k-th class	13,632	27,180	53,711	25,008	40,873	43,595	30,905	29,039	17,495	5,218
$P(k)$	0,62611	0,29611	0,26477	0,96931	0,43197	0,082989	0,84848	0,61715	0,82677	0,73406
Total num. of Degrees of freedom	304									
$\chi^2$ observed total value	286,658214806045									
Total null-hyp. Probability	0,754892580055093									
Maximum Likelihood Ratio Criterion (MLRC)										
k-th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for k-th class	16	24	48	40	40	32	40	32	24	8
Observed MLRC value for k-th class	15,615	32,164	56,604	23,082	42,413	37,896	31,514	28,607	17,204	6,851
$P(k)$	0,4801	0,12297	0,1847	0,9852	0,3674	0,2182	0,8288	0,6390	0,8398	0,5527
Total num. of Degrees of freedom	304									
MLRC observed total value	291,951112417279									
Total null-hyp. Probability	0,680152272998658									

Innovative Small & Medium Enterprises (InnoSME), (N=115 residings firms)										
Total Production Value										
Chi Squared test										
<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	24	40	24	32	32	40	32	32	32	16
Observed $\chi^2$ value for <i>k</i> -th class	18,723	44,560	28,196	35,592	36,632	38,854	33,804	47,439	48,643	12,219
<i>P</i> ( <i>k</i> )	0,7666	0,2858	0,2518	0,3029	0,2625	0,5217	0,3804	0,03871	0,02998	0,7287
Total num. of Degrees of freedom	304									
$\chi^2$ observed total value	344,663339557483									
Total null-hyp. Probability	0,0539623272157999									
Maximum Likelihood Ratio Criterion (MLRC)										
<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	24	40	24	32	32	40	32	32	32	16
Observed MLRC value for <i>k</i> -th class	12,728	45,230	31,940	36,802	37,807	33,109	40,478	46,584	39,149	15,570
<i>P</i> ( <i>k</i> )	0,9704	0,2627	0,1285	0,2562	0,2211	0,7715	0,1445	0,04618	0,1797	0,4833
Total num. of Degrees of freedom	304									
MLRC observed total value	339,398300603004									
Total null-hyp. Probability	0,0792460771501770									

## A2.4 Null hypothesis of a time-homogeneous 1<sup>st</sup>-order chain against the alternative of a time homogeneous 2<sup>nd</sup>-order one for Innovative Small & Medium Enterprises (InnoSME)

Innovative Small & Medium Enterprises (InnoSME), (N=115 residings firms)										
Total Sales (Revenues from Sales and Services)										
Chi Squared test										
k-th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for k-th class	8	16	20	20	20	30	42	15	20	6
Observed $\chi^2$ value for k-th class	114,869	164,152	19,970	48,133	47,369	59,981	161,804	18,729	54,695	28,558
$P(k)$	3,791e-21	1,231e-26	0,45977	0,000407	0,000521	0,000925	5,7385e-16	0,22629	4,560e-05	7,373e-05
Total num. of Degrees of freedom	197									
$\chi^2$ observed total value	718,261814779685									
Total null-hyp. Probability	2,12483605044216e-60									
Maximum Likelihood Ratio Criterion (MLRC)										
k-th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for k-th class	8	16	20	20	20	30	42	15	20	6
Observed MLRC value for k-th class	28,014	34,205	21,876	36,409	28,190	37,873	54,268	16,662	27,209	18,291
$P(k)$	0,000471	0,005097	0,347265	0,013761	0,104986	0,153026	0,097160	0,339432	0,129483	0,005543
Total num. of Degrees of freedom	197									
MLRC observed total value	302,999396981637									
Total null-hyp. Probability	1,78407256399169e-06									

Innovative Small & Medium Enterprises (InnoSME), (N=115 residings firms)										
Total Assets										
Chi Squared test										
<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	2	8	20	16	16	10	25	12	6	4
Observed $\chi^2$ value for <i>k</i> -th class	4,395	9,797	37,617	29,269	25,004	16,041	17,364	13,603	10,704	0,416
<i>P</i> ( <i>k</i> )	0,1110	0,2795	0,009858	0,02218	0,06974	0,09846	0,8682	0,3268	0,09796	0,9811
Total num. of Degrees of freedom	119									
$\chi^2$ observed total value	164,212080872291									
Total null-hyp. Probability	0,00382815611702117									
Maximum Likelihood Ratio Criterion (MLRC)										
<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	2	8	20	16	16	10	25	12	6	4
Observed MLRC value for <i>k</i> -th class	3,330	15,064	25,156	21,160	20,012	16,257	14,052	13,774	10,652	0,768
<i>P</i> ( <i>k</i> )	0,1892	0,0579	0,1955	0,1724	0,2196	0,0925	0,9608	0,3154	0,0997	0,9426
Total num. of Degrees of freedom	119									
MLRC observed total value	140,226579702595									
Total null-hyp. Probability	0,0894240788779651									

Innovative Small & Medium Enterprises (InnoSME), (N=115 residings firms)										
Total Debts (Total Payables)										
Chi Squared test										
k-th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for k-th class	4	9	24	25	20	20	30	12	12	2
Observed $\chi^2$ value for k-th class	12,084	12,551	61,833	46,498	36,346	41,450	29,130	6,916	8,951	19,266
$P(k)$	0,01674	0,18401	3,504e-05	0,005615	0,01400	0,003259	0,5107	0,8631	0,7071	6,552e-05
Total num. of Degrees of freedom	158									
$\chi^2$ observed total value	275,026260741549									
Total null-hyp. Probability	2,34716442385896e-08									
Maximum Likelihood Ratio Criterion (MLRC)										
k-th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for k-th class	4	9	24	25	20	20	30	12	12	2
Observed MLRC value for k-th class	10,553	10,323	43,686	30,049	29,035	36,852	25,016	8,682	9,072	12,461
$P(k)$	0,03207	0,32500	0,008297	0,22245	0,08706	0,01219	0,7243	0,7298	0,6968	0,001968
Total num. of Degrees of freedom	158									
MLRC observed total value	215,729109868958									
Total null-hyp. Probability	0,00155841795348171									

Innovative Small & Medium Enterprises (InnoSME), (N=115 residings firms)										
Total Production Value										
Chi Squared test										
<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	2	12	12	16	20	30	20	16	16	4
Observed $\chi^2$ value for <i>k</i> -th class	3,412	28,846	43,620	19,505	35,142	73,701	26,701	19,858	50,870	29,521
<i>P</i> ( <i>k</i> )	0,1816	0,004153	1,772e-05	0,2433	0,01936	1,524e-05	0,1439	0,2267	1,664e-05	6,126e-06
Total num. of Degrees of freedom	148									
$\chi^2$ observed total value	331,176835902855									
Total null-hyp. Probability	4,71687897813637e-16									
Maximum Likelihood Ratio Criterion (MLRC)										
<i>k</i> -th Interdecile class (Markov state)	1	2	3	4	5	6	7	8	9	10
Degrees of freedom for <i>k</i> -th class	2	12	12	16	20	30	20	16	16	4
Observed MLRC value for <i>k</i> -th class	2,537	19,624	26,680	17,125	18,132	36,272	27,532	18,392	20,480	19,943
<i>P</i> ( <i>k</i> )	0,2812	0,07453	0,008589	0,3775	0,5787	0,1993	0,1209	0,3014	0,1994	0,0005124
Total num. of Degrees of freedom	148									
MLRC observed total value	206,718510832678									
Total null-hyp. Probability	0,00102996417282318									



## Appendix A3

In the last appendix are tabulated all the values of the nine transition-matrix based mobility indices implemented in '*MarkovInfer\_MobInd.m*'. They are computed through both the 1<sup>st</sup>-order Markov Chains (the stationary and the not-stationary ones), in the mono-period (or date-by-date, or one-step) version, as well as in the aggregated-in-time compound version, for every one of the 28 combinations.

However, mobility values for Italian Innovative Start-ups have not any scientific trustworthiness, due to the too much exiguous number of permanent firms in the database for the epoch 2010-2015: they exclusively provide an instance of misleading application about inferential procedures for Markov Chains.

Instead, the other tables must be considered an example of the scientific outputs which can be extracted from the workspace produced by our tool and could be subjected to further examinations.

### A3.1 Mono-period (or one-step, or date-by-date) mobility measures for both the 1<sup>st</sup>-order Markov Chains

ATECO C10 Food and Beverages: Total Sales										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,01095	0,01024	-0,00204	0,00404	0,00278	-0,00126	-0,00537	-0,00387	-0,00566	0,00108
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,01913	0,01860	0,01610	0,01568	0,01537	0,01487	0,01602	0,01559	0,01486	0,01571
Trace Index (I3)	0,26468	0,26030	0,23440	0,22603	0,22205	0,21470	0,21727	0,21582	0,20496	0,22888
Gener. Determinant Index (b=1,5) (I4)	0,98802	0,98808	0,98012	0,97640	0,97395	0,97134	0,97191	0,97083	0,96459	0,97769
Predictability Index (I5)	5,82558	5,87883	6,12043	6,27990	6,37209	6,41492	6,37473	6,37412	6,54922	6,21790
2nd Index of Bartholomew (I6)	3,13156	3,03437	2,66332	2,59612	2,48478	2,50197	2,65029	2,60208	2,47187	2,68465
2nd Eigenvalue Index (I7)	0,04486	0,03125	0,02954	0,01830	0,01737	0,01460	0,01558	0,01588	0,01295	0,01807
All-Eigenvalues Index (I8)	0,26426	0,26028	0,23440	0,22603	0,22205	0,21470	0,21727	0,21582	0,20496	0,22888
Asymptotic-half-life Index (I9) [**]	2,764E-07	3,295E-10	9,170E-11	5,049E-17	6,616E-18	3,387E-21	6,774E-20	1,559E-19	8,002E-24	3,083E-17

ATECO C10 Food and Beverages: Total Assets										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,00795	0,01355	0,00242	0,00606	0,00588	-0,00041	-0,00134	0,00078	0,00110	0,00406
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,01578	0,01992	0,01301	0,01433	0,01348	0,01215	0,01219	0,01207	0,01286	0,01371
Trace Index (I3)	0,23425	0,28459	0,19959	0,21510	0,20580	0,18764	0,18262	0,18148	0,19650	0,21042
Gener. Determinant Index (b=1,5) (I4)	0,97926	0,99268	0,95928	0,96935	0,96412	0,94967	0,94432	0,94332	0,95755	0,96711
Predictability Index (I5)	6,19617	5,62048	6,59628	6,45833	6,59160	6,77277	6,84045	6,86625	6,67416	6,46764
2nd Index of Bartholomew (I6)	2,56728	3,26251	2,12001	2,31354	2,11737	1,98382	1,98406	1,97900	2,09440	2,28038
2nd Eigenvalue Index (I7)	0,03511	0,05125	0,01937	0,03004	0,02485	0,01020	0,01439	0,00994	0,00725	0,01925
All-Eigenvalues Index (I8)	0,23425	0,28459	0,19959	0,21507	0,20580	0,18764	0,18262	0,18148	0,19650	0,21042
Asymptotic-half-life Index (I9) [**]	3,785E-09	1,897E-06	4,053E-16	1,349E-10	1,085E-12	4,308E-30	1,706E-21	7,196E-31	4,423E-42	3,263E-16

[\*]: 1-step MI for S-TH-MCh  $\equiv$  mono-periodal value of the corresponding mobility index according to the Simple (1<sup>st</sup>-order), Time Homogeneous, Markov Chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C10 Food and Beverages: Total Debts										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,01144	0,00391	0,00181	0,00712	0,00753	-0,00250	-0,00224	0,00035	-0,00207	0,00290
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02281	0,02011	0,01918	0,02100	0,01932	0,01829	0,01821	0,01859	0,01786	0,01915
Trace Index (I3)	0,33250	0,29880	0,29057	0,31230	0,29076	0,27973	0,27288	0,27339	0,26926	0,29144
Gener. Determinant Index (b=1,5) (I4)	0,99786	0,99542	0,99440	0,99674	0,99440	0,99318	0,99177	0,99169	0,99183	0,99471
Predictability Index (I5)	5,01699	5,32872	5,41491	5,26104	5,50299	5,59559	5,65822	5,63677	5,74930	5,43052
2nd Index of Bartholomew (I6)	3,82410	3,44670	3,22203	3,52702	3,22523	3,05758	3,04514	3,16725	3,02364	3,28959
2nd Eigenvalue Index (I7)	0,04184	0,02328	0,01967	0,02931	0,02804	0,01594	0,02269	0,01853	0,01137	0,01895
All-Eigenvalues Index (I8)	0,33250	0,29880	0,29057	0,31230	0,29076	0,27973	0,27288	0,27339	0,26926	0,29144
Asymptotic-half-life Index (I9) [**]	9,067E-08	1,667E-13	7,024E-16	7,571E-11	2,614E-11	1,846E-19	7,619E-14	8,084E-17	4,763E-27	1,833E-16

ATECO C10 Food and Beverages: Total Production Monetary Value										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,01128	0,00879	-0,00188	0,00412	0,00274	-0,00144	-0,00485	-0,00358	-0,00416	0,00126
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,01947	0,01778	0,01666	0,01612	0,01499	0,01492	0,01590	0,01551	0,01472	0,01574
Trace Index (I3)	0,27253	0,24767	0,24478	0,23836	0,21832	0,21696	0,21906	0,21794	0,21052	0,23143
Gener. Determinant Index (b=1,5) (I4)	0,98973	0,98415	0,98366	0,98192	0,97188	0,97232	0,97231	0,97167	0,96802	0,97860
Predictability Index (I5)	5,72322	6,00549	5,97251	6,15897	6,40106	6,37617	6,32682	6,33244	6,46689	6,17444
2nd Index of Bartholomew (I6)	3,19346	2,90349	2,74313	2,67090	2,44587	2,50616	2,65921	2,63012	2,44268	2,69440
2nd Eigenvalue Index (I7)	0,04714	0,02672	0,03463	0,01808	0,01802	0,01651	0,01501	0,01554	0,01284	0,01754
All-Eigenvalues Index (I8)	0,27218	0,24767	0,24478	0,23836	0,21831	0,21696	0,21906	0,21794	0,21052	0,23143
Asymptotic-half-life Index (I9) [**]	5,826E-07	7,672E-12	2,876E-09	3,192E-17	2,777E-17	8,346E-19	1,247E-20	5,937E-20	5,122E-24	9,753E-18

[\*]: 1-step MI for S-TH-MCh  $\equiv$  mono-periodal value of the corresponding mobility index according to the Simple (1<sup>st</sup>-order), Time Homogeneous, Markov Chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C13 Textiles: Total Sales										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,00461	-0,00654	-0,02573	0,00649	0,00189	-0,01174	-0,00350	-0,00648	-0,00920	-0,00601
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02137	0,01992	0,02798	0,02584	0,01806	0,02081	0,01878	0,01734	0,01777	0,01964
Trace Index (I3)	0,28627	0,27159	0,38195	0,33988	0,26298	0,28954	0,25942	0,23907	0,24725	0,28523
Gener. Determinant Index (b=1,5) (I4)	0,99385	0,99031	0,99938	0,99813	0,98941	0,99440	0,98843	0,98301	0,98557	0,99407
Predictability Index (I5)	5,48062	5,64973	4,88592	5,10916	5,83981	5,54521	5,79813	6,12262	5,98607	5,50177
2nd Index of Bartholomew (I6)	3,55149	3,20443	4,45910	4,18127	2,99873	3,36084	3,16626	2,87577	2,90915	3,38814
2nd Eigenvalue Index (I7)	0,02800	0,05833	0,11359	0,04012	0,01963	0,05300	0,01966	0,00744	0,02176	0,02618
All-Eigenvalues Index (I8)	0,28627	0,27159	0,38195	0,33932	0,26298	0,28953	0,25942	0,23907	0,24725	0,28523
Asymptotic-half-life Index (I9) [**]	2,512E-11	9,797E-06	3,187E-03	4,446E-08	6,595E-16	2,962E-06	6,898E-16	4,781E-41	2,083E-14	4,504E-12

ATECO C13 Textiles: Total Assets										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,00336	0,00814	-0,00487	0,00777	0,00077	-0,00469	-0,00286	-0,00246	-0,00300	0,00024
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,01817	0,02365	0,01699	0,01749	0,01528	0,01437	0,01500	0,01510	0,01419	0,01622
Trace Index (I3)	0,27401	0,34626	0,25142	0,25761	0,22939	0,21778	0,22564	0,22183	0,21009	0,24836
Gener. Determinant Index (b=1,5) (I4)	0,99186	0,99868	0,98596	0,98759	0,97813	0,97226	0,97668	0,97459	0,96787	0,98537
Predictability Index (I5)	5,65450	4,79461	5,92505	5,91180	6,19673	6,36462	6,25610	6,30778	6,46889	5,93709
2nd Index of Bartholomew (I6)	2,97132	3,97646	2,77978	2,86812	2,48977	2,36105	2,50296	2,53568	2,38700	2,76605
2nd Eigenvalue Index (I7)	0,02515	0,03272	0,03752	0,03565	0,02010	0,02342	0,01215	0,00967	0,01086	0,01768
All-Eigenvalues Index (I8)	0,27401	0,34626	0,25142	0,25761	0,22939	0,21778	0,22564	0,22183	0,21009	0,24836
Asymptotic-half-life Index (I9) [**]	1,519E-12	8,956E-10	1,344E-08	5,108E-09	1,491E-15	1,987E-13	2,326E-25	1,056E-31	2,773E-28	1,340E-17

[\*]: 1-step MI for S-TH-MCh  $\equiv$  mono-periodal value of the corresponding mobility index according to the Simple (1<sup>st</sup>-order), Time Homogeneous, Markov Chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C13 Textiles: Total Debts										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,00210	-0,00564	-0,00839	0,01486	0,00126	-0,00662	-0,00013	-0,00298	-0,00289	-0,00093
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02448	0,02521	0,02463	0,02555	0,02171	0,02075	0,01898	0,02042	0,02115	0,02194
Trace Index (I3)	0,36125	0,37722	0,35382	0,37005	0,32269	0,31244	0,29034	0,29332	0,30428	0,33146
Gener. Determinant Index (b=1,5) (I4)	0,99912	0,99954	0,99886	0,99930	0,99749	0,99703	0,99472	0,99505	0,99633	0,99813
Predictability Index (I5)	4,62601	4,53920	4,72303	4,70881	5,03912	5,24150	5,44442	5,41384	5,28488	4,95595
2nd Index of Bartholomew (I6)	4,18845	4,23594	4,08090	4,28885	3,67466	3,42597	3,16919	3,42989	3,58707	3,78992
2nd Eigenvalue Index (I7)	0,03093	0,04004	0,05173	0,05509	0,02611	0,03859	0,02039	0,02328	0,02441	0,02620
All-Eigenvalues Index (I8)	0,36125	0,37722	0,35382	0,37005	0,32269	0,31244	0,29034	0,29332	0,30428	0,33146
Asymptotic-half-life Index (I9) [**]	2,626E-10	4,307E-08	2,150E-06	4,869E-06	4,173E-12	2,245E-08	2,453E-15	1,661E-13	6,593E-13	4,603E-12

ATECO C13 Textiles: Total Production Monetary Value										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,00461	-0,00448	-0,02492	0,00922	0,00090	-0,01098	-0,00350	-0,00377	-0,01009	-0,00535
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02156	0,02146	0,02824	0,02456	0,01820	0,02038	0,01903	0,01736	0,01857	0,01980
Trace Index (I3)	0,30601	0,29593	0,39016	0,34311	0,26620	0,30299	0,26985	0,25335	0,26325	0,29051
Gener. Determinant Index (b=1,5) (I4)	0,99571	0,99390	0,99943	0,99807	0,98977	0,99550	0,99063	0,98719	0,98915	0,99452
Predictability Index (I5)	5,25078	5,30115	4,67663	5,00713	5,75525	5,30375	5,64679	5,90916	5,72829	5,41357
2nd Index of Bartholomew (I6)	3,61156	3,51614	4,50166	4,04546	3,05268	3,36699	3,21038	2,90743	3,10257	3,43513
2nd Eigenvalue Index (I7)	0,02952	0,06917	0,14915	0,05051	0,02772	0,05257	0,02844	0,02026	0,03142	0,02683
All-Eigenvalues Index (I8)	0,30601	0,29593	0,39016	0,34191	0,26620	0,30299	0,26985	0,25335	0,26325	0,29051
Asymptotic-half-life Index (I9) [**]	8,995E-11	6,318E-05	1,368E-02	1,554E-06	1,950E-11	2,667E-06	3,697E-11	1,948E-15	3,712E-10	8,557E-12

[\*]: 1-step MI for S-TH-MCh  $\equiv$  mono-periodal value of the corresponding mobility index according to the Simple (1<sup>st</sup>-order), Time Homogeneous, Markov Chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C20 Chemicals: Total Sales										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,00813	0,00357	-0,00815	0,00404	0,00720	-0,00728	-0,00697	-0,00029	-0,00389	-0,00041
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02020	0,01745	0,02398	0,02223	0,01717	0,01553	0,01650	0,01485	0,01538	0,01716
Trace Index (I3)	0,25972	0,24772	0,33223	0,29839	0,23026	0,22720	0,22212	0,21170	0,21402	0,24967
Gener. Determinant Index (b=1,5) (I4)	0,98804	0,98525	0,99767	0,99439	0,97856	0,97849	0,97488	0,96989	0,97166	0,98619
Predictability Index (I5)	5,92345	5,97525	5,09192	5,52515	6,29474	6,29248	6,31954	6,48676	6,46422	5,95293
2nd Index of Bartholomew (I6)	3,13549	2,81436	3,74311	3,45883	2,70293	2,53884	2,67354	2,44217	2,49971	2,89413
2nd Eigenvalue Index (I7)	0,06520	0,02090	0,09698	0,05157	0,02664	0,02046	0,01125	0,01169	0,00685	0,01951
All-Eigenvalues Index (I8)	0,25918	0,24772	0,33154	0,29805	0,23022	0,22720	0,22212	0,21170	0,21402	0,24967
Asymptotic-half-life Index (I9) [**]	3,427E-05	5,614E-15	1,120E-03	2,065E-06	7,070E-12	2,753E-15	2,417E-27	2,476E-26	1,546E-44	5,242E-16

ATECO C20 Chemicals: Total Assets										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,00602	0,01244	0,00142	0,00632	0,00489	-0,00216	0,00007	0,00267	0,00140	0,00375
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,01676	0,02059	0,01547	0,01485	0,01387	0,01179	0,01275	0,01311	0,01289	0,01429
Trace Index (I3)	0,23782	0,30171	0,22551	0,22294	0,21069	0,17651	0,19555	0,19968	0,19912	0,21933
Gener. Determinant Index (b=1,5) (I4)	0,98223	0,99579	0,97570	0,97449	0,96782	0,93824	0,95842	0,96189	0,96071	0,97329
Predictability Index (I5)	6,22277	5,44664	6,24309	6,37597	6,49429	6,94061	6,70114	6,65243	6,64551	6,36253
2nd Index of Bartholomew (I6)	2,62760	3,33440	2,50394	2,34260	2,25184	1,89836	2,08538	2,10065	2,08877	2,36695
2nd Eigenvalue Index (I7)	0,02545	0,05935	0,02458	0,03177	0,02122	0,01690	0,01329	0,00960	0,01494	0,02130
All-Eigenvalues Index (I8)	0,23758	0,30171	0,22551	0,22294	0,21069	0,17651	0,19555	0,19968	0,19912	0,21933
Asymptotic-half-life Index (I9) [**]	2,096E-12	1,201E-05	7,977E-13	4,749E-10	9,290E-15	2,178E-18	3,119E-23	6,023E-32	9,940E-21	1,038E-14

[\*]: 1-step MI for S-TH-MCh  $\equiv$  mono-periodal value of the corresponding mobility index according to the Simple (1<sup>st</sup>-order), Time Homogeneous, Markov Chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C20 Chemicals: Total Debts										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,00625	0,00172	-0,00296	0,00568	0,00489	-0,00339	-0,00256	0,00066	-0,00206	0,00099
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02138	0,02139	0,02249	0,01997	0,01979	0,01872	0,01795	0,01782	0,01884	0,01928
Trace Index (I3)	0,31170	0,32195	0,32409	0,29822	0,29356	0,28014	0,27038	0,26201	0,28009	0,29402
Gener. Determinant Index (b=1,5) (I4)	0,99662	0,99800	0,99779	0,99577	0,99499	0,99295	0,99112	0,98920	0,99348	0,99510
Predictability Index (I5)	5,22010	5,15309	5,06485	5,44932	5,43504	5,57084	5,68095	5,77361	5,59683	5,39223
2nd Index of Bartholomew (I6)	3,48075	3,58331	3,68287	3,29974	3,29363	3,06070	2,95739	2,97564	3,15939	3,28380
2nd Eigenvalue Index (I7)	0,02841	0,02028	0,04215	0,02520	0,02616	0,01686	0,01722	0,01622	0,01758	0,02093
All-Eigenvalues Index (I8)	0,31170	0,32195	0,32409	0,29822	0,29356	0,28014	0,27038	0,26201	0,28009	0,29402
Asymptotic-half-life Index (I9) [**]	3,583E-11	2,013E-15	1,021E-07	1,604E-12	4,401E-12	1,991E-18	4,640E-18	3,938E-19	1,064E-17	5,908E-15

ATECO C20 Chemicals: Total Production Monetary Value										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,00912	0,00535	-0,01262	0,00602	0,00616	-0,00763	-0,00702	-0,00034	-0,00336	-0,00039
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,01992	0,01715	0,02367	0,02308	0,01723	0,01691	0,01676	0,01524	0,01471	0,01739
Trace Index (I3)	0,26345	0,24661	0,33219	0,31674	0,24170	0,24833	0,22813	0,21780	0,21075	0,25664
Gener. Determinant Index (b=1,5) (I4)	0,98841	0,98480	0,99764	0,99662	0,98349	0,98554	0,97800	0,97303	0,96912	0,98811
Predictability Index (I5)	5,84742	5,98958	5,10436	5,39478	6,14695	5,98021	6,23264	6,39892	6,49340	5,86208
2nd Index of Bartholomew (I6)	3,14486	2,78591	3,76003	3,60752	2,79517	2,77341	2,73199	2,50937	2,42702	2,95677
2nd Eigenvalue Index (I7)	0,06320	0,02113	0,09069	0,06291	0,02243	0,02205	0,01367	0,01120	0,00717	0,01835
All-Eigenvalues Index (I8)	0,26283	0,24661	0,33156	0,31493	0,24169	0,24833	0,22813	0,21780	0,21075	0,25664
Asymptotic-half-life Index (I9) [**]	2,448E-05	7,992E-15	6,815E-04	2,329E-05	5,342E-14	3,165E-14	1,372E-22	1,856E-27	1,567E-42	5,547E-17

[\*]: 1-step MI for S-TH-MCh  $\equiv$  mono-periodal value of the corresponding mobility index according to the Simple (1<sup>st</sup>-order), Time Homogeneous, Markov Chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C22 Rubbers & Plastics: Total Sales										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,01173	0,00034	-0,02774	0,01498	0,00358	-0,01001	-0,00509	-0,00212	-0,00404	-0,00217
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02138	0,01836	0,03422	0,02690	0,01849	0,01872	0,01726	0,01751	0,01745	0,02001
Trace Index (I3)	0,30489	0,26963	0,45327	0,37898	0,26581	0,26715	0,24119	0,25217	0,24106	0,29767
Gener. Determinant Index (b=1,5) (I4)	0,99540	0,99122	0,99990	0,99931	0,98997	0,99050	0,98353	0,98716	0,98345	0,99565
Predictability Index (I5)	5,36621	5,69074	4,43263	4,90808	5,85781	5,81671	6,07328	5,94487	6,09539	5,36805
2nd Index of Bartholomew (I6)	3,44851	3,04430	5,36476	4,26796	2,95602	3,05675	2,87799	2,93164	2,87734	3,42774
2nd Eigenvalue Index (I7)	0,07408	0,01874	0,16908	0,08256	0,03941	0,03404	0,01208	0,01430	0,01334	0,01966
All-Eigenvalues Index (I8)	0,30453	0,26963	0,44709	0,37686	0,26581	0,26715	0,24119	0,25217	0,24106	0,29767
Asymptotic-half-life Index (I9) [**]	1,227E-04	1,221E-16	2,370E-02	3,210E-04	3,253E-08	2,037E-09	1,682E-25	1,261E-21	3,838E-23	6,846E-16

ATECO C22 Rubbers & Plastics: Total Assets										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,00755	0,00963	-0,00308	0,00979	0,00216	-0,00083	-0,00025	0,00180	0,00205	0,00323
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,01660	0,02105	0,01649	0,01880	0,01372	0,01373	0,01303	0,01353	0,01414	0,01533
Trace Index (I3)	0,25139	0,31262	0,25317	0,27837	0,21165	0,20994	0,19563	0,20896	0,21525	0,23789
Gener. Determinant Index (b=1,5) (I4)	0,98592	0,99679	0,98715	0,99203	0,96874	0,96817	0,95834	0,96722	0,97109	0,98196
Predictability Index (I5)	5,98891	5,22854	5,91454	5,75369	6,45679	6,47275	6,69770	6,49176	6,43070	6,10620
2nd Index of Bartholomew (I6)	2,65325	3,51187	2,69234	2,95109	2,23700	2,26449	2,14265	2,21796	2,29572	2,55758
2nd Eigenvalue Index (I7)	0,03835	0,03007	0,02450	0,06370	0,01648	0,01405	0,00905	0,01392	0,01177	0,01777
All-Eigenvalues Index (I8)	0,25139	0,31262	0,25317	0,27836	0,21165	0,20994	0,19563	0,20896	0,21525	0,23789
Asymptotic-half-life Index (I9) [**]	2,005E-08	1,377E-10	7,334E-13	2,668E-05	7,638E-19	5,228E-22	7,875E-34	3,315E-22	3,722E-26	1,627E-17

[\*]: 1-step MI for S-TH-MCh  $\equiv$  mono-periodal value of the corresponding mobility index according to the Simple (1<sup>st</sup>-order), Time Homogeneous, Markov Chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C22 Rubbers & Plastics: Total Debts										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,00817	-0,00272	-0,00657	0,01305	0,00160	-0,00312	-0,00080	0,00073	0,00155	0,00137
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02276	0,02425	0,02416	0,02452	0,02056	0,02045	0,01800	0,01968	0,02003	0,02109
Trace Index (I3)	0,33973	0,35751	0,35573	0,36027	0,31485	0,30807	0,27192	0,29363	0,30058	0,32308
Gener. Determinant Index (b=1,5) (I4)	0,99861	0,99926	0,99900	0,99910	0,99718	0,99661	0,99158	0,99500	0,99588	0,99778
Predictability Index (I5)	4,99690	4,78817	4,74380	4,84657	5,18431	5,25966	5,66185	5,39805	5,33082	5,07353
2nd Index of Bartholomew (I6)	3,75559	4,06992	4,01434	4,02525	3,44028	3,37600	2,99585	3,29490	3,34058	3,59828
2nd Eigenvalue Index (I7)	0,04243	0,02902	0,05068	0,06362	0,01638	0,02948	0,01602	0,02075	0,01865	0,02108
All-Eigenvalues Index (I8)	0,33973	0,35751	0,35573	0,36027	0,31485	0,30807	0,27192	0,29363	0,30058	0,32308
Asymptotic-half-life Index (I9) [**]	1,141E-07	6,008E-11	1,627E-06	2,632E-05	5,928E-19	8,722E-11	2,275E-19	4,429E-15	1,014E-16	7,471E-15

ATECO C22 Rubbers & Plastics: Total Production Monetary Value										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,01268	-0,00019	-0,02856	0,01685	0,00388	-0,00959	-0,00434	-0,00263	-0,00178	-0,00161
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02116	0,01779	0,03381	0,02720	0,01859	0,01844	0,01690	0,01720	0,01677	0,01977
Trace Index (I3)	0,30358	0,26595	0,45343	0,38250	0,27073	0,26549	0,23772	0,25096	0,24010	0,29674
Gener. Determinant Index (b=1,5) (I4)	0,99512	0,99078	0,99990	0,99928	0,99105	0,99029	0,98255	0,98673	0,98293	0,99559
Predictability Index (I5)	5,39336	5,74786	4,43289	4,89972	5,78326	5,83969	6,12844	5,95387	6,10899	5,38377
2nd Index of Bartholomew (I6)	3,39145	2,93508	5,32084	4,28659	2,97622	3,00644	2,77269	2,88631	2,80973	3,38468
2nd Eigenvalue Index (I7)	0,07573	0,01900	0,15922	0,11899	0,05132	0,03597	0,01243	0,01592	0,01563	0,01946
All-Eigenvalues Index (I8)	0,30346	0,26595	0,45029	0,37951	0,27073	0,26549	0,23772	0,25096	0,24010	0,29674
Asymptotic-half-life Index (I9) [**]	1,505E-04	2,039E-16	1,837E-02	4,206E-03	1,935E-06	6,057E-09	8,722E-25	1,762E-19	7,861E-20	4,777E-16

[\*]: 1-step MI for S-TH-MCh  $\equiv$  mono-periodal value of the corresponding mobility index according to the Simple (1<sup>st</sup>-order), Time Homogeneous, Markov Chain

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ATECO C28 Machinery & Equipment: Total Sales										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,02201	0,00614	-0,03768	0,00950	0,00749	-0,00573	-0,00425	0,00173	-0,00256	-0,00063
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02960	0,02417	0,04293	0,03006	0,02670	0,02443	0,02415	0,02325	0,02446	0,02687
Trace Index (I3)	0,39766	0,35223	0,54854	0,42268	0,38249	0,35393	0,34102	0,33578	0,33860	0,38601
Gener. Determinant Index (b=1,5) (I4)	0,99964	0,99893	1,00000	0,99987	0,99957	0,99905	0,99856	0,99832	0,99844	0,99963
Predictability Index (I5)	4,43562	4,74251	3,59704	4,14611	4,54005	4,75148	4,84429	4,95471	4,92853	4,38298
2nd Index of Bartholomew (I6)	5,02273	4,18777	6,98569	5,16770	4,58529	4,26384	4,20997	4,04022	4,18398	4,73880
2nd Eigenvalue Index (I7)	0,07276	0,02859	0,22044	0,05827	0,04467	0,02768	0,03085	0,02753	0,03038	0,03461
All-Eigenvalues Index (I8)	0,39764	0,35223	0,54810	0,42268	0,38249	0,35393	0,34102	0,33578	0,33860	0,38601
Asymptotic-half-life Index (I9) [**]	1,036E-04	4,186E-11	6,182E-02	9,690E-06	2,591E-07	1,894E-11	2,476E-10	1,644E-11	1,743E-10	2,836E-09

ATECO C28 Machinery & Equipment: Total Assets										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,01287	0,01213	-0,00776	0,00972	0,00393	-0,00099	0,00138	0,00386	0,00247	0,00425
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02256	0,02453	0,02209	0,02016	0,01824	0,01714	0,01713	0,01732	0,01741	0,01932
Trace Index (I3)	0,32831	0,36141	0,32727	0,30110	0,27816	0,26360	0,26230	0,26326	0,26511	0,29523
Gener. Determinant Index (b=1,5) (I4)	0,99772	0,99920	0,99772	0,99565	0,99315	0,99036	0,98988	0,98970	0,99056	0,99532
Predictability Index (I5)	5,09440	4,75162	5,08333	5,41087	5,63959	5,78873	5,80644	5,79416	5,80322	5,40728
2nd Index of Bartholomew (I6)	3,77212	4,14493	3,68395	3,33980	3,09025	2,90007	2,87897	2,89782	2,93169	3,30422
2nd Eigenvalue Index (I7)	0,05208	0,04255	0,05138	0,04251	0,01910	0,01657	0,01556	0,02088	0,01614	0,02179
All-Eigenvalues Index (I8)	0,32831	0,36141	0,32727	0,30110	0,27816	0,26360	0,26230	0,26326	0,26511	0,29523
Asymptotic-half-life Index (I9) [**]	2,354E-06	1,195E-07	1,962E-06	1,176E-07	2,475E-16	9,620E-19	6,290E-20	5,440E-15	3,164E-19	2,176E-14

[\*]: 1-step MI for S-TH-MCh  $\equiv$  mono-periodal value of the corresponding mobility index according to the Simple (1<sup>st</sup>-order), Time Homogeneous, Markov Chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C28 Machinery & Equipment: Total Debts										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,01370	0,00022	-0,01663	0,01302	0,00547	-0,00535	0,00046	0,00255	0,00265	0,00179
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02835	0,02721	0,03246	0,02863	0,02539	0,02519	0,02412	0,02419	0,02416	0,02623
Trace Index (I3)	0,40617	0,40037	0,45254	0,40614	0,37440	0,37197	0,35571	0,35588	0,35470	0,38684
Gener. Determinant Index (b=1,5) (I4)	0,99980	0,99979	0,99996	0,99979	0,99949	0,99948	0,99909	0,99916	0,99911	0,99967
Predictability Index (I5)	4,27653	4,27920	3,93653	4,27748	4,56046	4,58889	4,70922	4,74222	4,74651	4,39820
2nd Index of Bartholomew (I6)	4,87113	4,72550	5,48923	4,94557	4,41330	4,35681	4,16704	4,19043	4,18742	4,60063
2nd Eigenvalue Index (I7)	0,04924	0,02800	0,10262	0,05046	0,02432	0,03594	0,02787	0,02727	0,02745	0,03001
All-Eigenvalues Index (I8)	0,40617	0,40037	0,45254	0,40614	0,37440	0,37197	0,35571	0,35588	0,35470	0,38684
Asymptotic-half-life Index (I9) [**]	1,094E-06	2,516E-11	1,658E-03	1,537E-06	5,965E-13	5,955E-09	2,238E-11	1,292E-11	1,533E-11	1,323E-10

ATECO C28 Machinery & Equipment: Total Production Monetary Value										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,02108	0,00517	-0,03655	0,00921	0,00811	-0,00663	-0,00292	0,00128	-0,00059	-0,00049
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02779	0,02273	0,04103	0,02812	0,02429	0,02238	0,02203	0,02086	0,02212	0,02482
Trace Index (I3)	0,37886	0,33886	0,53478	0,40574	0,35763	0,33120	0,32480	0,31150	0,32196	0,36615
Gener. Determinant Index (b=1,5) (I4)	0,99932	0,99851	1,00000	0,99975	0,99907	0,99810	0,99785	0,99680	0,99750	0,99932
Predictability Index (I5)	4,64506	4,90529	3,73558	4,33036	4,82352	4,98259	5,04304	5,22326	5,10850	4,59345
2nd Index of Bartholomew (I6)	4,67013	3,89053	6,59305	4,74147	4,11425	3,85728	3,81569	3,55285	3,78193	4,33689
2nd Eigenvalue Index (I7)	0,06951	0,02536	0,23362	0,05984	0,04217	0,03175	0,02578	0,02592	0,02898	0,03009
All-Eigenvalues Index (I8)	0,37886	0,33886	0,53227	0,40574	0,35763	0,33120	0,32480	0,31150	0,32196	0,36615
Asymptotic-half-life Index (I9) [**]	6,635E-05	1,899E-12	7,390E-02	1,324E-05	1,029E-07	4,684E-10	2,977E-12	3,450E-12	5,828E-11	1,402E-10

[\*]: 1-step MI for S-TH-MCh  $\equiv$  mono-periodal value of the corresponding mobility index according to the Simple (1<sup>st</sup>-order), Time Homogeneous, Markov Chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

Outside ATECO, Innovative Small & Medium Enterprises: Total Sales										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,01450	0,00686	-0,01017	0,01930	0,01432	0,00130	0,01253	-0,00242	0,01676	0,00835
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02514	0,04658	0,04691	0,03736	0,03099	0,02632	0,03798	0,02952	0,02758	0,03076
Trace Index (I3)	0,35555	0,47898	0,47096	0,46293	0,42007	0,35303	0,40885	0,36019	0,33045	0,41169
Gener. Determinant Index (b=1,5) (I4)	0,99920	1,00000	1,00000	0,99998	0,99996	0,99903	0,99994	0,99938	0,99813	0,99981
Predictability Index (I5)	5,13902	4,25017	4,21430	4,01245	4,44820	5,03532	4,62905	5,12479	5,58001	4,19132
2nd Index of Bartholomew (I6)	3,70705	6,32818	6,51087	5,75842	4,74303	4,03704	5,15594	4,43110	3,90135	5,09020
2nd Eigenvalue Index (I7)	0,13249	0,03932	0,09478	0,08640	0,05485	0,01512	0,03833	0,00440	0,05036	0,05800
All-Eigenvalues Index (I8)	0,35555	0,47638	0,47058	0,46261	0,41825	0,35303	0,40885	0,35913	0,33045	0,41168
Asymptotic-half-life Index (I9) [**]	7,620E-03	3,129E-08	9,482E-04	4,663E-04	4,610E-06	1,744E-20	1,984E-08	4,937E-69	1,495E-06	9,163E-06

Outside ATECO, Innovative Small & Medium Enterprises: Total Assets										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,01820	0,02442	0,00093	0,02316	0,01291	0,00671	0,00888	0,01439	0,02073	0,01417
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02356	0,02789	0,02324	0,03028	0,02479	0,01951	0,02047	0,02163	0,02467	0,02223
Trace Index (I3)	0,32699	0,33496	0,30769	0,41001	0,32997	0,27320	0,26532	0,27946	0,29328	0,31820
Gener. Determinant Index (b=1,5) (I4)	0,99741	0,99797	0,99999	0,99976	0,99921	0,99224	0,99015	0,98994	0,99501	0,99669
Predictability Index (I5)	5,73268	5,63053	6,19596	5,11063	5,39103	5,87910	6,04075	5,84200	5,81354	5,24490
2nd Index of Bartholomew (I6)	3,42077	4,07995	3,49573	4,32823	3,79630	2,89684	2,98597	3,07053	3,49196	3,57547
2nd Eigenvalue Index (I7)	0,08077	0,00000	0,00000	0,15385	0,04318	0,02860	0,02907	0,06845	0,03858	0,06752
All-Eigenvalues Index (I8)	0,32699	0,32939	0,30733	0,41001	0,32997	0,27320	0,26532	0,27946	0,29328	0,31820
Asymptotic-half-life Index (I9) [**]	2,664E-04	Inf	Inf	1,578E-02	1,511E-07	4,215E-11	6,262E-11	5,680E-05	2,230E-08	4,944E-05

[\*]: 1-step MI for S-TH-MCh  $\equiv$  mono-periodal value of the corresponding mobility index according to the Simple (1<sup>st</sup>-order), Time Homogeneous, Markov Chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

Outside ATECO, Innovative Small & Medium Enterprises: Total Debts										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,02013	0,01141	0,00673	0,02484	0,01820	0,00121	0,01438	0,01145	0,01042	0,01372
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02715	0,03012	0,03064	0,03427	0,03246	0,02687	0,03210	0,02551	0,03421	0,02853
Trace Index (I3)	0,33421	0,35638	0,36703	0,43835	0,42845	0,36353	0,40350	0,34276	0,46484	0,39771
Gener. Determinant Index (b=1,5) (I4)	0,99813	0,99967	0,99986	0,99994	1,00000	0,99922	0,99995	0,99932	0,99998	0,99966
Predictability Index (I5)	5,37734	5,38089	5,16420	4,37242	4,49839	4,78666	4,71666	5,55460	4,16806	4,31437
2nd Index of Bartholomew (I6)	3,94829	4,41252	4,48308	5,28149	4,97475	4,15752	5,01876	3,99718	5,12368	4,72154
2nd Eigenvalue Index (I7)	0,00000	0,04624	0,00873	0,06186	0,05309	0,04067	0,02996	0,00000	0,04469	0,05445
All-Eigenvalues Index (I8)	0,33421	0,35638	0,36703	0,43835	0,42721	0,36342	0,40350	0,34276	0,46345	0,39771
Asymptotic-half-life Index (I9) [**]	0,000E+00	4,375E-07	4,642E-35	1,931E-05	3,033E-06	5,616E-08	1,270E-10	Inf	2,602E-07	4,198E-06

Outside ATECO, Innovative Small & Medium Enterprises: Total Production Monetary Value										
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain									1-step MI for S-TH-MCh [*]
	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,02092	0,01877	-0,01421	0,02187	0,01388	-0,00176	0,02075	0,00398	0,01150	0,01012
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,02782	0,03216	0,03785	0,03225	0,02330	0,02711	0,03170	0,02779	0,02783	0,02691
Trace Index (I3)	0,34908	0,41014	0,44066	0,41447	0,32417	0,34259	0,33272	0,38028	0,38715	0,38061
Gener. Determinant Index (b=1,5) (I4)	0,99888	1,00000	0,99995	0,99985	0,99744	0,99823	0,99790	0,99996	0,99978	0,99947
Predictability Index (I5)	5,23885	5,45542	4,80767	4,88700	5,26661	5,25520	5,23388	5,01422	4,85348	4,46812
2nd Index of Bartholomew (I6)	4,10954	4,74240	5,34167	4,77428	3,56855	4,09933	4,24050	4,23830	4,15373	4,41354
2nd Eigenvalue Index (I7)	0,08855	0,00010	0,07493	0,06154	0,03491	0,00679	0,06903	0,00610	0,00585	0,05281
All-Eigenvalues Index (I8)	0,34908	0,40999	0,43996	0,41106	0,32417	0,33568	0,33245	0,38028	0,38715	0,38056
Asymptotic-half-life Index (I9) [**]	5,664E-04	0,000E+00	1,364E-04	1,823E-05	3,370E-09	6,876E-45	6,185E-05	5,946E-50	5,198E-52	2,830E-06

[\*]: 1-step MI for S-TH-MCh  $\equiv$  mono-periodal value of the corresponding mobility index according to the Simple (1<sup>st</sup>-order), Time Homogeneous, Markov Chain

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**The following mobility values for Italian Innovative Start-ups are not scientifically rigorous, owing to the exiguous number of companies residing in the database for the entire epoch 2010-2015: they are exclusively an example of rough and misleading inference.**

Outside ATECO, Innovative Start-Ups: Total Sales						
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain					1-step MI for S-TH-MCh [*]
	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,25855	-0,00911	0,01464	0,04879	0,08946	0,07863
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,18751	0,11368	0,03120	0,05645	0,08170	0,11526
Trace Index (I3)	1,15000	0,72917	0,42500	0,58333	0,45833	0,54167
Gener. Determinant Index (b=1,5) (I4)	1,00000	1,00000	1,00000	1,00000	1,00000	0,99916
Predictability Index (I5)	1,77500	1,19792	2,72500	1,80556	3,81944	3,00223
2nd Index of Bartholomew (I6)	4,75000	2,60417	0,87500	1,66667	2,29167	3,28869
2nd Eigenvalue Index (I7)	1,00000	0,10233	0,00000	0,16667	0,00000	0,09349
All-Eigenvalues Index (I8)	1,15000	0,72917	0,42500	0,58333	0,45833	0,50636
Asymptotic-half-life Index (I9) [**]	1,000E+00	1,628E-03	Inf	2,233E-02	Inf	8,571E-04

Outside ATECO, Innovative Start-Ups: Total Assets						
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain					1-step MI for S-TH-MCh [*]
	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,19400	0,08953	0,12625	-0,04325	0,08392	0,13790
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,12917	0,07132	0,08918	0,10567	0,08402	0,12852
Trace Index (I3)	1,05769	0,60000	0,62500	0,49583	0,62917	0,50256
Gener. Determinant Index (b=1,5) (I4)	1,00000	1,00000	1,00000	1,00000	1,00000	0,99595
Predictability Index (I5)	0,76183	3,05000	2,22222	2,12569	1,64792	2,56976
2nd Index of Bartholomew (I6)	3,07692	2,00000	2,50000	2,37500	2,39583	3,67837
2nd Eigenvalue Index (I7)	1,00000	0,00000	0,16667	0,00000	0,25000	0,11532
All-Eigenvalues Index (I8)	1,05769	0,60000	0,62500	0,46132	0,62917	0,45552
Asymptotic-half-life Index (I9) [**]	1,000E+00	Inf	2,233E-02	Inf	8,987E-02	3,493E-03

[\*]: 1-step MI for S-TH-MCh  $\equiv$  mono-periodal value of the corresponding mobility index according to the Simple (1<sup>st</sup>-order), Time Homogeneous, Markov Chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

**The following mobility values for Italian Innovative Start-ups are not scientifically rigorous, owing to the exiguous number of companies residing in the database for the entire epoch 2010-2015: they are exclusively an example of rough and misleading inference.**

Outside ATECO, Innovative Start-Ups: Total Debts						
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain					1-step MI for S-TH-MCh [*]
	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,08663	0,00407	0,22666	0,06312	0,11794	0,13531
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,06484	0,00371	0,16826	0,04938	0,07268	0,08721
Trace Index (I3)	1,01923	0,52083	0,81818	0,18750	0,25000	0,36250
Gener. Determinant Index (b=1,5) (I4)	1,00000	1,00000	1,00000	0,77036	0,87500	0,94191
Predictability Index (I5)	1,07249	2,30903	3,21281	3,90625	3,45486	2,55411
2nd Index of Bartholomew (I6)	1,63462	0,10417	4,20455	1,25000	1,97917	2,53333
2nd Eigenvalue Index (I7)	1,00000	0,00000	0,27273	0,00000	0,00000	0,20000
All-Eigenvalues Index (I8)	1,01923	0,52083	0,81818	0,18750	0,25000	0,36250
Asymptotic-half-life Index (I9) [**]	1,000E+00	Inf	1,134E-01	Inf	Inf	4,477E-02

Outside ATECO, Innovative Start-Ups: Total Production Monetary Value						
Transition period	Mobility Index value from $t$ to $t+1$ according to the 1 <sup>st</sup> -order not-stationary Markov chain					1-step MI for S-TH-MCh [*]
	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	
Norm. Directional Index (I1)	0,11824	-0,00890	-0,07702	0,02440	0,04895	0,05248
Norm. Alcalde-Unzu Index (alfa=1,3) (I2)	0,08285	0,10201	0,17205	0,06685	0,11293	0,12291
Trace Index (I3)	1,06818	0,81250	0,57143	0,37500	0,53333	0,50417
Gener. Determinant Index (b=1,5) (I4)	1,00000	1,00000	0,94601	0,95581	0,86229	0,99633
Predictability Index (I5)	0,71281	0,74219	4,43878	3,12500	3,64444	2,26297
2nd Index of Bartholomew (I6)	2,15909	2,50000	4,28571	1,87500	3,16667	3,53571
2nd Eigenvalue Index (I7)	1,00000	0,50000	0,00000	0,00000	0,00000	0,10519
All-Eigenvalues Index (I8)	1,06818	0,81250	0,31102	0,37500	0,26318	0,46331
Asymptotic-half-life Index (I9) [**]	1,000E+00	3,679E-01	Inf	Inf	Inf	1,957E-03

[\*]: 1-step MI for S-TH-MCh  $\equiv$  mono-periodal value of the corresponding mobility index according to the Simple (1<sup>st</sup>-order), Time Homogeneous, Markov Chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

## A3.2 Compound aggregated-in-time mobility measures for both the 1<sup>st</sup>-order Markov Chains

ATECO C10 Food and Beverages: Total Sales									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006-2007	2006-2008	2006-2009	2006-2010	2006-2011	2006-2012	2006-2013	2006-2014	2006-2015
Aggregated Norm. Directional Index (AI1)	0,01095	0,02336	0,02198	0,02884	0,03642	0,03716	0,03474	0,03249	0,02865
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01913	0,03268	0,04033	0,04843	0,05608	0,06179	0,06849	0,07388	0,07893
Aggregated Trace Index (AI3)	0,26468	0,43256	0,52320	0,59063	0,64177	0,67642	0,70637	0,73026	0,74899
Aggregated Gener. Determinant Index (AI4)	0,98802	0,98805	0,98584	0,98391	0,98228	0,98080	0,97973	0,97879	0,97755
Aggregated Index of Predictability (AI5)	5,82558	4,09695	3,21566	2,73278	2,42229	2,16637	1,96342	1,77952	1,64498
Aggregated 2nd Index of Bartholomew (AI6)	3,13156	5,57412	7,04343	8,47631	9,75823	10,74778	11,83843	12,70991	13,48853
Aggregated 2nd Eigenvalue Index (AI7)	0,04486	0,07650	0,07543	0,09246	0,11164	0,12097	0,13306	0,14131	0,14931
Aggregated All-Eigenvalues Index (AI8)	0,26426	0,43256	0,52320	0,59063	0,64177	0,67642	0,70637	0,73026	0,74899
Aggregated Asymptotic-half-life Index (AI9) [**]	2,764E-07	2,721E-08	3,051E-12	3,876E-13	1,927E-13	9,811E-15	1,748E-15	1,557E-16	1,764E-17

ATECO C10 Food and Beverages: Total Sales									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006-2007	2006-2008	2006-2009	2006-2010	2006-2011	2006-2012	2006-2013	2006-2014	2006-2015
Aggregated Norm. Directional Index (AI1)	0,00108	0,00315	0,00596	0,00933	0,01314	0,01730	0,02173	0,02638	0,03122
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01571	0,02749	0,03726	0,04577	0,05344	0,06047	0,06703	0,07319	0,07902
Aggregated Trace Index (AI3)	0,22888	0,38141	0,48707	0,56307	0,61970	0,66325	0,69769	0,72559	0,74866
Aggregated Gener. Determinant Index (AI4)	0,97769	0,97769	0,97769	0,97769	0,97769	0,97769	0,97769	0,97769	0,97769
Aggregated Index of Predictability (AI5)	6,21790	4,44420	3,47564	2,87680	2,47086	2,17692	1,95368	1,77803	1,63602
Aggregated 2nd Index of Bartholomew (AI6)	2,68465	4,81641	6,57909	8,08474	9,40382	10,58214	11,65052	12,63043	13,53730
Aggregated 2nd Eigenvalue Index (AI7)	0,01807	0,03581	0,05323	0,07033	0,08713	0,10362	0,11982	0,13572	0,15133
Aggregated All-Eigenvalues Index (AI8)	0,22888	0,38141	0,48707	0,56307	0,61970	0,66325	0,69769	0,72559	0,74866
Aggregated Asymptotic-half-life Index (AI9) [**]	3,083E-17	3,083E-17	3,083E-17	3,083E-17	3,083E-17	3,083E-17	3,083E-17	3,083E-17	3,083E-17

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C10 Food and Beverages: Total Assets									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,00795	0,02411	0,02829	0,03843	0,04897	0,04977	0,04905	0,05127	0,05583
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01578	0,03143	0,03854	0,04675	0,05380	0,05756	0,06128	0,06514	0,06971
Aggregated Trace Index (AI3)	0,23425	0,42842	0,51072	0,58113	0,63168	0,66273	0,68829	0,71061	0,73181
Aggregated Gener. Determinant Index (AI4)	0,97926	0,98768	0,98165	0,97914	0,97675	0,97355	0,97059	0,96807	0,96705
Aggregated Index of Predictability (AI5)	6,19617	4,15217	3,38823	2,91360	2,62066	2,36438	2,13732	1,97252	1,85134
Aggregated 2nd Index of Bartholomew (AI6)	2,56728	5,29555	6,57092	7,94977	9,08513	9,78307	10,45313	11,12629	11,86665
Aggregated 2nd Eigenvalue Index (AI7)	0,03511	0,08713	0,09916	0,12682	0,14881	0,14516	0,14899	0,15151	0,15581
Aggregated All-Eigenvalues Index (AI8)	0,23425	0,42842	0,51072	0,58113	0,63168	0,66273	0,68829	0,71061	0,73181
Aggregated Asymptotic-half-life Index (AI9) [**]	3,785E- 09	2,487E- 07	2,248E- 09	1,321E- 09	4,550E- 10	3,047E- 12	8,690E- 14	2,196E- 15	1,010E- 16

ATECO C10 Food and Beverages: Total Assets									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,00406	0,00902	0,01468	0,02088	0,02751	0,03450	0,04178	0,04930	0,05702
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01371	0,02408	0,03271	0,04026	0,04708	0,05338	0,05930	0,06491	0,07029
Aggregated Trace Index (AI3)	0,21042	0,35692	0,46184	0,53913	0,59766	0,64314	0,67933	0,70877	0,73317
Aggregated Gener. Determinant Index (AI4)	0,96711	0,96711	0,96711	0,96711	0,96711	0,96711	0,96711	0,96711	0,96711
Aggregated Index of Predictability (AI5)	6,46764	4,70203	3,71142	3,09549	2,67935	2,37980	2,15376	1,97712	1,83541
Aggregated 2nd Index of Bartholomew (AI6)	2,28038	4,12633	5,67274	7,00651	8,18472	9,24573	10,21590	11,11376	11,95266
Aggregated 2nd Eigenvalue Index (AI7)	0,01925	0,03813	0,05665	0,07481	0,09262	0,11009	0,12722	0,14402	0,16050
Aggregated All-Eigenvalues Index (AI8)	0,21042	0,35692	0,46184	0,53913	0,59766	0,64314	0,67933	0,70877	0,73317
Aggregated Asymptotic-half-life Index (AI9) [**]	3,263E- 16	3,263E- 16	3,263E- 16	3,263E- 16	3,263E- 16	3,263E- 16	3,263E- 16	3,263E- 16	3,263E- 16

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C10 Food and Beverages: Total Debts									
Transition period	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
	2006-2007	2006-2008	2006-2009	2006-2010	2006-2011	2006-2012	2006-2013	2006-2014	2006-2015
Aggregated Norm. Directional Index (AI1)	0,01144	0,01718	0,02017	0,03188	0,04454	0,04186	0,03869	0,04050	0,03966
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02281	0,03578	0,04515	0,05513	0,06362	0,06833	0,07288	0,07858	0,08338
Aggregated Trace Index (AI3)	0,33250	0,49101	0,58493	0,65512	0,70030	0,72690	0,74909	0,76945	0,78456
Aggregated Gener. Determinant Index (AI4)	0,99786	0,99687	0,99620	0,99634	0,99602	0,99564	0,99523	0,99489	0,99461
Aggregated Index of Predictability (AI5)	5,01699	3,47986	2,71945	2,28998	2,05126	1,83713	1,64535	1,50537	1,40405
Aggregated 2nd Index of Bartholomew (AI6)	3,82410	6,25049	7,93685	9,63524	10,98953	11,82071	12,60247	13,51528	14,27884
Aggregated 2nd Eigenvalue Index (AI7)	0,04184	0,05919	0,07288	0,09961	0,12336	0,12478	0,13451	0,14821	0,15613
Aggregated All-Eigenvalues Index (AI8)	0,33250	0,49101	0,58493	0,65512	0,70030	0,72690	0,74909	0,76945	0,78456
Aggregated Asymptotic-half-life Index (AI9) [**]	9,067E-08	1,355E-10	1,162E-12	3,345E-12	3,696E-12	2,805E-14	2,587E-15	9,705E-16	1,098E-16

ATECO C10 Food and Beverages: Total Debts									
Transition period	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
	2006-2007	2006-2008	2006-2009	2006-2010	2006-2011	2006-2012	2006-2013	2006-2014	2006-2015
Aggregated Norm. Directional Index (AI1)	0,00290	0,00660	0,01086	0,01552	0,02047	0,02566	0,03102	0,03651	0,04211
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01915	0,03215	0,04242	0,05114	0,05886	0,06588	0,07236	0,07843	0,08415
Aggregated Trace Index (AI3)	0,29144	0,45864	0,56215	0,63095	0,67963	0,71590	0,74407	0,76667	0,78530
Aggregated Gener. Determinant Index (AI4)	0,99471	0,99471	0,99471	0,99471	0,99471	0,99471	0,99471	0,99471	0,99471
Aggregated Index of Predictability (AI5)	5,43052	3,72943	2,89792	2,40475	2,07412	1,83460	1,65191	1,50745	1,39014
Aggregated 2nd Index of Bartholomew (AI6)	3,28959	5,66840	7,52438	9,05557	10,36947	11,52831	12,57047	13,52102	14,39724
Aggregated 2nd Eigenvalue Index (AI7)	0,01895	0,03754	0,05577	0,07366	0,09121	0,10843	0,12533	0,14190	0,15816
Aggregated All-Eigenvalues Index (AI8)	0,29144	0,45864	0,56215	0,63095	0,67963	0,71590	0,74407	0,76667	0,78530
Aggregated Asymptotic-half-life Index (AI9) [**]	1,833E-16	1,833E-16	1,833E-16	1,833E-16	1,833E-16	1,833E-16	1,833E-16	1,833E-16	1,833E-16

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C10 Food and Beverages: Total Production Monetary Value									
Transition period	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
	2006-2007	2006-2008	2006-2009	2006-2010	2006-2011	2006-2012	2006-2013	2006-2014	2006-2015
Aggregated Norm. Directional Index (AI1)	0,01128	0,02207	0,02089	0,02871	0,03588	0,03629	0,03392	0,03146	0,02940
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01947	0,03232	0,04029	0,04876	0,05607	0,06166	0,06798	0,07305	0,07787
Aggregated Trace Index (AI3)	0,27253	0,43055	0,52802	0,59857	0,64756	0,68233	0,71211	0,73540	0,75421
Aggregated Gener. Determinant Index (AI4)	0,98973	0,98724	0,98614	0,98519	0,98316	0,98171	0,98059	0,97965	0,97860
Aggregated Index of Predictability (AI5)	5,72322	4,07430	3,14122	2,67186	2,37482	2,10754	1,89763	1,72335	1,59684
Aggregated 2nd Index of Bartholomew (AI6)	3,19346	5,51424	7,03829	8,52991	9,76075	10,73688	11,79016	12,63396	13,37452
Aggregated 2nd Eigenvalue Index (AI7)	0,04714	0,07346	0,07930	0,09797	0,11434	0,12512	0,13544	0,14082	0,15121
Aggregated All-Eigenvalues Index (AI8)	0,27218	0,43055	0,52802	0,59857	0,64756	0,68233	0,71211	0,73540	0,75421
Aggregated Asymptotic-half-life Index (AI9) [**]	5,826E-07	1,285E-08	1,175E-11	2,099E-12	4,023E-13	3,076E-14	3,316E-15	1,358E-16	2,985E-17

ATECO C10 Food and Beverages: Total Production Monetary Value									
Transition period	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
	2006-2007	2006-2008	2006-2009	2006-2010	2006-2011	2006-2012	2006-2013	2006-2014	2006-2015
Aggregated Norm. Directional Index (AI1)	0,00126	0,00348	0,00639	0,00984	0,01371	0,01791	0,02237	0,02704	0,03189
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01574	0,02750	0,03721	0,04564	0,05320	0,06013	0,06658	0,07263	0,07836
Aggregated Trace Index (AI3)	0,23143	0,38552	0,49197	0,56824	0,62481	0,66811	0,70220	0,72972	0,75240
Aggregated Gener. Determinant Index (AI4)	0,97860	0,97860	0,97860	0,97860	0,97860	0,97860	0,97860	0,97860	0,97860
Aggregated Index of Predictability (AI5)	6,17444	4,38695	3,41902	2,82568	2,42600	2,13770	1,91917	1,74737	1,60852
Aggregated 2nd Index of Bartholomew (AI6)	2,69440	4,82849	6,58783	8,08624	9,39555	10,56259	11,61887	12,58630	13,48063
Aggregated 2nd Eigenvalue Index (AI7)	0,01754	0,03477	0,05170	0,06834	0,08468	0,10074	0,11651	0,13201	0,14723
Aggregated All-Eigenvalues Index (AI8)	0,23143	0,38552	0,49197	0,56824	0,62481	0,66811	0,70220	0,72972	0,75240
Aggregated Asymptotic-half-life Index (AI9) [**]	9,753E-18	9,753E-18	9,753E-18	9,753E-18	9,753E-18	9,753E-18	9,753E-18	9,753E-18	9,753E-18

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C13 Textiles: Total Sales									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,00461	-0,00290	-0,03566	-0,02511	-0,02029	-0,03719	-0,04032	-0,04744	-0,05825
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02137	0,03442	0,05078	0,05913	0,06565	0,07352	0,08054	0,08749	0,09367
Aggregated Trace Index (AI3)	0,28627	0,44885	0,60436	0,65902	0,69559	0,73184	0,75720	0,77471	0,79199
Aggregated Gener. Determinant Index (AI4)	0,99385	0,99228	0,99668	0,99712	0,99627	0,99600	0,99535	0,99453	0,99391
Aggregated Index of Predictability (AI5)	5,48062	3,69519	2,59028	2,01989	1,75570	1,51593	1,34096	1,25578	1,16199
Aggregated 2nd Index of Bartholomew (AI6)	3,55149	5,90129	8,69094	10,29451	11,45191	12,66421	13,75857	14,73508	15,59922
Aggregated 2nd Eigenvalue Index (AI7)	0,02800	0,07682	0,18933	0,14646	0,14425	0,17558	0,19128	0,19563	0,21326
Aggregated All-Eigenvalues Index (AI8)	0,28627	0,44885	0,60436	0,65902	0,69559	0,73184	0,75720	0,77471	0,79199
Aggregated Asymptotic-half-life Index (AI9) [**]	2,512E- 11	2,934E- 08	4,983E- 05	2,490E- 08	2,177E- 10	4,416E- 10	1,187E- 10	8,667E- 12	5,065E- 12

ATECO C13 Textiles: Total Sales									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	-0,00601	-0,01231	-0,01879	-0,02536	-0,03196	-0,03857	-0,04516	-0,05169	-0,05817
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01964	0,03328	0,04439	0,05407	0,06280	0,07084	0,07834	0,08540	0,09208
Aggregated Trace Index (AI3)	0,28523	0,45032	0,55433	0,62489	0,67579	0,71432	0,74462	0,76916	0,78949
Aggregated Gener. Determinant Index (AI4)	0,99407	0,99407	0,99407	0,99407	0,99407	0,99407	0,99407	0,99407	0,99407
Aggregated Index of Predictability (AI5)	5,50177	3,76391	2,87744	2,33664	1,96867	1,70037	1,49543	1,33360	1,20259
Aggregated 2nd Index of Bartholomew (AI6)	3,38814	5,88889	7,87798	9,54374	10,98795	12,26999	13,42702	14,48383	15,45779
Aggregated 2nd Eigenvalue Index (AI7)	0,02618	0,05168	0,07651	0,10069	0,12423	0,14716	0,16949	0,19123	0,21241
Aggregated All-Eigenvalues Index (AI8)	0,28523	0,45032	0,55433	0,62489	0,67579	0,71432	0,74462	0,76916	0,78949
Aggregated Asymptotic-half-life Index (AI9) [**]	4,504E- 12	4,504E- 12	4,504E- 12	4,504E- 12	4,504E- 12	4,504E- 12	4,504E- 12	4,504E- 12	4,504E- 12

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C13 Textiles: Total Assets									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,00336	0,01283	0,00668	0,01783	0,01894	0,01254	0,00949	0,00682	0,00392
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01817	0,03477	0,04242	0,05021	0,05582	0,06020	0,06516	0,07015	0,07464
Aggregated Trace Index (AI3)	0,27401	0,48585	0,56851	0,62894	0,66923	0,69776	0,72233	0,74310	0,75937
Aggregated Gener. Determinant Index (AI4)	0,99186	0,99672	0,99468	0,99342	0,99164	0,98979	0,98851	0,98731	0,98593
Aggregated Index of Predictability (AI5)	5,65450	3,45809	2,76819	2,34453	2,04572	1,83245	1,66685	1,52859	1,42700
Aggregated 2nd Index of Bartholomew (AI6)	2,97132	6,00618	7,42454	8,79901	9,77562	10,55366	11,40255	12,23948	12,96519
Aggregated 2nd Eigenvalue Index (AI7)	0,02515	0,05960	0,07263	0,09526	0,11056	0,11837	0,13076	0,13784	0,14669
Aggregated All-Eigenvalues Index (AI8)	0,27401	0,48585	0,56851	0,62894	0,66923	0,69776	0,72233	0,74310	0,75937
Aggregated Asymptotic-half-life Index (AI9) [**]	1,519E- 12	1,591E- 10	1,056E- 12	9,358E- 13	1,424E- 13	4,601E- 15	9,188E- 16	5,794E- 17	8,334E- 18

ATECO C13 Textiles: Total Assets									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,00024	0,00070	0,00131	0,00204	0,00287	0,00376	0,00471	0,00570	0,00673
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01622	0,02783	0,03711	0,04497	0,05189	0,05814	0,06387	0,06918	0,07417
Aggregated Trace Index (AI3)	0,24836	0,40649	0,51206	0,58586	0,63970	0,68050	0,71245	0,73817	0,75937
Aggregated Gener. Determinant Index (AI4)	0,98537	0,98537	0,98537	0,98537	0,98537	0,98537	0,98537	0,98537	0,98537
Aggregated Index of Predictability (AI5)	5,93709	4,14616	3,20172	2,62628	2,23703	1,95380	1,73676	1,56405	1,42264
Aggregated 2nd Index of Bartholomew (AI6)	2,76605	4,87287	6,56120	7,97031	9,18386	10,25411	11,21507	12,08975	12,89434
Aggregated 2nd Eigenvalue Index (AI7)	0,01768	0,03505	0,05212	0,06888	0,08534	0,10152	0,11740	0,13301	0,14834
Aggregated All-Eigenvalues Index (AI8)	0,24836	0,40649	0,51206	0,58586	0,63970	0,68050	0,71245	0,73817	0,75937
Aggregated Asymptotic-half-life Index (AI9) [**]	1,340E- 17	1,340E- 17	1,340E- 17	1,340E- 17	1,340E- 17	1,340E- 17	1,340E- 17	1,340E- 17	1,340E- 17

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C13 Textiles: Total Debts									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,00210	-0,00442	-0,01568	0,00388	0,00515	-0,00560	-0,00547	-0,00904	-0,01305
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02448	0,03964	0,05157	0,06053	0,06814	0,07425	0,07944	0,08577	0,09198
Aggregated Trace Index (AI3)	0,36125	0,54583	0,64227	0,69857	0,73678	0,76284	0,78280	0,79975	0,81525
Aggregated Gener. Determinant Index (AI4)	0,99912	0,99936	0,99923	0,99925	0,99904	0,99884	0,99856	0,99832	0,99817
Aggregated Index of Predictability (AI5)	4,62601	2,97072	2,24402	1,84564	1,57177	1,38097	1,24848	1,13383	1,02961
Aggregated 2nd Index of Bartholomew (AI6)	4,18845	6,97937	9,04980	10,67421	11,95415	12,92281	13,74509	14,72567	15,65810
Aggregated 2nd Eigenvalue Index (AI7)	0,03093	0,06357	0,10717	0,11054	0,13447	0,15766	0,17581	0,19388	0,21324
Aggregated All-Eigenvalues Index (AI8)	0,36125	0,54583	0,64227	0,69857	0,73678	0,76284	0,78280	0,79975	0,81525
Aggregated Asymptotic-half-life Index (AI9) [**]	2,626E- 10	6,808E- 10	1,081E- 08	5,251E- 11	3,779E- 11	2,969E- 11	1,264E- 11	6,699E- 12	5,049E- 12

ATECO C13 Textiles: Total Debts									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	-0,00093	-0,00209	-0,00337	-0,00473	-0,00612	-0,00752	-0,00892	-0,01031	-0,01169
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02194	0,03613	0,04724	0,05667	0,06499	0,07253	0,07946	0,08590	0,09195
Aggregated Trace Index (AI3)	0,33146	0,50462	0,60581	0,67107	0,71665	0,75045	0,77669	0,79775	0,81511
Aggregated Gener. Determinant Index (AI4)	0,99813	0,99813	0,99813	0,99813	0,99813	0,99813	0,99813	0,99813	0,99813
Aggregated Index of Predictability (AI5)	4,95595	3,29303	2,49995	2,02731	1,70790	1,47488	1,29600	1,15357	1,03703
Aggregated 2nd Index of Bartholomew (AI6)	3,78992	6,40677	8,40695	10,04272	11,43987	12,66748	13,76705	14,76545	15,68122
Aggregated 2nd Eigenvalue Index (AI7)	0,02620	0,05172	0,07657	0,10077	0,12433	0,14727	0,16962	0,19138	0,21257
Aggregated All-Eigenvalues Index (AI8)	0,33146	0,50462	0,60581	0,67107	0,71665	0,75045	0,77669	0,79775	0,81511
Aggregated Asymptotic-half-life Index (AI9) [**]	4,603E- 12	4,603E- 12	4,603E- 12	4,603E- 12	4,603E- 12	4,603E- 12	4,603E- 12	4,603E- 12	4,603E- 12

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C13 Textiles: Total Production Monetary Value									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,00461	-0,00141	-0,03313	-0,01984	-0,01634	-0,03188	-0,03526	-0,03855	-0,04950
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02156	0,03522	0,05170	0,05876	0,06542	0,07237	0,07889	0,08433	0,09008
Aggregated Trace Index (AI3)	0,30601	0,47901	0,63453	0,68689	0,72178	0,75843	0,78294	0,80001	0,81841
Aggregated Gener. Determinant Index (AI4)	0,99571	0,99489	0,99754	0,99768	0,99688	0,99669	0,99616	0,99553	0,99507
Aggregated Index of Predictability (AI5)	5,25078	3,41993	2,32925	1,82762	1,55973	1,32183	1,13336	1,02701	0,90128
Aggregated 2nd Index of Bartholomew (AI6)	3,61156	6,11705	8,84650	10,28743	11,45585	12,52645	13,54093	14,34354	15,14973
Aggregated 2nd Eigenvalue Index (AI7)	0,02952	0,08034	0,19548	0,14869	0,15376	0,18758	0,20991	0,21884	0,24338
Aggregated All-Eigenvalues Index (AI8)	0,30601	0,47901	0,63453	0,68689	0,72178	0,75843	0,78294	0,80001	0,81841
Aggregated Asymptotic-half-life Index (AI9) [**]	8,995E- 11	6,472E- 08	7,047E- 05	3,313E- 08	9,652E- 10	2,020E- 09	1,139E- 09	1,775E- 10	1,930E- 10

ATECO C13 Textiles: Total Production Monetary Value									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	-0,00535	-0,01103	-0,01692	-0,02293	-0,02900	-0,03511	-0,04121	-0,04728	-0,05332
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01980	0,03359	0,04475	0,05439	0,06303	0,07095	0,07831	0,08521	0,09173
Aggregated Trace Index (AI3)	0,29051	0,45899	0,56471	0,63591	0,68681	0,72502	0,75481	0,77878	0,79852
Aggregated Gener. Determinant Index (AI4)	0,99452	0,99452	0,99452	0,99452	0,99452	0,99452	0,99452	0,99452	0,99452
Aggregated Index of Predictability (AI5)	5,41357	3,64962	2,76368	2,23105	1,87256	1,61308	1,41575	1,26029	1,13450
Aggregated 2nd Index of Bartholomew (AI6)	3,43513	5,96218	7,96178	9,62777	11,06594	12,33824	13,48345	14,52735	15,48795
Aggregated 2nd Eigenvalue Index (AI7)	0,02683	0,05295	0,07836	0,10309	0,12715	0,15057	0,17337	0,19555	0,21713
Aggregated All-Eigenvalues Index (AI8)	0,29051	0,45899	0,56471	0,63591	0,68681	0,72502	0,75481	0,77878	0,79852
Aggregated Asymptotic-half-life Index (AI9) [**]	8,557E- 12	8,557E- 12	8,557E- 12	8,557E- 12	8,557E- 12	8,557E- 12	8,557E- 12	8,557E- 12	8,557E- 12

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C20 Chemicals: Total Sales									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,00813	0,01291	0,00259	0,01179	0,02350	0,01540	0,01027	0,01233	0,01110
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02020	0,03142	0,04184	0,05241	0,06005	0,06472	0,07125	0,07646	0,08151
Aggregated Trace Index (AI3)	0,25972	0,41556	0,54669	0,62278	0,66508	0,69401	0,71984	0,74030	0,75707
Aggregated Gener. Determinant Index (AI4)	0,98804	0,98672	0,99257	0,99307	0,99131	0,98990	0,98849	0,98702	0,98585
Aggregated Index of Predictability (AI5)	5,92345	4,16434	2,90353	2,37609	2,12250	1,88652	1,72481	1,60980	1,51675
Aggregated 2nd Index of Bartholomew (AI6)	3,13549	5,29765	7,29001	9,09196	10,38140	11,21124	12,22681	13,02323	13,79854
Aggregated 2nd Eigenvalue Index (AI7)	0,06520	0,07604	0,08914	0,11043	0,13170	0,13458	0,14560	0,15647	0,16301
Aggregated All-Eigenvalues Index (AI8)	0,25918	0,41556	0,54669	0,62278	0,66508	0,69401	0,71984	0,74030	0,75707
Aggregated Asymptotic-half-life Index (AI9) [**]	3,427E- 05	2,439E- 08	2,124E- 10	5,130E- 11	2,195E- 11	3,195E- 13	4,057E- 14	7,035E- 15	5,948E- 16

ATECO C20 Chemicals: Total Sales									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	-0,00041	0,00001	0,00102	0,00247	0,00426	0,00633	0,00861	0,01107	0,01367
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01716	0,02947	0,03949	0,04815	0,05590	0,06298	0,06954	0,07570	0,08151
Aggregated Trace Index (AI3)	0,24967	0,40665	0,51102	0,58415	0,63786	0,67892	0,71138	0,73773	0,75961
Aggregated Gener. Determinant Index (AI4)	0,98619	0,98619	0,98619	0,98619	0,98619	0,98619	0,98619	0,98619	0,98619
Aggregated Index of Predictability (AI5)	5,95293	4,20325	3,27465	2,70033	2,30673	2,01800	1,79606	1,61956	1,47553
Aggregated 2nd Index of Bartholomew (AI6)	2,89413	5,11623	6,91771	8,43980	9,76544	10,94570	12,01356	12,99133	13,89480
Aggregated 2nd Eigenvalue Index (AI7)	0,01951	0,03863	0,05739	0,07578	0,09381	0,11148	0,12882	0,14581	0,16247
Aggregated All-Eigenvalues Index (AI8)	0,24967	0,40665	0,51102	0,58415	0,63786	0,67892	0,71138	0,73773	0,75961
Aggregated Asymptotic-half-life Index (AI9) [**]	5,242E- 16	5,242E- 16	5,242E- 16	5,242E- 16	5,242E- 16	5,242E- 16	5,242E- 16	5,242E- 16	5,242E- 16

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C20 Chemicals: Total Assets									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,00602	0,02119	0,02381	0,03431	0,04271	0,04122	0,04331	0,04871	0,05249
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01676	0,03198	0,03969	0,04766	0,05424	0,05782	0,06214	0,06663	0,07075
Aggregated Trace Index (AI3)	0,23782	0,43676	0,52394	0,58990	0,63702	0,66499	0,69141	0,71349	0,73331
Aggregated Gener. Determinant Index (AI4)	0,98223	0,99136	0,98780	0,98533	0,98283	0,97875	0,97661	0,97514	0,97384
Aggregated Index of Predictability (AI5)	6,22277	4,13990	3,29230	2,83275	2,51190	2,25558	2,05783	1,90737	1,76172
Aggregated 2nd Index of Bartholomew (AI6)	2,62760	5,35268	6,80762	8,14875	9,24298	9,90682	10,64335	11,39693	12,04983
Aggregated 2nd Eigenvalue Index (AI7)	0,02545	0,08078	0,09112	0,12069	0,14106	0,14539	0,15356	0,15937	0,17325
Aggregated All-Eigenvalues Index (AI8)	0,23758	0,43676	0,52394	0,58990	0,63702	0,66499	0,69141	0,71349	0,73331
Aggregated Asymptotic-half-life Index (AI9) [**]	2,096E- 12	7,120E- 08	3,532E- 10	4,348E- 10	1,263E- 10	3,192E- 12	2,292E- 13	1,342E- 14	5,750E- 15

ATECO C20 Chemicals: Total Assets									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,00375	0,00843	0,01380	0,01970	0,02600	0,03263	0,03952	0,04662	0,05388
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01429	0,02483	0,03353	0,04110	0,04792	0,05421	0,06011	0,06570	0,07104
Aggregated Trace Index (AI3)	0,21933	0,36728	0,47090	0,54620	0,60282	0,64675	0,68177	0,71036	0,73419
Aggregated Gener. Determinant Index (AI4)	0,97329	0,97329	0,97329	0,97329	0,97329	0,97329	0,97329	0,97329	0,97329
Aggregated Index of Predictability (AI5)	6,36253	4,62191	3,65524	3,04867	2,63179	2,32596	2,09082	1,90373	1,75098
Aggregated 2nd Index of Bartholomew (AI6)	2,36695	4,24913	5,80987	7,14903	8,32937	9,39153	10,36264	11,26141	12,10107
Aggregated 2nd Eigenvalue Index (AI7)	0,02130	0,04214	0,06254	0,08251	0,10205	0,12117	0,13989	0,15820	0,17613
Aggregated All-Eigenvalues Index (AI8)	0,21933	0,36728	0,47090	0,54620	0,60282	0,64675	0,68177	0,71036	0,73419
Aggregated Asymptotic-half-life Index (AI9) [**]	1,038E- 14	1,038E- 14	1,038E- 14	1,038E- 14	1,038E- 14	1,038E- 14	1,038E- 14	1,038E- 14	1,038E- 14

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C20 Chemicals: Total Debts									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006-2007	2006-2008	2006-2009	2006-2010	2006-2011	2006-2012	2006-2013	2006-2014	2006-2015
Aggregated Norm. Directional Index (AI1)	0,00625	0,00908	0,00617	0,01533	0,02328	0,01878	0,01642	0,01713	0,01562
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02138	0,03453	0,04533	0,05383	0,06160	0,06688	0,07210	0,07744	0,08285
Aggregated Trace Index (AI3)	0,31170	0,48621	0,59003	0,65077	0,69583	0,72631	0,75044	0,77082	0,78831
Aggregated Gener. Determinant Index (AI4)	0,99662	0,99740	0,99754	0,99718	0,99684	0,99639	0,99589	0,99536	0,99518
Aggregated Index of Predictability (AI5)	5,22010	3,52595	2,63365	2,21927	1,91708	1,68585	1,51199	1,36385	1,25240
Aggregated 2nd Index of Bartholomew (AI6)	3,48075	5,98945	7,96138	9,46159	10,78909	11,70258	12,55860	13,42717	14,27025
Aggregated 2nd Eigenvalue Index (AI7)	0,02841	0,04521	0,07818	0,09698	0,11993	0,12625	0,14266	0,15650	0,17185
Aggregated All-Eigenvalues Index (AI8)	0,31170	0,48621	0,59003	0,65077	0,69583	0,72631	0,75044	0,77082	0,78831
Aggregated Asymptotic-half-life Index (AI9) [**]	3,583E-11	9,683E-14	8,069E-12	1,570E-12	1,652E-12	4,138E-14	2,040E-14	7,089E-15	4,281E-15

ATECO C20 Chemicals: Total Debts									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006-2007	2006-2008	2006-2009	2006-2010	2006-2011	2006-2012	2006-2013	2006-2014	2006-2015
Aggregated Norm. Directional Index (AI1)	0,00099	0,00240	0,00411	0,00604	0,00813	0,01034	0,01264	0,01502	0,01746
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01928	0,03212	0,04220	0,05073	0,05826	0,06507	0,07135	0,07719	0,08268
Aggregated Trace Index (AI3)	0,29402	0,46126	0,56456	0,63337	0,68226	0,71883	0,74733	0,77026	0,78918
Aggregated Gener. Determinant Index (AI4)	0,99510	0,99510	0,99510	0,99510	0,99510	0,99510	0,99510	0,99510	0,99510
Aggregated Index of Predictability (AI5)	5,39223	3,67629	2,82538	2,31382	1,96764	1,71511	1,52127	1,36693	1,24062
Aggregated 2nd Index of Bartholomew (AI6)	3,28380	5,64499	7,48335	8,99880	10,29825	11,44309	12,47107	13,40690	14,26768
Aggregated 2nd Eigenvalue Index (AI7)	0,02093	0,04143	0,06150	0,08114	0,10038	0,11921	0,13765	0,15571	0,17338
Aggregated All-Eigenvalues Index (AI8)	0,29402	0,46126	0,56456	0,63337	0,68226	0,71883	0,74733	0,77026	0,78918
Aggregated Asymptotic-half-life Index (AI9) [**]	5,908E-15	5,908E-15	5,908E-15	5,908E-15	5,908E-15	5,908E-15	5,908E-15	5,908E-15	5,908E-15

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C20 Chemicals: Total Production Monetary Value									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,00912	0,01596	0,00135	0,01278	0,02325	0,01502	0,00978	0,01189	0,01114
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01992	0,03122	0,04162	0,05203	0,05971	0,06472	0,07093	0,07618	0,08093
Aggregated Trace Index (AI3)	0,26345	0,41836	0,54936	0,62701	0,66946	0,70114	0,72625	0,74592	0,76174
Aggregated Gener. Determinant Index (AI4)	0,98841	0,98672	0,99253	0,99387	0,99253	0,99166	0,99042	0,98910	0,98776
Aggregated Index of Predictability (AI5)	5,84742	4,12383	2,88190	2,37400	2,12277	1,87171	1,70049	1,59511	1,51055
Aggregated 2nd Index of Bartholomew (AI6)	3,14486	5,28935	7,28826	9,11587	10,40619	11,28855	12,26787	13,08539	13,79426
Aggregated 2nd Eigenvalue Index (AI7)	0,06320	0,07765	0,08382	0,10346	0,12240	0,12859	0,13972	0,14898	0,15521
Aggregated All-Eigenvalues Index (AI8)	0,26283	0,41836	0,54936	0,62701	0,66946	0,70114	0,72625	0,74592	0,76174
Aggregated Asymptotic-half-life Index (AI9) [**]	2,448E- 05	3,560E- 08	4,831E- 11	9,431E- 12	2,967E- 12	7,543E- 14	9,952E- 15	1,180E- 15	8,661E- 17

ATECO C20 Chemicals: Total Production Monetary Value									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	-0,00039	0,00003	0,00102	0,00244	0,00419	0,00619	0,00840	0,01078	0,01329
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01739	0,02971	0,03967	0,04823	0,05587	0,06283	0,06928	0,07531	0,08100
Aggregated Trace Index (AI3)	0,25664	0,41578	0,52030	0,59279	0,64559	0,68570	0,71723	0,74274	0,76385
Aggregated Gener. Determinant Index (AI4)	0,98811	0,98811	0,98811	0,98811	0,98811	0,98811	0,98811	0,98811	0,98811
Aggregated Index of Predictability (AI5)	5,86208	4,11698	3,20641	2,64910	2,26950	1,99203	1,77914	1,60998	1,47195
Aggregated 2nd Index of Bartholomew (AI6)	2,95677	5,20148	7,00544	8,51947	9,83135	10,99470	12,04393	13,00220	13,88581
Aggregated 2nd Eigenvalue Index (AI7)	0,01835	0,03636	0,05404	0,07140	0,08843	0,10516	0,12158	0,13769	0,15352
Aggregated All-Eigenvalues Index (AI8)	0,25664	0,41578	0,52030	0,59279	0,64559	0,68570	0,71723	0,74274	0,76385
Aggregated Asymptotic-half-life Index (AI9) [**]	5,547E- 17	5,547E- 17	5,547E- 17	5,547E- 17	5,547E- 17	5,547E- 17	5,547E- 17	5,547E- 17	5,547E- 17

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C22 Rubbers & Plastics: Total Sales									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,01173	0,01321	-0,01984	0,00177	0,00997	-0,00361	-0,00702	-0,00680	-0,00780
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02138	0,03269	0,04744	0,05461	0,06158	0,06738	0,07365	0,07961	0,08592
Aggregated Trace Index (AI3)	0,30489	0,45550	0,61037	0,66341	0,69756	0,72424	0,74541	0,76453	0,78020
Aggregated Gener. Determinant Index (AI4)	0,99540	0,99364	0,99842	0,99872	0,99806	0,99748	0,99670	0,99609	0,99541
Aggregated Index of Predictability (AI5)	5,36621	3,77720	2,57122	2,11016	1,89702	1,69391	1,55683	1,44015	1,36305
Aggregated 2nd Index of Bartholomew (AI6)	3,44851	5,59530	8,22272	9,65035	10,81442	11,76315	12,74392	13,64810	14,55577
Aggregated 2nd Eigenvalue Index (AI7)	0,07408	0,05689	0,10172	0,08681	0,10601	0,11450	0,12742	0,14033	0,15267
Aggregated All-Eigenvalues Index (AI8)	0,30453	0,45550	0,61037	0,66341	0,69756	0,72424	0,74541	0,76453	0,78020
Aggregated Asymptotic-half-life Index (AI9) [**]	1,227E- 04	5,263E- 11	3,813E- 09	5,510E- 14	3,704E- 14	1,405E- 15	3,472E- 16	1,183E- 16	4,426E- 17

ATECO C22 Rubbers & Plastics: Total Sales									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	-0,00217	-0,00375	-0,00492	-0,00577	-0,00638	-0,00677	-0,00700	-0,00708	-0,00703
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02001	0,03344	0,04411	0,05323	0,06133	0,06870	0,07550	0,08183	0,08778
Aggregated Trace Index (AI3)	0,29767	0,46421	0,56627	0,63418	0,68252	0,71882	0,74720	0,77007	0,78897
Aggregated Gener. Determinant Index (AI4)	0,99565	0,99565	0,99565	0,99565	0,99565	0,99565	0,99565	0,99565	0,99565
Aggregated Index of Predictability (AI5)	5,36805	3,67882	2,83940	2,33207	1,98831	1,73814	1,54694	1,39546	1,27210
Aggregated 2nd Index of Bartholomew (AI6)	3,42774	5,88855	7,80850	9,39545	10,75871	11,96047	13,03895	14,01935	14,91925
Aggregated 2nd Eigenvalue Index (AI7)	0,01966	0,03892	0,05781	0,07633	0,09449	0,11229	0,12973	0,14684	0,16361
Aggregated All-Eigenvalues Index (AI8)	0,29767	0,46421	0,56627	0,63418	0,68252	0,71882	0,74720	0,77007	0,78897
Aggregated Asymptotic-half-life Index (AI9) [**]	6,846E- 16	6,846E- 16	6,846E- 16	6,846E- 16	6,846E- 16	6,846E- 16	6,846E- 16	6,846E- 16	6,846E- 16

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C22 Rubbers & Plastics: Total Assets									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,00755	0,01908	0,01602	0,03060	0,03494	0,03452	0,03601	0,03989	0,04525
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01660	0,03199	0,03908	0,04833	0,05373	0,05800	0,06232	0,06678	0,07150
Aggregated Trace Index (AI3)	0,25139	0,45253	0,53988	0,61411	0,65331	0,68226	0,70476	0,72651	0,74539
Aggregated Gener. Determinant Index (AI4)	0,98592	0,99328	0,99166	0,99175	0,98923	0,98710	0,98475	0,98322	0,98217
Aggregated Index of Predictability (AI5)	5,98891	3,86221	3,07294	2,60795	2,33615	2,08870	1,94035	1,79281	1,68268
Aggregated 2nd Index of Bartholomew (AI6)	2,65325	5,43513	6,79428	8,33206	9,25740	10,02966	10,78234	11,50377	12,25245
Aggregated 2nd Eigenvalue Index (AI7)	0,03835	0,06639	0,05923	0,10215	0,11548	0,11818	0,12422	0,13615	0,14676
Aggregated All-Eigenvalues Index (AI8)	0,25139	0,45253	0,53988	0,61411	0,65331	0,68226	0,70476	0,72651	0,74539
Aggregated Asymptotic-half-life Index (AI9) [**]	2,005E- 08	1,722E- 09	1,617E- 15	6,691E- 12	5,422E- 13	4,356E- 15	1,297E- 16	3,505E- 17	8,516E- 18

ATECO C22 Rubbers & Plastics: Total Assets									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,00323	0,00726	0,01189	0,01696	0,02239	0,02809	0,03401	0,04011	0,04636
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01533	0,02630	0,03514	0,04271	0,04945	0,05560	0,06133	0,06671	0,07181
Aggregated Trace Index (AI3)	0,23789	0,39178	0,49596	0,56968	0,62399	0,66548	0,69819	0,72467	0,74660
Aggregated Gener. Determinant Index (AI4)	0,98196	0,98196	0,98196	0,98196	0,98196	0,98196	0,98196	0,98196	0,98196
Aggregated Index of Predictability (AI5)	6,10620	4,35993	3,42862	2,85811	2,47170	2,19089	1,97641	1,80657	1,66841
Aggregated 2nd Index of Bartholomew (AI6)	2,55758	4,52844	6,12515	7,47177	8,64327	9,68670	10,63273	11,50211	12,30941
Aggregated 2nd Eigenvalue Index (AI7)	0,01777	0,03523	0,05237	0,06921	0,08575	0,10200	0,11796	0,13363	0,14903
Aggregated All-Eigenvalues Index (AI8)	0,23789	0,39178	0,49596	0,56968	0,62399	0,66548	0,69819	0,72467	0,74660
Aggregated Asymptotic-half-life Index (AI9) [**]	1,627E- 17	1,627E- 17	1,627E- 17	1,627E- 17	1,627E- 17	1,627E- 17	1,627E- 17	1,627E- 17	1,627E- 17

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C22 Rubbers & Plastics: Total Debts									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006-2007	2006-2008	2006-2009	2006-2010	2006-2011	2006-2012	2006-2013	2006-2014	2006-2015
Aggregated Norm. Directional Index (AI1)	0,00817	0,00575	-0,00288	0,01551	0,01876	0,01350	0,01285	0,01437	0,01756
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02276	0,03656	0,04751	0,05660	0,06386	0,06950	0,07461	0,08035	0,08569
Aggregated Trace Index (AI3)	0,33973	0,51405	0,61543	0,67673	0,71497	0,74287	0,76374	0,78312	0,79941
Aggregated Gener. Determinant Index (AI4)	0,99861	0,99899	0,99899	0,99902	0,99879	0,99856	0,99815	0,99791	0,99774
Aggregated Index of Predictability (AI5)	4,99690	3,32369	2,48599	2,06024	1,80625	1,59455	1,44046	1,30918	1,19774
Aggregated 2nd Index of Bartholomew (AI6)	3,75559	6,42097	8,39030	9,97740	11,19972	12,14182	12,97108	13,87586	14,71158
Aggregated 2nd Eigenvalue Index (AI7)	0,04243	0,04216	0,07105	0,08991	0,10534	0,12213	0,13680	0,15421	0,17013
Aggregated All-Eigenvalues Index (AI8)	0,33973	0,51405	0,61543	0,67673	0,71497	0,74287	0,76374	0,78312	0,79941
Aggregated Asymptotic-half-life Index (AI9) [**]	1,141E-07	1,056E-14	5,586E-13	1,655E-13	3,008E-14	1,362E-14	4,740E-15	4,177E-15	2,965E-15

ATECO C22 Rubbers & Plastics: Total Debts									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006-2007	2006-2008	2006-2009	2006-2010	2006-2011	2006-2012	2006-2013	2006-2014	2006-2015
Aggregated Norm. Directional Index (AI1)	0,00137	0,00301	0,00486	0,00686	0,00898	0,01120	0,01349	0,01584	0,01823
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02109	0,03452	0,04495	0,05373	0,06147	0,06846	0,07488	0,08086	0,08646
Aggregated Trace Index (AI3)	0,32308	0,49298	0,59295	0,65777	0,70319	0,73696	0,76321	0,78433	0,80176
Aggregated Gener. Determinant Index (AI4)	0,99778	0,99778	0,99778	0,99778	0,99778	0,99778	0,99778	0,99778	0,99778
Aggregated Index of Predictability (AI5)	5,07353	3,42970	2,64130	2,17051	1,85171	1,61860	1,43929	1,29629	1,17914
Aggregated 2nd Index of Bartholomew (AI6)	3,59828	6,07603	7,96655	9,51095	10,82948	11,98827	13,02696	13,97118	14,83851
Aggregated 2nd Eigenvalue Index (AI7)	0,02108	0,04172	0,06193	0,08171	0,10107	0,12002	0,13857	0,15674	0,17452
Aggregated All-Eigenvalues Index (AI8)	0,32308	0,49298	0,59295	0,65777	0,70319	0,73696	0,76321	0,78433	0,80176
Aggregated Asymptotic-half-life Index (AI9) [**]	7,471E-15	7,471E-15	7,471E-15	7,471E-15	7,471E-15	7,471E-15	7,471E-15	7,471E-15	7,471E-15

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C22 Rubbers & Plastics: Total Production Monetary Value									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,01268	0,01369	-0,01991	0,00366	0,01214	-0,00095	-0,00359	-0,00416	-0,00246
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02116	0,03191	0,04635	0,05354	0,06048	0,06587	0,07181	0,07748	0,08304
Aggregated Trace Index (AI3)	0,30358	0,45167	0,60666	0,66141	0,69751	0,72370	0,74450	0,76379	0,77992
Aggregated Gener. Determinant Index (AI4)	0,99512	0,99329	0,99834	0,99866	0,99804	0,99744	0,99663	0,99600	0,99530
Aggregated Index of Predictability (AI5)	5,39336	3,82290	2,60441	2,12827	1,89924	1,69172	1,55656	1,43752	1,34807
Aggregated 2nd Index of Bartholomew (AI6)	3,39145	5,45033	8,05493	9,47395	10,64067	11,53419	12,46509	13,34715	14,18912
Aggregated 2nd Eigenvalue Index (AI7)	0,07573	0,05893	0,10271	0,08632	0,10840	0,11576	0,12857	0,14350	0,15680
Aggregated All-Eigenvalues Index (AI8)	0,30346	0,45167	0,60666	0,66141	0,69751	0,72370	0,74450	0,76379	0,77992
Aggregated Asymptotic-half-life Index (AI9) [**]	1,505E- 04	1,224E- 10	4,647E- 09	4,585E- 14	7,615E- 14	2,086E- 15	4,876E- 16	2,840E- 16	1,302E- 16

ATECO C22 Rubbers & Plastics: Total Production Monetary Value									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	-0,00161	-0,00263	-0,00325	-0,00357	-0,00366	-0,00355	-0,00329	-0,00290	-0,00240
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01977	0,03294	0,04338	0,05229	0,06020	0,06739	0,07403	0,08021	0,08603
Aggregated Trace Index (AI3)	0,29674	0,46265	0,56434	0,63202	0,68023	0,71646	0,74480	0,76768	0,78660
Aggregated Gener. Determinant Index (AI4)	0,99559	0,99559	0,99559	0,99559	0,99559	0,99559	0,99559	0,99559	0,99559
Aggregated Index of Predictability (AI5)	5,38377	3,70074	2,86367	2,35699	2,01288	1,76178	1,56938	1,41661	1,29198
Aggregated 2nd Index of Bartholomew (AI6)	3,38468	5,81052	7,70157	9,26419	10,60669	11,79064	12,85384	13,82113	14,70979
Aggregated 2nd Eigenvalue Index (AI7)	0,01946	0,03853	0,05724	0,07558	0,09357	0,11121	0,12850	0,14546	0,16208
Aggregated All-Eigenvalues Index (AI8)	0,29674	0,46265	0,56434	0,63202	0,68023	0,71646	0,74480	0,76768	0,78660
Aggregated Asymptotic-half-life Index (AI9) [**]	4,777E- 16	4,777E- 16	4,777E- 16	4,777E- 16	4,777E- 16	4,777E- 16	4,777E- 16	4,777E- 16	4,777E- 16

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C28 Machinery & Equipment: Total Sales									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,02201	0,03115	-0,01556	-0,00124	0,01267	0,00440	-0,00049	0,00542	0,00790
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02960	0,04456	0,05986	0,07065	0,08012	0,08814	0,09568	0,10258	0,11029
Aggregated Trace Index (AI3)	0,39766	0,55743	0,69221	0,74166	0,77395	0,79756	0,81691	0,83162	0,84438
Aggregated Gener. Determinant Index (AI4)	0,99964	0,99938	0,99991	0,99990	0,99987	0,99981	0,99975	0,99968	0,99962
Aggregated Index of Predictability (AI5)	4,43562	2,98612	1,90609	1,51442	1,30669	1,14275	1,01247	0,92003	0,84816
Aggregated 2nd Index of Bartholomew (AI6)	5,02273	7,77683	10,48326	12,35608	13,87336	15,10908	16,22522	17,21941	18,27203
Aggregated 2nd Eigenvalue Index (AI7)	0,07276	0,08932	0,14118	0,15074	0,17033	0,19252	0,21719	0,23699	0,25885
Aggregated All-Eigenvalues Index (AI8)	0,39764	0,55743	0,69221	0,74166	0,77395	0,79756	0,81691	0,83162	0,84438
Aggregated Asymptotic-half-life Index (AI9) [**]	1,036E- 04	3,678E- 07	1,164E- 06	4,267E- 08	8,692E- 09	3,578E- 09	2,481E- 09	1,249E- 09	9,030E- 10

ATECO C28 Machinery & Equipment: Total Sales									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	-0,00063	-0,00042	0,00023	0,00116	0,00228	0,00354	0,00490	0,00633	0,00782
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02687	0,04395	0,05738	0,06880	0,07890	0,08803	0,09640	0,10415	0,11137
Aggregated Trace Index (AI3)	0,38601	0,56263	0,65895	0,71916	0,76063	0,79115	0,81471	0,83353	0,84898
Aggregated Gener. Determinant Index (AI4)	0,99963	0,99963	0,99963	0,99963	0,99963	0,99963	0,99963	0,99963	0,99963
Aggregated Index of Predictability (AI5)	4,38298	2,82017	2,10073	1,67722	1,39367	1,18848	1,03207	0,90833	0,80770
Aggregated 2nd Index of Bartholomew (AI6)	4,73880	7,84335	10,17555	12,06984	13,67914	15,08462	16,33482	17,46147	18,48671
Aggregated 2nd Eigenvalue Index (AI7)	0,03461	0,06801	0,10027	0,13140	0,16146	0,19048	0,21849	0,24554	0,27165
Aggregated All-Eigenvalues Index (AI8)	0,38601	0,56263	0,65895	0,71916	0,76063	0,79115	0,81471	0,83353	0,84898
Aggregated Asymptotic-half-life Index (AI9) [**]	2,836E- 09	2,836E- 09	2,836E- 09	2,836E- 09	2,836E- 09	2,836E- 09	2,836E- 09	2,836E- 09	2,836E- 09

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C28 Machinery & Equipment: Total Assets									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,01287	0,02856	0,01931	0,03384	0,04137	0,04080	0,04453	0,05201	0,05862
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02256	0,03974	0,04727	0,05641	0,06360	0,06857	0,07386	0,07940	0,08489
Aggregated Trace Index (AI3)	0,32831	0,52341	0,60580	0,66400	0,70161	0,72693	0,74878	0,76841	0,78407
Aggregated Gener. Determinant Index (AI4)	0,99772	0,99865	0,99839	0,99794	0,99738	0,99674	0,99617	0,99567	0,99528
Aggregated Index of Predictability (AI5)	5,09440	3,34873	2,59076	2,21585	1,97415	1,77462	1,62921	1,50899	1,42223
Aggregated 2nd Index of Bartholomew (AI6)	3,77212	6,80849	8,29802	9,79789	10,98553	11,82801	12,69366	13,54809	14,38286
Aggregated 2nd Eigenvalue Index (AI7)	0,05208	0,09419	0,07372	0,10612	0,12349	0,13072	0,14454	0,16233	0,17650
Aggregated All-Eigenvalues Index (AI8)	0,32831	0,52341	0,60580	0,66400	0,70161	0,72693	0,74878	0,76841	0,78407
Aggregated Asymptotic-half-life Index (AI9) [**]	2,354E- 06	8,204E- 07	1,611E- 12	1,846E- 11	3,808E- 12	1,281E- 13	3,178E- 14	2,538E- 14	1,119E- 14

ATECO C28 Machinery & Equipment: Total Assets									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,00425	0,00964	0,01580	0,02250	0,02959	0,03697	0,04456	0,05232	0,06019
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,01932	0,03240	0,04276	0,05161	0,05949	0,06668	0,07336	0,07963	0,08557
Aggregated Trace Index (AI3)	0,29523	0,46179	0,56411	0,63207	0,68032	0,71644	0,74465	0,76741	0,78624
Aggregated Gener. Determinant Index (AI4)	0,99532	0,99532	0,99532	0,99532	0,99532	0,99532	0,99532	0,99532	0,99532
Aggregated Index of Predictability (AI5)	5,40728	3,73414	2,91339	2,42232	2,09101	1,85016	1,66618	1,52063	1,40247
Aggregated 2nd Index of Bartholomew (AI6)	3,30422	5,67762	7,52962	9,06285	10,38473	11,55638	12,61510	13,58512	14,48305
Aggregated 2nd Eigenvalue Index (AI7)	0,02179	0,04311	0,06396	0,08436	0,10432	0,12384	0,14293	0,16161	0,17988
Aggregated All-Eigenvalues Index (AI8)	0,29523	0,46179	0,56411	0,63207	0,68032	0,71644	0,74465	0,76741	0,78624
Aggregated Asymptotic-half-life Index (AI9) [**]	2,176E- 14	2,176E- 14	2,176E- 14	2,176E- 14	2,176E- 14	2,176E- 14	2,176E- 14	2,176E- 14	2,176E- 14

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C28 Machinery & Equipment: Total Debts									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,01370	0,01510	-0,00658	0,01150	0,02043	0,01145	0,01247	0,01750	0,02286
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02835	0,04402	0,05766	0,06845	0,07733	0,08459	0,09132	0,09793	0,10433
Aggregated Trace Index (AI3)	0,40617	0,57530	0,67803	0,73050	0,76347	0,78865	0,80823	0,82390	0,83731
Aggregated Gener. Determinant Index (AI4)	0,99980	0,99979	0,99988	0,99986	0,99982	0,99978	0,99973	0,99969	0,99965
Aggregated Index of Predictability (AI5)	4,27653	2,81150	2,01648	1,63898	1,43566	1,24615	1,11099	1,00762	0,91890
Aggregated 2nd Index of Bartholomew (AI6)	4,87113	7,77535	10,14172	11,96608	13,38846	14,52693	15,55597	16,52947	17,45736
Aggregated 2nd Eigenvalue Index (AI7)	0,04924	0,06250	0,10281	0,12600	0,14489	0,16880	0,19198	0,21333	0,23458
Aggregated All-Eigenvalues Index (AI8)	0,40617	0,57530	0,67803	0,73050	0,76347	0,78865	0,80823	0,82390	0,83731
Aggregated Asymptotic-half-life Index (AI9) [**]	1,094E- 06	4,697E- 10	4,736E- 09	1,145E- 09	2,419E- 10	1,702E- 10	1,302E- 10	9,195E- 11	7,340E- 11

ATECO C28 Machinery & Equipment: Total Debts									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,00179	0,00393	0,00632	0,00890	0,01160	0,01440	0,01725	0,02014	0,02305
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02623	0,04246	0,05505	0,06567	0,07503	0,08346	0,09119	0,09834	0,10501
Aggregated Trace Index (AI3)	0,38684	0,56052	0,65423	0,71261	0,75285	0,78259	0,80565	0,82418	0,83947
Aggregated Gener. Determinant Index (AI4)	0,99967	0,99967	0,99967	0,99967	0,99967	0,99967	0,99967	0,99967	0,99967
Aggregated Index of Predictability (AI5)	4,39820	2,88090	2,18385	1,77004	1,49027	1,28608	1,12933	1,00461	0,90264
Aggregated 2nd Index of Bartholomew (AI6)	4,60063	7,55597	9,75443	11,53290	13,04245	14,36195	15,53788	16,60026	17,56984
Aggregated 2nd Eigenvalue Index (AI7)	0,03001	0,05913	0,08737	0,11476	0,14133	0,16710	0,19210	0,21635	0,23987
Aggregated All-Eigenvalues Index (AI8)	0,38684	0,56052	0,65423	0,71261	0,75285	0,78259	0,80565	0,82418	0,83947
Aggregated Asymptotic-half-life Index (AI9) [**]	1,323E- 10	1,323E- 10	1,323E- 10	1,323E- 10	1,323E- 10	1,323E- 10	1,323E- 10	1,323E- 10	1,323E- 10

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

ATECO C28 Machinery & Equipment: Total Production Monetary Value									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,02108	0,02878	-0,01570	-0,00177	0,01236	0,00297	-0,00026	0,00478	0,00865
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02779	0,04162	0,05620	0,06549	0,07374	0,08079	0,08742	0,09313	0,09965
Aggregated Trace Index (AI3)	0,37886	0,53890	0,67862	0,72784	0,75976	0,78465	0,80434	0,81930	0,83309
Aggregated Gener. Determinant Index (AI4)	0,99932	0,99899	0,99983	0,99981	0,99974	0,99964	0,99953	0,99940	0,99930
Aggregated Index of Predictability (AI5)	4,64506	3,17641	2,03370	1,62975	1,42115	1,23469	1,09803	1,00111	0,91488
Aggregated 2nd Index of Bartholomew (AI6)	4,67013	7,24545	9,81861	11,50285	12,86285	13,97642	14,99599	15,84400	16,78102
Aggregated 2nd Eigenvalue Index (AI7)	0,06951	0,08298	0,14002	0,13734	0,15067	0,17428	0,19585	0,21331	0,23353
Aggregated All-Eigenvalues Index (AI8)	0,37886	0,53890	0,67862	0,72784	0,75976	0,78465	0,80434	0,81930	0,83309
Aggregated Asymptotic-half-life Index (AI9) [**]	6,635E- 05	1,121E- 07	1,030E- 06	7,070E- 09	6,071E- 10	3,699E- 10	2,151E- 10	9,166E- 11	6,507E- 11

ATECO C28 Machinery & Equipment: Total Production Monetary Value									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	-0,00049	-0,00025	0,00039	0,00127	0,00231	0,00349	0,00475	0,00608	0,00746
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02482	0,04056	0,05289	0,06335	0,07259	0,08095	0,08863	0,09575	0,10240
Aggregated Trace Index (AI3)	0,36615	0,54173	0,63975	0,70167	0,74450	0,77611	0,80054	0,82008	0,83614
Aggregated Gener. Determinant Index (AI4)	0,99932	0,99932	0,99932	0,99932	0,99932	0,99932	0,99932	0,99932	0,99932
Aggregated Index of Predictability (AI5)	4,59345	2,99899	2,25634	1,81744	1,52280	1,30905	1,14571	1,01615	0,91048
Aggregated 2nd Index of Bartholomew (AI6)	4,33689	7,22530	9,40585	11,18093	12,69229	14,01572	15,19648	16,26403	17,23884
Aggregated 2nd Eigenvalue Index (AI7)	0,03009	0,05927	0,08758	0,11503	0,14166	0,16749	0,19254	0,21683	0,24040
Aggregated All-Eigenvalues Index (AI8)	0,36615	0,54173	0,63975	0,70167	0,74450	0,77611	0,80054	0,82008	0,83614
Aggregated Asymptotic-half-life Index (AI9) [**]	1,402E- 10	1,402E- 10	1,402E- 10	1,402E- 10	1,402E- 10	1,402E- 10	1,402E- 10	1,402E- 10	1,402E- 10

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

Outside ATECO, Innovative Small & Medium Businesses: Total Sales									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,01450	0,02462	0,00905	0,04776	0,07315	0,07548	0,08230	0,07938	0,11106
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02514	0,06003	0,07539	0,08981	0,10066	0,10550	0,11339	0,12030	0,13129
Aggregated Trace Index (AI3)	0,35555	0,62334	0,71954	0,78636	0,82318	0,83756	0,85736	0,86382	0,87872
Aggregated Gener. Determinant Index (AI4)	0,99920	0,99999	1,00000	1,00000	1,00000	0,99999	0,99999	0,99998	0,99997
Aggregated Index of Predictability (AI5)	5,13902	3,03988	1,98466	1,36892	1,19048	1,12612	0,93430	0,83740	0,88597
Aggregated 2nd Index of Bartholomew (AI6)	3,70705	8,99267	11,86670	14,58639	16,27705	17,05669	18,21109	19,19080	20,37887
Aggregated 2nd Eigenvalue Index (AI7)	0,13249	0,17985	0,30854	0,35379	0,35437	0,36345	0,39920	0,38198	0,42734
Aggregated All-Eigenvalues Index (AI8)	0,35555	0,61823	0,71935	0,78551	0,82191	0,83736	0,85730	0,86379	0,87869
Aggregated Asymptotic-half-life Index (AI9) [**]	7,620E- 03	9,193E- 04	3,567E- 03	1,747E- 03	3,629E- 04	1,003E- 04	7,314E- 05	9,902E- 06	1,380E- 05

Outside ATECO, Innovative Small & Medium Businesses: Total Sales									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,00835	0,01928	0,03145	0,04432	0,05758	0,07102	0,08448	0,09786	0,11108
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,03076	0,04924	0,06413	0,07709	0,08881	0,09963	0,10975	0,11929	0,12832
Aggregated Trace Index (AI3)	0,41169	0,59294	0,68956	0,74942	0,79057	0,82091	0,84438	0,86316	0,87857
Aggregated Gener. Determinant Index (AI4)	0,99981	0,99981	0,99981	0,99981	0,99981	0,99981	0,99981	0,99981	0,99981
Aggregated Index of Predictability (AI5)	4,19132	2,67755	1,98318	1,57470	1,30308	1,10942	0,96517	0,85448	0,76771
Aggregated 2nd Index of Bartholomew (AI6)	5,09020	8,40889	10,92948	13,01104	14,81000	16,40602	17,84524	19,15709	20,36181
Aggregated 2nd Eigenvalue Index (AI7)	0,05800	0,11264	0,16411	0,21259	0,25826	0,30129	0,34181	0,37999	0,41595
Aggregated All-Eigenvalues Index (AI8)	0,41168	0,59293	0,68955	0,74942	0,79057	0,82091	0,84438	0,86316	0,87857
Aggregated Asymptotic-half-life Index (AI9) [**]	9,163E- 06	9,163E- 06	9,163E- 06	9,163E- 06	9,163E- 06	9,163E- 06	9,163E- 06	9,163E- 06	9,163E- 06

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

Outside ATECO, Innovative Small & Medium Businesses: Total Assets									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,01820	0,05087	0,05575	0,09800	0,12389	0,13424	0,15167	0,17944	0,20954
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02356	0,04663	0,05603	0,07461	0,08850	0,09530	0,10445	0,11831	0,13127
Aggregated Trace Index (AI3)	0,32699	0,53308	0,60920	0,70872	0,75416	0,78341	0,80438	0,84259	0,86442
Aggregated Gener. Determinant Index (AI4)	0,99741	0,99771	0,99964	0,99968	0,99961	0,99936	0,99905	0,99873	0,99852
Aggregated Index of Predictability (AI5)	5,73268	4,09064	3,37073	2,65031	2,42715	2,09225	2,02668	1,77307	1,80357
Aggregated 2nd Index of Bartholomew (AI6)	3,42077	7,13178	8,78173	11,71012	13,62421	14,67174	15,86848	17,67076	19,29929
Aggregated 2nd Eigenvalue Index (AI7)	0,08077	0,14286	0,09364	0,22111	0,25364	0,28603	0,30699	0,43441	0,47180
Aggregated All-Eigenvalues Index (AI8)	0,32699	0,53233	0,60889	0,70872	0,75416	0,78341	0,80438	0,84259	0,86442
Aggregated Asymptotic-half-life Index (AI9) [**]	2,664E- 04	1,243E- 04	6,533E- 10	1,518E- 05	7,162E- 06	4,356E- 06	1,793E- 06	5,944E- 05	5,693E- 05

Outside ATECO, Innovative Small & Medium Businesses: Total Assets									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,01417	0,03237	0,05335	0,07631	0,10072	0,12616	0,15230	0,17888	0,20569
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02223	0,03861	0,05315	0,06680	0,07994	0,09274	0,10529	0,11763	0,12976
Aggregated Trace Index (AI3)	0,31820	0,50204	0,61673	0,69337	0,74765	0,78795	0,81903	0,84371	0,86377
Aggregated Gener. Determinant Index (AI4)	0,99669	0,99669	0,99669	0,99669	0,99669	0,99669	0,99669	0,99669	0,99669
Aggregated Index of Predictability (AI5)	5,24490	3,61259	2,85307	2,42434	2,15537	1,97782	1,85876	1,78002	1,73055
Aggregated 2nd Index of Bartholomew (AI6)	3,57547	6,36102	8,71188	10,79901	12,70942	14,48990	16,16700	17,75660	19,26867
Aggregated 2nd Eigenvalue Index (AI7)	0,06752	0,13049	0,18920	0,24395	0,29500	0,34260	0,38699	0,42838	0,46698
Aggregated All-Eigenvalues Index (AI8)	0,31820	0,50204	0,61673	0,69337	0,74765	0,78795	0,81903	0,84371	0,86377
Aggregated Asymptotic-half-life Index (AI9) [**]	4,944E- 05	4,944E- 05	4,944E- 05	4,944E- 05	4,944E- 05	4,944E- 05	4,944E- 05	4,944E- 05	4,944E- 05

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

Outside ATECO, Innovative Small & Medium Businesses: Total Debts									
Transition period	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
	2006-2007	2006-2008	2006-2009	2006-2010	2006-2011	2006-2012	2006-2013	2006-2014	2006-2015
Aggregated Norm. Directional Index (AI1)	0,02013	0,03779	0,05124	0,09250	0,12100	0,12201	0,14540	0,16940	0,18356
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02715	0,04689	0,06143	0,07961	0,09446	0,09905	0,11213	0,12210	0,12953
Aggregated Trace Index (AI3)	0,33421	0,52740	0,64237	0,73752	0,79355	0,81397	0,83721	0,85543	0,87473
Aggregated Gener. Determinant Index (AI4)	0,99813	0,99921	0,99955	0,99973	0,99991	0,99988	0,99989	0,99987	0,99989
Aggregated Index of Predictability (AI5)	5,37734	3,42701	2,66054	2,02129	1,79594	1,54139	1,60785	1,46994	1,28246
Aggregated 2nd Index of Bartholomew (AI6)	3,94829	7,32259	9,68355	12,77024	14,80575	15,68586	17,34693	18,70836	19,77280
Aggregated 2nd Eigenvalue Index (AI7)	0,00000	0,14304	0,19328	0,23991	0,29838	0,29648	0,32536	0,35982	0,39448
Aggregated All-Eigenvalues Index (AI8)	0,33421	0,52740	0,64237	0,73743	0,79328	0,81396	0,83719	0,85542	0,87473
Aggregated Asymptotic-half-life Index (AI9) [**]	0,000E+00	1,258E-04	6,241E-05	4,079E-05	5,657E-05	7,309E-06	4,425E-06	3,984E-06	3,976E-06

Outside ATECO, Innovative Small & Medium Businesses: Total Debts									
Transition period	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
	2006-2007	2006-2008	2006-2009	2006-2010	2006-2011	2006-2012	2006-2013	2006-2014	2006-2015
Aggregated Norm. Directional Index (AI1)	0,01372	0,03133	0,05113	0,07230	0,09430	0,11680	0,13952	0,16227	0,18489
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02853	0,04653	0,06119	0,07422	0,08632	0,09780	0,10881	0,11944	0,12974
Aggregated Trace Index (AI3)	0,39771	0,58228	0,68150	0,74242	0,78394	0,81443	0,83802	0,85692	0,87248
Aggregated Gener. Determinant Index (AI4)	0,99966	0,99966	0,99966	0,99966	0,99966	0,99966	0,99966	0,99966	0,99966
Aggregated Index of Predictability (AI5)	4,31437	2,83048	2,19163	1,83107	1,59937	1,44022	1,32697	1,24515	1,18607
Aggregated 2nd Index of Bartholomew (AI6)	4,72154	7,85149	10,25434	12,27090	14,04977	15,66320	17,15026	18,53431	19,83054
Aggregated 2nd Eigenvalue Index (AI7)	0,05445	0,10593	0,15461	0,20064	0,24416	0,28532	0,32423	0,36102	0,39581
Aggregated All-Eigenvalues Index (AI8)	0,39771	0,58228	0,68150	0,74242	0,78394	0,81443	0,83802	0,85692	0,87248
Aggregated Asymptotic-half-life Index (AI9) [**]	4,198E-06	4,198E-06	4,198E-06	4,198E-06	4,198E-06	4,198E-06	4,198E-06	4,198E-06	4,198E-06

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

Outside ATECO, Innovative Small & Medium Businesses: Total Production Monetary Value									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] not-stationary Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,02092	0,04747	0,02500	0,05870	0,08784	0,08577	0,10843	0,11600	0,13180
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02782	0,05344	0,06059	0,07546	0,08551	0,09068	0,10209	0,10812	0,11660
Aggregated Trace Index (AI3)	0,34908	0,60708	0,68492	0,75305	0,78585	0,79828	0,83005	0,84746	0,86542
Aggregated Gener. Determinant Index (AI4)	0,99888	0,99999	0,99999	0,99998	0,99994	0,99989	0,99984	0,99986	0,99986
Aggregated Index of Predictability (AI5)	5,23885	3,32645	2,37013	1,84286	1,56241	1,44845	1,20358	1,12100	1,12724
Aggregated 2nd Index of Bartholomew (AI6)	4,10954	8,15655	9,81914	12,18469	13,77257	14,62600	16,20123	17,11041	18,12948
Aggregated 2nd Eigenvalue Index (AI7)	0,08855	0,24229	0,19105	0,27684	0,30741	0,27527	0,34781	0,33977	0,37858
Aggregated All-Eigenvalues Index (AI8)	0,34908	0,60699	0,68481	0,75221	0,78553	0,79823	0,83005	0,84746	0,86542
Aggregated Asymptotic-half-life Index (AI9) [**]	5,664E-04	6,762E-03	5,503E-05	1,927E-04	7,985E-05	2,455E-06	1,175E-05	1,582E-06	2,019E-06

Outside ATECO, Innovative Small & Medium Businesses: Total Production Monetary Value									
	Aggregated Mobility Index value from 2006 to 2006 + $n$ according to the simple [*] time-homogeneous Markov chain								
Transition period	2006- 2007	2006- 2008	2006- 2009	2006- 2010	2006- 2011	2006- 2012	2006- 2013	2006- 2014	2006- 2015
Aggregated Norm. Directional Index (AI1)	0,01012	0,02294	0,03732	0,05268	0,06868	0,08508	0,10170	0,11842	0,13511
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,02691	0,04366	0,05713	0,06895	0,07975	0,08985	0,09941	0,10852	0,11725
Aggregated Trace Index (AI3)	0,38061	0,56352	0,66510	0,72907	0,77337	0,80614	0,83151	0,85181	0,86842
Aggregated Gener. Determinant Index (AI4)	0,99947	0,99947	0,99947	0,99947	0,99947	0,99947	0,99947	0,99947	0,99947
Aggregated Index of Predictability (AI5)	4,46812	2,89590	2,17599	1,75565	1,47964	1,28595	1,14407	1,03703	0,95460
Aggregated 2nd Index of Bartholomew (AI6)	4,41354	7,37142	9,64041	11,53128	13,18390	14,66887	16,02602	17,28007	18,44759
Aggregated 2nd Eigenvalue Index (AI7)	0,05281	0,10283	0,15021	0,19509	0,23760	0,27786	0,31600	0,35212	0,38634
Aggregated All-Eigenvalues Index (AI8)	0,38056	0,56345	0,66505	0,72905	0,77335	0,80613	0,83151	0,85180	0,86842
Aggregated Asymptotic-half-life Index (AI9) [**]	2,830E-06	2,830E-06	2,830E-06	2,830E-06	2,830E-06	2,830E-06	2,830E-06	2,830E-06	2,830E-06

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

**The following mobility values for Italian Innovative Start-ups are not scientifically rigorous, owing to the exiguous number of companies residing in the database for the entire epoch 2010-2015: they are exclusively an example of rough and misleading inference.**

Outside ATECO, Innovative Start-Ups: Total Sales					
	Aggregated Mobility Index value from 2010 to 2010 + $n$ for the simple [*] not-stationary Markov chain				
Transition period	2010-2011	2010-2012	2010-2013	2010-2014	2010-2015
Aggregated Norm. Directional Index (AI1)	0,25855	0,23452	0,26359	0,34819	0,42318
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,18751	0,20294	0,19047	0,22501	0,24955
Aggregated Trace Index (AI3)	1,15000	1,16667	1,14167	1,13333	1,15833
Aggregated Gener. Determinant Index (AI4)	1,00000	1,00000	1,00000	1,00000	1,00000
Aggregated Index of Predictability (AI5)	1,77500	0,05833	-0,04167	-0,12292	-0,16015
Aggregated 2nd Index of Bartholomew (AI6)	4,75000	5,87500	5,54167	6,20139	6,80671
Aggregated 2nd Eigenvalue Index (AI7)	1,00000	0,72571	0,90531	0,95147	0,97628
Aggregated All-Eigenvalues Index (AI8)	1,15000	1,02952	1,09432	1,10907	1,14647
Aggregated Asymptotic-half-life Index (AI9) [**]	1,00000	0,34243	0,41387	0,39997	0,39602

Outside ATECO, Innovative Start-Ups: Total Sales					
	Aggregated Mobility Index value from 2010 to 2010 + $n$ for the simple [*] time-homogeneous Markov chain				
Transition period	2010-2011	2010-2012	2010-2013	2010-2014	2010-2015
Aggregated Norm. Directional Index (AI1)	0,07863	0,14066	0,21626	0,29084	0,35967
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,11526	0,16938	0,21261	0,24607	0,27405
Aggregated Trace Index (AI3)	0,54167	0,69482	0,78200	0,82696	0,85134
Aggregated Gener. Determinant Index (AI4)	0,99916	0,99916	0,99916	0,99916	0,99916
Aggregated Index of Predictability (AI5)	3,00223	1,85812	1,57981	1,55546	1,61856
Aggregated 2nd Index of Bartholomew (AI6)	3,28869	5,17578	6,41367	7,23849	7,83015
Aggregated 2nd Eigenvalue Index (AI7)	0,09349	0,17824	0,25506	0,32471	0,38784
Aggregated All-Eigenvalues Index (AI8)	0,50636	0,66135	0,74526	0,79587	0,82876
Aggregated Asymptotic-half-life Index (AI9) [**]	8,571E-04	8,571E-04	8,571E-04	8,571E-04	8,571E-04

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

Outside ATECO, Innovative Start-Ups: Total Assets					
	Aggregated Mobility Index value from 2010 to 2010 + $n$ for the simple [*] not-stationary Markov chain				
Transition period	2010-2011	2010-2012	2010-2013	2010-2014	2010-2015
Aggregated Norm. Directional Index (AI1)	0,19400	0,23685	0,33749	0,30118	0,36384
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,12917	0,15207	0,17692	0,16510	0,18903
Aggregated Trace Index (AI3)	1,05769	1,13462	1,15385	1,17735	1,18162
Aggregated Gener. Determinant Index (AI4)	1,00000	1,00000	1,00000	1,00000	1,00000
Aggregated Index of Predictability (AI5)	0,76183	0,39201	-0,19149	-0,44315	-0,61083
Aggregated 2nd Index of Bartholomew (AI6)	3,07692	3,75000	4,61538	4,49786	5,00000
Aggregated 2nd Eigenvalue Index (AI7)	1,00000	1,00000	1,00000	0,81883	0,92618
Aggregated All-Eigenvalues Index (AI8)	1,05769	1,13462	1,15385	1,08677	1,14471
Aggregated Asymptotic-half-life Index (AI9) [**]	1,00000	1,00000	1,00000	0,19731	0,26451

Outside ATECO, Innovative Start-Ups: Total Assets					
	Aggregated Mobility Index value from 2010 to 2010 + $n$ for the simple [*] time-homogeneous Markov chain				
Transition period	2010-2011	2010-2012	2010-2013	2010-2014	2010-2015
Aggregated Norm. Directional Index (AI1)	0,13790	0,22538	0,31244	0,39190	0,46317
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,12852	0,18907	0,23993	0,28087	0,31514
Aggregated Trace Index (AI3)	0,50256	0,63773	0,74096	0,80604	0,84803
Aggregated Gener. Determinant Index (AI4)	0,99595	0,99595	0,99595	0,99595	0,99595
Aggregated Index of Predictability (AI5)	2,56976	1,79784	1,62825	1,65376	1,77053
Aggregated 2nd Index of Bartholomew (AI6)	3,67837	5,42338	6,78139	7,77364	8,51521
Aggregated 2nd Eigenvalue Index (AI7)	0,11532	0,21734	0,30760	0,38745	0,45809
Aggregated All-Eigenvalues Index (AI8)	0,45552	0,61815	0,71455	0,77865	0,82307
Aggregated Asymptotic-half-life Index (AI9) [**]	3,493E-03	3,493E-03	3,493E-03	3,493E-03	3,493E-03

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

Outside ATECO, Innovative Start-Ups: Total Debts					
	Aggregated Mobility Index value from 2010 to 2010 + $n$ for the simple [*] not-stationary Markov chain				
Transition period	2010-2011	2010-2012	2010-2013	2010-2014	2010-2015
Aggregated Norm. Directional Index (AI1)	0,08663	0,09038	0,23559	0,26758	0,33593
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,06484	0,06637	0,14528	0,16301	0,19123
Aggregated Trace Index (AI3)	1,01923	1,03846	1,09615	1,13462	1,19231
Aggregated Gener. Determinant Index (AI4)	1,00000	1,00000	1,00000	1,00000	1,00000
Aggregated Index of Predictability (AI5)	1,07249	0,90976	0,52515	0,40680	0,25888
Aggregated 2nd Index of Bartholomew (AI6)	1,63462	1,73077	3,55769	4,03846	4,80769
Aggregated 2nd Eigenvalue Index (AI7)	1,00000	1,00000	1,00000	1,00000	1,00000
Aggregated All-Eigenvalues Index (AI8)	1,01923	1,03846	1,09615	1,13462	1,19231
Aggregated Asymptotic-half-life Index (AI9) [**]	1,00000	1,00000	1,00000	1,00000	1,00000

Outside ATECO, Innovative Start-Ups: Total Debts					
	Aggregated Mobility Index value from 2010 to 2010 + $n$ for the simple [*] time-homogeneous Markov chain				
Transition period	2010-2011	2010-2012	2010-2013	2010-2014	2010-2015
Aggregated Norm. Directional Index (AI1)	0,13531	0,26907	0,39004	0,49494	0,58392
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,08721	0,15860	0,22238	0,27872	0,32796
Aggregated Trace Index (AI3)	0,36250	0,57438	0,70403	0,78725	0,84313
Aggregated Gener. Determinant Index (AI4)	0,94191	0,94191	0,94191	0,94191	0,94191
Aggregated Index of Predictability (AI5)	2,55411	1,99132	1,92905	2,06661	2,30605
Aggregated 2nd Index of Bartholomew (AI6)	2,53333	4,55375	6,15394	7,42146	8,42834
Aggregated 2nd Eigenvalue Index (AI7)	0,20000	0,36000	0,48800	0,59040	0,67232
Aggregated All-Eigenvalues Index (AI8)	0,36250	0,57438	0,70403	0,78725	0,84313
Aggregated Asymptotic-half-life Index (AI9) [**]	0,04477	0,04477	0,04477	0,04477	0,04477

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$

**The following mobility values for Italian Innovative Start-ups are not scientifically rigorous, owing to the exiguous number of companies residing in the database for the entire epoch 2010-2015: they are exclusively an example of rough and misleading inference.**

Outside ATECO, Innovative Start-Ups: Total Production Monetary Value					
	Aggregated Mobility Index value from 2010 to 2010 + $n$ for the simple [*] not-stationary Markov chain				
Transition period	2010-2011	2010-2012	2010-2013	2010-2014	2010-2015
Aggregated Norm. Directional Index (AI1)	0,11824	0,11155	0,09301	0,13131	0,23343
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,08285	0,09017	0,10014	0,09029	0,12919
Aggregated Trace Index (AI3)	1,06818	1,10227	1,11526	1,05763	1,13961
Aggregated Gener. Determinant Index (AI4)	1,00000	1,00000	1,00000	1,00000	1,00000
Aggregated Index of Predictability (AI5)	0,71281	-0,12397	0,05334	-0,32462	-0,67022
Aggregated 2nd Index of Bartholomew (AI6)	2,15909	2,61364	2,84091	2,55682	3,73377
Aggregated 2nd Eigenvalue Index (AI7)	1,00000	0,93111	0,93544	0,94120	0,94743
Aggregated All-Eigenvalues Index (AI8)	1,06818	1,06783	1,08298	1,02823	1,11333
Aggregated Asymptotic-half-life Index (AI9) [**]	1,00000	0,59559	0,46820	0,37589	0,30834

Outside ATECO, Innovative Start-Ups: Total Production Monetary Value					
	Aggregated Mobility Index value from 2010 to 2010 + $n$ for the simple [*] time-homogeneous Markov chain				
Transition period	2010-2011	2010-2012	2010-2013	2010-2014	2010-2015
Aggregated Norm. Directional Index (AI1)	0,05248	0,12234	0,19423	0,26501	0,33260
Aggregated Norm. Alcalde-Unzu Index (AI2)	0,12291	0,15682	0,18902	0,21773	0,24525
Aggregated Trace Index (AI3)	0,50417	0,61864	0,70622	0,76383	0,80518
Aggregated Gener. Determinant Index (AI4)	0,99633	0,99633	0,99633	0,99633	0,99633
Aggregated Index of Predictability (AI5)	2,26297	1,65939	1,55487	1,58420	1,66661
Aggregated 2nd Index of Bartholomew (AI6)	3,53571	4,67662	5,62259	6,37532	7,01788
Aggregated 2nd Eigenvalue Index (AI7)	0,10519	0,19931	0,28354	0,35890	0,42634
Aggregated All-Eigenvalues Index (AI8)	0,46331	0,61864	0,70594	0,76383	0,80517
Aggregated Asymptotic-half-life Index (AI9) [**]	1,957E-03	1,957E-03	1,957E-03	1,957E-03	1,957E-03

Exponent “alfa” inside the Aggregated Alcalde-Unzu Index is fixed as  $\alpha = 1,3$

Exponent “b” inside the Aggregated Generalized Determinant Index is fixed as  $b = 1,5$

[\*]: Simple Markov chain  $\equiv$  1<sup>st</sup>-order Markov chain

[\*\*]: The scientific notation has been adopted, that is  $nE-m \equiv n \cdot 10^{-m}$