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Changes in Risky Benefits and in Risky Costs: A Question of the Right Order

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Abstract. We organize and extend findings on the comparative static effects of risk changes on optimal behavior in a unifying expected utility model. We determine restrictions on preferences for clear-cut results. Risk increases of a benefit are compensated by lowering exposure to risk. For risk increases of a cost, the response depends on the order of the risk change. This discrepancy arises because even-order risk increases of a cost raise the riskiness of the payoff distribution, whereas odd-order risk increases of a cost reduce it. We identify the stochastic dominance orders to resolve this discrepancy and discuss specific decision problems as applications.

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1. Introduction

How do risk changes affect behavior? In their seminal paper, Rothschild and Stiglitz (1970) introduced an increase in risk as a change in the distribution of a random variable that makes every risk-averse decision maker (henceforth DM) worse off. The same authors noted in a companion paper that an increase in risk may have counterintuitive comparative statics (Rothschild and Stiglitz 1971). In the expected utility model, risk-averse investors with a positive demand for a risky asset may rationally prefer to *increase* exposure to risk when the asset's return distribution becomes riskier. Starting from this puzzling observation, different strategies have been explored in the literature to obtain more consistent results. One can impose restrictions on the payoff function, limit the class of DMs, or place stronger restrictions on the risk change.[1]

In this paper, we provide a unifying expected utility framework to organize and extend results on the comparative static effects of risk on behavior. We use the so-called linear-payoff model of Dionne et al. (1993), which encompasses many applications as special cases including the standard portfolio problem and the coinsurance problem. In the spirit of Athey (2002) and Nocetti (2016), we work with monotone comparative statics to avoid unnecessarily constraining regularity assumptions and also, cover higher-order risk effects (see Eeckhoudt and Schlesinger 2006). Our unified treatment allows us to distinguish between risk taking and risk mitigation, depending on whether a higher level of the decision variable increases or reduces the variability of payoffs, and between a risky benefit and a risky cost, depending on whether higher realizations of the random variable increase or decrease the DM's welfare.

We contribute to the literature in several ways. First, we derive the monotone comparative statics of the Ekern (1980) Nth-degree risk increases in a general model of optimal behavior. Second, we find that the distinction between risk taking and risk mitigation is inconsequential in our analysis, whereas that between a risky benefit and a risky cost matters. Table 1 summarizes our results in nontechnical terms. When a benefit is risky, the DM compensates risk increases by reduced risk taking and increased risk mitigation, regardless of the order of the risk increase. When a cost is risky instead, we recoup these behavioral effects only for even-order risk increases but find the opposite when the order of the risk increase is odd. This is because the link between the riskiness of a cost and the riskiness of the payoff distribution depends on the

Table 1. Effect of Nth-Degree Risk Increases on Risk Taking and Risk Mitigation (See Propositions 1 and 2)

	Order of risk increase	Risk taking	Risk mitigation
Risky benefit	*Any*	Decreases	Increases
Risky cost	*Even*	Decreases	Increases
	Odd	Increases	Decreases

Notes. The top row means that a DM who faces a risky benefit and experiences any order of risk increase will decrease risk taking and increase risk mitigation. The bottom row shows that the order of the risk increase matters for optimal behavior in case of a risky cost.

parity of the risk increase, making its comparative static effects a question of its order. Third, we extend our results to stochastic dominance. The alternating pattern for a risky cost implies that different stochastic ordering assumptions need to be applied to a risky benefit than to a risky cost. Obtaining consistent results is yet again a question of the right order. Finally, we apply our results to specific decision problems that have been studied independently in the literature. This produces several new results and provides new insights into known results.

2. Classification of Decision Problems

We work with the so-called linear-payoff model suggested by Dionne et al. (1993). It contains several well-known decision problems as special cases (see Section 6). In this framework, the DM's payoff can be written as

$$z(X,b) = f(b)\cdot(X - x_0) + z_0. \tag{1}$$

X is the realization of a positive random variable $\tilde{X} \geq 0$, and x_0 and z_0 are constants. b is shorthand for the DM's behavior, and f links behavior to the payoff. The real-valued function $f: B \to C$ is continuous with compact domain $B \subset \mathbb{R}$ and codomain $C \subset \mathbb{R}$. The DM has an increasing and sufficiently differentiable utility function $U(z)$ and solves

$$\max_{b\in B} \mathbb{E}U(z(\tilde{X},b)). \tag{2}$$

The continuity of f ensures the existence of a solution, which may be set valued.[2] Properties of f allow us to classify decision problems into four categories.

Definition 1. Problem (2) represents

Risk taking with a risky benefit if f is increasing and $C \subset \mathbb{R}_0^+$;

Risk mitigation with a risky benefit if f is decreasing and $C \subset \mathbb{R}_0^+$;

Risk mitigation with a risky cost if f is increasing and $C \subset \mathbb{R}_0^-$;

Risk taking with a risky cost if f is decreasing and $C \subset \mathbb{R}_0^-$.

This taxonomy is motivated by the following observation. If f takes positive values, a higher realization of the random variable raises the DM's payoff, so $\tilde{X}$ is a benefit or return. If f takes negative values, a higher realization of the random variable lowers the DM's payoff, so $\tilde{X}$ is a cost or loss. The variance of the payoff distribution is given by

$$\mathrm{Var}[z(\tilde{X},b)] = f(b)^2 \cdot \mathrm{Var}[\tilde{X}]. \tag{3}$$

An increase in b raises the variance of payoffs if f takes positive (negative) values and is increasing (decreasing), consistent with risk taking. An increase in b lowers the variance of payoffs if f takes positive (negative) values and is decreasing (increasing), suggesting risk mitigation. The distinction is whether "doing more" leads to a more or less volatile payoff distribution. The values of f have a uniform sign, so Problem (2) only entails a trade-off if the random variable has realizations above and below x_0, each with strictly positive probability.[3]

3. Nth-Degree Risk Increases

We will start with the Ekern (1980) concept of an Nth-degree risk increase in our analysis. Consider two random variables $\tilde{X}_1$ and $\tilde{X}_2$ defined over the interval $[\underline{x},\bar{x}]$, and let F and G denote their cumulative distribution functions. Define $F^{(k)}$ on $[\underline{x},\bar{x}]$ inductively by $F^{(1)}(z) = F(z)$ and $F^{(k+1)}(z) = \int_{\underline{x}}^{z} F^{(k)}(t)\,\mathrm{d}t$ for $k \geq 1$ and likewise for G.

Definition 2. (Ekern 1980) $\tilde{X}_2$ is an Nth-degree risk increase over $\tilde{X}_1$ if

i. $F^{(k)}(\bar{x}) = G^{(k)}(\bar{x})$ for all $k = 1,\ldots,N$,

ii. $F^{(N)}(z) \leq G^{(N)}(z)$ for all $z \in [\underline{x},\bar{x}]$.

The strict version of this definition is obtained by requiring that (ii) holds strictly for some $z \in [\underline{x},\bar{x}]$. If $\tilde{X}_2$ is an Nth-degree risk increase over $\tilde{X}_1$, we can likewise call $\tilde{X}_1$ an Nth-degree risk decrease over $\tilde{X}_2$. An Nth-degree risk increase preserves the first $(N-1)$ moments of the distribution and increases the Nth one, sign adjusted by $(-1)^N$. Special cases include an increase in risk for $N = 2$ (Rothschild and Stiglitz 1970), a downside risk increase for $N = 3$ (Menezes et al. 1980), and an outer risk increase for $N = 4$ (Menezes and Wang 2005). Let $U^{(N)}(z)$ denote $\mathrm{d}^N U(z)/\mathrm{d}z^N$.

Lemma 1. (Ekern 1980) *The following two statements are equivalent:*

i. *$\tilde{X}_2$ is an Nth-degree risk increase over $\tilde{X}_1$.*

ii. $\mathbb{E}U(\tilde{X}_2) \leq \mathbb{E}U(\tilde{X}_1)$ *for any function $U(z)$ such that* $(-1)^{N+1}U^{(N)}(z) \geq 0$.

This is the main result of Ekern (1980). A risk increase of any given order can be equivalently characterized in terms of its distributional properties as stated in Definition 2 or by its impact on expected utility for an appropriately defined set of utility functions. Jouini et al. (2013) show that, if any Nth-degree risk increase makes the DM worse off, his utility function must necessarily satisfy sign condition (ii) at the corresponding order (see also lemma 1 in Nocetti 2016). This motivates the following definition.

Definition 3. (Ekern 1980) A DM is called Nth-degree risk averse (risk neutral, risk loving) if $(-1)^{N+1}U^{(N)}(z) \geq (=, \leq) 0$ on the relevant domain of his utility function.

Nth-degree risk aversion generalizes the concepts of risk aversion, prudence, and temperance to higher orders. Nth-degree risk increases are precisely those risk changes that make every Nth-degree risk averter worse off and every Nth-degree risk lover better off; they do not affect Nth-degree risk-neutral DMs. Before we proceed, we provide a simple but important result on how taking the mirror image affects Nth-degree riskiness.

Lemma 2. *Let $\tilde{X}_2$ be an Nth-degree risk increase over $\tilde{X}_1$. Then, $-\tilde{X}_2$ is an Nth-degree risk increase (decrease) over $-\tilde{X}_1$ if N is even (odd).*

Lemma 2 follows easily from Lemma 1. In an earlier working paper version, we provide an elementary proof where we verify Definition 2 directly. Intuitively, taking the mirror image preserves even moments because positive and negative deviations from the mean are treated equally. Odd moments instead are sign flipped because positive deviations from the mean become negative and vice versa. Odd-order risk increases are directional, whereas even-order risk increases are not; for example, a third-degree risk increase is a transfer of dispersion from higher to lower outcomes (Menezes et al. 1980). This explains why the ranking of $-\tilde{X}_1$ and $-\tilde{X}_2$ in terms of Nth-degree riskiness depends on the parity of N.

4. *N*th-Degree Risk Effects in the Linear-Payoff Model

Based on the taxonomy developed in Definition 1, we now apply the Topkis (1978) Monotonicity Theorem to determine the comparative static effects of Nth-degree risk increases on behavior. If $\tilde{X}_2$ is an Nth-degree risk increase over $\tilde{X}_1$, the DM's optimal behavior is given by

$$B_i^* = \arg\max_{b \in B} \mathbb{E}U(z(\tilde{X}_i, b)),\ i = 1, 2. \tag{4}$$

B_1^* and B_2^* are nonempty under our assumptions and may be set valued. We use the strong set order to compare the maximizers and write $\geq_S$ and $\leq_S$ for greater than and smaller than, respectively in the strong set order.[4] Lemma 3 holds.

Lemma 3. *Let $\tilde{X}_2$ be an Nth-degree risk increase over $\tilde{X}_1$. Then, $B_2^* \geq_S B_1^*$ ($B_2^* \leq_S B_1^*$) if*

$$(-1)^N f(b)^N U^{(N)}(f(b) \cdot (X - x_0) + z_0) \tag{5}$$

is increasing (decreasing) in b for all X.

Lemma 3 follows from condition 2 in proposition 1 of Nocetti (2016) when applied to Program (2). Our next result shows how the taxonomy in Definition 1 matters for the comparative statics of risk increases. A proof is given in Appendix A.

Proposition 1. *Consider a DM who is $(N+1)$th-degree risk neutral, and let $\tilde{X}_2$ be an Nth-degree risk increase over $\tilde{X}_1$.*

i. *In the risk-taking problem with a risky benefit, $B_2^* \leq_S B_1^*$ ($B_2^* \geq_S B_1^*$) for Nth-degree risk averters (lovers).*

ii. *In the risk-mitigation problem with a risky benefit, $B_2^* \geq_S B_1^*$ ($B_2^* \leq_S B_1^*$) for Nth-degree risk averters (lovers).*

iii. *In the risk-taking problem with a risky cost, $B_2^* \leq_S B_1^*$ ($B_2^* \geq_S B_1^*$) for Nth-degree risk averters (lovers) if N is even; the orderings are reversed if N is odd.*

iv. *In the risk-mitigation problem with a risky cost, $B_2^* \geq_S B_1^*$ ($B_2^* \leq_S B_1^*$) for Nth-degree risk averters (lovers) if N is even; the orderings are reversed if N is odd.*

In the risk-taking problem, an increase (decrease) in the decision variable raises (lowers) the variance of the payoff distribution. So, if the risk increase from $\tilde{X}_1$ to $\tilde{X}_2$ increases (decreases) the set of maximizers, this represents more (less) risk taking. Similarly, higher (lower) values of b correspond to a lower (higher) variance of the payoff distribution in the risk-mitigation problem. Hence, if the risk increase from $\tilde{X}_1$ to $\tilde{X}_2$ increases (decreases) the set of maximizers, this corresponds to more (less) risk mitigation. We summarize the results of Proposition 1 for Nth-degree risk averters in compact form in Table 1 (Section 1).[5]

Proposition 1 shows that what matters is the distinction between a risky benefit and a risky cost but not the distinction between risk taking and risk mitigation. For a risky benefit, a risk increase leads to less risk taking and more risk mitigation regardless of the order of the risk change. When a cost is risky, we observe a dichotomy in the DM's response. For an even-order risk increase, there is less risk taking and more risk mitigation just as in the case of a risky benefit. However, when the order of the risk increase is odd, we find more risk taking and less risk mitigation in response to the increased risk. The DM's behavioral response appears to be the opposite of what we find in case of a risky benefit.

To explain this seeming reversal, we recall Lemma 2. It showed that taking the mirror image preserves the ranking in terms of Nth-degree riskiness for N even but reverses it for N odd. An Nth-degree risk increase of a risky benefit always increases the Nth-degree riskiness of the payoff distribution. This lowers the welfare of an Nth-degree risk-averse DM, and he reacts by reducing exposure to risk.[6] For a risky cost, the Nth-degree risk increase raises the Nth-degree riskiness of the payoff distribution if N is even but lowers it if N is odd. The link between the Nth-degree riskiness of the cost and the Nth-degree riskiness of the payoff distribution now depends on the order of the risk change because of Lemma 2. An Nth-degree risk-averse DM is worse off for N even and thus, reduces exposure to risk, but it is better off for N odd, allowing him to increase exposure to risk.[7]

We will now relax the assumption of $(N+1)$ th-degree risk neutrality in Proposition 1.

Definition 4. (Eeckhoudt and Schlesinger 2008) When $(-1)^{N+1}U^{(N)}(z) > 0$, the DM's relative $(N+1)$ th-degree risk aversion is defined as $r_{N+1}(z) = -zU^{(N+1)}(z)/U^{(N)}(z)$.

The index $r_2(z)$ is conventional relative risk aversion, which dates back to Arrow (1963) and Pratt (1964). $r_3(z)$ and $r_4(z)$ denote relative prudence and relative temperance, respectively. Eeckhoudt and Schlesinger (2008) show that an Nth-degree risk increase in the interest rate raises precautionary saving if and only if relative Nth-degree risk aversion exceeds N.[8] Chiu et al. (2012) interpret this threshold condition in terms of a preference over simple lotteries. Our next result focuses on Nth-degree risk averters but could easily be extended to Nth-degree risk lovers.

Proposition 2. *Consider a DM who is Nth- and $(N+1)$th-degree risk averse with relative $(N+1)$th-degree risk aversion below N. Let $\tilde{X}_2$ be an Nth-degree risk increase over $\tilde{X}_1$.*

i. *For a risky benefit, $B_2^* \leq_S B_1^*$ in the risk-taking problem, and $B_2^* \geq_S B_1^*$ in the risk-mitigation problem.*

ii. *For a risky cost, $B_2^* \leq_S B_1^*$ in the risk-taking problem , and $B_2^* \geq_S B_1^*$ in the risk-mitigation problem if N is even; the orderings are reversed if N is odd.*

We provide a proof in Appendix B. Proposition 2 confirms that the distinction between a risky benefit and a risky cost matters. If f is differentiable, there is a simple direct proof, which helps uncover the economic intuition. The derivative of (5) with respect to b is

$$f'(b)f(b)^{N-1}\cdot\Big[f(b)\cdot(X-x_0)\cdot(-1)^{N+2}U^{(N+1)}(f(b)\cdot(X-x_0)+z_0)-N(-1)^{(N+1)}U^{(N)}(f(b)\cdot(X-x_0)+z_0)\Big]. \tag{7}$$

Setting $t = f(b)(X-x_0)+z_0$, we rearrange the term in the square bracket to

$$\underbrace{(-1)^{N+1}U^{(N)}(t+z_0)}_{\geq 0}\cdot\underbrace{\big[r_N(t+z_0)-N\big]}_{\leq 0} - \underbrace{z_0(-1)^{N+2}U^{(N+1)}(t+z_0)}_{\geq 0}. \tag{8}$$

This is negative under our assumptions. The monotonicity of (5) in b then depends entirely on the sign of $f'(b)f(b)^{N-1}$. For a risky benefit, the codomain of f is positive so $f(b)^{N-1}$ is also positive. In the risk-taking problem, f is increasing so (7) is negative and (5) is decreasing in b; in the risk-mitigation problem, f is decreasing so (7) is positive and (5) is increasing in b. The ordering of the maximizers then follows from Lemma 3. For a risky cost, the codomain of f is negative so $f(b)^{N-1}$ is negative for N even and positive for N odd. f is decreasing in the risk-taking problem and increasing in the risk-mitigation problem. Combining signs accordingly determines the sign of (7), which then provides the monotonicity of (5) in b. Finally, Lemma 3 ranks the maximizers. Table 1 presents these results intuitively.

The restriction on relative $(N+1)$ th-degree risk aversion balances two economic effects, which can be gleaned from the square bracket in (7). The second term in the square bracket is negative, representing a substitution effect. If the risk change lowers the DM's welfare, he has an incentive to substitute certain consumption for risky consumption. He can achieve this by lowering exposure to risk. The first term in the square bracket is, however, sign ambiguous because random variables take values above and below x_0 with strictly positive probability (see footnote 3). If the Nth-degree riskiness of the payoff distribution increases, $(N+1)$th-degree risk aversion introduces a precautionary effect to raise consumption in order to better cope with the increased riskiness. This precautionary motive can affect b in either direction because the link between b and the payoff is different for $X > x_0$ than for $X < x_0$. Relative $(N+1)$ th-degree risk aversion below N ensures that the conflicting portion of the precautionary effect is dominated.

5. Stochastic Dominance

We will now extend our results to stochastic dominance orders, which are more general than Nth-degree risk increases. To obtain consistent results, different stochastic ordering assumptions need to be used for a risky benefit and a risky cost. This finding reinforces and expands the relevance of our main distinction in this paper. We start with the following definition.

Definition 5. (Liu 2014) For integers $N \geq 1$ and $0 \leq M \leq N-1$, $\tilde{X}_2$ is smaller than $\tilde{X}_1$ in the M moments preserving Nth-order stochastic dominance order if

i. $F^{(k)}(\bar{x}) \leq G^{(k)}(\bar{x})$ for $k = 1, \ldots, N$ with equality for $k = 1, \ldots, M+1$,

ii. $F^{(N)}(z) \leq G^{(N)}(z)$ for all $z \in [\underline{x}, \bar{x}]$.

For brevity, we say that $\tilde{X}_2$ is smaller than $\tilde{X}_1$ in the (M/N) order. For $M = N-1$, $\tilde{X}_2$ is an Nth-degree risk increase over $\tilde{X}_1$ because all of the first $(N-1)$ moments are preserved. For $M = 0$, $\tilde{X}_2$ is dominated by $\tilde{X}_1$ by Nth-degree stochastic dominance because none of the lower moments need to coincide. The (M/N) order provides an umbrella over these risk changes and allows for intermediate cases where some but not all lower moments are preserved. We now state the characterization of the (M/N) order in terms of expected utility.

Lemma 4. (Liu 2014) *The following two statements are equivalent:*

i. $\tilde{X}_2$ *is smaller than* $\tilde{X}_1$ *in the* (M/N) *order.*

ii. $\mathbb{E}U(\tilde{X}_2) \le \mathbb{E}U(\tilde{X}_1)$ *for any function* $U(z)$ *such that* $(-1)^{k+1}U^{(k)}(z) \ge 0$ *for* $k = M+1,\dots,N$.

DMs with such preferences are called mixed risk averse from order $(M+1)$ to N. Caballé and Pomansky (1996) introduced the term mixed risk aversion for utility functions that are Nth-degree risk averse for all $N \in \mathbb{N}$. These DMs exhibit a preference for combining good with bad (Eeckhoudt and Schlesinger 2006). Mixed risk aversion ensures mutual aggravation of risks and greater mutual aggravation for greater risks (Ebert et al. 2018).[9]

Risk changes in the (M/N) order have clear-cut comparative static effects in linear-payoff problems with a risky benefit. Let $\tilde{X}_2$ be smaller than $\tilde{X}_1$ in the (M/N) order, and let B_i^* be the associated maximizers for $i = 1, 2$. The arguments in section 6 of Nocetti (2016) imply that $B_2^* \ge_S B_1^*$ ($B_2^* \le_S B_1^*$) if $(-1)^k f(b)^k U^{(k)}(f(b)\cdot(X-x_0)+z_0)$ is increasing (decreasing) in b for all X and all $k = M+1,\dots,N$. We obtain the following result.

Theorem 1. (Risky Benefit) *For* $N \ge 1$ *and* $0 \le M \le N-1$, *consider a DM who is mixed risk averse from order* $(M+1)$ *to* $(N+1)$ *with relative* $(k+1)$*th-degree risk aversion below* k *for* $k = M+1,\dots,N$. *If* $\tilde{X}_2$ *is smaller than* $\tilde{X}_1$ *in the* (M/N) *order, then* $B_2^* \le_S B_1^*$ *in the risk-taking problem, and* $B_2^* \ge_S B_1^*$ *in the risk-mitigation problem.*

Clearly, Proposition 2(i) is a special case for $M = N-1$. If a risky benefit becomes smaller in the (M/N) order, the DM is worse off and compensates the risk change by lowering exposure to risk. For a risky cost, the (M/N) order is not suitable to generalize our results because the ranking of the maximizers alternates as we move up the orders for Ekern (1980) risk increases. Lemma 2 provides the clue for a remedy and motivates the following definition.

Definition 6. For integers $N \ge 1$ and $0 \le M \le N-1$, we say that $\tilde{X}_2$ is larger than $\tilde{X}_1$ in the reversed M moments preserving Nth-order stochastic dominance order if $-\tilde{X}_2$ is smaller than $-\tilde{X}_1$ in the M moments preserving Nth-order stochastic dominance order.

For brevity, we say that $\tilde{X}_2$ is larger than $\tilde{X}_1$ in the reversed (M/N) order. Because of Lemma 4, this is the case if and only if $\mathbb{E}U(\tilde{X}_2) \ge \mathbb{E}U(\tilde{X}_1)$ for any function $U(z)$ such that $U^{(k)}(z) \ge 0$ for $k = M+1,\dots,N$. We can therefore think of the reversed (M/N) order as the counterpart to the Liu (2014) (M/N) order but for mixed risk lovers. It is the integral order generated by the collection of the corresponding utility functions (see Denuit et al. 1999). The terminology "reversed" conveys that the ordering of the mirror images of two random variables with respect to the (M/N) order informs about their ranking in the reversed (M/N) order.[10]

Several special cases are useful for intuition.[11] When $\tilde{X}_2$ is larger than $\tilde{X}_1$ in the reversed $(N-1/N)$ order, then $\tilde{X}_2$ is an Nth-degree risk increase (decrease) over $\tilde{X}_1$ when N is even (odd). When applied to a risky cost, this lowers the welfare of Nth-degree risk averters because of Lemma 2. If $\tilde{X}_2$ is larger than $\tilde{X}_1$ in the reversed $\tilde{X}_1$ order, then $\tilde{X}_2$ is larger than $\tilde{X}_1$ in the so-called Nth-increasing convex order (Denuit et al. 1998). When applied to a risky cost, it makes DMs worse off who are mixed risk averse up to order N.

Risk changes in the reversed (M/N) order have unambiguous comparative static effects in linear-payoff problems with a risky cost. For payoff function (1), define $g(b) = -f(b)$, $\tilde{Y} = (\bar{x} - \tilde{X})$ and $y_0 = (\bar{x} - x_0)$. Then, it holds that

$$z(\tilde{X}, b) = f(b)\cdot(\tilde{X} - x_0) + z_0 = g(b)\cdot(\tilde{Y} - y_0) + z_0. \quad (9)$$

Function g has the opposite monotonicity and sign of the codomain than f. $\tilde{X}$ is a positive random variable with realizations above and below x_0, each with strictly positive probability. However, then $\tilde{Y}$ is also a positive random variable with realizations above and below y_0, each with strictly positive probability. Let $\tilde{X}_2$ be larger than $\tilde{X}_1$ in the reversed (M/N) order; then, $\tilde{Y}_2 = (\bar{x} - \tilde{X}_2)$ is smaller than $\tilde{Y}_1 = (\bar{x} - \tilde{X}_1)$ in the (M/N) order. We apply Theorem 1 to the cases where g represents a risky benefit, which covers precisely those cases where f represents a risky cost. This yields our last result.

Theorem 2. (Risky Cost) *For* $N \ge 1$ *and* $0 \le M \le N-1$, *consider a DM who is mixed risk averse from order* $(M+1)$ *to* $(N+1)$ *with relative* $(k+1)$*th-degree risk aversion bounded by* k *for* $k = M+1,\dots,N$. *If* $\tilde{X}_2$ *is larger than* $\tilde{X}_1$ *by the reversed* (M/N) *order, then* $B_2^* \le_S B_1^*$ *in the risk-taking problem, and* $B_2^* \ge_S B_1^*$ *in the risk-mitigation problem.*

Proposition 2(ii) is a special case for $M = N-1$. If a risky cost becomes larger in the reversed (M/N) order, the DM is worse off and compensates for the risk change by lowering exposure to risk. Collectively, Theorems 1 and 2 demonstrate that obtaining consistent results for mixed risk-averse DMs is a question of the right order. For a risky benefit, the (M/N) order is suitable, whereas a risky cost requires the reversed (M/N) order instead.

6. Applications

In this section, we present five applications to illustrate the versatility of the linear-payoff model and to show the implications of the differences between a risky benefit and a risky cost in several well-known decision problems.

6.1. Portfolio Problem

We start with the standard portfolio problem, which dates back to Rothschild and Stiglitz (1971) and Fishburn and Porter (1976). An investor allocates his wealth w between a risk-free asset with return $r \geq 0$ and a risky asset with return $\tilde{X}$. Decision variable α denotes the amount invested in the risky asset and $(w-\alpha)$ the amount invested risk free. We assume $0 \leq \alpha \leq wr/(r-\underline{x})$ to exclude short selling and ensure positive consumption. The investor maximizes his expected utility by solving $\max_\alpha \mathbb{E}U(\alpha\tilde{X} + (w-\alpha)r)$. The first row of Table 2 classifies the portfolio problem as a risk-taking problem with a risky benefit, and Theorem 1 applies.

Corollary 1. *Let the investor be mixed risk averse from order $(M+1)$ to $(N+1)$ with relative $(k+1)$th-degree risk aversion below k for $k = M+1, \ldots, N$. If the asset's return becomes smaller in the (M/N) order, the investor reduces his optimal investment.*

Corollary 1 contains proposition 2 of Chiu et al. (2012) as a special case for $M = N-1$ or $M = 0$. It also covers some of the results of Hadar and Seo (1990). Specifically, under the assumptions made, an Nth-degree risk increase lowers the optimal investment.

6.2. Coinsurance Problem

A related problem is the coinsurance problem, which was introduced by Mossin (1968) and Smith (1968). An insuree has initial wealth w, which is subject to a random loss $\tilde{X}$. He can insure a share of this loss against payment of a premium. Parameter β denotes the coinsurance rate, and we assume $0 \leq \beta \leq 1$.[12] The premium is given by $\beta(1+\lambda)\mu$ where $\lambda \geq 0$ denotes a loading factor and $\mu = \mathbb{E}\tilde{X} > 0$ the expected loss. The insuree maximizes expected utility by solving $\max_\beta \mathbb{E}U(w - \beta(1+\lambda)\mu - (1-\beta)\tilde{X})$. The second row of Table 2 classifies the coinsurance problem as a risk-mitigation problem with a risky cost, and Theorem 2 applies.

Corollary 2. *Let the insuree be mixed risk averse from order $(M+1)$ to $(N+1)$ with relative $(k+1)$th-degree risk aversion below k for $k = M+1, \ldots, N$. If the loss becomes larger in the reversed (M/N) order, the insuree increases his optimal insurance demand.*

Dionne and Gollier (1992), Hadar and Seo (1992), and Meyer (1992) provide some results on first- and second-order risk effects in the coinsurance model. Corollary 2 contains their results as special cases and extends them to higher orders. In particular, under our assumptions, an Nth-degree risk increase in the loss distribution increases optimal insurance demand if N is even and reduces it if N is odd. The coinsurance problem has rarely been studied separately in the literature because of the commonly held belief that it is isomorphic to the standard portfolio problem.[13] A comparison of Corollaries 1 and 2 reveals that consistent results are only obtained when the stochastic dominance order is adjusted accordingly.

6.3. Output Choice

Sandmo (1971) introduced the problem of optimal output choice in a competitive market. w denotes the

Table 2. Examples of Linear-Payoff Problems and Their Classification

Problem	Type	x_0	z_0	$f(b)$	B	C
Portfolio problem	Risk taking, risky benefit	r	wr	$f(b) = b$ with $b = \alpha$	$\left[0, \frac{wr}{r-\underline{x}}\right]$	$\left[0, \frac{wr}{r-\underline{x}}\right]$
Coinsurance problem	Risk mitigation, risky cost	$(1+\lambda)\mu$	$w-(1+\lambda)\mu$	$f(b) = -(1-b)$ with $b = \beta$	$[0,1]$	$[-1,0]$
Output choice with risky price	Risk taking, risky benefit	c	$w-F$	$f(b) = b$ with $b = y$	$\left[0, \frac{w-F}{c-\underline{x}}\right]$	$\left[0, \frac{w-F}{c-\underline{x}}\right]$
Output choice with risky cost	Risk taking, risky cost	p	$w-F$	$f(b) = -b$ with $b = y$	$\left[0, \frac{w-F}{\bar{x}-p}\right]$	$\left[-\frac{w-F}{\bar{x}-p}, 0\right]$
Hedging a risky return	Risk mitigation, risky benefit	p	$w+p$	$f(b) = (1-b)$ with $b = \gamma$	$[0,1]$	$[-1,0]$

Notes. In the standard portfolio problem, w denotes wealth, r is the return of the risk-free asset, and α is the amount invested in the risky asset with random return $\tilde{X}$. In the coinsurance problem, w denotes wealth, λ is the loading factor, μ is the expected loss, and β is the coinsurance rate of the random loss $\tilde{X}$. In the output choice problem, w denotes initial wealth, F is a fixed cost of production, c is a variable cost of production, and p is the per-unit price of the output. $\tilde{X}$ denotes either the random price or the random variable cost, and y is the level of output. In the problem of hedging a risky return, w denotes wealth, p is a fixed price, and γ is the share of the random return $\tilde{X}$ that is being hedged.

firm owner's initial wealth and $y \geq 0$ the firm's output. Production involves a fixed cost F with $0 < F < w$ and a variable cost $c > 0$. The per-unit price of output on the market is $p > 0$. We distinguish between a risky price and a risky cost, depending on whether p or c is uncertain at the time the production decision is made. If $\tilde{X}$ denotes a risky price, the firm owner solves $\max_{y\geq 0} \mathbb{E}U(w + (\tilde{X} - c)y - F)$. The third row of Table 2 classifies this as a risk-taking problem with a risky benefit. If $\tilde{X}$ denotes a risky cost, the firm owner solves $\max_{y\geq 0}\mathbb{E}U(w + (p - \tilde{X})y - F)$. The fourth row of Table 2 classifies this as a risk-taking problem with a risky cost. Theorems 1 and 2 apply, respectively.

Corollary 3. *Let the firm owner be mixed risk averse from order* $(M+1)$ *to* $(N+1)$ *with relative* $(k+1)$*th-degree risk aversion below* k *for* $k = M+1, \ldots, N$. *If the output price becomes smaller in the* (M/N) *order, the firm owner reduces his optimal output.*

Corollary 4. *Let the firm owner be mixed risk averse from order* $(M+1)$ *to* $(N+1)$ *with relative* $(k+1)$*th-degree risk aversion below* k *for* $k = M+1, \ldots, N$. *If the variable cost becomes larger in the reversed* (M/N) *order, the firm owner reduces his optimal output.*

Corollary 3 contains proposition 3 of Chiu et al. (2012) as a special case for $M = N - 1$ or $M = 0$. It also covers the Cheng et al. (1987) result about production under price risk. Corollary 4 is new because the case of a risky cost has been neglected so far in the literature. As in the comparison of the portfolio problem and the coinsurance problem, consistent results require the use of the appropriate stochastic dominance order.

6.4. Hedging a Risky Return

Our final illustration covers the missing case in Definition 1, which is risk mitigation with a risky benefit. Consider the problem of hedging a risky return. Hedging output price risk for a given level of production is a possible example. Another one is a DM who anticipates a random return (for example, because he owns a patent) and considers selling a share of the return for a fixed price to hedge the risk. If $\tilde{X}$ denotes the random return, $p > 0$ the available fixed price, and γ the share the DM is willing to hedge with $0 \leq \gamma \leq 1$, he maximizes expected utility by solving $\max_{\gamma\in[0,1]} \mathbb{E}U(w + (1-\gamma)\tilde{X} + \gamma p)$. The fifth row of Table 2 classifies this problem as risk mitigation with a risky benefit, and Theorem 1 applies.

Corollary 5. *Let the DM be mixed risk averse from order* $(M+1)$ *to* $(N+1)$ *with relative* $(k+1)$*th-degree risk aversion below* k *for* $k = M+1, \ldots, N$. *If the random return becomes smaller in the* (M/N) *order, the DM increases his demand for hedging.*

7. Conclusion

In this paper, we provide a unifying expected utility framework to organize and extend the comparative static effects of risk on behavior. Specifically, we use the Dionne et al. (1993) linear-payoff model, which encompasses many applications. We derive the monotone comparative statics of Ekern (1980) Nth-degree risk increases and detect a discrepancy between problems with a risky benefit and problems with a risky cost. DMs compensate for the risk increase of a benefit by lowering exposure to risk. When facing a risky cost instead, DMs reduce exposure to risk for even-order risk increases but increase exposure to risk for odd-order risk increases. This alternating pattern comes from the fact that odd-order risk increases are directional, whereas even-order risk increases are not. Therefore, the link between the Nth-degree riskiness of a cost and the Nth-degree riskiness of the payoff depends on the order of the risk change. This also implies that different stochastic ordering assumptions need to be used for a risky benefit than a risky cost in order to obtain consistent results. The equivalence between problems with a risky benefit and problems with a risky cost is thus a question of the right (stochastic) order. Lastly, we apply our results to different decision problems under risk: the standard portfolio problem, the coinsurance problem, the problem of output choice with a risky price or a risky cost, and the problem of hedging a risky return.

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Appendix A. Proof of Proposition 1

To determine the monotonicity of (5) in b, take $b', b \in B$ with $b' \geq b$. It holds that

$$\begin{aligned}
&(-1)^N f(b')^N U^{(N)}(f(b')\cdot(X - x_0) + z_0)\\
&\quad - (-1)^N f(b)^N U^{(N)}(f(b)\cdot(X - x_0) + z_0)\\
&= (-1)^N f(b')^N U^{(N)}(f(b')\cdot(X - x_0) + z_0)\\
&\quad - (-1)^N f(b)^N U^{(N)}(f(b')\cdot(X - x_0) + z_0)\\
&\quad + (-1)^N f(b)^N U^{(N)}(f(b')\cdot(X - x_0) + z_0)\\
&\quad - (-1)^N f(b)^N U^{(N)}(f(b)\cdot(X - x_0) + z_0)\\
&= (-1)^N \Big[f(b')^N - f(b)^N\Big] U^{(N)}(f(b')\cdot(X - x_0) + z_0).
\end{aligned} \tag{A.1}$$

The second equality utilizes $U^{(N)}(f(b')\cdot(X - x_0) + z_0) = U^{(N)}(f(b)\cdot(X - x_0) + z_0)$, which follows from $U^{(N)}(z)$ being constant because of $(N+1)$ th-degree risk neutrality. We can now exploit the classification developed in Definition 1 to examine the various cases.

In the risk-taking problem with a risky benefit, f is increasing with positive codomain. So, $f(b')^N \geq f(b)^N$, and (A.1) is negative (positive) for Nth-degree risk averters (lovers). Then, (5) is decreasing (increasing) in b, implying $B_2^* \leq_S B_1^*$ ($B_2^* \geq_S B_1^*$). This demonstrates (i).

For (ii), recall that f is decreasing and has a positive codomain in the risk-mitigation problem with a risky benefit. In this case, $f(b')^N \leq f(b)^N$, so that (5) is increasing (decreasing) in b for Nth-degree risk averters (lovers), resulting in $B_2^* \geq_S B_1^*$ ($B_2^* \leq_S B_1^*$).

In the risk-taking problem with a risky cost, f is decreasing and has a negative codomain. So, $0 \geq f(b') \geq f(b)$, which implies $f(b')^N \geq (\leq) f(b)^N$ for N even (odd). The sign condition on $U^{(N)}(z)$ then makes (5) decreasing (increasing) in b for Nth-degree risk averters (lovers) when N is even. The monotonicity of (5) in b reverses when N is odd. This shows (iii).

For (iv), utilize that f is increasing and has a negative codomain. Then, $0 \geq f(b') \geq f(b)$ so that $f(b')^N \leq (\geq) f(b)^N$ for N even (odd). Combining this with the sign of $U^{(N)}$ allows us to establish whether (5) is increasing or decreasing in b, which completes the proof.

Appendix B. Proof of Proposition 2

According to Chiu et al. (2012, theorem 2 and lemma 2), relative $(N+1)$ th-degree risk aversion bounded by N implies a lottery preference of $\mathcal{L}_{dis}(w) = [0.5, k_1\tilde{X}_2 + w; 0.5, k_2\tilde{X}_1 + w]$ over $\mathcal{L}_{con}(w) = [0.5, k_2\tilde{X}_2 + w; 0.5, k_1\tilde{X}_1 + w]$ for any $w \geq 0, k_2 \geq k_1 \geq 0$ and any Nth-degree risk increase from $\tilde{X}_1$ to $\tilde{X}_2$. For $\mathcal{L}_{dis}$, the larger risk has the smaller scaling factor, and the smaller risk has the larger scaling factor. The risk increase and the scaling increase are discordant. Matters are reversed for $\mathcal{L}_{con}$ so the risk increase and the scaling increase are concordant. In expected utility terms, a preference of $\mathcal{L}_{dis}$ over $\mathcal{L}_{con}$ is equivalent to

$$\mathbb{E}U(k_1\tilde{X}_1 + w) - \mathbb{E}U(k_2\tilde{X}_1 + w) \leq \mathbb{E}U(k_1\tilde{X}_2 + w) - \mathbb{E}U(k_2\tilde{X}_2 + w), \tag{B.1}$$

so scaling down the larger risk is more valuable than scaling down the smaller risk. With the help of auxiliary function, $\Psi(X) = U(k_1X + w) - U(k_2X + w)$, Inequality (B.1) becomes $\mathbb{E}\Psi(\tilde{X}_1) \leq \mathbb{E}\Psi(\tilde{X}_2)$. By Denuit et al. (1999), this holds for any Nth-degree risk increase if and only if $(-1)^{N+1}\Psi^{(N)}(X) \leq 0$ (see also Jouini et al. 2013, lemma 1 and our explanation after Lemma 1). We rearrange the condition on Ψ as follows:

$$(-1)^N k_2^N U^{(N)}(k_2X + w) \leq (-1)^N k_1^N U^{(N)}(k_1X + w). \tag{B.2}$$

In the risk-taking problem with a risky benefit, $b' \geq b$ implies $f(b') \geq f(b) \geq 0$. Therefore, relative $(N+1)$ th-degree risk aversion bounded by N implies

$$\begin{aligned}&(-1)^N f(b')^N U^{(N)}(f(b')X - f(b')x_0 + z_0)\\ &\quad \leq (-1)^N f(b)^N U^{(N)}(f(b)X - f(b')x_0 + z_0).\end{aligned} \tag{B.3}$$

Furthermore, $f(b')x_0 \geq f(b)x_0$ and $(-1)^N U^{(N)}(z)$ is increasing in z because of $(N+1)$ th-degree risk aversion. As a result,

$$\begin{aligned}&(-1)^N f(b)^N U^{(N)}(f(b)X - f(b')x_0 + z_0)\\ &\quad \leq (-1)^N f(b)^N U^{(N)}(f(b)X - f(b)x_0 + z_0).\end{aligned} \tag{B.4}$$

This ensures that (5) is decreasing in b so that $B_2^* \leq_S B_1^*$ per Lemma 3. In case of risk mitigation with a risky benefit, the argument is identical except for the fact that $b' \geq b \geq 0$ now implies $f(b) \geq f(b') \geq 0$ so that (5) is increasing in b. This shows (i).

For (ii), we first need to revisit the lottery preference between $\mathcal{L}_{dis}$ and $\mathcal{L}_{con}$ for negative scaling factors $k_2 \leq k_1 \leq 0$. We use the Chiu et al. (2012) notation for comparability. Define $Q(k, \tilde{X}_1, \tilde{X}_2) = \mathbb{E}U(k\tilde{X}_1 + w) - \mathbb{E}U(k\tilde{X}_2 + w)$; the preference of $\mathcal{L}_{dis}$ over $\mathcal{L}_{con}$ is equivalent to $Q(k_2, \tilde{X}_1, \tilde{X}_2) \geq Q(k_1, \tilde{X}_1, \tilde{X}_2)$ and is thus characterized by the monotonicity of Q in k. Introducing auxiliary function $\Phi(X) = X \cdot U'(kX + w)$, we obtain

$$\begin{aligned}\frac{\partial Q(k, \tilde{X}_1, \tilde{X}_2)}{\partial k} &= \mathbb{E}\tilde{X}_1 U'(k\tilde{X}_1 + w) - \mathbb{E}\tilde{X}_2 U'(k\tilde{X}_2 + w)\\ &= \mathbb{E}\Phi(\tilde{X}_1) - \mathbb{E}\Phi(\tilde{X}_2).\end{aligned} \tag{B.5}$$

To sign this expression for any Nth-degree risk increase from $\tilde{X}_1$ to $\tilde{X}_2$, we determine the sign-adjusted Nth derivative of Φ as follows:

$$\begin{aligned}(-1)^{N+1}\Phi^{(N)}(X) &= (-1)^{N+1} N k^{N-1} U^{(N)}(kX + w)\\ &\quad + (-1)^{N+1} k^N X U^{(N+1)}(kX + w)\\ &= k^{N-1} \cdot \{(-1)^{N+1} U^{(N)}(kX + w) \cdot [N - r_N(kX + w)]\\ &\quad + w(-1)^{N+2} U^{(N+1)}(kX + w)\}.\end{aligned} \tag{B.6}$$

If relative $(N+1)$ th-degree risk aversion is below N, the curly bracket is positive for a DM who is Nth- and $(N+1)$ th-degree risk averse. k is negative so the sign of $(-1)^{N+1}\Phi^{(N)}(X)$ is entirely determined by the order of the risk change. If N is even, Φ is Nth-degree risk loving so that Q is decreasing in k, resulting in $\mathcal{L}_{dis}$ being preferred over $\mathcal{L}_{con}$. If N is odd, Φ is Nth-degree risk averse, which makes Q increasing in k so that $\mathcal{L}_{con}$ is preferred over $\mathcal{L}_{dis}$.

If N is even, we obtain Inequality (B.2) with a similar argument as before but now for $k_2 \leq k_1 \leq 0$. In the risk-taking problem with a risky cost, $b' \geq b$ implies $f(b') \leq f(b) \leq 0$. Rewrite the payoff function as follows:

$$z(X, b) = f(b) \cdot (X - \bar{x}) + f(b)(\bar{x} - x_0) + z_0. \tag{B.7}$$

Nth-degree risk changes are preserved under additive shifts,[14] and the preference between $\mathcal{L}_{con}$ and $\mathcal{L}_{dis}$ does not depend on whether $\tilde{X}_1$ and $\tilde{X}_2$ are positive or negative random variables. As a result, we obtain

$$\begin{aligned}&(-1)^N f(b')^N U^{(N)}(f(b')(X - x_0) + z_0)\\ &= (-1)^N f(b')^N U^{(N)}(f(b')(X - \bar{x}) + f(b')(\bar{x} - x_0) + z_0)\\ &\quad \leq (-1)^N f(b)^N U^{(N)}(f(b)(X - \bar{x}) + f(b')(\bar{x} - x_0) + z_0)\\ &\quad \leq (-1)^N f(b)^N U^{(N)}(f(b)(X - \bar{x}) + f(b)(\bar{x} - x_0) + z_0)\\ &= (-1)^N f(b)^N U^{(N)}(f(b)(X - x_0) + z_0).\end{aligned} \tag{B.8}$$

The first inequality follows from (B.2), and the second inequality follows from $f(b')(\bar{x} - x_0) \leq f(b)(\bar{x} - x_0)$ coupled with $(-1)^N U^{(N)}(z)$ being decreasing in z because of $(N+1)$ th-degree risk aversion. So, (5) is decreasing in b and $B_2^* \leq_S B_1^*$ per Lemma 3. In the risk-mitigation problem with a risky cost, the argument is identical and even simpler because the payoff function does not have to be rewritten. When N is odd, the lottery preference between $\mathcal{L}_{dis}$ and $\mathcal{L}_{con}$ is reversed, which reverses Inequality (B.2), and $(-1)^N U^{(N)}(z)$ being decreasing in z because of $(N+1)$ th-degree risk aversion implies for $(-1)^N f(b)^N U^{(N)}(z)$ to be increasing in z because $f(b)^N$ is negative. This completes the proof.

Endnotes

[1] We review some of this literature in an earlier working paper (see Menegatti and Peter 2020).

[2] If f is affine, concavity of U implies concavity of the objective function in b. In this case, the solution to (2) is unique. We do not presuppose concavity of U. Our results also apply to risk lovers who have been receiving increased attention recently (Crainich et al. 2013, Ebert 2013).

[3] If $\tilde{X} \geq x_0$ almost surely, the solution to Problem (2) is $\max_{b \in B} f(b)$ for f increasing and $\min_{b \in B} f(b)$ for f decreasing, independent of the specific distribution of $\tilde{X}$. We obtain a corner solution, and any risk change can only have one possible effect on the maximizers of expected utility. The same applies to $\tilde{X} \leq x_0$ almost surely.

[4] For any two subsets D and D' of $\mathbb{R}$, $D \geq_S D'$ if for any $d \in D$ and $d' \in D'$, it holds that $\max\{d, d'\} \in D$ and $\min\{d, d'\} \in D'$. Likewise, $D' \leq_S D$ is defined by $D \geq_S D'$.

[5] Scott and Horvath (1980) and Menegatti (2001) show that a utility function with domain $\mathbb{R}^+$ cannot be Nth-degree risk averse and $(N+1)$ th-degree risk neutral at the same time. Therefore, the results in Proposition 1 for Nth-degree risk averters require restriction of the utility function to a bounded domain.

[6] He cannot offset the welfare effect of the risk increase. For $b_i \in B_i^*$, $i = 1, 2$, an envelope argument yields

$$\mathbb{E}U(z(\tilde{X}_2, b_2)) \leq \mathbb{E}U(z(\tilde{X}_1, b_2)) \leq \mathbb{E}U(z(\tilde{X}_1, b_1)). \qquad (6)$$

The first inequality follows from Nth-degree risk aversion, and the second inequality follows from Equation (4). The first inequality is strict if the risk increase is strict, the DM strictly Nth-degree risk-averse, and $f(b_2) \neq 0$.

[7] Ebert (2020) discusses a similar reversal in a different context. Although payoff prudence relates to a preferences for positive skewness, he finds that discount prudence relates to a preference for negative skewness. The underlying reason is that higher payoffs are good for the DM, whereas longer delays are bad.

[8] As pointed out by Jouini et al. (2013), the assumption of zero labor income in the Eeckhoudt and Schlesinger (2008) model is not without loss of generality.

[9] Brockett and Golden (1987) were the first to study mixed risk-averse utility functions, albeit with a different terminology. See also Menegatti (2015) for some recent results.

[10] A similar terminology can be found in a different context. A random variable is larger than another one in the reversed hazard rate order if its mirror image is smaller than the mirror image of the other one in the hazard rate order (see Shaked and Shanthikumar 2007).

[11] If $\tilde{X}_2$ is larger than $\tilde{X}_1$ in the reversed (0/1) order, then $\tilde{X}_2$ is a first-degree risk decrease over $\tilde{X}_1$. When applied to a risky cost, the DM is worse off because the cost will be higher on average. If $\tilde{X}_2$ is larger than $\tilde{X}_1$ in the reversed (1/2) order, then $\tilde{X}_2$ is a Rothschild and Stiglitz (1970) increase in risk over $\tilde{X}_1$. When applied to a risky cost, a risk-averse DM will be worse off. If $\tilde{X}_2$ is larger than $\tilde{X}_1$ in the reversed (0/2) order, then $\tilde{X}_2$ is larger than $\tilde{X}_1$ in the increasing convex order (Liu and Meyer 2017). So, $\tilde{X}_2$ is "larger" and "more variable" than $\tilde{X}_1$, making it an undesirable change when applied to a risky cost for a DM with increasing and concave utility function.

[12] $\beta \geq 0$ rules out short selling of insurance, and $\beta \leq 1$ precludes overinsurance, which is known as the "principle of indemnity" in the insurance economics literature.

[13] Eeckhoudt et al. (2005) state that the standard portfolio problem is "formally equivalent to the program … describing the coinsurance problem." Similarly, Schlesinger (2013) refers to the "equivalence of the portfolio problem and the insurance problem" when discussing the comparative statics of risk on insurance demand. In a recent survey on stochastic dominance rules, Kim and Ryu (2020) write that the case of a risky cost "can be handled with appropriate modifications" without spelling out what these are.

[14] If $\tilde{X}_2$ is an Nth-degree risk increase over $\tilde{X}_1$, then $(\tilde{X}_2 - \bar{x})$ is an Nth-degree risk increase over $(\tilde{X}_1 - \bar{x})$.

References

Arrow KJ (1963) Liquidity preference. *Lecture Notes for Economics 285, The Economics of Uncertainty* (Stanford University, Stanford, CA), 33–53.

Athey S (2002) Monotone comparative statics under uncertainty. *Quart. J. Econom.* 117(1):187–223.

Brockett PL, Golden LL (1987) A class of utility functions containing all the common utility functions. *Management Sci.* 33(8):955–964.

Caballé J, Pomansky A (1996) Mixed risk aversion. *J. Econom. Theory* 71(2):485–513.

Cheng H-C, Magill MJ, Shafer WJ (1987) Some results on comparative statics under uncertainty. *Internat. Econom. Rev.* 28(2):493–507.

Chiu H, Eeckhoudt L, Rey B (2012) On relative and partial risk attitudes: Theory and implications. *Econom. Theory* 50(1):151–167.

Crainich D, Eeckhoudt L, Trannoy A (2013) Even (mixed) risk lovers are prudent. *Amer. Econom. Rev.* 103(4):1529–1535.

Denuit M, De Vylder E, Lefevre C (1999) Extremal generators and extremeal distributions for the continuous *s*-convex stochastic ordering. *Insurance Math. Econom.* 24(3):201–217.

Denuit M, Lefevre C, Shaked M (1998) The s-convex orders among real random variables, with applications. *Math. Inequalities Appl.* 1(4):585–613.

Dionne G, Gollier C (1992) Comparative statics under multiple sources of risk with applications to insurance demand. *Geneva Papers Risk Insurance Theory Issues Practice* 17(1):21–33.

Dionne G, Eeckhoudt L, Gollier C (1993) Increases in risk and linear payoffs. *Internat. Econom. Rev.* 34(2):309–319.

Ebert S (2013) Even (mixed) risk lovers are prudent. *Amer. Econom. Rev.* 103(4):1536–1537.

Ebert S (2020) Decision making when things are only a matter of time. *Oper. Res.* 68(5):1564–1575.

Ebert S, Nocetti DC, Schlesinger H (2018) Greater mutual aggravation. *Management Sci.* 64(6):2809–2811.

Eeckhoudt L, Schlesinger H (2006) Putting risk in its proper place. *Amer. Econom. Rev.* 96(1):280–289.

Eeckhoudt L, Schlesinger H (2008) Changes in risk and the demand for saving. *J. Monetary. Econom.* 55(7):1329–1336.

Eeckhoudt L, Gollier C, Schlesinger H (2005) *Economic and Financial Decisions Under Risk* (Princeton University Press, Princeton, NJ).

Ekern S (1980) Increasing *n*th degree risk. *Econom. Lett.* 6(4):329–333.

Fishburn P, Porter R (1976) Optimal portfolios with one safe and one risky asset: Effects of changes in rate of return and risk. *Management Sci.* 22(10):1064–1073.

Hadar J, Seo TK (1990) The effects of shifts in a return distribution on optimal portfolios. *Internat. Econom. Rev.* 31(3):721–736.

Hadar J, Seo TK (1992) A note on beneficial changes in random variables. *Geneva Papers Risk Insurance Theory Issues Practice* 17(2):171–179.

Jouini E, Napp C, Nocetti D (2013) Economic consequences of *n*th-degree risk increases and *n*th-degree risk attitudes. *J. Risk Uncertainty* 47(2):199–224.

Kim I, Ryu S (2020) General stochastic dominance rules. Hosoe M, Kim I, eds. *Applied Economic Analysis of Information and Risk* (Springer, Berlin), 127–140.

Liu L (2014) Precautionary saving in the large: *n*th degree deterioration in future income. *J. Math. Econom.* 52(2014):169–172.

Liu L, Meyer J (2017) The increasing convex order and the trade-off of size for risk. *J. Risk Insurance* 84(3):881–897.

Menegatti M (2001) On the conditions for precautionary saving. *J. Econom. Theory* 98(1):189–193.

Menegatti M (2015) New results on high-order risk changes. *Eur. J. Oper. Res.* 243(2):678–681.

Menegatti M, Peter R (2020) Risk effects on optimal decision making—a question of the right order. Preprint, submitted May 22, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3585380.

Menezes C, Geiss C, Tressler J (1980) Increasing downside risk. *Amer. Econom. Rev.* 70(5):921–932.

Menezes CF, Wang XH (2005) Increasing outer risk. *J. Math. Econom.* 41(7):875–886.

Meyer J (1992) Beneficial changes in random variables under multiple sources of risk and their comparative statics. *Geneva Papers Risk Insurance Theory Issues Practice* 17(1):7–19.

Mossin J (1968) Aspects of rational insurance purchasing. *J. Political Econom.* 46(4):553–568.

Nocetti D (2016) Robust comparative statics of risk changes. *Management Sci.* 62(5):1381–1392.

Pratt J (1964) Risk aversion in the small and in the large. *Econometrica* 32(1–2):122–136.

Rothschild M, Stiglitz J (1970) Increasing risk. I. A definition. *J. Econom. Theory* 2(3):225–243.

Rothschild M, Stiglitz J (1971) Increasing risk. II. Its economic consequences. *J. Econom. Theory* 3(1):66–84.

Sandmo A (1971) On the theory of the competitive firm under price uncertainty. *Amer. Econom. Rev.* 61(1):65–73.

Schlesinger H (2013) The theory of insurance demand. Dionne G, ed. *Handbook of Insurance* (Springer, Berlin), 167–184.

Scott RC, Horvath PA (1980) On the direction of preference for moments of higher order than the variance. *J. Finance* 35(4):915–919.

Shaked M, Shanthikumar JG (2007) *Stochastic Orders* (Springer Science & Business Media, New York).

Smith V (1968) Optimal insurance coverage. *J. Political Econom.* 76(1):68–77.

Topkis D (1978) Minimizing a submodular function on a lattice. *Oper. Res.* 26(2):305–321.