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EDITORIAL

Dear Reasoners,

The history of logic – and more specifically the history of logical reasoning, on which the current issue of *The Reasoner* focuses – is nowadays facing methodological and disciplinary challenges similar to those that other historical studies are also confronting. Many of these challenges have to do with a *reconsideration and redefinition of disciplinary boundaries*, and thus call into question the very idea of the history of logic as a discipline. To this purpose I have solicited contributions from friends and colleagues, highlighting new research directions in this exciting field.



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FEATURES: FOCUS ON THE HISTORY OF LOGICAL REASONING

Introducing the History of Logical Reasoning

The history of logic – and more specifically the history of logical reasoning, on which the current issue of *The Reasoner* focuses – is nowadays facing methodological and disciplinary challenges similar to those that other historical studies are also confronting. Many of these challenges have to do with a *reconsideration and redefinition of disciplinary the boundaries*, and thus call into question the very idea of the history of logic as a discipline.

In the first place, as is the case with the history of philosophy and the history of intellectual thought, the history of logic is pressingly required to become more inclusive and more multi-culturally oriented, and thus many scholars have recently made the effort of crossing geographic and linguistic boundaries, taking into account intellectual traditions other than the Western and European one. In the past few decades, the histories of Indian, Chinese, Arabic, Persian, and Byzantine logic (to mention just a few examples) have been investigated more extensively and systematically than ever before. Moreover, these traditions are now studied because of their intrinsic value and interest, rather than in light of the impact they had on the Western tradition (Marenbon, 2022: [Truth and Facts in Medieval Philosophy](#), conference lecture,). In her article, included in the present issue of *The Reasoner*, Silvia Di Vincenzo offers us a beautiful example of this research trend, presenting some theories of reasoning advanced by late antique and medieval authors in the Arabic tradition. She presents some highly sophisticated views concerning the different sorts of premises from which logical reasoning may stem, and particularly focuses on the kind of reasoning that is based on false and imaginative premises.

A second border that historians of logic are now asked to cross is the one between historical and contemporary debates: for several decades now, many scholars in ancient, me-

dieval and modern logic have applied contemporary logical terminology and conceptual categories as a ‘guide’ to rephrase historical discussions, with the twofold aim of making them more understandable and palatable to contemporary readers and, more importantly, of highlighting the (dis)continuities and (dis)similarities between intellectual contexts otherwise very distant from one another, both historically and linguistically. This form of ‘controlled and productive anachronism’ (König-Pralong, 2021: [Indiscipline in the Intellectual History](#), *Intersezioni*, 296) has proved very fruitful for revitalizing authors and texts in the history of logical reasoning that had been previously left at the margins. Some of the papers included in the present issue bear witness to this trend of bridging historical and contemporary logical debates: Jacob Archambault shows how essential aspects of the contemporary theory of consequence arose in the works of medieval authors. Enrico Donato begins his paper by presenting C.I. Lewis’s proof for the principle of explosion (according to which everything follows from a contradiction) and proceeds to show that, in fact, this principle and its proof had already been ‘discovered’ and extensively discussed in the early 12th century, a period in which many logicians were interested in how we can reason validly when starting from impossible premises. Costanza Larese offers an analysis of how some of the concepts advanced by Kant in his *Critique of Pure Reason*, and then reinterpreted by Hintikka, may be fruitfully applied to contemporary uses of first-order logic.

Finally, historians of logic have recently started to put into question borders that appeared even more insurmountable, namely the ones concerning the demarcation between what should count as a logical text and what should not. We all agree that works like Aristotle’s *Prior Analytics*, Ibn Sina’s *The Cure*, Ockham’s *Summa Logicae*, and Leibniz’s *Generales Inquisitiones* are texts on logic, and thus deserve a place in the history of the discipline. But while studying the different logical traditions, one encounters several texts that are more controversial, and difficult to categorize. First of all, contemporary historians of logic have questioned the demarcation between texts that were produced within and outside academic contexts. While traditional history of logic concentrated mainly on the former, recent research shows that, if we focus on logic and reasoning as ‘social practices’, we are forced to rethink historical developments in such a way that ‘the academic life [...] constitutes only a part of the whole picture’ (Brumberg-Chaumont 2020: [Social Uses of Logic in Medieval and Modern Contexts](#), *Annals of Cultural Studies*, 11, 4., 118). Moreover, even if we remain in the domain of texts produced by and for academics, we still have to deal with many works that may leave us in doubt: do the lecture reports written down by a student of logic in early-12th-century Paris – a student who mixes the logical theories heard from his master with his own errors and jokes – count as a logical text? Should the drawings left by a reader in the margins of a manuscript be included in the history of the discipline, if they were conceived by their author as visual devices to understand the logical content of the text? Should we open the history of logic to texts that are now categorized as literature, theology, or jurist legislation, but in which the authors take a stance on contemporary debates in logic or apply a specific logical theory? If we are willing to cross traditional boundaries, new voices are granted the right to be included in the history of logic, voices that would instead be left out if we approach the discipline with a more rigid attitude. In an article



Figure 1: Heloise arguing with Peter Abelard. Paris, BnF, Fr. 1560 (France, Normandie (?), 1300-1340, *Roman de la Rose*), f. 58B.

included in this issue, Georgina Rabassó explains how women scholars were traditionally excluded from the canon of the history of logic, because at least until the 17th century no woman composed or published a text that could be strictly considered as a treatise or monograph on logic. Moreover, for a long time, at least in our Western tradition, logic was seen as a discipline incompatible with the female soul and personality, since the practice of logic was often associated with the ideas of battle, of violent confrontation, of war even. However, if we are open to considering as ‘logical’ also texts in which reasoning and dialectical tools are intertwined with other disciplines – texts like novels, letters, dialogues, or moral and theological treatises – we will find that several women had in fact made use of logical tools, and put forward original reflections on theories of reasoning.

In another article also included in this issue, Mariken Teeuwen gives us a glimpse of how the study of notes and diagrams that can be found in the margins of medieval dialectical manuscripts offers invaluable help to discover new evidence on the transmission of dialectical works. On her website, [The Art of Reasoning in Medieval Manuscripts](#), created in collaboration with Irene O’Daly and Irene van Renwoude, Teeuwen examines this topic more broadly, offering fascinating pictures of highly sophisticated logical diagrams, which either the author or a reader drew to explain and comment on the logical view presented in a text. A famous diagram representing the logical relations holding among modal propositions, presented by John Buridan in his *Summulae de Dialectica* (Figure 2), well represents the sophistication that these logical graphic devices reached in the Western Late Middle Ages. On the role of diagrams and visual devices in manuscripts and printed texts on logic, see also Even-Ezra 2020: [Lines of Thought. Branching Diagrams and the Medieval Mind](#), Chicago University Press; Tarlazzi 2020: *The Latin Tradition of Studying Porphyry’s Isagoge*, ca 800-980. A [working catalogue of manuscripts, glosses and diagrams](#), AHLDMA, 87, 7-42.

The lines of research that I have briefly outlined in this in-

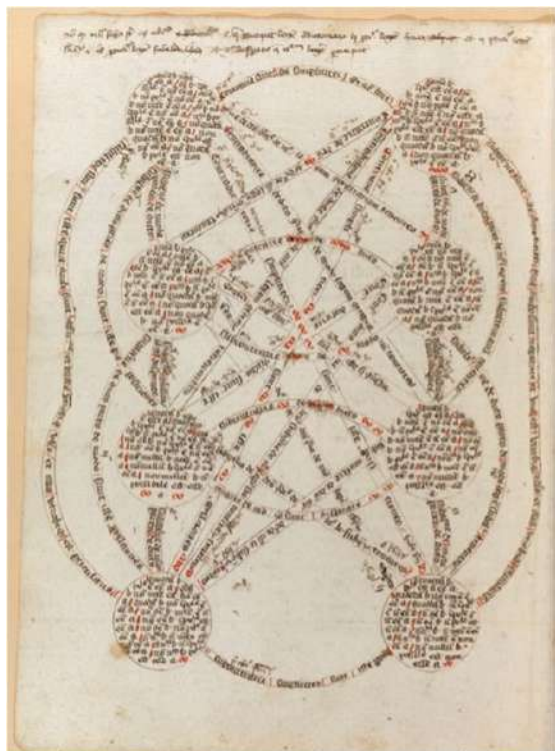


Figure 2: Paris, BnF, lat. 14716, f. 17v. The logical relations between different modal propositions according to John Buridan

roduction are not meant to be an exhaustive or thorough representation of contemporary research in the history of logic and logical reasoning. Far from it, in fact. I only wished to highlight some trends and new perspectives that I believe could be of interest to non-historians and non-specialists of this discipline. I suppose that many readers of *The Reasoner*, engaged as they are in contemporary debates on reasoning in its many forms, may not be entirely familiar with the historical theories on logical reasoning that will be presented in the articles of this issue. If this is indeed the case, I hope that reading these will provide them an occasion to transgress the usual disciplinary boundaries.

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Theories of Reasoning from Late Antiquity to the Medieval Arabic Tradition: the Syllogistic Arts

From what premises does human reasoning stem, and along which lines does it obtain new knowledge? Most philosophers and commentators from Antiquity to the Middle Ages regarded the demonstrative syllogism described in Aristotle's *Prior* and *Posterior Analytics* as the most powerful device to attain true and certain knowledge. Not all the premises that the human mind ordinarily uses are necessary, however, nor do all lines of reasoning yield conclusions as solid and reliable as demonstrative syllogism. Indeed, reasoning often starts from common beliefs and reaches probable conclusions. By the 6th century CE,

the Greek commentators active in the Neoplatonic school of Alexandria developed a classification of syllogistic arguments that included these different methods of reasoning. In his commentary on Aristotle's *Categories*, the Neoplatonic philosopher Elias identified five types of syllogism, which he arranged hierarchically in decreasing order of epistemic strength on the basis of the truth-values of their premises, as shown in Figure 3.

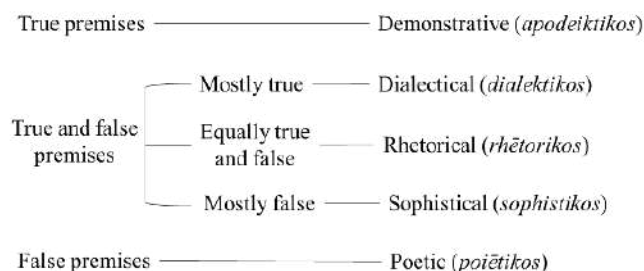


Figure 3: Elias' classification of the syllogisms based on their premises

Besides broadening the horizons of theory concerning the forms of argumentation, the classification in Fig. 3 paved the way for the formation of the so-called 'long *Organon*', that is, a new arrangement of Aristotelian writings according to which *Rhetoric* and *Poetics* became part of Aristotle's logical corpus.

This view would later become prevalent in the Arabic logical tradition. By "Arabic logical tradition," I refer here to the Hellenising philosophical trend named *falsafa* that, from the 8th century CE onwards, received and reworked the Aristotelian corpus in Arabic. This logico-philosophical tradition was not alone in developing a theory of argumentation in Arabo-Islamic contexts: jurists and theologians had also been using argumentative devices (such as analogy) to address the problems of interpreting the Coran and the *ḥadīth*. This latter tradition, however, is not considered here.

The Arabic tradition also individuated five types of syllogism. In contrast to the Greek tradition, however, the five arguments were categorised not so much according to the truth-values of their premises as the strength of the audience's assertion (*taṣdīq*) to their conclusions.

Key figures in the elaboration of a systematic theory of the types of syllogisms were al-Fārābī (d. ca. 950), the "second master" after Aristotle, and Ibn Sīnā (better known in the west as Avicenna, d. 1037). The most detailed account of the fivefold classification of syllogisms emerges from the major philosophical *summae* authored by Ibn Sīnā. The following reconstruction (Fig. 4) is based on the *Book of Healing* (especially *Madkhal* I.3 and *Burhān* I.1); the *Elements of Philosophy* ('*Uyūn al-Ḥikma*); the *Deliverance* (*Nağāt*); and the *Pointers and Reminders* (*Iṣārāt wa-Tanbīhāt*).

The syllogistic arts employ qualitatively different premises. Their cognitive outcomes vary accordingly, ranging from the highest degree of demonstrative certainty to the lowest degree of imagination.



Syllogism	Premises	Cognitive Outcomes	
Demonstrative	Primary (<i>awwalīyyāt</i>)	Certainty	Assertion
	Empirical (<i>taḡrībīyyāt</i>)		
	Sensible (<i>maḥsāsāt</i>)		
Dialectical	Generally accepted (<i>mašhūrāt</i>)	Assertion close to certainty	Assertion
Rhetorical	Received (<i>maqbulāt</i>)	Strong opinion, persuasion	
	Presumed (<i>maznūnāt</i>)		
Sophistical	Generally accepted (<i>mašhūrāt</i>)		No Assertion
	Specious (<i>mušabbāḥa</i>), resembling other premises	Fallacious assertion resembling certainty	
Poetic	Imaginative (or: figurative) statements (<i>mukhāyyalāt</i> , lit.: 'image-eliciting')	Imagination (<i>takhyīl</i>)	No Assertion

Figure 4: Ibn Sīnā's classification of syllogisms

(1) Demonstrative syllogism (*al-qiyās al-burhānī*) yields certain conclusions. It is the method par excellence, in relation to which the other syllogisms play an auxiliary role. Its premises are self-evident first principles as well as sensible, empirically evident propositions. (2) Dialectical syllogism (*al-qiyās al-ḡadalī*) results in an endoxic assertion that is close to certitude. It proceeds from generally accepted premises (*al-mašhūrāt*), equivalent to the *endoxa* of Aristotle's *Topics*; these are beliefs that are either universally accepted by all people, or by a number of people perceived as authoritative. (3) Rhetorical syllogism (*al-qiyās al-khiṭābī*, Aristotle's *enthymēma*) is also based on generally accepted opinions. Its premises include statements of varying degree of trustworthiness: received propositions (*al-maqbulāt*) are generally deemed to be trustworthy, as they come from an authoritative person (e.g., the *imām*), while presumed propositions (*al-maznūnāt*) do not result in a firm assertion. (4) Sophistical syllogism (*al-qiyās al-sūfistā'ī*), also called 'fallacious' (*al-muḡāliṭī*), moves from premises that are misleading on account of their apparent similarity to demonstrative or dialectical premises. (5) Poetic syllogism (*al-qiyās al-šī'rī*) includes as its premises and conclusion figurative statements. Its purpose is not to deceive the listener (as in sophistical syllogism), but to evoke an image in their mind. Its premises provoke a motion in the faculty of imagination (*al-takhyīl*) which affects the human soul in a way that is similar to certain assertion. This motion of the imagination can make the soul feel attraction or repulsion for a given thing. A classic example is that of honey, which can be erroneously taken for vomited bile due to their similarity in colour and complexion. The erroneous equation of honey and bile arguably derives from an argument with figurative statements as major premise and conclusion:

Honey is yellow;
 Everything yellow is vomited bile;
 Therefore, honey is vomited bile.

Poetic syllogism has been variably understood in the Arabic tradition. Al-Fārābī and, in his wake, Ibn Rušd (Averroes, d. 1198) considered it to be unproductive. As opposed to the Greek tradition, Avicenna did not entirely rule out the possibility that the premises and conclusions of poetic syllogisms may be true: in his last major work, the *Pointers*, he appears to criticise a classification of syllogisms similar to the one in Fig. 3, which left no room for such a possibility.

As argued by Black (1990: *Logic and Aristotle's Rhetoric and Poetics in Medieval Arabic Philosophy*, Brill), a theory of syllogisms including rhetorical and poetic arguments might have served in the Arabic tradition to account for the methods of reasoning employed in the arts that fall outside the domain of

theoretical and practical philosophical sciences. The developments and implications of this comprehensive theory of reasoning in the Arabic logical tradition are largely yet to be explored. The post-Avicennian logical tradition, which extends well into the 19th century CE, consists of a wealth of original texts, commentaries, and marginal annotations whose intellectual interest has only recently been acknowledged, and which await editing and study (for an overview, see El-Rouayheb 2019: *The Development of Arabic Logic, 1200-1800*, Schwabe Verlag). Historical and theoretical work on these materials promises to reveal in the near future an entirely new picture of the history of Arabic logic, and of the history of reasoning more generally.

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13th-14th Century Theories of Inference

In medieval logic, *consequence* refers to a relation between two parts of a hypothetical proposition, respectively called the *antecedent* and *consequent*, according to which what is stated in the consequent follows from what is stated in the antecedent, e.g. 'If Socrates is running, then he is moving'.

Consequence is arguably the core notion studied in formal logic today, where it has stood since Alfred Tarski's groundbreaking work on the topic in the 1930s, and also plays a major role in adjacent fields including computing and the philosophy of science. When compared to the intuitive notion of consequence that they aim to capture, however, the most widely-known theories of consequence today suffer from several deficiencies:



- Classical theories of consequence validate inference rules that are highly unintuitive. The best-known of these is *explosion*, which allows anything to be inferred from a contradiction.
- Natural language inference is *semantically closed*, making it possible for statements to refer, directly or indirectly, to themselves. The artificial languages studied in formal logic today, by contrast, tend to employ various devices to prevent semantic closure, leaving them less expressive than their natural counterparts.
- The inference schemata studied in modern logical systems tend, by design, to be indifferent to whatever content might be expressed in actual natural language inferences whose formalizations they capture. Particularly for novices, this can leave the use of these systems opaque.

Several essential aspects of the theory of consequence as we understand it today first arose during the later medieval period: the earliest extant treatises directly devoted to consequence, translated in Archambault (2017: *The development of the medieval Parisian account of formal consequence*, PhD thesis, Fordham University), were written at the turn of the 14th century, and the notion of *formal* consequence became a primary locus of attention shortly thereafter (Dutilh Novaes, 2020:

[Medieval Theories of Consequence](#), The Stanford Encyclopedia of Philosophy). This period therefore provides the backdrop for understanding, prior to the more mathematical aspects provided at the turn of the 20th century, some of the more general aims of research on consequence as it continues to be carried out today.

According to seminal research carried out through the 1980's, theories of consequence appear to have arisen out of two groups of sources.

The first group included treatises from the *logica modernorum* - works written on *syncategoremata* (i.e. terms roughly similar to today's logical constants) fallacies, and other subjects not fully addressed in Aristotle's organon (Green-Pedersen 1984: [The Tradition of the Topics in the Middle Ages. Commentaries on Aristotle's and Boethius' Topics.](#), Philosophia Verlag; Spruyt 2018: [Consequence and 'Cause': Thirteenth-Century Reflections on the Nature of Consequences.](#) Vivarium 56.3-4, 320–339). 13th century authors working on consequence in this manner include Nicholas of Paris, William of Sherwood, Lambert of Lagny and Peter of Spain.

The second was the body of work devoted to the theory of topics the medievals inherited from Aristotle, Cicero, Themistius, and Boethius, with the most conspicuous support for this being the placement of William of Ockham's treatise on consequences in his *Summa Logicae*, which sets out its subjects in an order corresponding to that of the books of Aristotle's logic, in the place normally reserved for topics (William of Ockham 1974: *Summa Logicae*. In: *Opera Philosophica*. Franciscan Institute; Stump 1982: [Topics: Their Development and Absorption into Consequences.](#) In: *The Cambridge History of Later Medieval Philosophy*. CUP, 273–299).

Traditional work on topics aimed to classify various properties or relations, e.g. between species and genera, parts and wholes, or causes and effects, which could ground the discovery of new conclusions about the entities to which those properties and relations applied. Early treatises on consequences, however, departed from their topical forebearers in grounding inferences not in a richer variety of arguably metaphysically robust relations, but almost entirely on thinner extensional relations pertaining to the distribution of terms as outlined in medieval theories of *supposition* (Hodges 2005: [Detecting the Logical Content. Burley's "Purity of Logic"](#). In: *We Will Show Them! Essays in Honour of Dov Gabbay.*, College Publications, 69–116; Read 2019: [Medieval Theories: Properties of Terms.](#) In: *The Stanford Encyclopedia of Philosophy*).

Compared with modern theories of consequence, the medieval theory's roots in the second set of influences leave it with a more concrete focus than its modern counterpart; its roots in the first group mentioned make it more interested in - and arguably more resilient to - natural language paradoxes, with solutions tending to be more likely to come from analyses of the meaning of particularly problematic terms than global rules applied to a language (Klima G., 2004: [Consequences of a closed, token-based semantics: the case of John Buridan.](#), *History and Philosophy of Logic* 25.2, 95–110; 2016: [Consequence.](#) In: *The Cambridge Companion to Medieval Logic.*, CUP, 316–341).

Through the 13th and early 14th century, consequences are commonly divided into *simple consequences*, which always hold, and *as-of-now* (*ut nunc*) consequences, which only hold at a given time. Simple consequences, in turn, are divided into natural and accidental, and natural consequences hold by virtue

of the antecedent in some way 'containing' the consequent - a stronger relationship than the always-good, but non-intrinsic relation of holding accidentally. One of the more striking aspects of this division is that it lays bare not merely a different way of doing what modern logic does, but a different goal: medieval logic in this period was interested not primarily in the classification of valid forms of inference, but in the classification and discovery of *sound* ones. With Walter Burley being an especially important transitional figure (Archambault 2018: [Consequence and formality in the logic of Walter Burley](#), *Vivarium* 56.3-4, 292–319), the division between natural and accidental consequence gives way to one between formal and material consequence. Particularly in the British Isles and later in Italy, the containment criterion mentioned above continues to be appealed to in discussions of formal consequence by figures like Thomas Bradwardine and Paul of Venice; while figures including Buridan, Albert of Saxony and Marsilius of Inghen on the European continent prefer to discuss formal consequences in terms of the impossibility of the antecedent holding with the consequent not holding for all consequences sharing the same form, i.e. those for which all sentences generated by substitutions of like terms for like also result in a good consequence. It is within this latter tradition especially that we see increased attention to formal consequence in the sense of one holding schematically, providing the prototype for the more general aims of logical research from Tarski to today.

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The Principle of Explosion in the Twelfth Century: Friends and Enemies

Nearly a century ago, C. I. Lewis recognised that the truth-preservation account of validity had some unintuitive consequences. For if a necessary and sufficient condition for validity is that it is impossible for the premises to be true and the conclusion false, then a contradiction entails anything (henceforth 'PE', from 'Principle of Explosion') and anything entails a necessary truth.



Long before Lewis, in the second half of the twelfth century, a proof for PE was discovered by the so-called *Parvipontani*, the followers of Adam of the Little Bridge. Alexander Neckam (1157–1217), an Englishman who moved to Paris to study under Adam in the 1170s, reports a proof for PE in his *De naturis rerum* (ca. 1200). The structure of Adam's proof is as follows:

1. $(p \wedge \neg p) \rightarrow p$ (Simplification)
2. $p \rightarrow (p \vee q)$ (Addition)
3. $(p \wedge \neg p) \rightarrow (p \vee q)$ (1, 2, Transitivity of Entailment)
4. $(p \wedge \neg p) \rightarrow \neg p$ (Simplification)
5. $(p \wedge \neg p) \rightarrow (p \vee q) \wedge \neg p$ (3, 4, Adjunction)
6. $(p \wedge \neg p) \rightarrow q$ (5, Disjunctive Syllogism)

The *Parvipontani*, just like Lewis, had no quarrel with **PE**. Other twelfth-century logicians, however, granted it either only partially or not at all. Peter Abelard (1079–1142), for instance, regarded truth-preservation as a necessary and sufficient condition for the validity of argument, and thus conceded that **PE** holds for enthymemes and categorical syllogisms. As far as the truth of conditional inference is concerned, however, he required that in addition to truth-preservation there be a relevant connection between antecedent and consequent. More precisely, Abelard maintained that the antecedent is relevant to the consequent only if the latter is “understood” or “contained” in the former. Consequently, **PE** cannot hold for conditionals, for it is not the case that the understanding of a contradiction contains the understandings of any consequent whatever.

A more explicit rejection of **PE** is found in the so-called ‘*Ars Meliduna*’, a most interesting treatise on logic written in the last quarter of the century by some followers of Robert of Melun (1100–1167), also known as the *Melidunenses*. The authors of the *Ars Meliduna* argued that **PE** must be dropped because its acceptance leads to the violation of three other logical principles. To wit: the principle that (i) no statement can imply its own negation, that (ii) a statement cannot entail another statement as well as the negation of that statement, and that (iii) no statement can imply another statement whose truth is incompatible with the truth of the former. Below, I reconstructed one of these interesting arguments, which was devised by the authors to prove the incompatibility of **PE** and (i):

1. $p \rightarrow \neg q$ (Assumption)
2. $\neg q \rightarrow \neg (p \wedge q)$ (Simplification, Contraposition)
3. $p \rightarrow \neg (p \wedge q)$ (1, 2, Transitivity of Entailment)
4. $(p \wedge q) \rightarrow p$ (Simplification)
5. $(p \wedge q) \rightarrow \neg (p \wedge q)$ (4, 3, Transitivity of Entailment)

Thanks to similar arguments, the authors were able to show that **PE** is far from being a free lunch. Unfortunately, no explanation is forthcoming as to why (i), (ii) and (iii) should be held in higher regard than **PE**. At any rate, the authors of the *Ars Meliduna* complemented this meta-logical stance with a thesis that is the direct opposite of **PE**, and namely that nothing follows from a falsehood. They tried to justify this thesis by appealing to certain epistemological considerations involved in the process of *inferring*. In order to infer something, they argued, one must be convinced of the truth of that from which one infers. But clearly, one can never assume a falsehood *qua* falsehood while at the same time being convinced of its truth. Therefore, the authors conclude, a falsehood is inferentially ineffective both when it occurs as a premise in an argument and when it is used as the antecedent of a conditional. But the thesis that nothing follows from a falsehood is not a free lunch either. Quite to the contrary, one might wonder how the authors of the *Ars Meliduna* would account for counterfactual reasoning, *modus tollens*, and transitivity (the latter failing whenever some falsehood is involved). Later, those medieval logicians who were also unhappy with **PE** tended to deploy different and less costly strategies, i.e. leveraging the distinction between material and formal validity or rejecting Disjunctive Syllogism. But

by then, the boldness – or better, temerity – that characterised twelfth-century logicians will have partly faded.

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Medieval Women Reasoners: On Dialectics

During the Middle Ages, access to *scholae* and universities - and thus to study of the liberal arts - was forbidden to women, and this led to an epistemic gender gap. Lack of training in grammar, rhetoric and dialectics had a profound impact, both individual and socio-political, on women. Any statement not grounded in reasoning and logical argumentation can easily be discounted as mere opinion, and thus the author of the statement can likewise be deprived of legitimacy and authority. In this way, scholasticism severely obstructed the validation of women’s discourse. Despite this, medieval women authors still wrote dialogues and debates, and although these were not strictly dialectically structured, neither were they completely alien to the art of dialectics. Héloïse d’Argenteuil (ca. 1092-ca. 1164) and Marguerite Porete (ca. 1250/60-1310), for example, were two women thinkers (amongst others) who taught themselves to use dialectical tools. In different ways and in different contexts, many women writers showed an implicit or explicit interest in this art, even though its use was seen as both personally and socially improper or inappropriate, since it was difficult to imagine a woman entering into verbal combat and defending an opinion with reasoned arguments.



The Augustinian abbess and canoness Herrad of Hohenbourg (ca. 1125-ca. 1195) mentions the liberal arts in her *Hortus deliciarum*, an encyclopaedic theological and didactic treatise that evidences the practice of the *cura monialium* in the late medieval reformed female monastery. A famous miniature in the *Hortus* (f. 32r) depicts the seven arts as female figures, each framed in the arch of a rose window, surrounding Philosophy at the centre with Socrates and Plato at her feet. The left hand of Dialectic holds a dog’s head with open mouth and bared teeth. Next to the animal we read the inscriptions “dog’s head” (*caput canis*) and *Argumenta sino concurrere more canino*, i.e. “I allow arguments to follow each other (or: engage in battle) in the manner of a dog.” According to Wirth (1979), *more canino* means “growling and barking (*latratus*),” which in turn “could be interpreted in *malam partem* as loud, aggressive, rude, and sophisticated but also in *bonam partem* as symbolising the zealous and vigilant orator and preacher fighting valiantly for the truth in the *duellum logicae*.” The reference is also to the ancient Cynics or “dogs,” among them Hipparchia of Maroneia, who is said to have left the loom for philosophy and to whom numerous anecdotes and phrases are attributed that show the biting nature of her intelligence and her philosophical retorts.

In various writings, Hildegard of Bingen (1098-1179) develops a debate between the forces of good and evil that takes on a theological and cosmological as well as moral tone. She

portrays this debate as a confrontation between two opposing forces trying to win over the free soul. These forces do not seek to convince each other, however, since they both represent unchanging essences; thus they endeavour only to vindicate themselves while disqualifying their opponent. One special feature of Hildegard's writing is that she interweaves the arguments by means of allegories that convey poetically each verbal foray. Diabolus, characterised as *illusor* and *fallax*, and apparently modelled on the figure of the scholastic master, challenges the Virtues by questioning their essence and existence (undifferentiated from God's being and existence), their power and faith, and even their virginity. The responses to these attacks pivot on the same idea: that the superiority of good is both unquestionable and invincible because it emanates from God, and even if the Devil strives to impose himself from the abyss of defeat, he has no chance of breaching this ontological gap. Hildegard regards humility as the most appropriate disposition towards knowledge, an attitude she does not attribute to the *philosophus* or the *magister scholarum*. Yet at the same time she describes humility as a *gloriosa regina* who combats evil with words, reason and argumentation. Throughout her texts there are also many references to the intellectual activity of the Virtues: *scire*, *perfectum studium*, *magno studio*, *speculata*, etc.

In *La Cité des dames*, Christine de Pizan (ca. 1366-ca. 1431) uses the rhetorical trope (*figure de grammaire*) of antiphrasis, "which consists of interpreting something that is negative in a positive light, or vice versa" (I, II). The statement she refutes is: "Women are by nature fearful creatures, having weak, frail bodies and lacking physical strength. [...] If a person's body is defective in some way, this undermines and diminishes that person's moral qualities [*vertu*]" and consequently the individual "is less worthy of praise." De Pizan's main argument is that physical weakness brings benefits to women because it exempts them from violence. Moreover, this "imperfection" has a recompense, because "when Nature fails to make a body which is as perfect as others she has created [...] she very often compensates for it by giving that body some greater quality than the one she has taken away" (II, XIV). The gift that Nature has given to women is "a most virtuous disposition," and for this reason the nature of women cannot be, as the sages erroneously claim, "wholly given up to vice," but quite the opposite. De Pizan's conclusion is that "women who don't act like this are going against their own nature."

The practice of reasoning, questioning, arguing and refuting is as important in the *Querelle des femmes* as the defence of women's dignity. The dialectical structure of the *Querelle*, which De Pizan and other women authors created by formulating antitheses to prior theses, helped them incorporate dialectics and rhetoric into their discourse, and these became crucial elements in the genealogy of the struggle for women's rights and in their bid to occupy the public and political spheres.

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Reasoning in the margin: the processing of knowledge in the medieval manuscript

In the Middle Ages, the art of reasoning was deemed a crucial part of one's education. Reasoning was intrinsically tied to understanding, explaining and convincing – three processes that were also crucial for a Christian life. Reading and writ-

ing, moreover, were (for the larger part of the Middle Ages) a prerogative of the clergy. Books were produced in and for the monastery or the church, often in the language of the church: Latin. Education was also the domain of the church, first the monastery and then also the cathedral, later also city schools and the university. Being taught meant first to learn Latin, in order to be able to learn the liturgy and read the Bible, but second also to get introduced into an inherited learned tradition, which encompassed both religious and secular learning.

The 'secular' curriculum was often presented as a building consisting of seven pillars or building blocks: the *trivium*, three arts of language, and the *quadrivium*, three arts of number. For the art of reasoning, the three arts of language were crucial: *grammatica* or the understanding of the (Latin) language itself; *rhetorica* or the art of (public) speaking and persuasion; and *dialectica* or the art of argumentation and logic reasoning. For each of these arts, authorities were read, studied and processed.



Grammatica, for example, was studied using the handbooks of Donatus and Priscianus, but also the many summaries, commentaries and secondary works that were produced in their slipstream. For *rhetorica*, Cicero's *De Inventione* and the *Rhetorica ad Herennium* (unknown author but generally attributed to Cicero in the Middle Ages) were key texts for the study of rhetoric during the entire Middle Ages (Copeland, Sluiter (eds.), 2009: [Medieval Grammar and Rhetoric. Language Arts and Literary Theory](#), OUP). For *dialectica*, the main body of learning was formed by texts from Aristotle, in several translations and re-workings, for example by Boethius (Dutilh No-vaes, Read (eds.), 2016: [Cambridge Companion to Medieval Logic](#), CUP). In the process of reading and studying these texts, medieval scribes and scholars left traces in the margin: texts that offered guidance to the reader, in order to follow or understand better, or texts that referred to other, sometimes different authorities, in order to compare and make connections. These voices in the margin, as has been argued elsewhere, are a rich source for understanding the intellectual climate of the medieval reader: they show us not only what was read in the period, but also how it was read and why, sometimes also giving clues about the who and where (see, e.g., Teeuwen, 2017: [Voices from the Edge: Annotating Books in the Carolingian Period](#), In: *The Annotated Book in the Early Middle Ages: Practices of Reading and Writing*, Brepols, 13-36).

Let me offer two examples to illustrate this. Manuscript Leiden, University Library, GRO 22, made in the first half of the twelfth century in France, contains three texts on rhetoric: Cicero's *De inventione*, the *Letter to Aristotle* attributed to Alexander the Great and the *Rhetorica ad Herennium* (O'Daly). In the first work, Cicero explains how people are defined by features such as their sex, their age, their place of birth. One can be Greek, Cicero writes, or barbarian, from Athens or from Lacedaemon. In this copy of the text, however, a medieval scribe added a diagram that visualizes these features and puts them in context: the concept "nature" can be split up into two categories: "divine" or "mortal". "Mortal", then, can be subdivided into a list of features that define mortals: "sex",

“country”, “place”, “age”. Examples are given: for “country”, this reader suggests “England” or “France”, for “place” he suggests “York” or “London”, for “age” “adolescent” or “old man”. As has been argued by others, the references to English place names may suggest that the glossator came from England. More in general, however, the use of the diagram shows how concepts were embedded in larger structures, and how they functioned to give other concepts a place in a large unit. They show a systematized, organised technique of understanding. The use of diagrams as a structuring and also mnemotechnical device is neatly illustrated.



Figure 5: Leiden, UL, GRO 22, f. 11r (detail)

Manuscript Leiden, UL, BPL 88 was made in the third quarter of the ninth century. It contained the late-antique encyclopedia of the seven liberal arts (*trivium* and *quadrivium*) of the Latin author Martianus Capella, *On the marriage of Mercury and Philology* (see Teeuwen). In it, we find not only useful introductions to each of these disciplines, but also a neo-Platonic framework that gives meaning to the ancient knowledge tradition. The margins of medieval copies of this text are often filled with notes and remarks, which help the reader make sense of the difficult language and style of this author, and guide the reader to other, clearer authorities. These are not always contemporary with the making of the manuscript. In this ninth-century manuscript, for example, an author from the eleventh century added material on the flyleaves preceding the text of Martianus Capella (fols. 1v-2r): a set of diagrams which illustrate the method of creating affirmations, negations, oppositions, contradictions and consequences – the basic building blocks of argumentation. This material is, in fact, not taken from Martianus Capella’s text, but rather a diagrammed version of a text from Aristotle, as commented upon by Boethius (see O’Daly). It must have been added here to provide means to understand the art of dialectic better and perhaps practice with it.



Figure 6: Leiden, UL, BPL 88, f. 2r (detail)

So, we can learn several things from these additions to the manuscript by a later reader: 1. the manuscript was still studied a few centuries later; 2. the reader was interested in logical

reasoning, and added material that was useful to learn and train this; 3. he used a method that did not only use text, but also the visual aid of the diagram; 4. he connected the authority which was already present in the manuscript (Martianus) with a different one (Aristotle, as translated by Boethius and ‘diagrammised’ by an unknown medieval author). To conclude: medieval manuscripts are not only vessels of text, they are also fertile ground to find the traces of readers, their techniques of reading and learning, and their ways of transforming material into something that was their own. The voices in the margin, for a long time neglected as anonymous and therefore unimportant witnesses of readership and scholarship, cry out for a thorough exploration!

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***Ecthesis*, constructions and existential instantiation. A Kantian tale**

According to Proclus (1992: A Commentary on the First Book of Euclid’s *Elements*, Princeton University Press), the proof of a theorem and the solution of a problem in Euclid’s *Elements* comprise six main parts. The second of them is called *ecthesis* and immediately follows the general enunciation of the proposition in question. It consists of the exhibition of a particular figure that sets out the geometrical entities with which the general enunciation deals. This step (together with further determinations and constructions) allows the geometer to carry out the proof proper or *apodeixis* on that particular figure and to conclude by extending the result to the general case, given that the particular determinations of the specific figure are utterly indifferent to the proof of the proposition.



In his book *Logic, Language-games and Information* (1973: OUP), Jaakko Hintikka suggests that the geometrical method described by Proclus, and *ecthesis* in particular, acts as a model for Immanuel Kant’s conception of the synthetic *a priori*. In the *Critique of Pure Reason*, Kant introduces his celebrated analytic-synthetic distinction in these terms: “In all judgments in which the relation of a subject to the predicate is thought [...] this relation is possible in two different ways. Either the predicate B belongs to the subject A as something that is (covertly) contained in this concept A; or B lies entirely outside the concept A, though to be sure it stands in connection

with it. In the first case I call the judgment analytic, in the second synthetic” (Kant, 1998: Critique of Pure Reason, CUP, A6-7/B10).

As this quotation clarifies, the concept of the predicate of synthetic judgments is not contained in the concept of the subject. The connection between the two concepts involved, which is necessary for grounding the truth of the judgment, cannot be but indirect in that it must link the two concepts to one another by connecting them to a third element. The third element that is always necessary for the truth of synthetic judgments is an object, and the relation between concepts and objects must always be mediated by intuitions, which can only be sensible for all human beings. As a result, in Kant’s picture, intuition dependence turns out to be the main feature of synthetic judgments as opposed to the analytic ones.

According to Hintikka’s reading, synthetic methods might be seen, for Kant, as a kind of generalisation of geometrical methods. This conclusion is reached through the following interpretational steps: 1. judgments are synthetic if synthetic methods must be essentially used in their proof; 2. methods are synthetic if they make use of constructions (thus, Kant is “an heir to the constructional sense of analysis”, cf. Hintikka 1973, 205); 3. constructions are a priori exhibition of intuitions; 4. intuitions are singular representations. Although many of them might be challenged, these claims lead directly to the conclusion that, for Kant, the truth of *a priori* synthetic judgments is shown through a generalisation of the reasoning pattern for proving geometrical propositions. The latter are demonstrated using *ecthesis*, namely the introduction into the argument of geometrical figures, which are singular and individual objects; similarly, *a priori* synthetic judgments in general are proved using constructions, namely the introduction into the argument of intuitions, which are singular representations.

But Hintikka pushes this similarity further and applies (his interpretation of) Kant’s analytic-synthetic distinction to the logic of quantifiers. The result is the following definition: a step of a derivation in first-order logic is said to be synthetic if it introduces new individuals into the argument, that is, individuals that are not thought of in grasping the premises of that argument. The Kantian flavour of this notion should be clear enough: new individuals introduced into the argument are analogous to *a priori* intuitions understood as singular representations exhibited in constructions.

A new individual usually turns up in a derivation by applying the rule of existential instantiation. By this rule, a free individual symbol is introduced to replace the occurrences of a certain bound variable: this rule allows to infer from an existentially quantified sentence $\exists x F$ a sentence instantiating it, e.g. $F(x/a)$, where a is a free individual symbol and $F(x/a)$ the result of replacing x by a in F . The individual introduced in the proof through an application of this rule is new because of the requirement that the instantiating symbol a must be different from all the free individual symbols occurring earlier in the proof.

Thus, in Hintikka’s work, *ecthesis* turns out to be the paradigmatic reasoning pattern not only for the use of constructions in Kant’s synthetic judgments, but also for the application of the rule of existential instantiation in modern first-order logic. Hintikka devised this Kantian tale for the purpose of challenging the logical empiricists’ thesis, which soon became traditional after Quine’s *Two Dogmas of Empiricism*, that logic is analytical and tautological. Although his work did not en-

joy a considerable critical fortune, it has been recently resumed and provided inspiration for the construction of depth-bounded first-order logics (see D’Agostino, Larese, Modgil, 2021: Towards Depth-bounded Natural Deduction for Classical First-order Logic, *Journal of Applied Logics*, 8,2, 423-51), a new proof-theory for classical first-order logic that allows for a natural characterization of a notion of inferential depth.

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