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DEVELOPMENT OF A NONLINEAR
MODEL FOR RC/FRC ELEMENTS APPLIED
TO THE ANALYSIS OF TUNNEL
LININGS UNDER FIRE CONDITIONS

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Abstract

This work deals with the development of an existing nonlinear model for the analysis up to failure of reinforced concrete and fiber reinforced concrete (RC and FRC) elements, as well as its extension to the study of concrete lining behavior under fire conditions. The model, which belongs to the smeared approaches and applies a strain decomposition procedure in the cracked stage, is formulated in terms of a secant stiffness matrix to be implemented into a finite element (FE) code.

In more detail, the model – named 2D-PARC – is firstly revised with regard to the modeling of concrete contribution, so as to correctly describe the behavior of the material under a general biaxial state of stress, both before and after cracking development. The adopted formulation, based on isotropic non-linear elasticity, is able to provide a sophisticated representation of concrete behavior, being at the same time characterized by a high numerical feasibility. In this way, the computational efficiency of 2D-PARC is highly improved. Moreover, the significant phenomena of crushing and dilatation of concrete in compression are inserted in the algorithm.

Subsequently, the model is extended to include initial deformations of concrete, which were not considered in the original formulation. At first, shrinkage strains are inserted in the algorithm. To this aim, a new set of equilibrium and compatibility equations is written and the material secant stiffness matrix is consequently rearranged and implemented into the adopted FE code, by properly updating the internal convergence procedures. Shrinkage strains are computed as a function of the moisture field in the element, which is in turn obtained by performing an “equivalent” thermal analysis, by exploiting the analogy between the equations governing the moisture and the thermal problem.

Extensive comparisons with data from the literature are provided to validate first the enhanced concrete modeling and subsequently the inclusion of shrinkage strains. Plain concrete specimens, as well as RC and SFRC elements belonging to different structural typologies are considered, so as to validate the proposed model over a wide range of conditions.

Finally, the model is further extended and applied to the structural assessment of concrete linings under fire conditions. Concrete lining behavior is then analyzed by performing 3D FE simulations where mechanical nonlinearity is taken into account through 2D-PARC constitutive law. To this aim, thermal strains are inserted into the algorithm by following the same procedure adopted

for shrinkage deformations. At the same time, the decay of strength and stiffness of concrete during heating is considered, by inserting the dependence of the main mechanical properties on temperature. In order to realistically represent the soil excavation phase and the subsequent lining installation, a step-by-step procedure is followed; whereas a sequentially coupled thermo-mechanical procedure is applied for the fire stage. A heat-transfer analysis is carried out so as to obtain the temperature field under an assigned fire scenario; then the simulation of tunnel behavior under fire conditions is performed on the basis of the obtained results. The stress-strain field is then evaluated by taking into account both the temperature-induced effects and the ground-structure interaction. The effectiveness of the proposed model is verified through comparisons with analytical closed-form solutions. The accuracy of the proposed approach in simulating the excavation and lining installation phases is verified by comparing FE results to well-known analytical relations available in the literature. Moreover, a simplified analytical model is developed on purpose, so as to validate the simulation of concrete lining under fire conditions. Finally, a parametric study is also performed in order to better highlight the influence exerted by different parameters on the structural response of the lining during fire.

Keywords:

Reinforced concrete; Constitutive modeling; Shrinkage; Tunnel lining; Fire

Table of Contents

INTRODUCTION	1
CHAPTER 1 2D-PARC MODEL	5
1.1 Introduction	5
1.2 State of the art	6
1.2.1 Reinforcing steel modeling	7
1.2.2 Cracking modeling.....	8
1.2.2.1 <i>Discrete crack approach</i>	8
1.2.2.2 <i>Smeared crack approach</i>	10
1.2.3 Overview of some smeared models for RC elements.....	13
1.2.3.1 <i>Modified Compression Field Theory (MCFT)</i>	13
1.2.3.2 <i>Disturbed Stress Field Model (DSFM)</i>	16
1.2.3.3 <i>Fixed-Angle Softened Truss Model (FA-STM)</i>	21
1.2.3.4 <i>Soltani, An and Maekawa's model</i>	23
1.3 Description of 2D-PARC model	26
1.3.1 Origin of 2D-PARC model	26
1.3.2 Basic hypotheses	28
1.3.3 Uncracked stage	30
1.3.3.1 <i>Steel stiffness matrix</i>	31
1.3.3.2 <i>Concrete stiffness matrix</i>	32
1.3.3.3 <i>Fiber contribution</i>	37
1.3.4 Cracked stage	37
1.3.4.1 <i>Stiffness matrix of the concrete between two adjacent cracks</i>	40
1.3.4.2 <i>Stiffness matrix of the steel between two adjacent cracks</i>	40
1.3.4.3 <i>Crack stiffness matrix: concrete contribution</i>	41
1.3.4.4 <i>Crack stiffness matrix: steel contribution</i>	42
1.3.4.5 <i>Fiber contribution</i>	49

1.3.5	Implementation of 2D-PARC into a commercial FE code	58
1.3.5.1	<i>Description of UMAT subroutine</i>	59
CHAPTER 2	REVISION OF CONCRETE CONTRIBUTION	63
2.1	Introduction	63
2.2	State of the art	65
2.2.1	Concrete behavior under uniaxial loading.....	66
2.2.2	Concrete behavior under biaxial loading.....	68
2.2.3	Failure criteria.....	71
2.2.3.1	<i>Failure criterion by Kupfer and Gerstle</i>	73
2.2.4	Stress-strain models.....	75
2.3	A different approach to model concrete contribution into 2D-PARC... 77	
2.3.1	Evaluation of the nonlinearity index	79
2.3.2	Adoption of a proper biaxial strength envelope.....	80
2.3.3	Evaluation of the secant value of the Young modulus in the uncracked stage.....	81
2.3.3.1	<i>Proper modifications in case of crushing</i>	83
2.3.4	Evaluation of the secant value of the Poisson coefficient in the uncracked stage.....	85
2.3.4.1	<i>Proper modifications in case of crushing</i>	85
2.3.5	Adaptation of the formulation to the cracked stage.....	86
2.3.6	Implementation of revised concrete model into 2D-PARC.....	88
2.3.6.1	<i>Convergence of stress-strain relation for concrete for the evaluation of the Young modulus and the Poisson coefficient.....</i>	91
2.4	Comparison between numerical and experimental results	92
2.4.1	Concrete panels subjected to biaxial stresses.....	93
2.4.2	RC beams without shear reinforcement.....	98
2.4.2.1	<i>RC shear critical beams tested by Vecchio and Shim</i>	98
2.4.2.2	<i>RC shear critical beams tested by Pogdorniak-Stanik</i>	103
2.4.3	SFRC beams without shear reinforcement	109
2.5	Concluding remarks	115

CHAPTER 3 INCLUSION OF SHRINKAGE STRAIN	117
3.1 Introduction	117
3.2 State of the art	119
3.2.1 Description of the physical phenomenon of shrinkage	120
3.2.1.1 <i>Plastic shrinkage</i>	121
3.2.1.2 <i>Thermal shrinkage</i>	121
3.2.1.3 <i>Autogenous shrinkage</i>	121
3.2.1.4 <i>Carbonation shrinkage</i>	122
3.2.1.5 <i>Drying shrinkage</i>	122
3.2.2 Effects of shrinkage on the behavior of structural elements	123
3.2.2.1 <i>Unreinforced elements</i>	123
3.2.2.2 <i>Reinforced elements</i>	125
3.2.3 Numerical modeling of shrinkage effects	128
3.3 Inclusion of shrinkage effects into 2D-PARC model.....	130
3.3.1 Model A	130
3.3.1.1 <i>Uncracked stage</i>	130
3.3.1.2 <i>Cracked stage</i>	131
3.3.1.3 <i>Fiber contribution</i>	133
3.3.1.4 <i>Model implementation and convergence checks in the cracked stage</i>	134
3.3.2 Model B	136
3.3.2.1 <i>Adopted modeling strategy</i>	139
3.4 Comparison between numerical and experimental results	140
3.4.1 Model A	140
3.4.1.1 <i>RC and SFRC tension members tested by Deluce</i>	141
3.4.1.2 <i>RC beams tested by Sato et al.</i>	147
3.4.1.3 <i>RC beams tested by Gribniak</i>	149
3.4.2 Model B	153
3.4.2.1 <i>Prismatic concrete specimens tested by Kim and Lee</i>	154
3.4.2.2 <i>RC tension members tested by Wu and Gilbert</i>	157
3.4.2.3 <i>RC beams tested by Wu</i>	163
3.4.2.4 <i>RC and SFRC tension members tested by Deluce</i>	168
3.5 Concluding remarks	170

CHAPTER 4 NLFEA OF CONCRETE TUNNEL LININGS SUBJECTED TO FIRE.....	173
4.1 Introduction	173
4.2 State of the art.....	175
4.2.1 Characteristics of fires in tunnels	177
4.2.2 Effects of fire on concrete linings	179
4.3 A numerical procedure to simulate tunnel lining behavior	185
4.3.1 Modeling choices.....	186
4.3.2 Thermal analysis	188
4.3.2.1 <i>Mathematical formulation</i>	188
4.3.2.2 <i>Fire curves</i>	189
4.3.2.3 <i>Thermal properties</i>	192
4.3.3 Structural analysis	193
4.3.3.1 <i>Excavation and lining installation phases</i>	194
4.3.3.2 <i>Fire phase</i>	195
4.3.3.3 <i>Material properties</i>	195
4.4 Validation of the FE model.....	203
4.4.1 Definition of the reference case study.....	203
4.4.2 Validation of the thermal analysis	204
4.4.3 Validation of the structural analysis: excavation and lining installation phases.....	205
4.4.4 Validation of the structural analysis: fire phase.....	209
4.5 Parametric study.....	213
4.5.1 Type of fire.....	213
4.5.2 Ground properties and tunnel depth	215
4.6 Concluding remarks	221
CONCLUSIONS	223
REFERENCES	227
LIST OF PUBLICATIONS	241

Introduction

The modeling of reinforced concrete (RC) structures is a quite complicated task, since their behavior is often nonlinear even starting from low loads. Reinforced concrete is indeed a composite material characterized by a very complex behavior, mainly due to the heterogeneous nature of concrete itself and to its interaction with the reinforcement, especially after cracking development. Although RC structures are widespread all over the world from more than a century and intensively studied from the middle eighties, their behavior is not yet fully understood and the investigations in this direction are still relevant. To correctly predict the response of RC structures up to failure, a refined approach is required and nowadays the finite element (FE) method is the most applied formulation, due to its attractive features. However, the accuracy of the simulations is strictly related to the constitutive laws adopted for the modeling of material behavior and considerable efforts are still being made by researchers in this sense. The work presented in this thesis lies in this line of research.

An existing nonlinear model, named 2D-PARC, is revised and extended in several aspects. This comprehensive model, which considers both the reinforcement and the cracking as smeared within the element, is formulated in terms of a secant stiffness matrix describing the behavior of reinforced and fiber reinforced concrete (RC/FRC) elements subjected to in-plane stresses, from the beginning of the loading history up to failure. 2D-PARC treats the material as an equivalent continuum, whose properties are derived by following a so called "localized stress-field approach": each mechanical phenomenon is indeed individually analyzed by using a proper constitutive relation and then the corresponding contribution is inserted into the material stiffness matrix, obtaining a realistic representation of fracture mechanisms in RC/FRC elements. In the cracked stage a strain decomposition procedure is adopted, so allowing the inclusion to in the algorithm of all the resistant contributions taking place at crack surfaces in a transparent manner. Moreover, this approach permits to easily handle the case of multi-directional cracking. Nevertheless, even if this model is proven to provide reliable results for a wide range of structures, some limitations are noticeable.

In particular, the modeling of concrete contribution under a general biaxial state of stress is managed through an orthotropic formulation, which complicates the algorithm and requires high computational effort, considerably lengthening, in some cases, the calculational time. Moreover, the adopted

procedure does not allow the description of the post-peak behavior of concrete in compression and consequently crushing failures cannot be predicted.

Another limitation of the model is that concrete prestrains, such as shrinkage or thermal expansion, are not included in its original formulation.

In this work, attempts are made to address these deficiencies in order to apply 2D-PARC model to the analysis of tunnel linings under fire conditions.

Initially, 2D-PARC model is revised with regard to the modeling of concrete contribution, so as to describe accurately and efficiently the behavior of the concrete under a general biaxial state of stress both before and after cracking development. A careful simulation of concrete behavior in compression, able to deal with crushing and dilatation, is indeed crucial for the analysis of concrete linings, since they are subjected to high biaxial compressive stresses in case of fire. Moreover, FE analyses on tunnels, which are very computational demanding by nature, require an extremely efficient numerical algorithm. The enhanced formulation provided herein is able to reduce the computational efforts as well as to take into account the effects of crushing and dilatation of concrete in compression with satisfactory accuracy.

Subsequently, the model is extended to include initial deformations of concrete, by initially considering shrinkage strains, as an initial step of a more extensive work devoted to the inclusion of all concrete prestrains. The fundamental equations governing the model are properly rewritten and the material stiffness matrix consequently rearranged. The modeling of concrete shrinkage is treated in a quite rigorous manner, since its non-uniform distribution within the element, related to the moisture gradient, is considered. To this aim an "equivalent" thermal analysis is performed, by exploiting the analogy between the equations governing the moisture and the thermal problem, in order to obtain the moisture field within the element to be subsequently employed to perform the structural analysis.

Extensive comparisons with data from the literature are provided to validate first the enhanced concrete modeling and subsequently the inclusion of shrinkage strains. Plain concrete specimens, as well as RC and FRC elements belonging to different structural typologies are considered, so as to validate the proposed formulation over a wide range of conditions.

Finally, the model is further extended and applied to the structural assessment of concrete linings under fire conditions. Concrete lining behavior is indeed analyzed by performing 3D FE simulations where mechanical nonlinearity is taken into account through 2D-PARC constitutive law. To this aim, thermal strains are inserted in the algorithm by following the same procedure adopted for shrinkage. Moreover, the model is further extended to take into account the decay of strength and stiffness of concrete during heating, by inserting the dependence of the main mechanical properties on temperature. Hence, the stress-strain field in concrete lining is evaluated accounting for both ground-structure interaction and temperature-induced effects, by performing a sequentially coupled thermo-mechanical analysis that links a heat transfer analysis with a nonlinear structural simulation. The effectiveness of the proposed approach is verified by comparing the obtained results with analytical closed-form

solutions and, in particular, a simplified analytical model is developed on purpose, so as to validate the simulation of concrete lining behavior under fire conditions. Finally, a parametric study is provided so as to better highlight the influence exerted by different parameters on the structural response of the lining, especially during fire. In more detail, different ground mechanical properties, tunnel depths and fire curves are considered to evaluate their effects on the stress and strain fields of tunnel lining.

Outline of the thesis

The thesis is structured in four main chapters.

Chapter 1 describes the line of research followed in this work, by defining its starting point. At first, an overview of the current state of the art relative to bi-dimensional models for RC elements is presented. It includes a short description of the most common approaches followed for the simulation of both cracking and reinforcement as well as a review of some well-known models that are relevant to this study. In particular, the ones that treat the material as an equivalent continuum, by smearing both cracking and reinforcement within the element, are selected. Subsequently, 2D-PARC model in its original form is detailed described. The basic hypotheses, as well as its formulation both before and after cracking development are presented, together with a description of the main steps required for its implementation in the adopted FE code.

Chapter 2 deals with the modeling of concrete contribution. At first, the peculiar characteristics of concrete behavior under uniaxial and biaxial loading are presented, together with a brief review of some well-known failure criteria and stress-strain models for concrete available in the literature. Then, the revised version of 2D-PARC model with regards to concrete modeling is presented. The adopted formulation is described in detail together with the convergence procedure proposed. Finally, the obtained numerical results are compared to several experimental data available in technical literature. To this end, plain concrete panels subjected to different biaxial stress states are first analyzed, as a sort of benchmark example, so as to prove the capability of the revised formulation in describing concrete behavior under various loading conditions, also in the post-crushing region. Then the model is applied to the analysis of RC beams where concrete modeling assumes particular relevance both before and after cracking onset. Finally, in order to verify the applicability of the proposed procedure also in case of fiber addition to the concrete mix, FRC beams without shear reinforcement are simulated, so proving the effectiveness of the model over a wide range of conditions.

Chapter 3 deals with shrinkage modeling. After a brief description of shrinkage phenomenon, the effects of concrete shrinkage on the short-term behavior of RC elements are illustrated together with a discussion on their inclusion in previously published formulations available in the literature.

Subsequently the extended formulation of 2D-PARC model herein developed to account for shrinkage prestrains is presented. For sake of simplicity, shrinkage strains are considered as uniform within the element, so developing a first simplified formulation, termed as Model A. Then a more refined approach is proposed, named Model B, where shrinkage is computed as a function of the moisture gradient within the element; thus enabling to account for self-equilibrated stresses related to differential shrinkage. Finally, extensive comparisons with data from the literature are provided to validate both formulations, by considering plain concrete specimens, as well as RC and SFRC elements belonging to different structural typologies.

Chapter 4 treats the behavior of concrete tunnel linings under fire conditions. At first, the most important characteristics of fires in tunnels are discussed, highlighting the distinguishing features from fires in traditional buildings and providing an overview on the effects of fire on concrete itself and on the behavior of the whole tunnel. Then, the main aspects of the sequentially coupled thermo-mechanical adopted approach are outlined, with special focus on the further improvements required by 2D-PARC model to properly simulate the behavior of concrete during fire. After the validation of the proposed procedure by comparing the obtained FE results with analytical formulations, the work is concluded by performing a parametric study and providing some considerations on the effects of fire on the stress and strain fields in concrete linings.

Chapter 1

2D-PARC Model

1.1 Introduction

In this chapter the basic concepts concerning the formulation of 2D-PARC model (two-dimensional Physical Approach for Reinforced Concrete) will be described in detail.

Reinforced concrete is a material characterized by a very complex behavior, mainly due to the heterogeneous nature of concrete itself and to the complicated interaction between the reinforcement and the surrounding concrete. Moreover, the complexity of the problem increases even more after cracking development. As a matter of fact, although reinforced concrete (RC) structures are widespread all over the world from more than a century and intensively studied from the middle eighties, their behavior is not yet fully understood and the investigations in this direction are still relevant. The work presented in this thesis lies in this line of research. To correctly predict the response of RC structures up to failure, due to the complexity and highly nonlinear characteristics of the problem, a refined approach is required and nowadays the finite element (FE) method is the most applied formulation, due to its attractive features. In this work the attention is focused on bi-dimensional modeling and in more detail, the existing 2D-PARC constitutive model is analyzed and improved.

This macroscopic model, proposed by Cerioni and co-workers in 2008, [1] belongs to the approaches that smear both the reinforcement and cracking within the element. It aims to formulate a stiffness matrix for reinforced concrete subjected to in-plane stresses from the beginning of the loading history up to failure, considering all the main physical phenomena governing RC behavior. A localized stress field approach is followed: not only the mean response of the element is evaluated (providing the average stress and strain field in concrete and steel) but also the local characteristics at crack location. The model is characterized by a modular structure which allows to separate the contributions related to crack formation: each mechanical phenomenon is individually analyzed and then the corresponding contribution is inserted into the material stiffness matrix. This approach permits to apply actual constitutive laws directly linked with physical reality, thus ensuring a more refined description of the behavior of RC structures both before and after cracking.

It should be pointed out that 2D-PARC model was originally developed for ordinary reinforced concrete elements, but then extended to the case of fiber reinforced concrete (SFRC) elements and this last version represents the starting point of this work.

This chapter, which describes the line of research followed in this thesis, consists of two major parts. Initially, an overview of the current state of the art about bi-dimensional modeling methods for RC elements is presented. It includes the discussion of the different cracking and reinforcement modeling techniques as well as a review of the most known models that treat the material as an equivalent continuum for both cracking and reinforcement inclusion. This choice is related to the approach followed in this work, that is the same. The second part mainly deals with the model applied in this work, 2D-PARC, focusing on the fundamental concepts describing its formulation both in the uncracked and in the cracked stage, as well as on its implementation.

1.2 State of the art

The modeling of reinforced concrete (RC) structures is a quite complex task, since their behavior is often nonlinear even starting from low loads. Reinforced concrete is indeed a composite material: the heterogeneous nature of plain concrete itself and the phenomena related to its constituent materials, especially after cracking, give rise to a highly nonlinear behavior. The structural response is indeed further complicated when cracking occurs. A considerable redistribution of stresses within the intact concrete together with the stress transfer from concrete to steel reinforcement take place as well as the appearance of additional nonlinear phenomena. Thus, in a RC structure various nonlinear mechanisms happen at the same time and often interact with each other, namely concrete cracking and crushing, aggregate interlock, bond-slip, dowel action and yielding of reinforcement. Therefore, a refined approach is required. The finite element method is the most adopted numerical tool to simulate the nonlinear behavior of reinforced concrete structures and considerable efforts are still made by researchers in order to gain a deeper understanding of the material behavior and to consequently obtain a reliable description. In this line of research lies the scope of this thesis.

Before describing the model applied in this work, in this section, an overview of the current state of the art about bi-dimensional modeling methods for RC elements is presented. Initially, the main formulations that can be followed to include reinforcement (discrete, embedded and smeared) and cracks (discrete and smeared) in modeling are discussed. Finally, some of the most important and well-known numerical models available in technical literature for the analysis of RC structure that apply the smeared approach for both the reinforcement and cracking are presented. The choice of models that treat the material as an equivalent continuum for both cracking and reinforcement is related to the approach followed in this work, that is the same.

1.2.1 Reinforcing steel modeling

Three different approaches can be followed to include steel reinforcement in RC modeling: discrete, embedded and smeared formulations [2–5].

In the discrete approach, concrete and rebars are modeled through separated elements: two different meshes are created and superimposed, each one containing its own elements (Figure 1.1a). Following this approach and applying a bi-dimensional modeling, concrete is simulated by bi-dimensional elements while for steel one-dimensional elements as truss (when the reinforcement is assumed to carry axial load only) or beam elements are usually applied. However, some Authors (see, e.g., [6]) adopt the same elements for concrete and rebars. The bar elements are forced to share their nodes with concrete elements, so assuming the hypothesis of perfect bond, or alternatively the node displacements of the two materials are related by applying a proper constitutive law. One of the main advantages of this formulation is indeed that possible displacements between the reinforcement and the surrounding concrete can be taken into account. This is usually achieved by inserting appropriate interface bond-slip elements that constrain the node of the bar elements to the concrete ones. However, the discrete approach presents several deficiencies. The concrete mesh is tied up to the position and the direction of the reinforcements and this is not convenient, especially for large structures or RC elements characterized by a complex spatial distribution of the bars. Moreover, the elements referring to concrete and steel occupy the same region, thus, the volume of steel is not subtracted to that of concrete.

In the embedded steel formulation, each reinforcing bar is considered as a one-dimensional element incorporated into the parent concrete element by imposing compatible displacements between the two materials (Figure 1.1b). The principle of virtual work is applied and the stiffness of embedded steel and concrete elements is evaluated separately. The main benefit of the embedded steel formulation is that rebar elements can be defined independently from the mesh shape and size of the concrete elements. This feature is very advantageous, especially for complex reinforcement arrangements. However, higher order isoparametric elements should be used, increasing the computational time.

In the distributed steel formulation, the reinforcing bars are assumed to be uniformly spread in the concrete elements (in the region of the FE mesh corresponding to the reinforcement location), see Figure 1.1c. The orientation of the bars must be defined as well as the reinforcement ratio. The RC element is modeled as a continuum equivalent material characterized by properties that account for both the two material constituents and their interaction. When the reinforcement is smeared, a full compatibility between steel and concrete is naturally enforced, however suitable numerical corrections that account for bond-slip can be included. The total stiffness of the RC element is obtained by superposing different contributions. The stress field within the element is indeed given by the sum of the concrete contribution and the steel one (smeared through the smeared reinforcing ratio). This approach is very widespread and

probably it is the most used method [7], since the exact definition of every single reinforcing bar can be avoided and the material can be simply treated as an equivalent continuum.

Moreover, even if this type of formulation was not originally conceived for elements whose behavior is significantly influenced by the reinforcement (e.g. tension ties), some studies demonstrated its capability also in this cases [8,9]. Since the smeared steel formulation is the approach followed in this thesis, in the following among the several well-known bi-dimensional models available in technical literature for the analysis of RC elements, only those adopting the distributed reinforcement formulation will be briefly outlined.

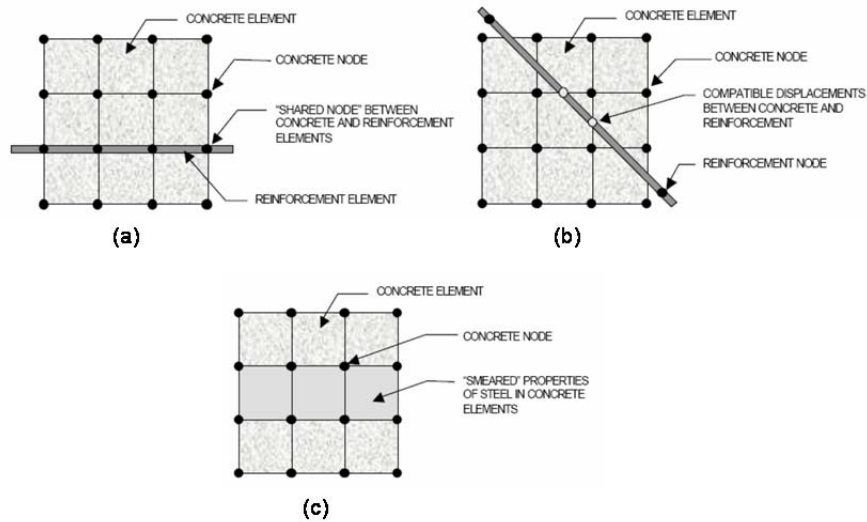


Figure 1.1 Different modeling formulations for reinforcing steel: (a) discrete; (b) embedded; (c) smeared, [3]

1.2.2 Cracking modeling

The two main formulations available in the literature for crack modeling in concrete structures are the discrete crack approach and the smeared crack approach.

1.2.2.1 Discrete crack approach

The discrete crack formulation simulates cracking as a discontinuity in the mesh. Two different methods can be classified within this approach, depending if crack propagates by separation of the edges of the element, namely the inter-element crack model, or if crack is allowed to propagate through the finite element, namely the intra-element crack model [10].

In the inter-element crack model, when cracking occurs there is a separation of the mesh providing new degrees of freedom [6,11], see Figure 1.2a. Interface elements (lumped or continuous), with zero thickness and variable stiffness are incorporated within the original mesh (where cracks are expected to arise) to simulate the crack opening. The initial stiffness of these elements is set large enough to simulate the uncracked stage, so linking rigidly the superimposed nodes. Once cracking occurs the stiffness is updated by applying a proper constitutive relation. This approach imitates the physical reality very closely; however, it suffers from serious drawbacks. The crack trajectory is forced to follow a predefined path along the element edges. This implies to know the position of cracks in the mesh definition phase making this approach suitable only in particular cases where great information about the fracture position and propagation are available. To overcome this problem, algorithms of automatic remeshing [12] were developed, but so lengthening the calculational times. In addition, the discrete crack model does not fit the original idea of the FE method because continue changes in the nodal connections are required.

In the intra-element crack model the crack can propagate within the element (Figure 1.2b); different formulations are available in technical literature. Special elements with embedded discontinuities to represent cracks inside the element can be applied: new displacements arise in the element by incorporating additional localization modes to the standard shape functions of the FE [13]. Alternatively, discontinuous shape functions can be used and the displacement of the cracks are represented by additional degrees of freedom at the exiting nodes [14].

The discrete crack approach may appear appealing, since the crack is simulated by a discontinuity and this representation is closer to the natural conception of fracture. As a matter of fact, it is often the most attractive tool to study element characterized by localized cracking, even if also smeared crack formulations, that better suit the FE method, can be used [15]. Moreover, a lot of structure, such as RC panels, shear walls or the tension side of RC beams are characterized by a diffuse cracking so obviously better fit the smeared crack concept [10].

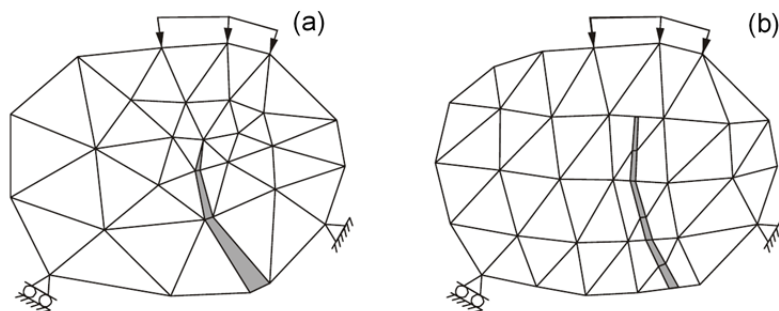


Figure 1.2 Discrete cracking approach: for reinforcing steel: (a) intra-element and (b) intra-element crack model [10]

1.2.2.2 Smeared crack approach

The smeared crack formulation represents the cracked material as an equivalent continuum by distributing cracking in a predefined volume of the material (Figure 1.3). This model permits a description in terms of stress-strain laws: when cracking occurs, at the corresponding integration point of the element the material stiffness is reduced with the introduction of proper constitutive laws. The initial isotropic stress-strain relation is turned to an orthotropic formulation with the axes of orthotropy directed as the principal stress directions. The constitutive matrix of the cracked material is first written in the local co-ordinate system of the crack and then rotated to the global one by adopting proper transformation matrices.

This approach, originally developed by Rashid in 1968 [16], is now the most widely adopted method for cracking modeling in structural engineering problems [10]. The smeared crack formulation is indeed very attractive since it is numerically feasible and offers great convenience in numerical implementation, allowing to maintain the same mesh topology for the whole analysis. Moreover, the cracks are allowed to propagate in any direction; the orientation is indeed not restricted since it coincides with the orthotropic axis.

As drawback, the assumption of displacement continuity conflicts with the nature of a fracture. However, if cracking is characterized by a distributed pattern, such as in case of densely distributed reinforcements, a physical foundation for the smeared concept is provided. The supporters of this formulation state that, even in case of localized fracture, the smeared crack approach is more realistic, because the fracture process zone of concrete is densely filled with microcracks [17]. However, the smeared crack formulation, when dealing with localized cracking, can suffer from other deficiencies. The shear strength of some structural elements can be overestimated; to this aim proper smeared formulations have been developed in order to overcome this problem, as discussed in the following. On the contrary a general drawback of the smeared crack methods is the danger of stress-locking that, in a small number of cases, leads to an underestimation of the relief occurring at the side of the crack.

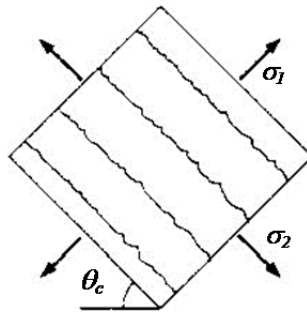


Figure 1.3 Smeared cracking approach, being σ_1 and σ_2 the two orthotropic directions and θ_c the orientation angle [18]

Smeared crack formulations can be subdivided into three main categories: fixed, multi-directional and rotating, depending on the variation of crack orientation during the loading process.

Fixed crack model

In the standard fixed crack approach, the crack initiates normal to the major principal stress direction and the orientation remains the same for the whole analysis even if the principal directions change. When a bi-dimensional simulation is considered, the stress-strain field is expressed as:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} E_{11} & E_{12} & 0 \\ E_{21} & E_{22} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}, \quad (1.1)$$

being 1 and 2 respectively the local axis normal and tangential to crack direction.

Initially, in the earliest development of this approach the terms on the main diagonal (E_{11} , E_{22} , G_{12}) were assumed equal to zero; thus, setting after crack formation both normal σ_1 and tangential τ_{12} stress equal to zero [16,19]. Nevertheless, this approximation was not in agreement with experimental evidences; moreover, the sudden drop of stiffness may lead to numerical difficulties. For all these reasons, the terms E_{11} , E_{22} , G_{12} were reinserted in Equation (1.1), but modified with a proper reduction factor. Various formulations were proposed, among these one of the most widespread is the incremental orthotropic formulation proposed by Rots et al. [15]:

$$\begin{Bmatrix} \Delta\sigma_1 \\ \Delta\sigma_2 \\ \Delta\tau_{12} \end{Bmatrix} = \begin{bmatrix} \frac{\mu E}{1-\nu^2\mu} & \frac{\nu\mu E}{1-\nu^2\mu} & 0 \\ \frac{\nu\mu E}{1-\nu^2\mu} & \frac{E}{1-\nu^2\mu} & 0 \\ 0 & 0 & \frac{\beta E}{2(1+\nu)} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_1 \\ \Delta\varepsilon_2 \\ \Delta\gamma_{12} \end{Bmatrix}, \quad (1.2)$$

where E is the Elastic modulus, ν the Poisson coefficient, while β and η represent the “shear” and the “axial reduction factor” respectively.

Nevertheless, principal stress directions change during the loading process leading in some cases (such as in case of high shear stresses) to a great discrepancy between the axes of the principal stresses and the ones of crack. Consequently, the standard fixed cracked model can provide stiffer responses if compared to experimental evidences.

A more refined approach, that is also followed in this thesis, is based on the strain decomposition: the total strain vector $\{\varepsilon\}$ is subdivided into two components, respectively the (intact) concrete $\{\varepsilon_c\}$ and the crack strain $\{\varepsilon_{cr}\}$. This formulation, proposed in [20], has gained a lot of popularity over the years. Following this approach, the crack constitutive laws, which usually are based on crack strain rather than on total strain, can be incorporated in a transparent

manner [15]. In this way, the shear factor, usually evaluated arbitrarily without referring to physical models that consider the aggregate-interlock effects, can be suppressed, so bridging the gap between smeared formulations and refined constitutive crack laws. This formulation is closer to discrete crack approach where intact concrete is separated from crack by creating a discontinuity in the FE mesh. The total strain can be written as follows:

$$\{\varepsilon\} = \{\varepsilon_c\} + \{\varepsilon_{cr}\} , \quad (1.3)$$

where the crack strain $\{\varepsilon_{cr}\}$ can be first computed in the crack local co-ordinate system 1-2 of the crack and then transferred to the global one, while concrete strain $\{\varepsilon_c\}$ can be even evaluated directly in the global co-ordinate system, exploiting the isotropic directions. The strain decomposition approach, formulated in the incremental form, can also account for closing and re-opening of cracks. Moreover, this approach can be easily extended to allow multiple cracking, as better described in the following paragraph.

Multi-directional crack model

A refined version of the fixed crack model based on the strain decomposition is the multi-directional cracking approach. The material is allowed to crack more than once by adding the strain of the subsequent cracks to the total strain vector. Each crack is allowed to open in any direction that remains fixed after the formation. In this way, a correct simulation of the changing in the principal directions during loading is provided while maintaining fixed the orientation of the cracks as demonstrated by experimental evidences.

The model is based on the sub-decomposition of the crack strain referring to multi-directional cracks that simultaneously can occur at the same integration point, leading to the following formulation:

$$\{\varepsilon\} = \{\varepsilon_c\} + \{\varepsilon_{cr1}\} + \{\varepsilon_{cr2}\} + \dots . \quad (1.4)$$

This model represents an intermediate option between the fixed and the rotating model; it is not only closer to physical reality (maintaining fixed crack orientation but varying the stress principal directions) but it also provides many advantages in modeling, because it offers all the benefits described for the fixed approach. One of the main drawbacks is related to the complication arising from opening and closing of cracks that may happen simultaneously. The computational procedure related to this model can indeed be quite complicated, but adopting special strategies (e.g. [21]) this issue can be overcome. Moreover, even adopting a formulation that allows secondary (or multiple) cracks non-orthogonal to the primary ones, the angle between them seems minor to that experimentally reported [22].

Rotating crack model

In the rotating crack formulation, the orientation of cracks changes according to principal directions during the entire loading process. The orthotropic axes are assumed coincident to the principal strain directions; thus the orthotropic constitutive law for cracked concrete varies according to crack directions. Consequently, Equation (1.1) can be rewritten in the principal directions instead of considering the local 1-2 co-ordinate system of the crack.

This approach, originally proposed by Cope et al. [23], is easier to be used and implemented in FE codes compared to the fixed and multi-directional cracking formulations; hence, a significant development and a lot of applications have occurred since its appearance. Moreover, the shear retention factor (arbitrary and sensitive) is no longer required, making this method appealing from an engineering point of view. Even if this method is proven to be computationally convenient and it represents an efficient tool in modeling, some questions about its physical meaning arise. In the empirical reality, cracks are not free to change their orientations, while only principal stress directions do. The direct use of constitutive laws along the principal axes can be considered indeed completely inconsistent, unless proper transformation laws are included in the calculation of the tangential stiffness moduli. Some procedures have been proposed to overcome this deficiency, such as the one proposed by Bazant [24] to assure coaxiality between the axes of principal strains and principal stresses.

Finally, it is worth noting that an evident parallelism applies between the rotating and multi-directional approach: the former can be seen as a limit case of the latter if some conditions apply, as detailed described in [15]. If the hypotheses are met the strain decomposition procedure can be employed, with the related advantages.

1.2.3 Overview of some smeared models for RC elements

In this section, among the several numerical formulations for the bi-dimensional analysis of RC elements available in technical literature, an overview is provided with only reference to the models that treat the material as an equivalent continuum, smearing both cracking and reinforcement. For brevity, only four models are herein briefly outlined: the MCFT [18], the DSFM [25], the FA-STM [26] and the model by Soltani et al. [27].

1.2.3.1 Modified Compression Field Theory (MCFT)

The MCFT was originally developed by Vecchio and Collins in 1986 [18] on the basis of a previous formulation, named the Compression Field Theory (CFT) [28,29], that treated the RC material as an equivalent continuum, characterized by proper stress-strain relationships.

As already said, the MCFT belongs to the smeared reinforcement formulations, so schematizing rebars as uniformly distributed in the element. A reinforced concrete membrane element, subjected to normal and shear stresses uniformly applied, is analyzed providing the average strains and stresses of

concrete and steel. The strain history is neglected; so a unique stress state for each strain state is given.

The MCFT refers also to the smeared crack concept: strains and stresses are averaged over a distance including several cracks and the cracked material is treated as an orthotropic solid continuum. Moreover, a rotating crack approach is followed: cracks are free to reorient during the whole loading process and coaxiality with the directions of the principal concrete stresses is guaranteed. The orientations of principal strains and stresses are assumed as coincident.

The model is based on equilibrium and compatibility equations together with constitutive laws that relate the average strains of compatibility conditions with the average stresses in the equilibrium relations, see Figure 1.4. Perfect bond is assumed between concrete and steel, shear stresses are assumed equal to zero in the reinforcement and separate constitutive laws are provided for concrete and steel. The equations governing the model are expressed by using averaged terms, namely mean prestresses and strains evaluated over a region large enough to comprehend a predefined crack pattern. In addition, also local stresses at crack surfaces are controlled, in this case a loss of tension in concrete corresponds to an increase in steel stresses.

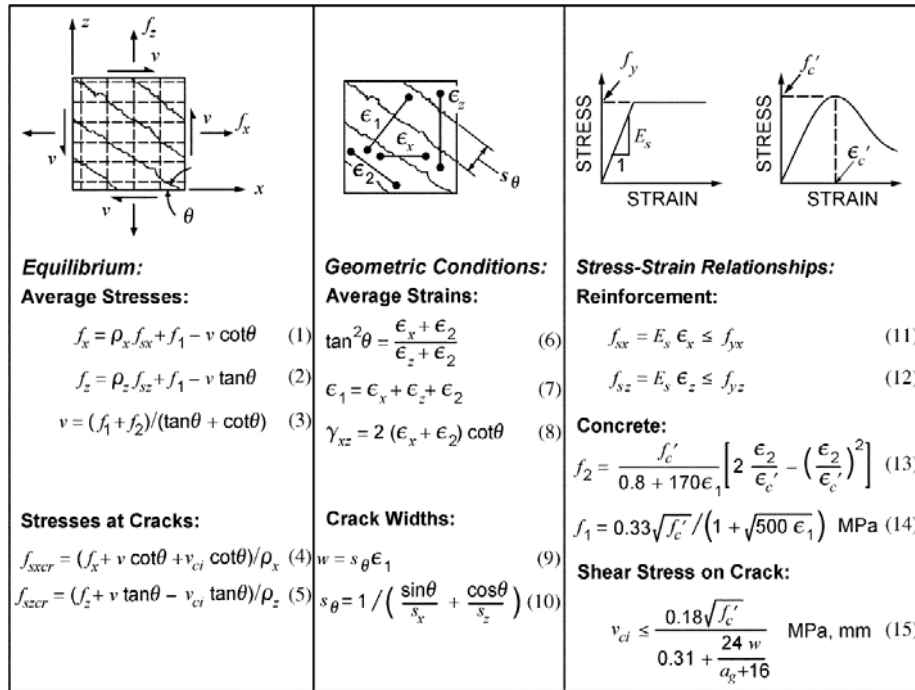


Figure 1.4 Summary of the main equations governing the MCFT [30]

Equilibrium equations are reported in the first column of Figure 1.4: the total stresses applied in the normal directions f_x, f_z , are balanced by the sum of the concrete f_{cx}, f_{cz} and the steel f_{sx}, f_{sz} average stresses; these latter obviously smeared through the reinforcing ratios ρ_x, ρ_z . The assumption that the reinforcement does not exhibit dowel action is made; it follows that the applied shear stresses v are entirely absorbed by shear stresses in concrete. The concrete stresses f_{cx}, f_{cz} are evaluated considering cracked concrete as an orthotropic material and applying the Mohr's circle. Therefore, they can be written as a function of average principal concrete stresses f_1, f_2 and the crack orientation angle θ (this latter set equal, as already said, to the average orientation of the principal tensile strain and stress direction).

As regards compatibility conditions, reported in the second column of Figure 1.4, the assumption of perfect bond is made; thus, the average concrete and steel strains are set equal to each other and to the total strain vector. Once again applying the Mohr's circle the principal tensile and compressive strains are provided.

The constitutive relationships are finally reported in the third column of Figure 1.4. As regards the reinforcement, both in tension and compression, a bilinear law is applied to relate the average steel stresses f_{sx}, f_{sz} to the average steel strains, assumed coincident to the global one $\varepsilon_x, \varepsilon_y$. The constitutive laws for cracked concrete (both in tension and compression) were instead developed by Vecchio and Collins [18] by performing in 1986 an extensive experimental program on panels subjected to in-plane stresses. The compressive law relates the concrete principal compressive stress f_2 to principal compressive strain ε_2 . It takes also into account the decrease of strength and stiffness that concrete exhibits as the principal tensile strain ε_1 increases, as suggested by the experimental tests on panels. ε_1 is utilized as damage indicator of the compression softening to modify the relation based on the Hognestad parabola for normal strength concrete under uniaxial compression. As regards concrete in tension it is assumed to behave liner-elastically until the attainment of the concrete tensile strength (assumed equal to $0.33\sqrt{f'_c}$ in absence of other information); then a softening relation is provided. After cracking a portion of tensile stresses is assumed to be still carried by concrete due to bond action between the rebar and the surrounding concrete, namely the tension stiffening mechanism. The relation proposed for cracked concrete in tension by the MCFT evaluates then the concrete principal tensile stress f_1 as a function of the principal tensile strain ε_1 of cracked concrete.

The model not only provides the average stress and strain field but also the local one at the crack location, providing also the crack width. This latter is computed as shown in Figure 1.4 as the product between the concrete principal tensile strain ε_1 and the mean crack spacing s_b in turn computed starting from the bond properties and the arrangement of the reinforcement [31].

Moreover, it is important to control the local yielding of the reinforcement at the crack or the crack sliding which may lead to shear failure. To this aim, the MCFT limits both the average concrete tensile stress f_1 and the local shear stress

at the crack location, v_{ci} . Equilibrium conditions at the free crack surfaces are considered; it results that in order to transmit the average tensile stress across the crack, the reinforcement stress and strain must increase locally. Therefore, it is at crack location that the reinforcement first yields, while its average stress value still remains below the yielding strength. Thus, the average tensile concrete stress must be limited in order not to be overestimated, because of the yielding of the reinforcement at the crack.

If average stresses between cracks are considered shear stresses are absent, but at crack surfaces local stresses v_{ci} are present. The value of v_{ci} can be inferred by considering the static equilibrium between the average and local stresses in the crack tangential direction. The so obtained value is then limited on the basis of the aggregate interlock effect, following the formulation proposed by Walraven [32] that considers the decreasing effect due to increasing crack width w and decreasing maximum aggregate size a_g .

1.2.3.2 Disturbed Stress Field Model (DSFM)

The DSFM can be seen as an extension and an improvement of the MCFT. It was indeed developed by Vecchio in 2000 [25] to overcome the main drawbacks of the MCFT: the forced coaxiality between the main directions of stress and strain, as well as the approximations related to the evaluation of shear stresses at crack location.

As a matter of fact, the MCFT has been proven to provide incorrect predictions when describing certain structural typologies and loading configurations. For example it tends to underestimate the shear stiffness and strength of elements characterized by reinforcement arrangements and load conditions that allow a limited rotations of the principal stress and strain fields, such as panels that are heavily reinforced in both directions or panels subject to high biaxial compressions acting together with shear stress. Conversely, it generally tends to overestimate the shear stiffness and strength of lightly reinforced elements where the shear slip across the crack is quite large and the rotation of principal stress axes lags the higher one of the principal strain field; such in case of beams without (or few) shear reinforcement and panel lightly reinforced in the transversal direction.

The DSFM tends to address the deficiencies of MCFT, by including the crack shear slip deformation in the compatibility relationship and uncoupling the orientation that the principal stress axes form with that of the principal strain. The MCFT considers the equilibrium at crack location only in order to foresee the possibility of sliding shear failure along the crack (hence, limiting the value that crack shear stress can assume) and of local yielding of the reinforcement (hence, limiting the value of the concrete tensile stress). However, for the respect of the congruence relations adopted, it does not consider the crack shear slip in the evaluation of the average total strain; moreover the principal stress and strain fields remain coaxial.

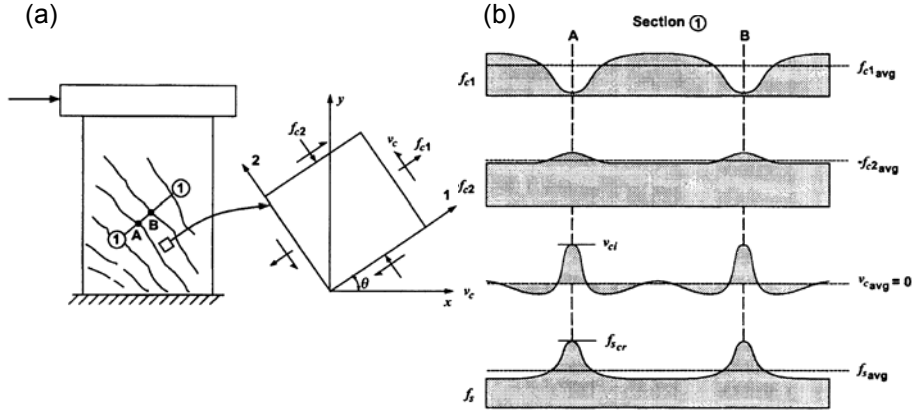


Figure 1.5 Nature of "disturbed" stress in cracked reinforced concrete: a) stress field; b) its variations [25]

Therefore, the DSFM while maintaining a general approach very similar to that of the MCFT, overcomes both these deficiencies. It distinguishes the strain due to shear slip from the strain of concrete continuum due to stress; moreover it allows the principal strain field to change inclination with a different rate respect to the principal stress field, thus permitting a differential lag between them. This phenomenon, experimentally confirmed, is attributable to the manner by which strain and stress are evaluated. The measured strains represent the total ones, which are attributable to straining of the concrete continuum in response to applied stresses combined with the deformation due to crack shear slip. Meanwhile, concrete stresses are attributable only to the continuum straining in response to applied stresses [33]. The DSFM considers that the local conditions at crack location determine a "disturb" of the average stress fields considered in the general problem formulation, so significantly influencing the behavior of the entire element (Figure 1.5). In this way, the concrete behavior is more precisely described and it is possible to eliminate the control of the crack shear stress, introduced by the MCFT.

Following these assumptions, an appropriate set of equilibrium and compatibility equations are written, in large extent, on the basis of the MCFT ones. Moreover, constitutive relationships for concrete and steel are refined. The main equations describing the DSFM are reported in Figure 1.6; in the following the main differences with MCFT are presented.

As can easily be seen from Figure 1.6, the main novelty in the DSFM lies in the formulation of the equation of compatibility, since the total (or apparent) deformation of the element, $\{\varepsilon\}$, is obtained by superposition of different aliquots: the net concrete strain (that is the continuum stress induced strain), $\{\varepsilon_c\}$, and the deformation related to sliding along crack surfaces, $\{\varepsilon^s\}$. Also the concrete strain

offset $\{\varepsilon^0\}$ is considered, whose application is treated in the manner suggested in [34]. The relation can be written in the vector form as:

$$\{\varepsilon\} = \{\varepsilon_c\} + \{\varepsilon^s\} + \{\varepsilon^0\} . \quad (1.5)$$

This implicitly means that there is no longer coincidence between the main directions of tension and deformation, since the average stresses are related, by means of the constitutive laws, only to the net concrete strain $\{\varepsilon_c\}$ and not to the total deformation $\{\varepsilon\}$. This rotation lag, $\Delta\theta$, can be expressed as follows:

$$\Delta\theta = \theta_\varepsilon - \theta_\sigma , \quad (1.6)$$

being θ_ε and θ_σ respectively the orientation angles of the total (apparent) strain and of the concrete stress (thus that of the equivalent continuum).

The average strains in the reinforcement are computed starting from the total strains and considering the bar orientations.

It is worth noting that the crack slip strains, $\{\varepsilon^s\}$, can be obtained, by means of the Mohr's circle, starting from the crack slip δ_s and the average crack spacing s . The crack slip δ_s was originally evaluated as reported in Figure 1.6 by applying the relation based on the aggregate interlock effects proposed by Walraven [32]:

$$\delta_s = \frac{\nu_{ci}}{1.8w^{-0.8} + (0.234w^{-0.707} - 0.20) f_{cc}} , \quad (1.7)$$

being w the crack opening, ν_{ci} the local shear stress and f_{cc} the concrete cube strength. Following this approach, the crack slip shear strain γ_s^a results:

$$\gamma_s^a = \frac{\delta_s}{s} . \quad (1.8)$$

However, this formulation has two main problems. Firstly, crack slip δ_s results equal to zero for unreinforced members because in turn the local shear stress ν_{ci} is equal to zero (see the equilibrium equation that predicts the shear stress at crack); this means neglecting the shear stresses from aggregate interlock (effect instead important in unreinforced members). Secondly, the initial crack slip that occurs before contact areas have been developed between the rough crack surfaces, is not considered.

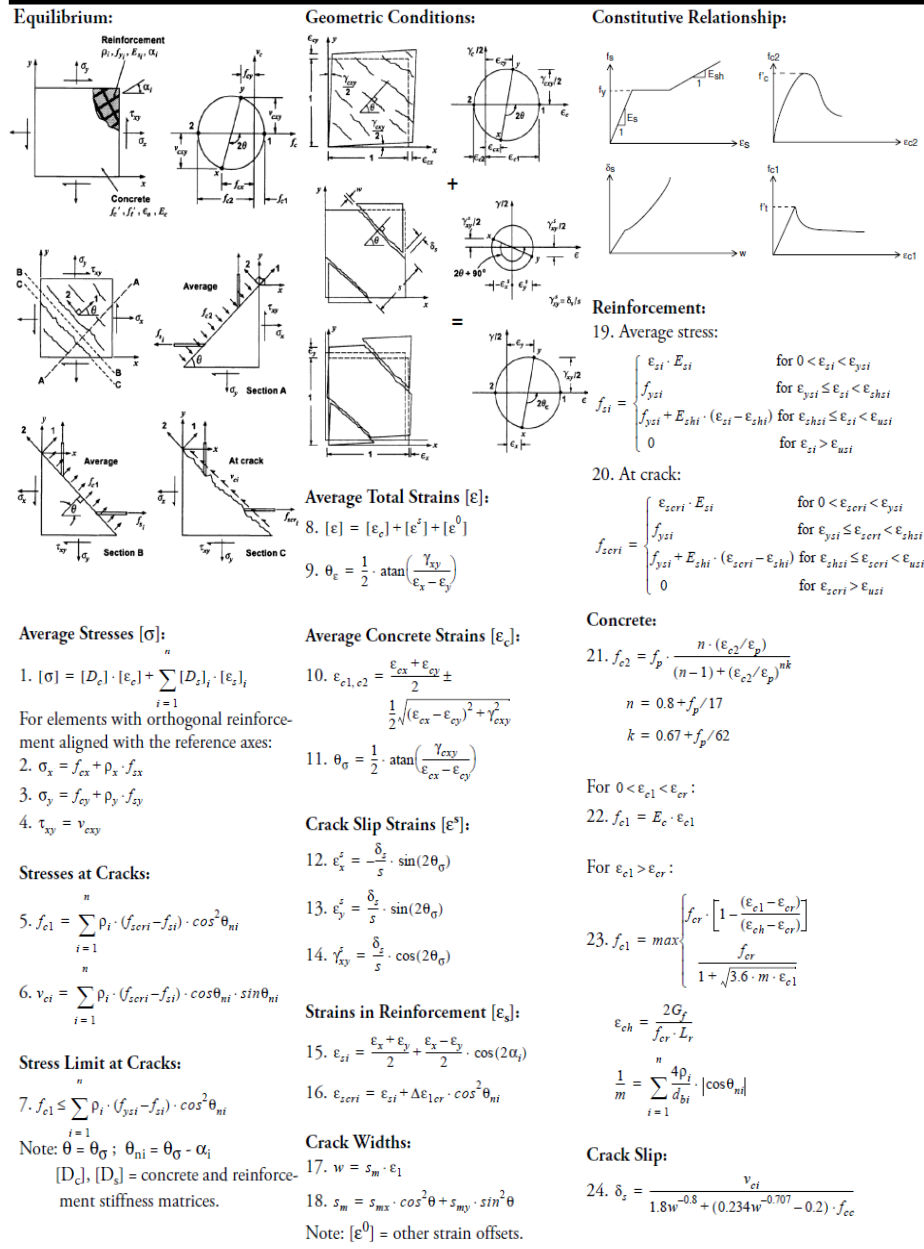


Figure 1.6 Summary of the main equations governing the DSFM [25], picture taken from [35]

An alternative relation was then developed to evaluate the crack slip shear strain γ_s^b , by opportunely computing the post-cracking orientation of the principal stress field. It is evaluated as the sum of its orientation at initial cracking, and its post-cracking rotation; this latter computed considering that the lag between the inclination of the axes of principal total strain and stress is established soon after cracking and it remains constant until the attainment of the yielding of the reinforcement whereupon it increases.

The two approaches are combined in a hybrid model to obtain the actual value of the shear slip strain γ_s :

$$\gamma_s = \max(\gamma_s^a, \gamma_s^b) . \quad (1.9)$$

It results that when the concrete element is unreinforced or when the local shear stress on the crack is small, the second approach rules the shear slip determination, reflecting the initial slip occurring prior to development of shear stresses at crack location. Conversely, when the shear stresses on the crack are large, the shear slip is predicted by the stress-based formulation.

The average stress equilibrium conditions of the DSFM are the same applied in the MCFT. However, the DSFM additionally incorporates the equilibrium relations for local stresses at crack location. Moreover, unlike the MCFT, it explicitly considers the deformations due to shear slip rather than limiting the concrete stress to the corresponding value related to shear slip failure.

As already said, also the constitutive relationship applied in the DSFM for concrete and steel are refined compared to the ones applied in the MCFT. The laws describing the response of concrete in tension and compression are both revised. The relationship describing the concrete behavior in compression becomes more general. For example, it is able to take into account the effect that the element slip distortion exerts on softening. For cracked concrete in tension, the tension softening effect is inserted in the formulation, computing the post-cracking principal tensile stress in concrete as the larger between the value considering tension stiffening and tension softening mechanism. As regards steel, a trilinear law both in tension and in compression is applied, in order to take into account the strain-hardening effect.

It can be concluded the DSFM, that improves the MCFT overcoming its main deficiencies, can be considered a smeared delayed rotating crack model [33]. In particular, it can be seen halfway between the fixed and the rotating crack approach. As a matter of fact, unlike the fixed crack formulation, a gradual and progressive re-orientation of the main directions of concrete stresses (and therefore of crack directions) is allowed, but, unlike rotating crack standard formulation, it is allowed a lag between the main directions of stress and strain.

1.2.3.3 Fixed-Angle Softened Truss Model (FA-STM)

The “Rotating-Angle Softened Truss Model” (RA-STM) [36], and the “Fixed-Angle Softened Truss Model” (FA-STM) [26,37–39], were developed by Hsu and co-workers in the late eighties and the early nineties after an extensive and significant experimental program on reinforced membrane elements subjected to plain stresses. The FA-STM is conceptually similar to the previous RA-STM, but it extends and improve the RA-STM in several aspects. Many of the basic assumptions of the two models are essentially the same. Both the models belong to the smeared reinforcement and cracking formulations: reinforced concrete is treated as a continuous material, characterized by proper stress-strain relationships, by formulating equilibrium, compatibility and constitutive laws in terms of average stress and strain.

The RA-STM model was developed following a rotating crack approach, so the direction of the cracks, which coincides with the direction of the main compressive stress, is variable throughout the loading history. The shear stress along the crack is neglected, so the model only requires three constitutive laws, describing concrete in tension, concrete in compression and the steel bar behavior (assumed equal in tension and compression). For this reason, it is not able to take into account the beneficial contribution that concrete exerts on the response of the element, that in some cases, such as for beams and panels characterized by low transverse reinforcement, is proven to be significant. This problem, also observed by Vecchio for the MCFT, has led, in this case, the development of a new model, the FA-STM, based on a fixed crack approach [26].

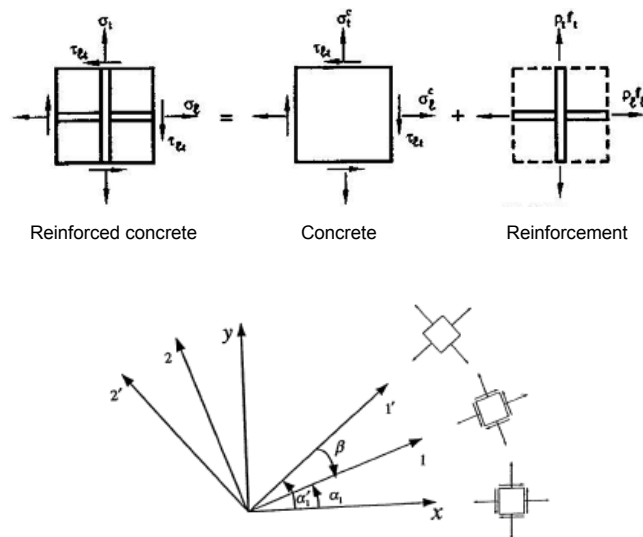


Figure 1.7 Schematization assumed in the model with the assumed principal co-ordinate system together with the definition of the fixed angle [39]

The FA-STM assumes that the crack direction is parallel to the direction assumed by the principal compressive stresses at the onset of cracking and it remains unchanged with the increasing of the applied load. In particular, when, after cracking, the loading increases, the principal directions of applied stresses will rotate from 1-2 to 1'-2' co-ordinate system and the principal angle will change. Therefore, since the cracking angle α_1' is assumed fixed to its value at the onset of cracking α_1 , the stresses in 1'-2' co-ordinate system are transformed back to the stresses in 1-2 co-ordinate through the angle β (see Figure 1.7).

Following this assumption nonzero shear stresses are provided at crack location, thus, the model can take into account the related concrete contribution, by simply adding an additional constitutive law (which refers to the concrete subjected to shear) to the three adopted for the RA-STM. In the original version of FA-STM, such a constitutive law for concrete in shear was based on a complex empirical relation, obtained starting from the results by the aforementioned experimental tests on reinforced concrete panels [26,38]. Later, with the implementation of the model in a finite element program (called FEAPRC), this empirical law was replaced by the introduction, in the stiffness matrix of the cracked concrete, of a more effective shear modulus, computed by solving the equilibrium and compatibility conditions. Before cracking concrete is schematize as an isotropic elastic material to pass an orthotropic nonlinear formulation after the cracking onset. The main equations describing the model are reported in Figure 1.8.

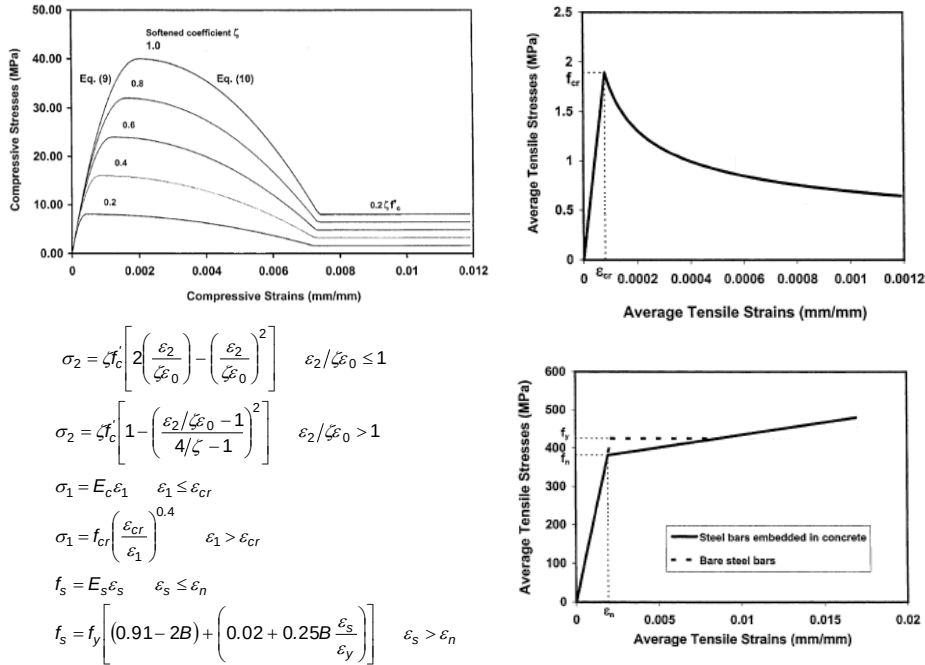


Figure 1.8 Main constitutive relationships adopted in FA-STM [39]

1.2.3.4 Soltani, An and Maekawa's model

The models described so far represent the true nature of the smeared formulation (or non-localized stress field approach, as termed by [27,40]). They provide the mean response of the analyzed elements, regardless of the specific contributions due to mechanical phenomena related to physical reality. This choice, mainly related to a higher computational simplicity, inevitably requires the introduction of some simplifications and the arbitrary assumption of some parameters. In this way, an incorrect description of the behavior of the analyzed element can be provided, especially, as already said, for some reinforcement arrangements and loading conditions.

For all these reasons, from the middle of the eighties, localized stress field approaches, able to describe the actual crack pattern of RC membrane elements as loading increases by assuming the crack opening and sliding as the main variables of the problem, begin to be developed. Such a formulation was seen since its appearance as an effective tool for the nonlinear analysis of RC structures, also easy to be used in conjunction with complex FE modeling (see [41,42]). The "Computational model for post cracking analysis of RC elements based on local stress-strain characteristics" developed by Soltani et al. in 2003, [27] can be included in this line of research. The response of RC membrane element in the cracked stage is formulated in terms of local stresses and strains at crack location, considering all the stress transmission mechanisms that are actually present in reinforced concrete structures (Figure 1.9).

The development of shear stresses at crack location, which is related to several phenomena, such as tension stiffening, dowel action and aggregate interlock, influences significantly the structural response and, thus the ultimate capacity of the element. Therefore, a careful comprehension and evaluation of all these mechanisms is crucial to correctly simulate the RC behavior.

The formulation proposed by Soltani et al. [27] is based on the application of equilibrium, compatibility and constitutive equations. Even if this model analyzes the local behavior of the material, it belongs to the smeared reinforcement and cracking formulations. In particular it follows the smeared fixed crack approach, since the crack orientation is maintained constant as loading (assumed applied monotonically) increases.

It is worth noting that the constitutive equations are based on the results of the intense research activity carried out at the Tokyo University, aimed at investigating the local response of both concrete and steel as well as their interaction. By applying the so determined constitutive relationships, the model provides the average stress and strain fields for both concrete and steel during the entire loading process, at the beginning of crack formation as well as in the stabilized cracking stage.

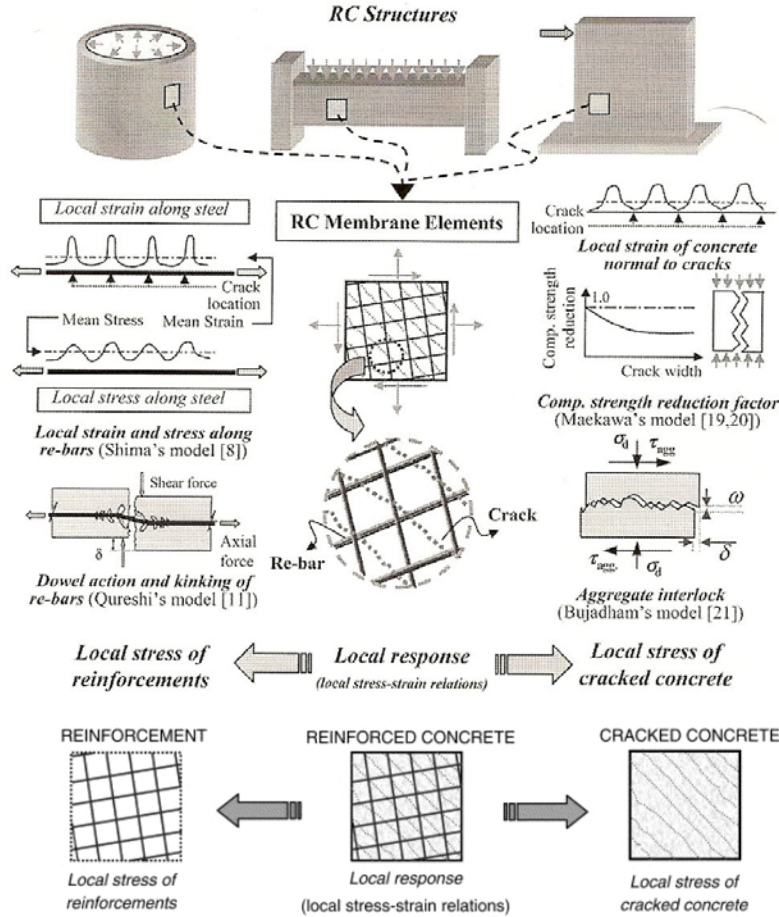


Figure 1.9 Local stress transfer mechanisms in RC membrane elements, according to Soltani et al. model [27,40]

In particular the model proposed by Soltani et al. [27] assumes the crack orientation at right angle to the tensile principal stress direction acting at the onset of cracking. After cracking development, the main variable describing the material behavior are the concrete strains and the local strains at crack location. These latter are computed starting from the crack opening w and the crack sliding δ , as follow:

$$\begin{aligned} w &= \varepsilon_1 S_{XY} \\ \delta &= \gamma_{12} S_{XY}, \end{aligned} \quad (1.10)$$

being S_{XY} the average crack spacing. Initially this value is assumed equal to the length of the sample in the direction of the principal tensile stress, S_0 , and

subsequently it is updated to consider the appearance of subsequent cracks, thus adopting $S_{XY} = S_0/N_{cr}$, where N_{cr} represents the number of generated cracks. As a matter of fact, whenever the local stress in concrete between two adjacent cracks exceeds the concrete tensile strength, the model allows the formation of a new crack, placed exactly halfway between the already existing cracks, until the attainment of the stabilized cracking stage.

The three components of the "local deformation" vector $\{\varepsilon_1, \varepsilon_2, \gamma_{12}\}$, are obtained by an incremental-iterative procedure. Then all the other variables necessary to solve the problem (i.e. crack spacing, crack opening and sliding, average stress and strain fields for concrete and steel) are evaluated by simply imposing the compatibility at crack location and the equilibrium between the applied and the local stresses (Figure 1.10).

This procedure proposed by Soltani et al. [27] can be considered as a sort of improved smeared approach, since the material stiffness is obtained by considering the local characteristics of the constituent materials (Figure 1.9). It is worth noting that the modeling of bond-slip plays a major role in the above described approach, since it rules the crack propagation within the element not only in terms of crack opening but also defining the crack distance.

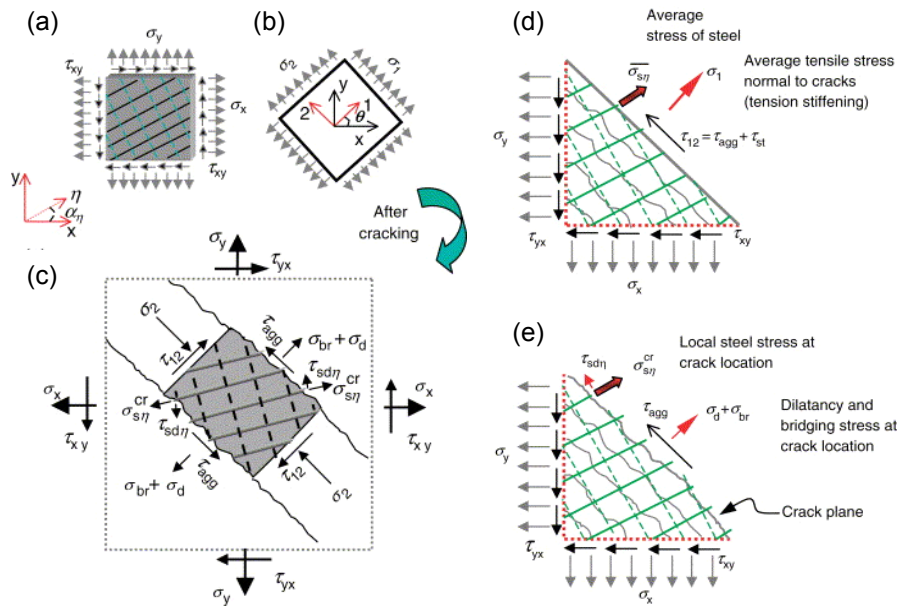


Figure 1.10 RC membrane element: (a) applied stress; (b) principle directions of applied stress, (c) local stress state at crack location, (d) equilibrium conditions for the average stresses; (e) equilibrium conditions for local stress state at crack plane [40]

The main advantage of such an approach is related to the possibility of applying constitutive relationships which better describe the physical behavior of the analyzed material. Moreover, they can be simply updated, once more refined formulations become available in the literature. In this sense, it can be reminded that Soltani and co-workers have carried out parametric studies aimed at investigating the influence exerted by both the reinforcement ratio and orientation (see [43]), as well as the scale effect (see [40]) on the behavior of RC elements after cracking, in order to contribute to the development of more general and refined constitutive laws.

1.3 Description of 2D-PARC model

1.3.1 Origin of 2D-PARC model

2D-PARC model represents an advanced version of a constitutive law for cracked reinforced concrete elements (so-called "PARC", i.e. Physical Approach for Reinforced Concrete), originally presented by Belletti et al. in 2001 [42], addressing its systematic deficiencies.

As a matter of fact, even if the PARC model is proven to be a useful tool for the analysis of RC elements subjected to in-plane stresses providing reliable results, some limitations are noticeable. Therefore, the basic formulation of PARC was revised to overcome its major limitations by formulating a more general model, so called 2D-PARC.

PARC model can be included in the smeared (for both reinforcement and cracking) formulations and it analyzes a reinforced concrete membrane element subjected to in-plane stresses. Two different constitutive behaviors are applied depending on the element is cracked or not. In the uncracked stage steel is assumed to behave as an elastic-perfectly plastic material, while concrete as an elastic, nonlinear and orthotropic material. The increase or decrease of the strength in a certain direction is computed starting from the stress acting in the corresponding transversal direction.

When the maximum principal stress reaches the concrete tensile strength, the transition to the cracked stage takes place. Cracks are assumed to develop equally spaced and with a fixed orientation, perpendicular to the principal tensile direction acting at the onset of cracking. Then, the variables that govern the problem become the crack opening w and sliding v , together with the deformation ε_{c2} of the concrete strut between two adjacent cracks, computed in the crack direction. The mechanical behavior of the cracked material is described by means of the constitutive laws of both concrete and steel in tension and compression; moreover, the stiffening contributions related to the mechanisms of aggregate bridging and interlock, tension stiffening, dowel action are inserted in the model. The final stiffness matrix is then obtained, both in uncracked and in cracked stage by summing up concrete and steel contributions.

It is worth noting that the transition from the uncracked to the cracked stage is abrupt, since no continuity is given between these two phases. The constitutive laws necessary to define the material stiffness matrix are indeed different if the element is cracked or not. However, this discontinuity cannot be simply removed because of the basic assumptions of the model. The contributions related to intact concrete between two adjacent cracks (so acting in the same manner as the whole material prior to cracking) cannot be separated from the one referring to the fracture zone; thus, not providing a reliable description of what happens in physical reality after crack formation. Moreover, the evolution of the crack pattern as loading increases cannot be precisely described, since PARC model does not allow the appearance of subsequent cracks with different orientation.

To overcome these deficiencies, in 2008, a new formulation was proposed by Cerioni and co-workers: 2D-PARC model [1]. This model, as the previous one, treats the material as an equivalent continuum: it can be included in the framework of smeared-fixed crack models [15] and it also smears the reinforcement into the concrete element.

2D-PARC model enables a better transition to the cracked stage by using both in the uncracked and in cracked phase the same 2D constitutive laws. This target is performed by writing new equilibrium and compatibility equations and applying the strain decomposition procedure in the cracked stage.

For both concrete and steel the same constitutive relationships are applied both before and after cracking development. Moreover, the behavior of RC between two cracks and the mechanisms that happen at crack surfaces are separately analyzed. 2D-PARC model has a modular structure which allows to separate the contributions related to crack formation: each mechanical phenomenon is individually analyzed by using a proper constitutive relation and then the corresponding contribution is inserted into the material stiffness matrix, obtaining a more realistic representation of fracture in RC elements. Thanks to the modular framework, the single contribution can be easily substituted if more reliable and effective formulations are available in technical literature.

Moreover 2D-PARC model provides an improved description of the cracking evolution, since the application of the strain decomposition procedure allows a simple generalization in case of secondary (or multiple) cracking; thus making this model a multi-directional fixed smeared formulation. The appearance of various cracks with different orientations is indeed allowed. When the concrete tensile strength is reached between two exiting cracks a new crack (perpendicularly to the principal current tensile stress) forms and the material matrix is obtained by simply adding this new stiffening contribution. It is also worth noting that 2D-PARC is characterized by a more refined formulation for tension stiffening, compared to PARC model.

2D-PARC was originally developed for ordinary RC elements and then simply extended to the case of fiber reinforced slabs and ties [44], thanks to its aforementioned modular structure. Moreover, in order to analyze the behavior of RC elements subjected to a tridimensional state of stress, an extended formulation, termed as 3D-PARC, was provided in the same years [45].

1.3.2 Basic hypotheses

2D-PARC is a constitutive model able to provide the mechanical behavior of a reinforced concrete membrane element (even with fibers inclusion in the concrete mix) subjected to general in-plane stresses (σ_x , σ_y , τ_{xy}) from the beginning of the loading history up to failure. As already said, it belongs to the models that smear both the reinforcement and the crack in the element.

In order to describe its theoretical formulation, a concrete membrane element with unit sides and thickness t and reinforced by various steel bars arranged in n layers must be considered (Figure 1.11a). Each layer is placed on its axis x_i forming an angle θ_i with respect to the global x -axis (Figure 1.11b) and it is smeared through the geometric steel ratio ρ_i :

$$\rho_i = \frac{A_{si}}{s_i t} \quad , \quad (1.11)$$

where A_{si} is the cross-section area and s_i the spacing of the bars within the layer.

In the uncracked stage concrete and steel are treated like two materials working in parallel, by assuming perfect bond between them; thus, the stiffness matrix is composed by summing up the concrete and the steel contributions.

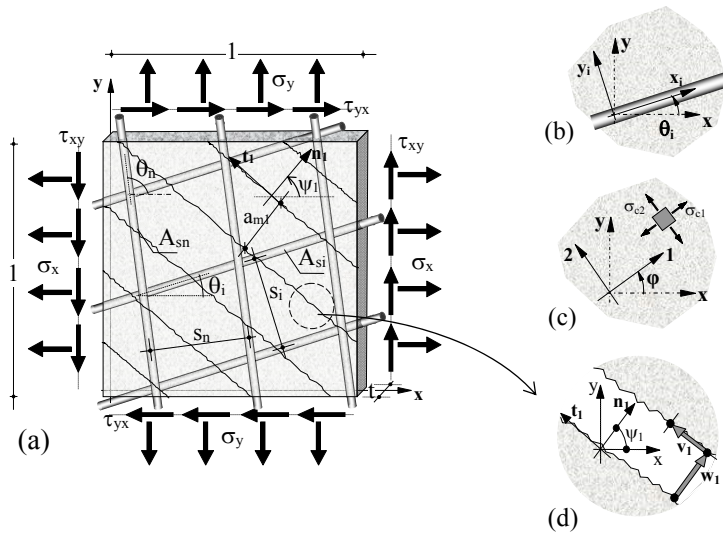


Figure 1.11 (a) Reinforced concrete membrane element: geometry and notation; (b) local co-ordinate system of the general i^{th} steel bar layer; (c) principal stress directions in concrete; (d) kinematics and local co-ordinate system of the crack

For steel an elastoplastic behavior is assumed, whereas concrete is modeled as an orthotropic, nonlinear elastic material subjected to a biaxial state of stress (with the orthotropic axes set equal to the principal stress directions). The axes of local co-ordinate system, 1-2, are assumed coincident with principal stress directions, oriented at an angle φ with respect to the global co-ordinate system x-y (Figure 1.11c). It is worth noting that in case of SFRC the same rules are followed, since fiber contribution is neglected because it does not influence the element behavior before the appearance of cracks.

When the maximum principal stress exceeds concrete tensile strength, the transition to the cracked stage takes place. As already said, the model follows the fixed crack approach and a strain decomposition procedure is adopted, by subdividing the total strain into two components, respectively related to the intact RC/SFRC material, even though damaged, between cracks and to all the resistant mechanisms of the fracture zone (i.e. aggregate bridging and interlock, tension stiffening, dowel action and fiber bridging in case of SFRC). Primary cracks arise in the element, at right angle with respect to principal tensile stress direction. Crack pattern is assumed as immediately fully developed with crack spacing a_{m1} , which remains constant during the loading process, as well as the crack direction. Crack spacing a_{m1} is evaluated on the basis of the transmission length, which varies depending on the element is un-reinforced, single or bi-reinforced and if fibers are added to the concrete mix. The following expressions are applied:

$$\left. \begin{array}{ll} a_{m1} = 3d_{\max}, & \text{without fibers} \\ a_{m1} = L_f, & \text{with fibers} \end{array} \right\} \text{no reinforcing bars} \quad (1.12)$$

$$\left. \begin{array}{ll} a_{m1} = s_j, & \text{if } \frac{s_i}{s_j} < 0.55 \\ a_{m1} = \frac{s_i + s_j}{2\sqrt{2}}, & \text{if } 0.55 \leq \frac{s_i}{s_j} \leq 1.80 \\ a_{m1} = s_j, & \text{if } \frac{s_i}{s_j} > 1.80 \end{array} \right\} \text{with reinforcing bars} \quad (1.13)$$

being $d_{\max}$ the maximum aggregate diameter and L_f the fiber length.

The axes of the local co-ordinate system of the crack, n_1-t_1 , are respectively perpendicular and parallel to the crack surfaces, n_1 forming an angle ψ_1 with respect to the x-axis of the global co-ordinate system x-y (Figure 1.11d). Cracks are assumed to develop perpendicular to the principal tensile stress direction, thus n_1-t_1 result coincident with the current 1-2 axes, and in turn ψ_1 angle with the current φ angle.

As already said, 2D-PARC model was also extended in order to allow the appearance of secondary (or multiple) cracks [1]; so it can be included in the multi-direction fixed cracking formulations. Secondary (or multiple) cracking takes place when the maximum principal tensile stress between two adjacent cracks exceeds the tensile strength of concrete. The secondary crack orientation is assumed to form at right angle with respect to the current principal tensile stress direction (current 1-axis described by the φ angle) and a new local n_2 – t_2 co-ordinate system of the crack described by the ψ_2 angle, is introduced. Also in this case the model can be applied also for the analysis of SFRC elements (see [46]).

As briefly described in §1.2.2.2, following the multi-direction fixed cracking approach the computational algorithm results quite complicated and the computational times very lengthened in some cases. Thus, for sake of simplicity and to reduce computational effort, in this thesis only primary cracking is considered.

1.3.3 Uncracked stage

In the uncracked stage perfect bond is assumed between concrete and steel. Hence, with reference to the global co-ordinate system, concrete and steel strains, respectively denoted as $\{\varepsilon_c\}$ and $\{\varepsilon_s\}$, are considered equal to each other and to the total strain vector $\{\varepsilon\}$:

$$\{\varepsilon_c\} = \{\varepsilon_s\} = \{\varepsilon\}. \quad (1.14)$$

For equilibrium, the total stress $\{\sigma\}$ can be calculated as the sum of the stress in concrete $\{\sigma_c\}$ and in steel reinforcement $\{\sigma_s\}$ – this latter smeared, because of the hypotheses of the model; as a consequence, concrete $[D_c]$ and steel $[D_s]$ stiffness matrices can be directly summed up to obtain the global stiffness matrix $[D]$:

$$[D] = [D_c] + [D_s] \quad (1.15)$$

and the related stress state as:

$$\{\sigma\} = \{\sigma_c\} + \{\sigma_s\} = [D_c]\{\varepsilon_c\} + [D_s]\{\varepsilon_s\} = ([D_c] + [D_s])\{\varepsilon\} = [D]\{\varepsilon\}. \quad (1.16)$$

It is worth noting that the concrete $[D_c]$ and steel $[D_s]$ stiffness matrices are first determined in their own local co-ordinate system and then rotated to the global one to fit the relation (1.16).

Moreover, it should be pointed out that in case of SFRC, fiber contribution is neglected in the uncracked stage; thus the same formulation is applied.

1.3.3.1 Steel stiffness matrix

The mechanical behavior of each steel reinforcing layer is described by an elastic-hardening constitutive law, symmetric in tension and compression, as reported in Figure 1.12.

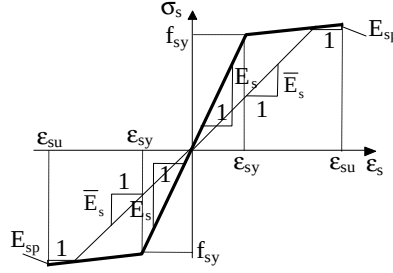


Figure 1.12 Assumed elastic-hardening constitutive law for ordinary steel, [1]

The total steel stiffness matrix $[D_s]$ is evaluated by summing up all the contributions of the n reinforcement layers (smeared through the geometric steel ratio), first evaluated the local x_i - y_i co-ordinate system of each i^{th} steel bar layer (Figure 1.11b) and then rotated to global x - y system:

$$[D_s] = \sum_{i=1}^n [D_{si}] = \sum_{i=1}^n [T_{si}]^t \left[D_{si}^{(x_i, y_i)} \right] [T_{si}], \quad (1.17)$$

where $[T_{si}]$ is the transformation matrix, function of the θ_i angle between the local x_i axis and the global one:

$$[T_{si}] = \begin{bmatrix} \cos^2 \theta_i & \sin^2 \theta_i & \cos \theta_i \sin \theta_i \\ -2 \cos \theta_i \sin \theta_i & 2 \cos \theta_i \sin \theta_i & \cos^2 \theta_i - \sin^2 \theta_i \end{bmatrix}. \quad (1.18)$$

The steel stiffness matrix $[D_{si}^{(x_i, y_i)}]$ of each reinforcing layer, expressed in its own local co-ordinate system, is evaluated by taking into account both the axial and shear stiffness:

$$[D_{si}^{(x_i, y_i)}] = \rho_{si} \begin{bmatrix} \bar{E}_{si} & 0 \\ 0 & \bar{G}_{si} \end{bmatrix}, \quad (1.19)$$

Where $\bar{E}_{si}$ and $\bar{G}_{si}$ are respectively the steel secant longitudinal and shear elastic modulus.

1.3.3.2 Concrete stiffness matrix

Concrete is modeled as an orthotropic (with the orthotropic axes set equal to the principal stress directions), nonlinear elastic material subjected to a biaxial state of stress, which is duplicated by means of two equivalent uniaxial curves, following a procedure originally proposed by Darwin and Pecknold [47].

The concrete stiffness matrix is first defined in the 1–2 local co-ordinate system, whose axes are directed along the principal maximum and minimum stress directions respectively (Figure 1.11c):

$$[D_c^{(1,2)}] = \frac{1}{1-\nu^2} \begin{bmatrix} \bar{E}_{c1} & \nu\sqrt{\bar{E}_{c1}\bar{E}_{c2}} & 0 \\ \nu\sqrt{\bar{E}_{c1}\bar{E}_{c2}} & \bar{E}_{c2} & 0 \\ 0 & 0 & (1-\nu^2)\bar{G}_{12} \end{bmatrix}, \quad (1.20)$$

being $\bar{E}_{c1}$ and $\bar{E}_{c2}$ the concrete secant elastic moduli in the two orthotropic directions, and $\bar{G}_{12}$ the concrete shear modulus. This latter can be evaluated by applying the following relation:

$$(1-\nu^2)\bar{G}_{12} = \frac{1}{4}(\bar{E}_{c1} + \bar{E}_{c2} - 2\nu\sqrt{\bar{E}_{c1}\bar{E}_{c2}}). \quad (1.21)$$

Then concrete stiffness matrix $[D_c^{(1,2)}]$ can be written in the global x–y co-ordinate system through the following:

$$[D_c] = [T_\varphi]^t [D_c^{(1,2)}] [T_\varphi] \quad (1.22)$$

by using the transformation matrix $[T_\varphi]$, function of the φ angle between the x-axis and the 1-axis (Figure 1.11c):

$$[T_\varphi] = \begin{bmatrix} \cos^2 \varphi & \sin^2 \varphi & \cos \varphi \sin \varphi \\ \sin^2 \varphi & \cos^2 \varphi & -\cos \varphi \sin \varphi \\ -2\cos \varphi \sin \varphi & 2\cos \varphi \sin \varphi & \cos^2 \varphi - \sin^2 \varphi \end{bmatrix}. \quad (1.23)$$

The material secant elastic moduli $\bar{E}_{c1}$, $\bar{E}_{c2}$ and the equivalent Poisson coefficient ν (being $\nu^2 = \nu_1\nu_2$), to be inserted into concrete stiffness matrix $[D_c^{(1,2)}]$ consider the actual biaxial state of stress by means of two equivalent uniaxial states of stress.

Evaluation of $\bar{E}_{c1}$ and $\bar{E}_{c2}$ secant elastic moduli

The two elastic moduli are determined through a procedure originally proposed by Darwin and Pecknold [47] by considering the effective biaxial state of stress equivalent to two uniaxial states of stress by using the equivalent uniaxial strain ε_{ci}^u :

$$\varepsilon_{ci}^u = \frac{\sigma_{ci}}{\bar{E}_{ci}}, \quad (1.24)$$

where the subscript i is 1 or 2, referring to the principal stress directions.

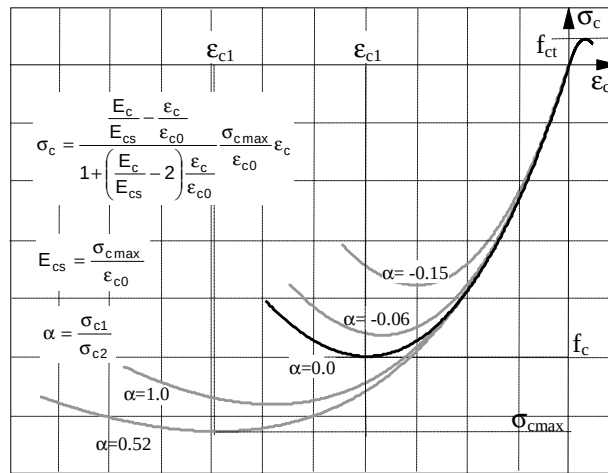


Figure 1.13 Assumed equivalent uniaxial curves for tension and compression

From the equivalent uniaxial curves (Figure 1.13) - different for compression and tension - is possible to obtain for each loading increment the two secant elastic moduli to be inserted in (1.20) relative to the current state of stress as $\bar{E}_{ci} = \sigma_{ci} / \varepsilon_{ci}^u$. To properly describe the equivalent uniaxial curves is necessary to define the peak stress $\sigma_{ci,max}$ and the corresponding peak strain ε_{c0}^* on the basis of the applied biaxial failure envelope. Subsequently, it is required to determine the correct equivalent uniaxial strain ε_{ci}^u : the procedure suggested in [48] for a nonlinear material is followed; thus obtaining:

$$\begin{Bmatrix} \varepsilon_{c1}^u \\ \varepsilon_{c2}^u \end{Bmatrix} = \frac{1}{1-\nu^2} \begin{bmatrix} 1 & \nu \sqrt{E_{c1}/E_{c2}} \\ \nu \sqrt{E_{c1}/E_{c2}} & 1 \end{bmatrix} \begin{Bmatrix} \varepsilon_{c1} \\ \varepsilon_{c2} \end{Bmatrix}, \quad (1.25)$$

where ε_{c1} and ε_{c2} are respectively the maximum and the minimum principal strain.

Uniaxial curves for compression-compression and tension-compression regions

In case of compression-compression or tension-compression, the Sargin law suggested in Model Code 1990 [49] (MC90 in the following) is applied (Figure 1.13):

$$\sigma_{ci} = \frac{\frac{E_c}{E_{cs}} - \frac{\varepsilon_{ci}^u}{\varepsilon_{c0}^*}}{1 + \left(\frac{E_c}{E_{cs}} - 2 \right) \frac{\varepsilon_{ci}^u}{\varepsilon_{c0}^*}} \frac{\sigma_{ci, \max}}{\varepsilon_{c0}^*} \varepsilon_{ci}^u, \quad (1.26)$$

where E_c and E_{cs} are respectively the initial value of the elastic modulus (thus, tangent to the origin of the axes) and the secant value of the elastic modulus corresponding to the peak stress $\sigma_{ci, \max}$, thus being $\varepsilon_{c0}^* = \sigma_{ci, \max} / E_{cs}$. It is worth noting that the peak stress $\sigma_{ci, \max}$ takes into account the different strength that concrete exhibits under biaxial state of stress than under uniaxial loading. It is computed as a function of the ratio between the maximum and the minimum principal stress of each increment, $\alpha = \sigma_1 / \sigma_2$, by applying the adopted biaxial strength envelope (Figure 1.14). In particular the one proposed by Kupfer et al. [50] is adopted with slight modifications.

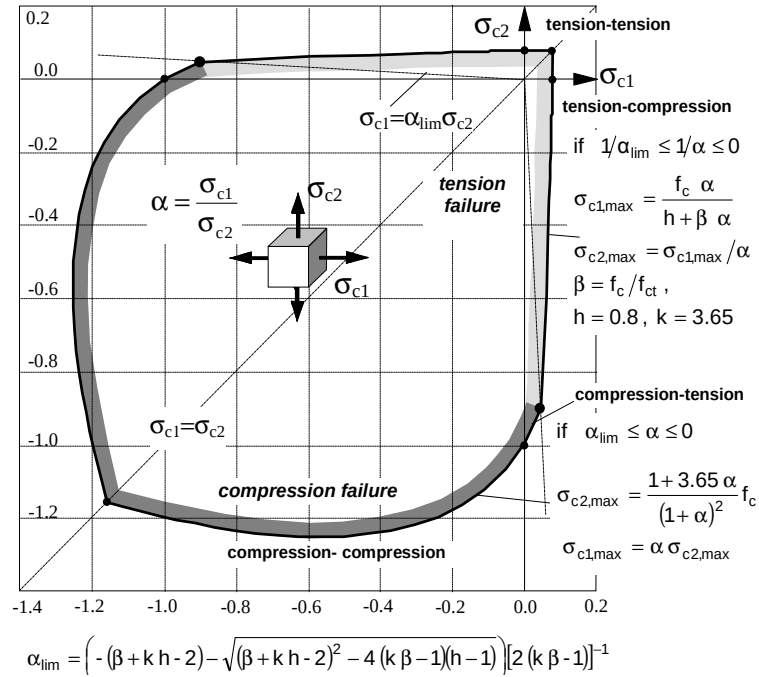


Figure 1.14 Adopted biaxial failure envelope

In case of biaxial compression the expression proposed by Kupfer and Gerstle [51] is applied:

$$\left(\frac{\sigma_{c1max}}{f_c} + \frac{\sigma_{c2max}}{f_c} \right)^2 - \frac{\sigma_{c2max}}{f_c} - 3.65 \frac{\sigma_{c1max}}{f_c} = 0 ; \quad (\sigma_{c1} > \sigma_{c2}) \quad (1.27)$$

thus obtaining the principal stresses on the failure envelope equal to:

$$\begin{aligned} \sigma_{c1max} &= \alpha \sigma_{c2max} , \\ \sigma_{c2max} &= \frac{1 + 3.65\alpha}{(1 + \alpha)^2} f_c . \end{aligned} \quad \text{if } 0 \leq \alpha \leq 1 \quad \text{being } \alpha = \sigma_1 / \sigma_2 \quad (1.28)$$

As regards the tension-compression region, it is subdivided by the value α_{lim} into two sub-regions, depending on tension or compression is the prevalent stress, where α_{lim} is computed as:

$$\alpha_{lim} = \left(-(\beta + kh - 2) - \sqrt{(\beta + kh - 2)^2 - 4(k\beta - 1)(h - 1)} \right) [2(k\beta - 1)]^{-1} \quad (1.29)$$

being $\beta = f_c / f_{ct}$, $k = 3.65$ and $h = 0.8$. The principal stresses on the failure envelope where tension is prevalent are evaluated as:

$$\begin{aligned} \sigma_{c1max} &= \frac{f_c \alpha}{h + \beta \alpha} , \\ \sigma_{c2max} &= \sigma_{c1max} / \alpha ; \end{aligned} \quad \text{if } \frac{1}{\alpha_{lim}} \leq \frac{1}{\alpha} \leq 0 \quad (1.30)$$

whereas for the compression-tension region (thus, for $\alpha_{lim} \leq \alpha \leq 0$), due to its limited extension, the same expression (1.28) as in case of compression-compression is applied.

To obtain the secant value of the elastic moduli $\bar{E}_{c1}$ and $\bar{E}_{c2}$ is necessary to first attain the value of the peak strain. The relation suggested in [47] is applied:

$$\begin{aligned} \varepsilon_{c0}^* &= \varepsilon_{c0} \left(3 \frac{\sigma_{ci max}}{f_c} - 2 \right) , & \text{if } |\sigma_{ci max}| > |f_c| \\ \varepsilon_{c0}^* &= \varepsilon_{c0} \left[-1.6 \left(\frac{\sigma_{ci max}}{f_c} \right)^3 + 2.25 \left(\frac{\sigma_{ci max}}{f_c} \right)^2 + 0.35 \left(\frac{\sigma_{ci max}}{f_c} \right) \right] & \text{if } |\sigma_{ci max}| < |f_c| \end{aligned} \quad (1.31)$$

being ε_{c0} and f_c respectively the uniaxial compressive peak strain and stress.

Uniaxial curve for tension-tension region

In the biaxial tension region, concrete is modeled as a brittle linear elastic material by adopting the bilinear MC90 law (Figure 1.15). The original curve is slightly modified by applying the power formula by Devalapura and Tadros [52] to smooth the discontinuity point between the two linear branches, so to ensure better convergence.

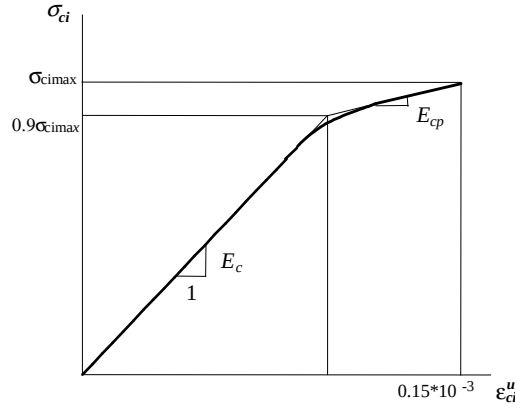


Figure 1.15 Adopted constitutive law for tension [22]

With reference to Figure 1.15 the following expression are obtained:

$$\sigma_{ci} = \bar{E}_{ci} \varepsilon_{ci}^u, \quad (1.32)$$

where:

$$\bar{E}_{ci} = A + \frac{B}{\left\{1 + \left(C \varepsilon_{ci}^u\right)^D\right\}^{1/D}}, \quad (1.33)$$

being:

$$A = E_{cp} = \frac{(\sigma_{ci \max} - f_{s0})}{(\varepsilon_{ct1} - \varepsilon_{ct0})}, \quad B = E_c - E_{cp}, \quad C = \frac{E_c}{f_{s0}}, \quad D = 5 \quad (1.34)$$

,

and:

$$f_{s0} = 0.9 \sigma_{ci \max}, \quad \varepsilon_{ct1} = 0.00015, \quad \varepsilon_{ct0} = f_{s0} / E_c. \quad (1.35)$$

It is worth noting that the maximum peak stress $\sigma_{ci, \max}$ for all the tension-tension region (Figure 1.14) is assumed equal to f_{ct} .

Evaluation of the Poisson coefficient ν

As suggested in [47] the Poisson coefficient is assumed equal to 0.2 for the tension-tension region whereas in the compression-compression and in the tension-compression regions, to take into account the dilatation that occurs in compression for high stresses, the following expression is applied:

$$\nu = 0.20 + 0.6 \left(\frac{\sigma_{c2}}{f_c} \right)^4 + 0.4 \left(\frac{\sigma_{c1}}{f_{ct}} \right)^4 \quad (1.36)$$

1.3.3.3 Fiber contribution

It is worth noting that the uncracked stiffness matrix for SFRC elements is assumed coincident with that of plain concrete because the fiber contribution plays a significant role only in the post-cracking stage.

1.3.4 Cracked stage

As already said, when the principal maximum stress exceeds concrete tensile strength, the transition to the cracked stage takes place. After cracking development at a given integration point (which is representative of the average behavior of a tributary region surrounding it), the adopted constitutive matrix of reinforced concrete is properly modified so as to correctly represent the softening effect resulting from the cracking process. In this way, the fracture zone is distributed over a certain width of the finite element and a discontinuity is imposed in the stress field, while the displacement field remains continuous.

A fixed, smeared crack approach is followed with a crack pattern characterized by a constant crack spacing a_{m1} . Moreover, the strain decomposition procedure is adopted, which allows to explicitly consider all the specific contributions related to physical reality, so overcoming the main disadvantages typical of classic smeared formulations. As known, these latter provide the mean response of the element, without making any distinction between the cracks and the solid material between them. Consequently, specific crack laws, which are commonly related to the crack strain rather than to the total strain, cannot be incorporated in classic smeared models in a transparent manner. As a matter of fact classical smeared formulations adopt in the cracked stage empirical formulations, based on experimental results [15].

On the contrary in 2D-PARC model the construction of the global stiffness matrix relative to the cracked stage is performed by separately calculating the contribution related to the uncracked RC (or SFRC) material between cracks, having a solid condition even if damaged, and the contribution related to all the kinematics developed at crack location. Afterwards, thanks to the modular framework of the model, these two main contributions can be in turn evaluated by summing up all the mechanisms referring to the two material components (i.e. concrete and steel).

The strain decomposition procedure allows to subdivide the total strain $\{\varepsilon\}$ into two components $\{\varepsilon_c\}$ and $\{\varepsilon_{cr1}\}$, respectively relative to the concrete between two adjacent cracks and to all the resistant mechanisms which develop at crack surfaces; thus the compatibility condition results:

$$\{\varepsilon\} = \{\varepsilon_c\} + \{\varepsilon_{cr1}\} . \quad (1.37)$$

The vector relative to the fracture zone $\{\varepsilon_{cr1}\}$, is first evaluated in the local co-ordinate system of the crack, n_1 - t_1 , respectively perpendicular and parallel to crack direction (Figure 1.11d), as a function of the two main variables, that are crack opening w_1 and sliding v_1 :

$$\{\varepsilon_{cr1}^{(n_1, t_1)}\} = \begin{Bmatrix} \frac{w_1}{a_m} & \frac{v_1}{a_m} \end{Bmatrix} . \quad (1.38)$$

The local crack strain vector is then transferred to the global x-y co-ordinate system:

$$\{\varepsilon_{cr1}\} = [T_{\psi_1}]^{-1} \{\varepsilon_{cr1}^{(n_1, t_1)}\} , \quad (1.39)$$

being $[T_{\psi_1}]$ the transformation matrix:

$$[T_{\psi_1}] = \begin{bmatrix} \cos^2 \psi_1 & \sin^2 \psi_1 & \cos \psi_1 \sin \psi_1 \\ -2 \cos \psi_1 \sin \psi_1 & 2 \cos \psi_1 \sin \psi_1 & \cos^2 \psi_1 - \sin^2 \psi_1 \end{bmatrix} . \quad (1.40)$$

Both the stress field of the fracture zone and that of RC (or SFRC) between two cracks are assumed in equilibrium with the external applied stresses, resulting coincident with each other.

It is worth noting that the concept of strain decomposition is applied in 2D-PARC model in a different manner with respect to other constitutive models available in technical literature (e.g., among others, [53–55]). Following the most common formulations, the strain decomposition procedure in the cracked stage usually refers only to concrete, while the interaction with steel rebars is often taken into account by simply modifying the constitutive relations adopted for the cracked concrete itself. In other cases, steel rebars and their interaction with surrounding concrete are separately modeled by introducing specific finite elements, generally embedded in the concrete ones. On the contrary, the peculiarity of 2D-PARC model is that both the resistant contributions related to the cracks and to the solid material between them are decomposed into two “fractions”, respectively related to concrete and to the smeared reinforcement, so allowing a more clear representation of all the mechanisms that are responsible for stress transfer in RC elements [1].

According to model hypotheses, the stresses field in RC between adjacent cracks and that in the crack are assumed coincident to each other, being in equilibrium with the external applied stresses.

The equilibrium in the crack, with reference to global x-y co-ordinate system, can be written as:

$$\{\sigma\} = \{\sigma_{cr1}\} = \{\sigma_{c,cr1}\} + \{\sigma_{s,cr1}\} = ([D_{c,cr1}] + [D_{s,cr1}]) \{\varepsilon_{cr1}\} = [D_{cr1}] \{\varepsilon_{cr1}\}, \quad (1.41)$$

where $\{\sigma_{c,cr1}\}$, $\{\sigma_{s,cr1}\}$ and $[D_{c,cr1}]$, $[D_{s,cr1}]$ represent the crack stresses and stiffness matrices relatively to concrete and steel in the global co-ordinate system, respectively. As a matter of fact, in cracked concrete crack opening w and slip v activate several resistant mechanisms which provide strength and stiffness; some due to concrete, in particular to the aggregates acting upon the crack surfaces, whereas others due to steel bars which cross the crack. Hence, the total crack stiffness matrix $[D_{cr1}]$ is formed by adding the matrix $[D_{c,cr1}]$ relative to concrete resistant contributions - i.e. due to aggregate bridging and interlock (with the addition of fiber bridging in case of SFRC elements) to matrix $[D_{s,cr1}]$, related to reinforcement mechanisms - i.e. tension stiffening and dowel action.

The equilibrium condition in the uncracked RC (or SFRC) between two adjacent cracks, similarly to the uncracked stage, can be expressed as the sum of the stresses in the two materials, that are concrete and steel. The expression, with reference to global x-y co-ordinate system, can be written as:

$$\{\sigma\} = \{\sigma_c\} + \{\sigma_s\} = [D_c] \{\varepsilon_c\} + [D_s] \{\varepsilon_s\} \quad (1.42)$$

Concrete and steel stresses $\{\sigma_c\}$, $\{\sigma_s\}$ are evaluated as the product between the correspondent stiffness matrices $[D_c]$ and $[D_s]$, slightly modified with reference to the uncracked stage, and their strains $\{\varepsilon_c\}$ and $\{\varepsilon_s\}$. As a consequence, in case of SFRC elements, the fiber contribution in the intact material between cracks is still neglected as in the uncracked stage, while it is explicitly considered in the construction of crack stiffness matrix. It should be also pointed out that the hypothesis of perfect bond is no longer valid, and therefore the two strain vectors, $\{\varepsilon_c\}$ and $\{\varepsilon_s\}$ cannot be assumed coincident. Nevertheless, as the average strain of steel between two contiguous cracks $\{\varepsilon_s\}$ is characterized by values little lower than the average strain $\{\varepsilon\}$ of element, to solve the problem $\{\varepsilon_s\}$ is assumed equal to $\{\varepsilon\}$.

The total stress vector $\{\sigma\}$ and stiffness matrix $[D]$ can be obtained with reference to global x-y co-ordinate system, by firstly deducing the two strain vectors $\{\varepsilon_{cr1}\}$ and $\{\varepsilon_c\}$ respectively inverting Equations (1.41) and (1.42):

$$\begin{aligned} \{\varepsilon_{cr1}\} &= [D_{cr1}]^{-1} \{\sigma\} \quad , \\ \{\varepsilon_c\} &= [D_c]^{-1} (\{\sigma\} - [D_s] \{\varepsilon\}) \quad , \end{aligned} \quad (1.43)$$

and then by substituting their values into the compatibility relation (1.37):

$$\{\varepsilon\} = [D_c]^{-1}(\{\sigma\} - [D_s] \{\varepsilon\}) + [D_{cr1}]^{-1}\{\sigma\} = ([D_c]^{-1} + [D_{cr1}]^{-1})\{\sigma\} - [D_c]^{-1}[D_s] \{\varepsilon\} \quad (1.44)$$

and finally re-writing the relation (1.44):

$$\{\sigma\} = ([D_c]^{-1} + [D_{cr1}]^{-1})^{-1}([I] + [D_c]^{-1}[D_s]) \{\varepsilon\} = [D] \{\varepsilon\} \quad (1.45)$$

being $[I]$ the identity matrix it can be obtain:

$$[D] = ([D_c]^{-1} + [D_{cr1}]^{-1})^{-1}([I] + [D_c]^{-1}[D_s]) \quad (1.46)$$

As can be seen by Equation (1.46) the total stiffness matrix $[D]$ of a RC (or SFRC) element in the cracked stage is defined by the matrix relative to two separate contributions: the former relative to the uncracked RC (or SFRC) between two cracks and the latter relative to the crack kinematics. As, already said, these contributions are in turn subdivided due to the presence of the two constitutive materials: concrete and steel.

It should be also pointed out that all the four matrices $[D_c]$, $[D_s]$ for the uncracked RC (or SFRC) and $[D_{c,cr1}]$, $[D_{s,cr1}]$ for the fracture zone are first evaluate in their local co-ordinate system and then rotated to the global one.

1.3.4.1 Stiffness matrix of the concrete between two adjacent cracks

The concrete matrix $[D_c]$ is directly obtained from the one assumed in the uncracked stage, as expressed by Equations (1.20)-(1.23), by introducing a damage coefficient ζ (see [18,36,56]), which accounts for the reduction of resistant cross-section due to the degraded material near the crack and to the irregularities in crack spacing. This damage coefficient is assumed equal to:

$$\zeta = \left(1 + 200 \frac{w_1}{a_{m1}}\right)^{-0.5} \quad (1.47)$$

being a_{m1} and w_1 crack spacing and opening. It affects both strength and stiffness of concrete, since it multiplies all terms of matrix $[D_c^{(1,2)}]$, reported in (1.20).

1.3.4.2 Stiffness matrix of the steel between two adjacent cracks

The steel matrix $[D_s]$ is evaluated as in the uncracked stage, as expressed by Equations (1.17)-(1.19), but assuming that the reinforcing bars retain axial but not shear stiffness. This latter indeed rapidly drops due to the loss of perfect bond between steel bars and the surrounding concrete. Hence, for sake of simplicity, the shear stiffness is neglected from the appearance of the first crack.

1.3.4.3 Crack stiffness matrix: concrete contribution

Matrix $[D_{c,cr1}]$ accounts for all the resistant mechanisms that happen across crack faces related to concrete: that are aggregate bridging and aggregate interlock (Figure 1.16). In case of SFRC elements this matrix accounts also for the resistant contributions due to the presence of fibers, as detailed explained in §1.3.4.5.

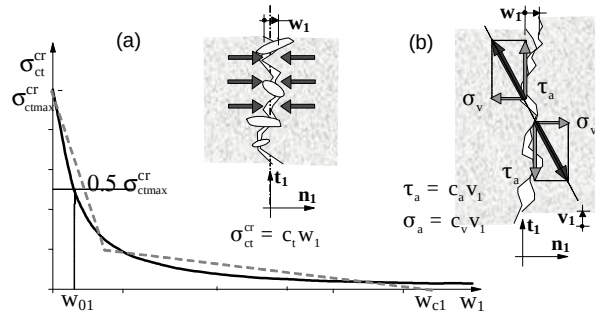


Figure 1.16 (a) Aggregate bridging; (b) aggregate interlock actions, [1]

The aggregate bridging action σ_{ct}^{cr} is expressed as a function of crack opening w_1 , through an empirical relation calibrated on the basis of several experimental data:

$$\sigma_{ct}^{cr}(w_1) = \frac{\sigma_{ct\max}}{1 + (w_1/w_{01})^p} = c_{b1} \frac{w_1}{a_m} = c_{b1} \varepsilon_1 \quad (1.48)$$

where c_{b1} is the bridging coefficient, w_{01} is the crack opening corresponding to $\sigma_{ct} = 0.5 \sigma_{ct\max}$, being $\sigma_{ct\max}$ the maximum bridging stress due to aggregate action at $w_1=0$ (thus, assumed equal to concrete tensile strength f_{ct}), while p is a coefficient defining the shape of the curve (Figure 1.16). The parameters w_{01} and p are chosen according to the bilinear law proposed in Model Code 2010 [57] (MC2010 in the following) by imposing the same area under the curves, namely the fracture energy, in the range from 0 to w_{c1} , which is the crack opening corresponding to zero stress.

Aggregate interlock activates shear and normal stresses due to the slip between crack surfaces. This contribution is modeled by applying the relations proposed by Gambarova (see [58,59]):

$$\begin{aligned} \sigma_{a1} &= -c_{01} \gamma_1 \quad , \\ \tau_{a1} &= c_{a1} \gamma_1 \quad . \end{aligned} \quad (1.49)$$

The aggregate interlock c_{a1} and confinement c_{01} coefficients are defined as functions of the crack opening w_1 and sliding v_1 :

$$c_{01} = \frac{a_1 a_2}{w_1^{2q}} \left(1 + \left(\frac{v_1}{w_1} \right)^2 \right)^{-q} v_1 c_{a1}$$

$$c_{a1} = \tau^* \left(1 - \sqrt{\frac{2w_1}{D_{max}}} \right) \frac{a_3 + a_4 \left| \frac{v_1}{w_1} \right|^3}{1 + a_4 \left(\frac{v_1}{w_1} \right)^4} \frac{a_m}{w_1} \quad (1.50)$$

where $\tau^* = 0.27 f_c$; $q = 0.25$; $a_1 a_2 = 0.62$; $a_3 = \frac{2.45}{\tau^*}$; $a_4 = 2.44 \left(1 - \frac{4}{\tau^*} \right)$.

The concrete stiffness matrix is then expressed, in the local co-ordinate system of the crack n_1 - t_1 , in the following form:

$$[D_{c,cr1}^{(n_1,t_1)}] = \begin{bmatrix} c_{b1} & -c_{01} \\ 0 & c_{a1} \end{bmatrix}, \quad (1.51)$$

and then transferred to the global one by the transformation matrix $[T_{\psi_1}]$ (1.40):

$$[D_{c,cr1}] = [T_{\psi_1}]^t [D_{c,cr1}^{(n_1,t_1)}] [T_{\psi_1}]. \quad (1.52)$$

1.3.4.4 Crack stiffness matrix: steel contribution

Matrix $[D_{s,cr1}]$ accounts for all the resistant mechanisms that happen across crack faces related to steel, that are tension stiffening and dowel action. In the crack, the forces in the steel bars due to the axial stiffness and to the dowel action are modeled in the local steel co-ordinate system (Figure 1.17a) and then smeared so as to obtain the static equivalent stresses (Figure 1.17b).

The dowel action contribution is modeled according to [32], applying the following equation:

$$S_{di} = 10 \eta_{i1}^{0.36} \phi_i^{1.75} f_{cc}^{0.38} (\delta_{i1} + 0.2)^{-1} = 10 \eta_{i1}^{0.36} \phi_i^{1.75} \left(\frac{f_c}{0.83} \right)^{0.38} (\delta_{i1} + 0.2)^{-1}$$

$$= 10.73 \eta_{i1}^{0.36} \phi_i^{1.75} f_c^{0.38} (\delta_{i1} + 0.2)^{-1}, \quad (1.53)$$

where δ_{i1} and η_{i1} are respectively the axial and the transversal component, with respect to the bar axis (local x_i - y_i co-ordinate system), of the crack displacement

vector (Figure 1.17a) having indicated with f_{cc} and f_c respectively the cubic and cylindrical concrete compressive strength and with ϕ the bar diameter. The equivalent shear stress τ_{sit} (Figure 1.17b), obtained by smearing S_{di} , results:

$$\begin{aligned} \tau_{sit} &= \frac{S_{di}}{s_i t} = \frac{10.73 \eta_{i1}^{0.36} \phi_i^{1.75} f_c^{0.38}}{s_i t (\delta_{i1} + 0.2)} = \frac{\pi \phi_i^2}{4 s_i t \pi} \frac{4 \cdot 10.73 \eta_{i1}^{-0.64} \phi_i^{-0.25} f_c^{0.38}}{(\delta_{i1} + 0.2)} \eta_{i1} \\ &= 13.66 \rho_{si} \frac{\eta_{i1}^{-0.64} \phi_i^{-0.25} f_c^{0.38} l_{si}}{(\delta_{i1} + 0.2)} \frac{\eta_{i1}}{l_{si}} = \rho_{si} d_i \frac{\eta_{i1}}{l_{si}}, \end{aligned} \quad (1.54)$$

so obtaining the dowel action contribution d_i equals to:

$$d_i = 13.66 f_c^{0.38} \phi_i^{-0.25} \frac{l_{si}}{\eta_{i1}^{0.64} (\delta_{i1} + 0.2)}, \quad (1.55)$$

where l_{si} is the length of the i^{th} bar between two adjacent cracks and it is evaluated as follows:

$$l_{si} = \frac{a_m}{\cos(\theta_i - \psi_1)}. \quad (1.56)$$

Tension stiffening effect is taken into account by implementing a proper bond-slip law into a numerical procedure based on the finite difference method. The stiffening contribution due to steel-concrete bond between two adjacent cracks provides a non-uniform distribution of strains in the reinforcement (Figure 1.18a), which can be numerically evaluated in a finite number of points along bar axis.

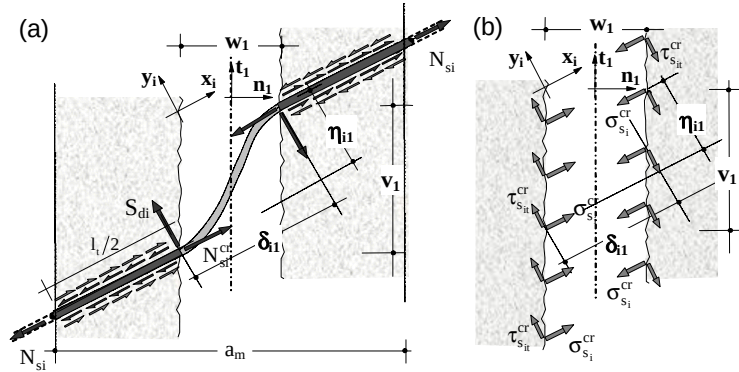


Figure 1.17 (a) Tension stiffening and dowel action of steel reinforcement; (b) their smeared equivalent effect on crack surfaces, [1].

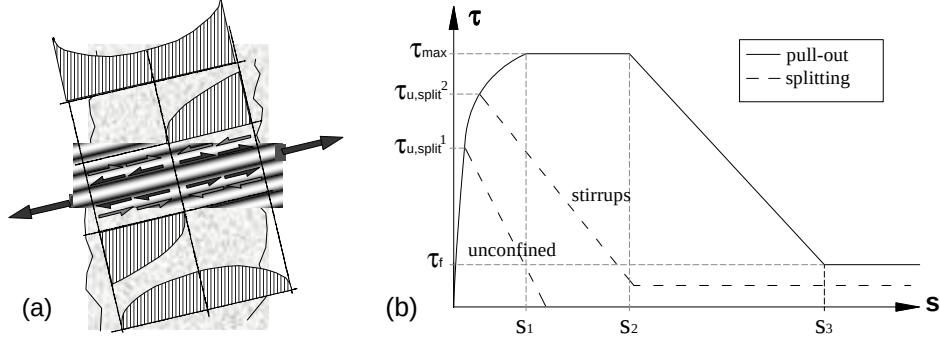


Figure 1.18 (a) Stabilized cracking stage: shear bond stresses, tensile stresses in concrete and in steel bar; (b) adopted bond-slip law, based on MC2010 law

The problem is solved by imposing the equilibrium equations for the whole section (Figure 1.19a), for concrete (Figure 1.19b) and for steel bar (Figure 1.19c):

$$\begin{aligned} \frac{d\sigma_c}{dx} + \rho \frac{d\sigma_s}{dx} &= 0, \\ \frac{d\sigma_c}{dx} &= -\rho \frac{4}{\phi} \tau(s(x)), \\ \frac{d\sigma_s}{dx} &= \frac{4}{\phi} \tau(s(x)), \end{aligned} \quad (1.57)$$

as well as the compatibility equation:

$$s(x_i) = u_s - u_c, \quad (1.58)$$

where s is the slip, defined as the difference of the displacements, u_s and u_c , between two initially overlapping points respectively belonging to steel and concrete. By derivation with respect to x_i , Equation (1.58) can be rewritten in the form:

$$\frac{ds}{dx_i} = \varepsilon_s - \varepsilon_c, \quad (1.59)$$

being ε_s and ε_c the normal strains of steel and concrete along bar axis.

The solving equation can be then obtained by simply combining Equations (1.57) and (1.59), so resulting:

$$\frac{d^2 s}{dx_i^2} = \frac{4}{\phi E_s} \left(1 + \frac{E_s}{E_c} \rho \right) \tau(s(x_i)) \quad (1.60)$$

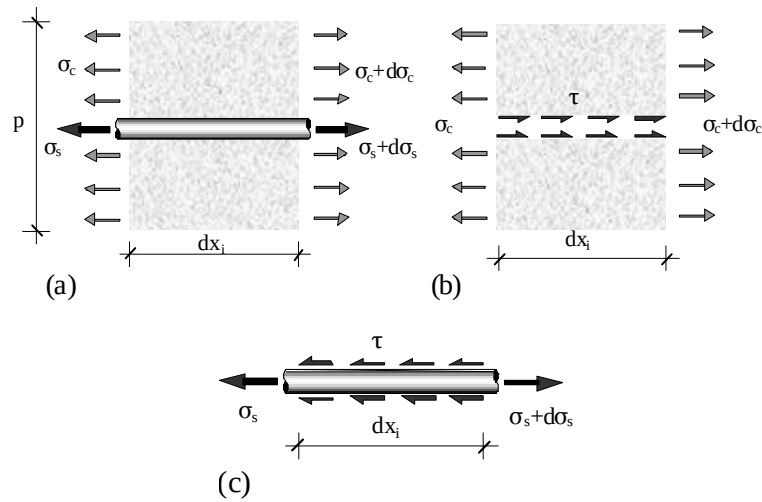


Figure 1.19 Equilibrium conditions (a) for a portion of the RC element, (b) for concrete and (c) for the steel bar, [60].

The differential equation is solved by applying the bond-slip law suggested in MC2010 [57] for RC elements, whereas if fibers are added to the concrete mix the one proposed by Harajli et al. (see [61,62]) is adopted, as detailed explained in §1.3.4.5. According MC2010 law, the bond stress τ is evaluated as a function of slip s , so obtaining the curve reported in Figure 1.18b:

$$\tau = \begin{cases} \tau_{max} \left(\frac{s}{s_1} \right)^\alpha & \text{se } 0 \leq s \leq s_1 \\ \tau_{max} & \text{se } s_1 \leq s \leq s_2 \\ \tau_{max} - (\tau_{max} - \tau_f) \frac{s - s_2}{s_3 - s_2} & \text{se } s_2 \leq s \leq s_3 \\ \tau_f & \text{se } s_3 \leq s \end{cases}, \quad (1.61)$$

whose parameters are function of bond conditions (Table 1.1).

	Pull out failure		Splitting failure			
	Good bond condition	All other bond condition	Good bond condition		All other bond condition	
			unconfined	stirrups	unconfined	stirrups
α	0.4	0.4	0.4	0.4	0.4	0.4
$\tau_{\max}$ [MPa]	$2.5 \sqrt{f_c}$	$1.25 \sqrt{f_c}$	$7.0 (f_c/25)^{0.25}$	$8.0 (f_c/25)^{0.25}$	$5.0 (f_c/25)^{0.25}$	$5.5 (f_c/25)^{0.25}$
τ_f [MPa]	$0.40 \tau_{\max}$	$0.40 \tau_{\max}$	0	$0.40 \tau_{\max}$	0	$0.40 \tau_{\max}$
s_1 [mm]	1.0	1.8	$S(\tau_{\max})$	$S(\tau_{\max})$	$S(\tau_{\max})$	$S(\tau_{\max})$
s_2 [mm]	2.0	3.6	s_1	s_1	s_1	s_1
s_3 [mm]	C_{clear}^*	C_{clear}^*	$1.2 s_1$	$0.5 C_{\text{clear}}^*$	$1.2 s_1$	$0.5 C_{\text{clear}}^*$

* C_{clear} is the clear distance between ribs

Table 1.1 Parameters defining the mean bond stress-slip relationship of ribbed bars, [57]

The shear stress can be subsequently modified to account for reinforcement yielding and transverse pressure, by introducing proper correction factors Ω_y and $\Omega_{p,tr}$ (again evaluated according to MC2010, [57]):

$$\tau_{bm} = \tau_0 \Omega_y \Omega_{p,tr} , \quad (1.62)$$

where τ_0 and τ_{bm} represent respectively the original and the modified bond stress.

The problem is solved by using the Finite Difference Method (FDM). Equation (1.60) is solved in a finite number of points, equidistant from each other, placed along the bar axis (Figure 1.20):

$$\frac{s_{j-1} - 2s_j + s_{j+1}}{\Delta x_j^2} = k \tau_j , \quad (1.63)$$

where:

$$k = \frac{4}{\phi E_s} \left(1 + \frac{E_s}{E_c} \rho \right) . \quad (1.64)$$

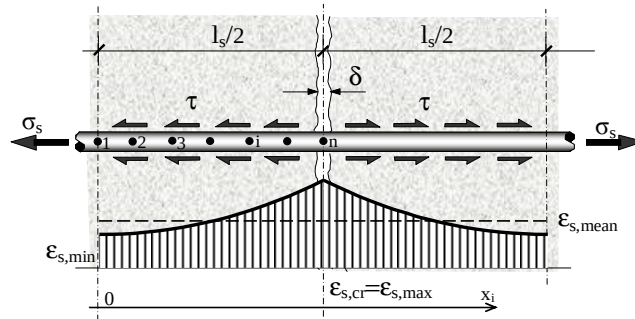


Figure 1.20 FD discretization, bond shear stresses and non uniform distribution of steel strains along the bar [60]

By applying the two boundary conditions $s(0) = 0$ and $s(l_s/2) = \delta/2$ (Figure 1.20) the following system can be written:

$$\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & -2 & 1 & \dots & 0 \\ & & \dots & & \\ 0 & \dots & 1 & -2 & 1 \\ 0 & \dots & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} s_1 \\ s_2 \\ \dots \\ s_{n-1} \\ s_n \end{Bmatrix} = \begin{Bmatrix} 0 \\ k \Delta x_i^2 \tau_2 \\ \dots \\ k \Delta x_i^2 \tau_{n-1} \\ \delta / 2 \end{Bmatrix}, \quad (1.65)$$

which is solved by means of an iterative procedure. A first time an attempt value of the slip s must be chosen and it is set equal to the one obtained as the close form solution of Equation (1.60) with the hypothesis of a linear distribution for the bond shear stresses τ . After having determined the slip $s(x_i)$, all the others variables can be evaluated through a similar procedure. The stresses in the concrete are computed through the integration of the concrete equilibrium equation by adopting, once again, the finite difference method:

$$\sigma_{c,j} - \sigma_{c,j+1} = \rho \frac{4}{\phi} \Delta x_i \tau_j, \quad (1.66)$$

so leading to the following solving system:

$$\begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & \dots & 0 \\ & & \dots & & \\ 0 & \dots & 0 & 1 & -1 \\ 0 & \dots & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \sigma_{c,1} \\ \sigma_{c,2} \\ \dots \\ \sigma_{c,n-1} \\ \sigma_{c,n} \end{Bmatrix} = \begin{Bmatrix} \rho \frac{4}{\phi} \Delta x_i \tau_1 \\ \rho \frac{4}{\phi} \Delta x_i \tau_2 \\ \dots \\ \rho \frac{4}{\phi} \Delta x_i \tau_{n-1} \\ \sigma_{ct}^{cr} \end{Bmatrix}. \quad (1.67)$$

System (1.67) can be solved by applying the approximate boundary condition $\sigma(l_s/2) = \sigma_{ct}^{cr}$, being σ_{ct}^{cr} the known value of concrete stress in the crack (due to aggregate bridging and interlock) on the bar direction. The concrete strains are evaluated from the corresponding stresses – in turn obtained by solving system (1.67) - on the basis of the adopted constitutive law for concrete in tension (Figure 1.15); then the steel strains can be obtained from Equation (1.59), properly rewritten in the form:

$$\varepsilon_{s,j} = \varepsilon_{c,j} + \frac{1}{2\Delta x_j} (s_{j+1} - s_{j-1}) \quad (1.68)$$

By imposing symmetry conditions in $x_i = 0$ and $x_i = l_s/2$, the related system results:

$$\begin{Bmatrix} \varepsilon_{s,1} \\ \varepsilon_{s,2} \\ \dots \\ \varepsilon_{s,n-1} \\ \varepsilon_{s,n} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{c,1} \\ \varepsilon_{c,2} \\ \dots \\ \varepsilon_{c,n-1} \\ \varepsilon_{c,n} \end{Bmatrix} + \frac{1}{2\Delta x_j} \begin{bmatrix} 0 & 2 & 0 & \dots & 0 \\ -1 & 0 & 1 & \dots & 0 \\ & & \dots & & \\ 0 & \dots & -1 & 0 & 1 \\ 0 & \dots & 0 & -2 & 0 \end{bmatrix} \begin{Bmatrix} s_1 \\ s_2 \\ \dots \\ s_{n-1} \\ s_n \end{Bmatrix} \quad (1.69)$$

Afterwards, it is necessary to ensure global compatibility by imposing that the mean value of steel strain computed between two adjacent cracks from the tension stiffening formulation $\varepsilon_{s,mean}$ (Figure 1.20):

$$\varepsilon_{s,mean} = \frac{1}{n-1} \left(\frac{\varepsilon_{s,1}}{2} + \sum_{i=2}^{n-1} \varepsilon_{s,i} + \frac{\varepsilon_{s,n}}{2} \right) \quad (1.70)$$

is equal to the steel strain evaluated in the global procedure. When this condition is satisfied, the effective steel strain distribution $\varepsilon_{si,cr}$ along the relative i^{th} reinforcement layer is known. Then, the stiffening coefficient g_{i1} results:

$$g_{i1} = \frac{\varepsilon_{si,cr}}{\delta_{i1}} l_{si} \quad (1.71)$$

The axial force in the bar smeared along crack surfaces is obtained as:

$$\sigma_{si}^{cr} = \frac{N_{si}^{cr}}{s_i t} = \frac{\bar{E}_{si}^{cr1} \pi \phi_i^2}{4 s_i t} = \rho_{si} \bar{E}_{si}^{cr1} \varepsilon_{si,cr} = \rho_{si} \bar{E}_{si}^{cr1} g_{i1} \frac{\delta_{i1}}{l_{si}} \quad (1.72)$$

where $\bar{E}_{si}^{cr1}$ is the secant elastic modulus in correspondence of the steel axial strain $\varepsilon_{si,cr}$ at the crack.

Therefore, the matrix $[D_{si,cr1}^{(x_i, y_i)}]$ of each reinforcement layer, evaluated in its own local co-ordinate system x_i-y_i , which contains the effects of tension stiffening and dowel action, is:

$$\begin{bmatrix} D_{si,cr1}^{(x_i, y_i)} \end{bmatrix} = \rho_{si} \begin{bmatrix} \bar{E}_{si}^{cr1} g_{i1} & 0 \\ 0 & d_{i1} \end{bmatrix} . \quad (1.73)$$

Summing up all the contributions of the n reinforcement layers (smeared through the geometric steel ratio) rotated to global x-y system, the total crack steel stiffness matrix $[D_{s,cr1}]$, that accounts for all the resistant mechanisms that happen across crack faces related to steel, results:

$$[D_{s,cr1}] = \sum_{i=1}^n [T_{si}]^t [D_{si,cr1}^{(x_i, y_i)}] [T_{si}] . \quad (1.74)$$

1.3.4.5 Fiber contribution

2D-PARC model is structured in a modular framework. All the contributions of the involved mechanical phenomena are analyzed separately on the basis of their properties and afterwards assembled together to create an equivalent, nonlinear, continuum material. In this way, fiber role can be easily taken into account by adopting proper constitutive laws and modifying the related matrix contributions but leaving unchanged the whole structure of the model.

Fiber contribution, as already said, is neglected in the evaluation of the matrix relative to the intact material between cracks, as done in the uncracked stage. On the contrary, the crack stiffness matrix $[D_{cr1}]$ is properly modified so to account for fiber influence. The main mechanisms that change with the addition of fibers in the concrete mix are tension softening and tension stiffening. As detailed explained in the following, for the evaluation of tension softening, fiber bridging is directly added to the bridging effect offered by aggregates; whereas the influence exerted by fibers on tension stiffening is taken into account indirectly, by modifying the bond-slip law between ordinary steel bars and concrete.

Tension stiffening

The tension stiffening coefficient g_{i1} to be inserted in matrix $[D_{s,cr1}]$ (1.73), which accounts for steel contribution at crack location, is evaluated as in case of plain concrete (detailed explained in §1.3.4.4) but applying the bond-slip law suggested in [61,62], specifically derived for SFRC elements, (Figure 1.21), instead of that of MC2010, [57], adopted for traditional RC elements. It is also worth noting that boundary condition $\sigma(l_s/2) = \sigma_{ct}^{cr}$ to solve system (1.67) accounts for the higher concrete stress in the crack provided by fiber bridging.

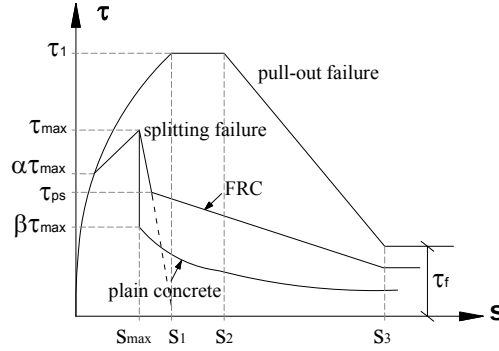


Figure 1.21 Adopted bond-slip law [44]

The role exerted by fibers in the concrete matrix is similar to that of transversal reinforcement, as can be seen by comparing Figure 1.18b and Figure 1.21. They both reduce the development of splitting cracks and consequently provide an increase in bond stresses in the post-splitting stage, so preventing the brittle failure which characterizes ordinary unconfined concrete. This effect is taken into account by explicitly introducing – in the governing equations – a sort of "confinement parameter", which depends on fiber volume percentage V_f in the concrete matrix as well as on fiber geometry (namely diameter d_f and length L_f).

In case of pull-out failure the influence of fibers is less significant and the local response in terms of bond stress-slip is quite similar to that of plain concrete; as a consequence, also the governing equations are quite similar – even if not identical – to those proposed in MC2010 (see [57]) for ordinary reinforced concrete. It is also worthwhile noting that, in presence of pull-out failure, the law proposed in [61,62] is independent from the fiber content V_f , while, as already said, this latter influences the trend of the curve in case of splitting failure.

More in detail, the ascending branch of Figure 1.21 is given by the equation:

$$\tau = \tau_1 \left(s/s_1 \right)^{0.3} , \quad (1.75)$$

where $\tau_1 = 2.57\sqrt{f_c}$ and $s_1 = 0.15 c_0$, being c_0 the distance between bar ribs, (otherwise, it can be assumed $s_1 = 1.5$ mm, as suggested in [61,62]). As far as the pull-out curve is concerned, it can be also assumed $s_2 = 0.35 c_0$ and $s_3 = c_0$ (or rather $s_2 = 3.5$ mm and $s_3 = 10$ mm), while τ_f can be assumed equal to $0.35 \tau_1$.

As regards splitting failure, the first ascending branch - Equation (1.75) - is the same describe for pull-out, however, in this case, in this case it is followed by a subsequent linear ascending branch characterized by a reduced slope, included between $\alpha \tau_{max}$ and τ_{max} , being $\alpha = 0.7$ and:

$$\tau_{max} = 0.78 \sqrt{f_c} \left(\frac{c + K_c}{\phi} \right)^{2/3}, \quad (1.76)$$

where c is the minimum concrete cover and K_c the "confinement parameter" defined as: $K_c = 0.45 c V_f L_f / d_f$. The maximum bond stress τ_{max} corresponds to a slip value s_{max} , which is assumed, for hooked-ended fibers, equal to:

$$s_{max} = s_1 e^{1.8 \left[(\tau_{max} / \tau_1)^2 - 1 \right]}. \quad (1.77)$$

The τ - s curve is then characterized by a descending branch, until the residual post-splitting bond stress τ_{ps} is reached:

$$\tau_{ps} = \tau_{max} (0.5 + K_{cs}) \leq \tau_{max}. \quad (1.78)$$

being $K_{cs} = 0.20 V_f L_f / d_f$. Finally, the last part of the curve is characterized by a progressive strength reduction.

Tension softening

The influence exerted by fibers on tension softening contribution is inserted into 2D-PARC model by properly modifying the stiffness matrix $[D_{c,cr1}]$ (1.51), which takes into account the contribution of concrete at crack location. Among the several semi-empirical formulations that can be found in the literature to model the transmission of tensile stresses across the crack three relationships are implemented in the model: the micro-mechanical model proposed by Li et al. [63,64], the semi-empirical law proposed by Barros [65–67] and the MC2010 law [57], suitable also for design practice.

Li et al. law [63,64]

Following the micro-mechanical model proposed by Li in 1992 [63] and subsequently refined by Li, Stang and Krenchel [64], the transmission of tensile stresses across the crack due to aggregates and fibers is separately modeled. As a consequence, in 2D-PARC model the fiber bridging coefficient c_{f1} is added to aggregate bridging coefficient c_{b1} (that is in turn evaluated through Expression (1.48) as in case of plain concrete); thus, the crack concrete matrix $[D_{c,cr1}]$, expressed in the local co-ordinate system of the crack (n_1, t_1) , assumes the following form:

$$[D_{c,cr1}^{(n_1, t_1)}] = \begin{bmatrix} c_{b1} + c_{f1} & -c_{01} \\ 0 & c_{a1} + c_{f1} \end{bmatrix}, \quad (1.79)$$

where c_{01} and c_{a1} schematize aggregate interlock effect (see §1.3.4.3).

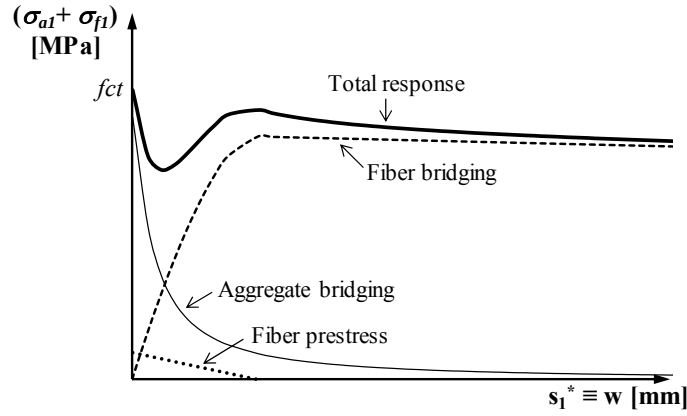


Figure 1.22 Aggregate and fiber roles on SFRC softening behavior

The model describes the interaction between the fibers and the concrete mix by summing up their effects, as can be seen in Figure 1.22, which reports the relation between normal stresses (related to plain concrete, σ_{a1} , and to fibers, σ_{f1}), and crack opening w_1 , in case of crack sliding v_1 equal to zero.

To calculate the fiber bridging coefficient c_{f1} is necessary to consider a proper law that links fiber stress σ_f with the total displacement at crack location s_1 . Fiber contribution σ_f can be evaluated as the sum of fiber bridging effect σ_{b1} , developed by the fibers themselves in the fracture region, and fiber prestress, σ_{ps1} , which characterizes the fibers before the opening of a crack:

$$\sigma_f(s_1) = \sigma_{b1}(s_1) + \sigma_{ps1}(s_1) \quad (1.80)$$

The bridging effect σ_{b1} and the fiber prestress σ_{ps1}^0 are computed following the approach proposed in [63,64].

The fiber bridging stress σ_{b1} is expressed as a function of an appropriate crack opening δ_1 , related to the interfacial frictional debonding, by integrating the contribution of each fiber loaded with a force P and located at a certain distance z from the crack with a casual orientation Φ :

$$\sigma_{b1}(\delta_1) = \frac{4 \cdot V_f}{\pi \cdot d_f^2} \int_{\Phi=0}^{\pi/2} \int_{z=0}^{(L_f/2) \cos \Phi} P(\delta_1) p(\Phi) p(z) dz d\Phi \quad (1.81)$$

being $p(\Phi)$ and $p(z)$ the probability density function, that, for a uniform random distribution, can be assumed equal to $\sin \Phi$ and $2/L_f$, respectively. The term $P(\delta_1)$ represents the bridging force exerted by the single fiber with embedded length l and its expression can be found in [64].

The resulting $\sigma_{b1} - \delta_1$ relation (Figure 1.23) is characterized by an ascending branch until the attainment of δ_1^* , corresponding to the complete deboning along the full length of the embedded fiber segment; then the descending branch begins. This value is computed according to [64]; thus assuming $\delta_1^* = \tau L_f^2 / [(1+\eta) E_f d_f]$, being $\eta = V_f E_f / V_m E_m$ and where V_f , E_f and V_m , E_m represent the volume fraction and Young modulus respectively of fibers and concrete. The expression of interfacial bond strength τ can be still be found in [64] as a function of the constants, τ_0 , a_1 , a_2 and the local slippage. The stress corresponding to peak, $\sigma_0 = \sigma_{b1}(\delta_1^*)$, is defined as $\sigma_0 = g \tau V_f / [L_f / (2 d_f)]$ being g the “snubbing factor” defined as $g = 2 (1 + e^{-\pi f/2}) / (4 + f^2)$ where f represents the snubbing coefficient, that takes into account for the increasing of the pull-out force with the fiber inclination Φ , the so called snubbing effect.

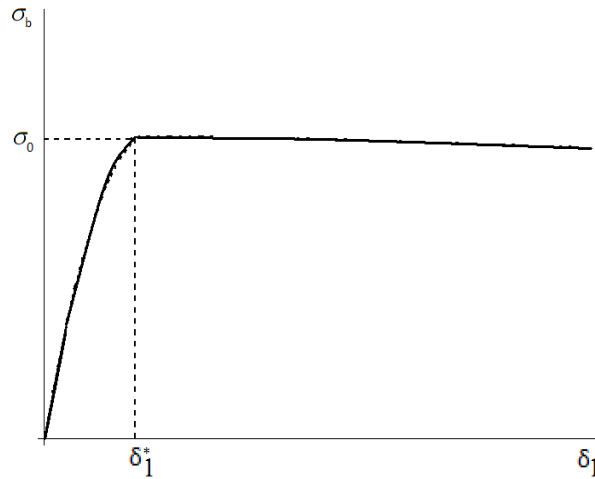


Figure 1.23 Assumed $\sigma_{b1} - \delta_1$ relation, [64]

It is worth noting that for the snubbing coefficient f and the interfacial bond strength constant τ_0 a proper expression is not given in the literature and only a range of variability of their values is generally provided. These parameters can be better calibrated for the specific case studies through the modelling of experimental three-point bending tests on small notched beams without rebars – when available – produced with the same concrete mix and the same fibre content as the structural elements of interest. As known, the post-cracking behaviour of the notched beams is basically ruled by the tension softening law inserted into the adopted constitutive matrix; as a consequence, the aforementioned parameters can be varied so to obtain the best fitting of the experimental results.

The fibre bridging stress, according to [64], is then expressed as:

$$\sigma_{b1}(\delta_1) = \sigma_0 \left[2 \left(\frac{\delta_1}{\delta_0} \right)^{1/2} - \left(\frac{\delta_1}{\delta_1^*} \right) \right] \quad \text{if } 0 \leq \delta_1 \leq \delta_1^* \quad (1.82)$$

$$\sigma_{b1}(\delta_1) = \frac{4\sigma_0}{L_f^2} \left[\frac{L_f}{2} - (\delta_1 - \delta_1^*) \right] \quad \text{if } \delta_1^* \leq \delta_1 \leq \frac{L_f}{2} .$$

The appropriate crack opening δ_1 , to be related to the total crack displacement s_1 , must be added to the displacement δ_{cg1} due to the Cook-Gordon effect (that is a premature fiber/matrix interphase deboning normal to the fiber axis), so obtaining $s_1 = \delta_1 + \delta_{cg1}$. By following the simple analytical computations reported in [68], Equations (1.82) can be rewritten, so obtaining σ_{b1} as a function of the total crack displacement s :

$$\sigma_{b1}(s_1) = \frac{\sigma_0}{(\delta_1^* - \beta)^2} \left[-s_1(\delta_1^* - \beta) - 2\beta\delta_1^* + 2\delta_1^* \sqrt{s_1(\delta_1^* - \beta) + \beta^2} \right] \quad \text{if } 0 \leq s_1 \leq s_1^* \quad (1.83)$$

$$\sigma_{b1}(s_1) = \frac{\sigma_0}{\beta} \left[s_1 - \left(\delta_1^* + \frac{L_f}{2} \right) + \frac{L_f^2}{8\beta} - \sqrt{\frac{L_f^4}{64\beta^2} + \frac{L_f^2}{4\beta} \left(s_1 - \left(\delta_1^* - \frac{L_f}{2} \right) \right)} \right] \quad \text{if } s_1^* \leq s_1 \leq \frac{L_f}{2}$$

where $\beta = 2\alpha g_f L_f / (E_f d_f)$ and $s_1^* = \delta_1^* + \delta_{cg1}^*$ with $\delta_{cg1}^* = 4\alpha\sigma_0 / (V_f E_f)$ being α the Cook-Gordon parameter as defined in [64].

The second term of Equation (1.80) σ_{ps1}^0 , according to [64], is evaluated by linearly reducing the stress state σ_{ps1}^0 which characterizes the fibres before the opening of the crack:

$$\sigma_{ps1}(s_1) = \sigma_{ps}^0 \frac{(s_1^* - s_1)}{s_1^*} \quad \text{if } 0 \leq s_1 \leq s_1^* \quad (1.84)$$

$$\sigma_{ps1}(s_1) = 0 \quad \text{if } s_1 \geq s_1^*$$

being $\sigma_{ps1}^0 = \eta_0 \eta_1 \varepsilon_{cr} V_f E_f$, where ε_{cr} is the first crack strain and η_0, η_1 are the orientation and the length efficiency factors, assumed respectively equals to 1 and to $(1 - \varepsilon_{cr} d_f E_f / 4 L_f \tau_0)$, according to [64].

After the evaluation of the terms of Equation (1.80) by following the above described procedure, the fiber contribution σ_f is in turn subdivided into two components, respectively parallel and perpendicular to crack direction; σ_f is indeed expressed as a function of the total displacement s_1 across the crack itself, representing the resultant of crack opening w_1 and sliding v_1 : $\vec{s}_1 = \vec{w}_1 + \vec{v}_1$.

Therefore, the normal σ_{f1} and the tangential τ_{f12} components of σ_f are expressed as:

$$\begin{aligned}\sigma_{f1}(s_1) &= \sigma_f(s_1) \cos \omega_1 = \sigma_f(s_1) \frac{w_1}{\sqrt{w_1^2 + v_1^2}} = c_{f1} \varepsilon_1, \\ \tau_{f12}(s_1) &= \sigma_f(s_1) \sin \omega_1 = \sigma_f(s_1) \frac{v_1}{\sqrt{w_1^2 + v_1^2}} = c_{f1} \gamma_{12}\end{aligned}\quad (1.85)$$

where c_{f1} represents the fiber coefficient, while $\varepsilon_1 = w_1/a_{m1}$ and $\gamma_{12} = v_1/a_{m1}$. Finally, taking advantage of the stress decomposition of Equations (1.85), the fiber coefficient c_{f1} is inserted in matrix (1.79).

Barros et al. law [65–67]

The approach proposed in [65–67] schematizes the post-cracking behavior of the whole composite material, so providing a unique tension softening contribution for SFRC. Hence, the concrete crack stiffness matrix $[D_{c,cr1}]$, in its local coordinate system (n_1, t_1) , is modified by considering a single coefficient c_{t1} , representing both aggregate and fiber bridging:

$$[D_{c,cr1}^{(n_1, t_1)}] = \begin{bmatrix} c_{t1} & -c_{01} \\ 0 & c_{a1} \end{bmatrix}, \quad (1.86)$$

so replacing the corresponding (1,1) term ($c_{b1} + c_{f1}$) of matrix (1.79).

Following the method proposed in [65–67], the total bridging effect of the SFRC material can be schematized by assuming the shape of the tensile strain softening diagram as tri-linear with quite good approximation. This curve was calibrated by Barros et al. [65–67] on the basis of several four point bending tests carried out under displacement control on SFRC notched beams, characterized by different fiber contents, concrete compression strengths and fiber aspect ratios.

The adopted expressions (see [67]) are here rewritten as function of crack opening w_1 :

$$\sigma_n(w_1) = \begin{cases} f_{ct} + S_{l,1}^{cr} w_1 & 0 < w_1 < \xi_1 w_{1u} \\ \alpha_1 f_{ct} + S_{l,2}^{cr} (w_1 - \xi_1 w_{1u}) & \xi_1 w_{1u} < w_1 < \xi_2 w_{1u} \\ \alpha_2 f_{ct} + S_{l,3}^{cr} (w_1 - \xi_2 w_{1u}) & \xi_2 w_{1u} < w_1 < w_{1u} \\ 0 & w_1 > w_{1u} \end{cases}, \quad (1.87)$$

where

$$S_{l,i}^{cr} = -k_i \frac{f_{ct}^2}{G_f} \quad , \quad i = 1, 2, 3 \quad (1.88)$$

being:

$$w_{1u} = k_4 \frac{G_f}{f_{ct}} \quad . \quad (1.89)$$

The expressions of coefficients k_1 , k_2 , k_3 and k_4 appearing in Equations (1.88) and (1.89) can be found in [67], as function of four parameters α_1 , α_2 , ξ_1 , ξ_2 defining the characteristic points of the softening diagram of SFRC (reported in Figure 1.24). Moreover, the four parameters α_1 , α_2 , ξ_1 , ξ_2 , whose values can be still found in [67], depend on fiber type, content W_f and aspect ratio L_f/d_f . It is worth noting that the SFRC fracture energy G_f reported in Expressions (1.88) and (1.89) can be derived from the empirical law proposed in [66] which relates G_f to the fracture energy of corresponding plain concrete G_{f0} and to the fiber weight percentage in the mixture W_f , with difference laws for different fiber types and aspect ratios L_f/d_f .

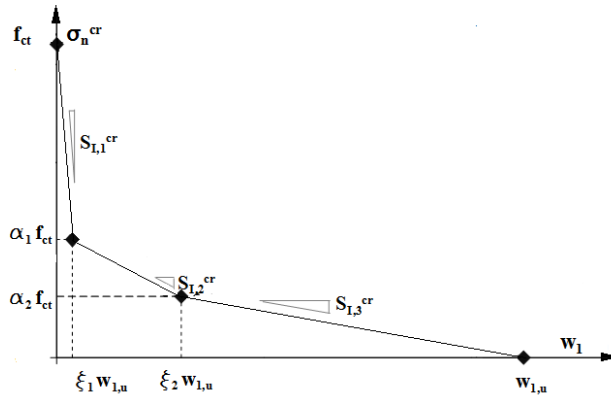


Figure 1.24 Post-peak tensile response: Barros et al. [65–67] law

Once the value of tension softening σ_n of SFRC is known, the bridging coefficient c_{t1} to be inserted in matrix (1.86) can be evaluated as:

$$c_{t1} = \frac{\sigma_n(w_1)}{w_1} a_{m1} \quad . \quad (1.90)$$

MC2010 law [57]

Since the MC2010 relation, as the approach follows by Barros et al. [65–67], provides the post-cracking behavior of the whole SFRC material, it does not allow for the separation of contributions due to aggregates and fibers, but a unique tension softening coefficient c_{t1} is applied. As a consequence, the concrete crack stiffness matrix $[D_{c,cr1}]$, in the local co-ordinate system (n_1, t_1) of the crack can be expressed by Equation (1.86).

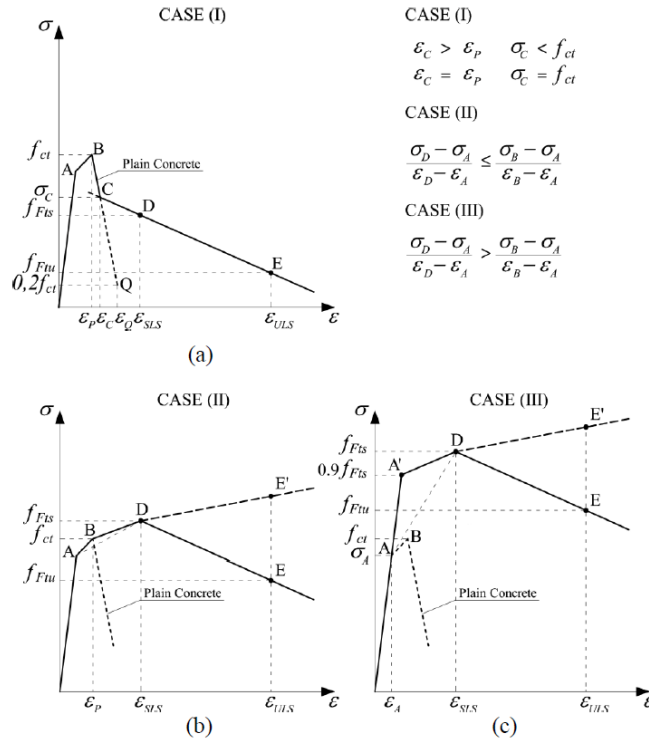


Figure 1.25 Post-peak tensile response for (a) softening and (b), (c) softening or hardening behavior of FRC [57]

The MC2010 law (Figure 1.25) – which is formulated in terms of stress σ_n vs. strain ϵ_1 – is rewritten as function of crack opening w_1 , by assuming $\epsilon_1 = w_1/a_{m1}$, and hypnotizing the structural characteristic length l_{cs} equal to the distance between cracks a_{m1} . Both the values of strain ϵ_p , ϵ_q (which define the first branch of the $\sigma_n - w_1$ curves and depend on the characteristics of plain concrete) and of strain ϵ_{SLS} , ϵ_{ULT} (which are needed for the definition of the last branch of the same curves), are determined according to MC2010, [57]. It should be also pointed out that the serviceability f_{Fts} and ultimate f_{Ftu} residual strength are determined on the basis of the simplified post-cracking linear model proposed in

MC2010, [57], as a function of the nominal residual strengths which can be obtained by three-point bending tests on SFRC notched beams.

Once the stress σ_n in the crack is determined by applying the tension softening law (Figure 1.25) that suits the case of interest, the tension softening coefficient c_{t1} is finally calculated, by applying Equation (1.90), as for the softening relation proposed by Barros et al. [65–67].

1.3.5 Implementation of 2D-PARC into a commercial FE code

The above described stiffness matrices have been implemented into a commercial FE code in the form of a "user-material" subroutine, in order to perform numerical analyses on various RC/SFRC structural elements, able to account for material nonlinearity in the post-cracking stage. Among the several FE software, ABAQUS has been selected since it allows the user to define and implement a general constitutive model without changing the general source code.

The incorporation of 2D-PARC routine into a FE code, compared to a standalone specifically written, gives the model more flexibility enabling the analysis of larger and more complex structures and an easier output of the results. In this way, the presented constitutive model can be simply applied for the analyses of numerous structural typologies by employing various but appropriate FE from the ABAQUS library. The elements chosen must indeed meet the hypothesis of plain stresses of 2D-PARC model.

2D-PARC is incorporated into a User-defined MATerial subroutine (UMAT for short) provided by ABAQUS, written in FORTRAN language and recalled by the FE program for each integration point at the beginning of each loading increment or iteration within the loading increment. As a matter of fact, UMAT, which is characterized by a fixed list of input/output data arguments, provides to the main program the material constitutive relation to be used in the solution. In more detail for each iteration, starting from the current state of stress $\{\sigma\}$ and strain $\{\varepsilon\}$ at the beginning of the increment and from the incremental strain $\{\Delta\varepsilon\}$, the user subroutine provides ABAQUS the updated total stresses $\{\sigma\}$ and Jacobian matrix $[D]$ that are the output variables. All the subsequent operations required to solve the problem (integration, assembly, computation of forces/incremental displacements and equilibrium iteration for convergence) are then carried out by ABAQUS. It is worth noting that $\{\Delta\varepsilon\}$ represents the trial strain increment since, for each iteration, ABAQUS searches for the right increment to gain convergence.

The UMAT computes $[D]$ employing 2D-PARC model and so applying the relations extensively described in §1.3.3-1.3.4, i.e. in the uncracked stage by summing up the contribution due to concrete and steel – Equation (1.15), while in the cracked stage considering both the material, integer even if damaged, between two cracks and all the phenomena happening at crack location – Equation (1.46).

Since 2D-PARC model is formulated in terms of secant stiffness, the total stress vector to be passed to the main program is evaluated as:

$$\{\sigma\} = [D](\{\varepsilon\} + \{\Delta\varepsilon\}). \quad (1.91)$$

Then, the FE code computes for each element the internal forces $\{f_{int}\}$ by integrating the stresses $\{\sigma\}$ over the element and subsequently they are assembled to obtain the internal forces of the whole structure. The Jacobian matrix $[D]$ is instead used to evaluate the stiffness matrix $[k_e]$ of each element by integration and then, all the $[k_e]$ are assembled to obtain the stiffness matrix of the structure $[K]$. Then, for each loading increment (or iteration within the loading increment) the solving system can be evaluated starting from the global stiffness matrix $[K]$ and the residual forces, expressed as the difference between the external forces $\{f_{ext}\}$, computed from the total load in the current increment, and the internal ones $\{f_{int}\}$, which represent the reaction forces related to the internal stresses:

$$\{\Delta q\} = [K]^{-1}(\{f_{ext}\} - \{f_{int}\}). \quad (1.92)$$

Therefore, the residual is the rate of the external load not balanced by the internal stresses integrated over the whole structure. The balance equation is checked and the equilibrium iterations are performed until convergence; then the solver goes to the next loading increment. On the contrary if convergence is not achieved a new attempt value of strain $\{\Delta\varepsilon\}$ is provided, starting from increment displacement found $\{\Delta q\}$.

1.3.5.1 Description of UMAT subroutine

The communication between 2D-PARC constitutive law and ABAQUS is obtained, as already said, by means of UMAT that exchanges with the main program some important variables.

The variables, passed in for each integration point, are the material properties, together with the current state of stress and strain provided by adopted FE code. In particular, for concrete, the definition of the uniaxial compressive strength f_c together with corresponding peak strain ε_{c0} as well as of the uniaxial tensile stress f_{ct} , the initial value of the Young modulus E_c and the Poisson coefficient ν is required. Moreover, the number n of reinforcing layers must be defined. For each steel layer the user sets the angle defining the bars orientation, the equivalent bars diameter ϕ and spacing s as well as the reinforcement ratio ρ . Also the steel mechanical properties must be provided, that are the elastic E_s and strain-hardening E_{sp} modulus, the Poisson coefficient ν_s , the yield ε_{sy} and ultimate ε_{su} strain. Moreover, in case of fibrous reinforcement also the geometrical and mechanical parameters defining fiber behavior must be provided.

Then, the user material subroutine computes, on the basis of the current state of stress of the considered integration point, by applying 2D-PARC constitutive law (see §1.3.3 and §1.3.4), the output variables to be passed to the main program.

As already said, the main output variables are the updated stresses and the Jacobian matrix to be used by ABAQUS to calculate respectively the internal forces and the global stiffness matrix required to solve the problem and so for the execution of external convergence checks. In addition, the so called solution-dependent state variables (STATEV for short) can be passed out. They are defined in the subroutine and they can be visualized as the standard output variables of the main program. For the purpose of this work, STATEV are typically quantities of interest for the structural analysis of RC/SFRC elements, for example stresses and strains. However, 2D-PARC model allows to evaluate many other interesting variables highly significant in the evaluation of structural behavior and that a traditional commercial FE code cannot provide; e.g. steel stresses (between two adjacent cracks and at crack location), concrete stresses, crack opening and sliding or the single value of each resistance contribution that develops after crack formation (i.e. aggregate and fiber bridging, aggregate interlock, tension stiffening and dowel action).

Internal convergence due to strain decomposition

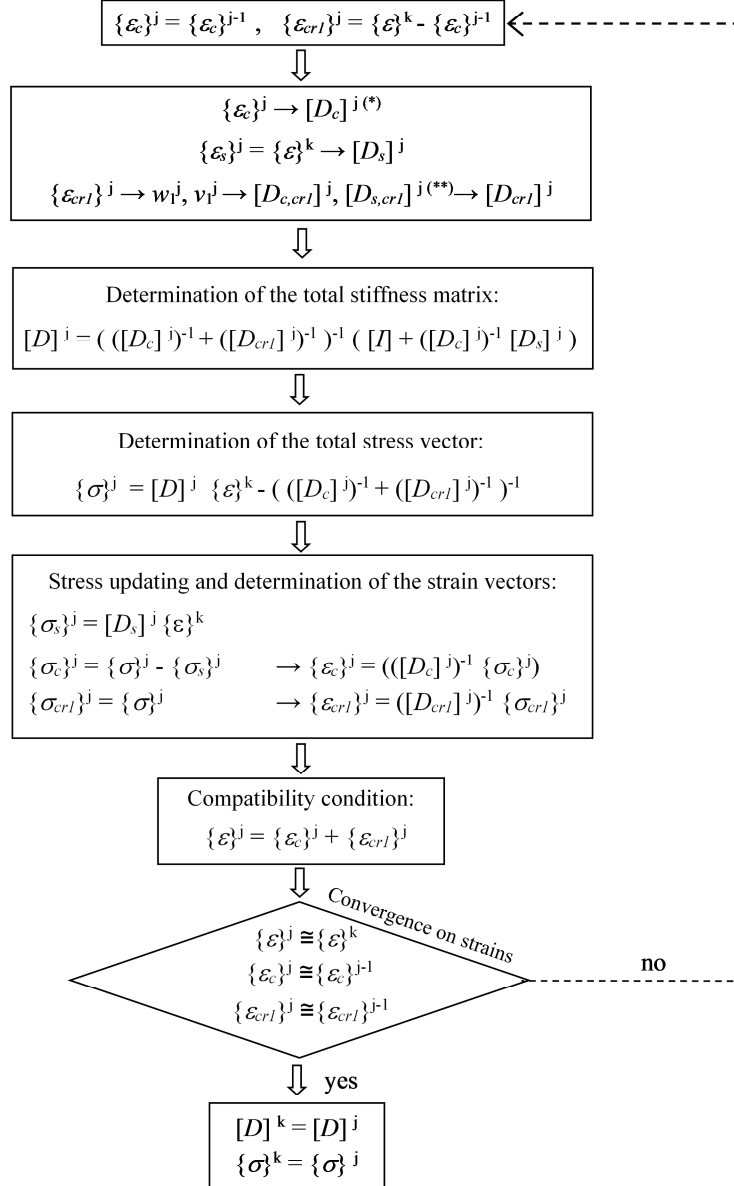
As already said, since RC behavior is markedly nonlinear, the solution strategy followed by ABAQUS is based on an incremental-iterative procedure; for this reason, the applied load is first subdivided into small increments and subsequently an iterative procedure is adopted within each loading increment to achieve convergence and restore equilibrium (so-called “external iterations” in the flow chart of Figure 1.26).

In addition, since the 2D-PARC model is based on a strain decomposition scheme in the cracked stage; within the subroutine an iterative “internal” procedure is required (so-called “internal iterations” in the flow chart of Figure 1.26). It is necessary to check the convergence on total strain: to attain the exact value of the strain relative to concrete between two adjacent cracks and of that relative to fracture zone, equilibrium and compatibility conditions must be fulfilled. Equilibrium condition is checked by posing the stress in the crack, Equation (1.41), equal to the total stress, Equation (1.42), while the compatibility is checked considering Equation (1.37).

As shown in the flow-chart of Figure 1.26 for the k^{th} external iteration within a fixed loading increment, ABAQUS provides the total strain vector $\{\varepsilon\}^k$ to the user-material subroutine. It is worth noting that $\{\varepsilon\}^k$ in the following is assumed as the sum of the strain of the current increment and the attempt value provided by the main-program external iteration.

The crack strain $\{\varepsilon_{cr1}\}^j$ can in turn be evaluated, as well as that of the intact material between adjacent cracks, $\{\varepsilon_c\}^j$, by initially assuming the latter coincident to the one obtained at the end of the last converged internal iteration, $\{\varepsilon_c\}^{j-1}$ (being j the current internal iteration in the user subroutine).

for the k -th external iteration and the j -th internal one



(*) The evaluation of matrix $[D_c]$ requires internal iterations; see [69] for further details

(**) The evaluation of matrix $[D_{s,crl}]$ requires internal iterations; see [60] for further details

Figure 1.26 Flow chart of the internal iterative procedure adopted in the cracked stage

Starting from these strain values, the corresponding stiffness matrix of steel and concrete between adjacent cracks, as well as that of the fracture zone are first separately determined and then combined together to form the global Jacobian matrix of the cracked element, $[D]^j$. Therefore the total stress field $\{\sigma\}^j$ is obtained and, starting from this value, the stresses of concrete and steel between two adjacent cracks and the crack stress vector, all relative to the current internal iteration, are computed.

The knowledge of these quantities allows in turn to determine the updated strain field relative to the intact, even if damaged, concrete between two adjacent cracks and to the crack itself.

The total strain $\{\epsilon\}^j$, obtained by summing up these updated values, should be then compared to the one provided by the FE code, $\{\epsilon\}^k$; if these two values are almost equal (within a specified tolerance), the global stiffness matrix $[D]^j$ and the corresponding global stress vector are passed to the FE code. If not, an iterative procedure begins by starting again the procedure with the updated value of the concrete strain, until convergence is achieved.

It is worth noting that within the subroutine two other internal convergence procedures respectively for the evaluation of the concrete $[D_c]$ and crack $[D_{cr}]$ stiffness matrix are required.

As already described in §1.3.3.2, concrete matrix $[D_c]$ is indeed related to the current state of strain-stress: the two elastic moduli $\bar{E}_{c1}$, $\bar{E}_{c2}$ are computed on the basis of the equivalent uniaxial strains ϵ_{ci}^u together the peak stress $\sigma_{ci,max}$ and strain ϵ_{c0}^* on the corresponding curves. $\sigma_{ci,max}$ and ϵ_{c0}^* are in turn determined as functions of the ratio between the maximum and the minimum principal stress of each increment, α . As better discussed in [69] an iterative procedure is performed by checking the convergence on α : if the difference between its current value and the previous one is less of a predefined tolerance $\bar{E}_{c1}$ and $\bar{E}_{c2}$ are inserted into matrix $[D_c]$, otherwise the search for convergence begins.

The latter internal iteration regards the evaluation of tension stiffening contribution g_{i1} , inserted in matrix $[D_{s,cr1}]$ that accounts for all the resistant mechanisms related to steel that happens across crack faces. In particular tension stiffening contribution g_{i1} is determined by considering the actual non-uniform distribution of the strains in the reinforcement. As described in §1.3.4.4, the finite difference method is applied at each integration point to solve the governing equations of the problem: equilibrium conditions for the whole section, for concrete and for steel, as well as compatibility condition. Finally it is necessary to ensure global compatibility by imposing that the mean value of steel strains computed between two adjacent cracks from the tension stiffening formulation is equal to the total strain evaluated in the global procedure, as better described in [60].

Chapter 2

Revision of concrete contribution

2.1 Introduction

In this chapter the constitutive model for the nonlinear analysis up to failure of reinforced concrete and steel fiber reinforced concrete (RC and SFRC) elements, named 2D-PARC, is revised with regard to the modeling of concrete contribution in order to improve its computational efficiency, as well as to incorporate the effects of crushing and dilatation of concrete in compression.

The model, developed at the University of Parma by Cerioni et al. in 2008 [1], belongs to the approaches that smear both the reinforcement and cracking within the element. It treats the material as an equivalent continuum, whose properties are derived through a so called “localized stress-field approach”: each mechanical phenomenon is individually analyzed, by using a proper constitutive law, and the corresponding contribution is then inserted into the material stiffness matrix. A comprehensive description of the original formulation of 2D-PARC model, is reported in Chapter 1.

In this chapter, a first improvement of the model is presented; more in detail, the constitutive relation adopted for the evaluation of concrete resistant contribution both before and after cracking development, is properly modified. This operation, thanks to the abovementioned modular structure of the model, can be easily performed by only changing the part of the general algorithm related to concrete behavior (i.e. the evaluation of concrete stiffness matrix in the uncracked and cracked stages) while leaving unchanged the rest.

In the original formulation of 2D-PARC model (see §1.3.3.2 for further details), concrete is modeled as an orthotropic material both in the uncracked and cracked stage. In more detail, by applying the method proposed by Darwin and Pecknold [47], the effective biaxial state of stress is turned into two uniaxial states of stress. However, this sophisticated formulation complicates the general algorithm and requires quite high computational effort, so considerably lengthening, in some cases, the calculational time. Moreover, the adopted procedure does not allow the description of the post-peak behavior of concrete and consequently crushing failures cannot be predicted.

For these reasons, this chapter discusses concrete contribution to be revised, by implementing into 2D-PARC model an alternative and simpler

formulation based on isotropic nonlinear elasticity, following the approach proposed by Ottosen [70,71]. Nonlinear stress-strain relations for concrete are provided and then the concrete matrix is calculated by only properly changing the secant values of Young modulus and Poisson ratio, on the basis of the actual state of stress. This formulation still provides a sophisticated representation of concrete behavior being at the same time characterized by an enhanced computational efficiency. Moreover, the post-crushing behavior can be satisfactorily handled.

The adoption of a refined model to simulate concrete behavior is very important. Several researches [50,51,72,73] have indeed pointed out that the adoption of a proper nonlinear stress-strain relationship, combined with an accurate failure criterion, is crucial for correctly simulating the structural behavior of RC elements, not only in terms of failure load, but also with reference to a realistic representation of crack initiation and development. As a matter of fact concrete under a biaxial (or triaxial) state of stress exhibits different stiffness, strength and ductility than under uniaxial loading. Consequently, the strength characteristics of concrete under a general multi-axial state of stress cannot be reproduced into a constitutive model by directly using experimental uniaxial stress-strain curves, but more complex approaches should be used. These aspects are extremely relevant especially for elements made of plain concrete or reinforced in a single direction whose behavior is mainly governed by concrete performances.

This chapter is subdivided into three main parts. At first, an overview of the current state of the art relative to the modeling of concrete contribution will be presented, secondly the revised formulation of 2D-PARC model will be outlined and in the last part of the chapter the presented model and its implementation will be verified by comparing FE results to experimental data available in technical literature relative to different structural typologies. At first, plain concrete panels [50] subjected to a biaxial stress state, as a sort of benchmark example, are analyzed. Subsequently the attention is focused on RC beams without shear reinforcement [74,75], which represent a quite difficult problem, particularly when studied with two-dimensional membrane analyses. Finally, in order to verify the applicability of the proposed procedure also in case of fiber addition to the concrete mix, SFRC beams [76] without shear reinforcement are analyzed. The selected cases point out the general flexibility of the revised model as well as its capability for correctly describing concrete behavior under different loading conditions, also in the post-crushing region.

2.2 State of the art

Concrete, especially in compression, exhibits a highly nonlinear behavior, even starting from low load levels. The complexity of its mechanical behavior lies principally in the particularity of the microstructure. Concrete is indeed a composite material characterized by the presence of aggregates in a brittle matrix, the hydrated cement paste.

It follows that the full understanding (and representation) of the problem is a difficult task. A lot of attempts were indeed made during the twentieth century to provide proper mathematical models able to correctly reproduce concrete behavior. Three levels of representation (macroscopic, mesoscopic and microscopic) related to increasing accuracy are recognized.

The more refined ones are the microscopic models. These studies (e.g. [77,78]), performed to understand the main mechanisms governing concrete behavior - such as the chemical reactions involved in the paste hardening, the bond between the aggregates and the mortar, the influence of the additives and the evolution of internal microcracks - are of main importance, but hard to be applied in the current modelling of RC structures, usually performed at a higher scale of representation.

The mesoscopic formulations (e.g. [79,80]) represent the intermediate approach. The cement paste and the aggregates are separately modeled, with the possibility of inserting also interfacial laws between them. The brittle matrix can be simulated as a viscoelastic porous material, whereas for aggregates a linear elastic behavior is generally assumed, since they are characterized by a higher strength compared to that of the cement paste. They provide a reliable description of concrete behavior; however, also in this case, they are difficult to incorporate in the current analyses of RC structures.

The macroscopic approaches are the most widespread and utilized models for the analyses of RC structures. In this case concrete is modeled as a nonlinear homogeneous material; thus, its actual heterogeneity, due to the presence of various components, and the existence of related zones of weakness are completely neglected. In this case, proper constitutive stress-strain relationships are provided, calibrated on the basis of experimental results.

In the following, concrete will be analyzed only in the macroscopic scale, since this is the approach followed in this thesis. In particular, the attention will be focused on the behavior of concrete under a general biaxial state of stress, since the goal of this work is to revise the modeling of concrete contribution into the existing bi-dimensional model, 2D-PARC.

Initially the experimental behavior of concrete under uniaxial and biaxial loading will be briefly outlined. To comprehend the concrete behavior under biaxial (or triaxial) stress state is indeed first necessary to analyze the response of the material subjected to uniaxial loading.

Subsequently the attention will be focused on both failure criteria and stress-strain models available in technical literature, with special attention to the ones adopted in this work. In the field of RC constitutive modeling, indeed only the

adoption of reliable concrete stress-strain relations in conjunction with accurate failure criteria allows realistic prediction of both deformability and failure load of structures.

2.2.1 Concrete behavior under uniaxial loading

Concrete exhibits a nonlinear behavior in compression, even under uniaxial loading. The nonlinearity arises from the internal microcracking, mainly localized at the interfaces between the cement paste and the aggregates, due to the stress localization. Concrete matrix is characterized by a lower stiffness compared to that of the aggregates. Hence, even if the sample is subjected to a uniform compressive stress, locally a multiaxial and not uniform state of stress arises. As a matter of fact, eigen stresses develop, at right angle with direction of the applied load.

The stress (σ_c) - strain (ε_c) curve for a concrete sample subjected to uniaxial compression is characterized by an ascending branch, until the attainment of the uniaxial compressive strength f_c (and the related peak strain ε_{c0}), then followed by a descending branch, related to the phenomenon of concrete crushing, as can be seen in Figure 2.1. In compression, concrete is characterized by a ductile failure, with the development of many small cracks and with the entire volume degraded. The stress-strain relationship can be considered nonlinear, even starting from a low stress level, usually identified as the 30% of the peak stress f_c [81].

Since the stress-strain curve is highly nonlinear, the elastic modulus E_c cannot be considered constant; its value during loading (expressed in terms of secant or tangent stiffness) results indeed very different from its initial value (E_{ci}). Thus, the elastic modulus E_c applied in simulation for describing concrete behavior should vary during the analysis and a reliable value should be adopted to correctly represent the nonlinear relationship between stress and strain.

In the ascending branch of the stress-strain compressive curve the following stages can be identified [82,83]:

- $0 \leq \sigma_c \leq 0.3 f_c$: the microcracking existing prior to loading remains nearly unchanged, since the energy required to crack propagation is higher than the available one; thus stress and strain are approximatively proportional;
- $0.3 f_c < \sigma_c \leq 0.5 f_c$: the existing microcracking begins to increase. The internal energy is almost equal to the one released in the fracture process and the stress localizations cause the propagation of the microcracks. These latter are equally distributed within the element and the propagation process is steady: zero load increment corresponds to no new cracks;
- $0.5 f_c < \sigma_c < 0.8 f_c$: microcracks tend to connect to each other. Even if no load increasing is provided cracks continue to propagate, with decreasing velocity, until the attainment of a stable configuration;
- $\sigma_c \geq 0.8 f_c$: the internal energy is higher than the one released in the fracture process; thus, even under constant load, cracks continue to propagate with an increasing rate. The limit between the stable and unstable crack propagation is indeed generally recognized for $\sigma_c = 0.8 f_c$.

Similar considerations can be applied for the volumetric strains $\Delta V/V$ (defined as $\Delta V/V = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$). Up to $\sigma_c = 0.3 f_c$ the volumetric strain - stress relationship is almost linear, while beyond this limit value the volume reduction increases with a higher rate. When σ_c reaches about $0.8 f_c$, which coincides with the beginning of major microcracking in the element, an inflection point in the curve can be recognized: further loading causes a volume increase, resulting eventually in positive value for the volumetric strains $\Delta V/V$.

The volumetric strain - stress law is strictly related to the value assumed by the Poisson coefficient. For a constant Poisson ratio the relationship is linear; hence a constant value for the Poisson coefficient can be certainly assumed until the attainment of the elastic limit; whereas it increases a lot beyond the point of inflection of the volumetric strain - stress curve, where the phenomenon of dilatation occurs.

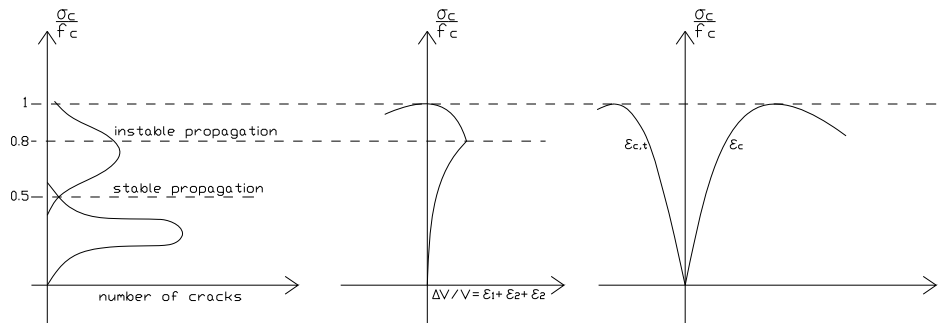


Figure 2.1 Concrete under uniaxial compression: (a) longitudinal (ε_c) and transversal ($\varepsilon_{c,t}$) strain, (b) volumetric strain ($\Delta V/V$) and (c) cracking propagation for increasing compressive stress (σ_c), adapted from [84]

Up to now, numerous analytical stress-strain relationships have been developed, among others can be mentioned the laws proposed by Hognestad in 1951 [85], Smith and Young in 1956 [86], Desayi and Krishnan in 1964 [87], Saenz [88], Sargin in 1971 [89], Popovics in 1973 [90], Wang et al. in 1978 [91], Carreira and Chu in 1985 [92], Thorenfeldt et al. in 1987 [93], Mander et al. 1988 [94], to which, for further details, reference is made.

The shape and the characteristics of the stress-strain curve, strictly related to the evolution of the microcracking process within the element, are similar in case of low, medium and high strength concrete. However, some differences arise. In more detail, the ascending part of the stress-strain curve becomes more linear and steeper the more the compressive strength of concrete increases; this phenomenon is related to the different microstructure of high strength concrete. In this case, concrete matrix is characterized by a comparable stiffness to that of aggregates, hence, the linear elastic behavior continues even for high stress-strain levels. Moreover, for high strength concretes, the slope of the curve is steeper also in descending part, compared to that of normal concretes. Thus, it

can be concluded that high strength concretes are characterized by a less ductile ultimate behavior.

As far as the Poisson coefficient is concerned, while for normal strength concrete it increases a lot beyond $\sigma_c \geq 0.8 f_c$, due to the development of the microcracking induced by the different deformability of concrete matrix and aggregates, for high strength concrete the Poisson coefficient can be considered constant up to $0.9-0.95 f_c$. This phenomenon is due to the higher mechanical uniformity of high strength concrete that provides an increase of the initial linear part of the curve, so allowing the Poisson ratio to be considered almost constant up to about the peak of the stress-strain curve.

As far as concrete in tension is concerned, its behavior is characterized by a brittle failure, with the initiation and growth of one dominant crack that causes the loss of strength perpendicular to the crack direction. The experimental analysis of concrete in tension is even more complex than under compressive loading, due to the brittleness of concrete behavior in tension; thus, various typologies of tests are nowadays applied to analyze concrete in tension.

The tensile strength f_{ct} is function of the same variables defining the compressive strength, namely the characteristics of the hydrated cement paste, of the aggregates, as well as the bond between the aggregates and the surrounding matrix. As in compression, the internal heterogeneity of the material and the internal microcracking at the interfaces between the cement paste and the aggregates, markedly influence the mechanical behavior of concrete in tension. Under a uniaxial uniform state of tension, microcracking does not significantly increase until the attainment of about $0.7f_{ct}$. Up to this stress level, the stress-strain curve is almost linear, whereas for higher loading, microcracks begin to propagate. Nevertheless, the stress-strain curve deviates little from the value calculated according to Hook law [50] and the behavior can be considered almost linear.

Also for high strength concrete the stress-strain curve in tension is almost linear, but they exhibit proportionately a less increase in the ratio between tensile and compressive strength.

2.2.2 Concrete behavior under biaxial loading

The behavior of the material, nonlinear even under uniaxial loading, becomes even more complex when a biaxial (or triaxial) state of stresses is involved. Experimental tests in this sense are quite complex and particular techniques and instrumentations must be applied to obtain reliable results and to overcome all the several issues affecting biaxial, and even more triaxial, problems.

Both strength and stiffness of concrete are closely dependent on the current state of stress: as it is known, concrete under biaxial (or triaxial) loading exhibits indeed a very different behavior than under uniaxial loading; for example, deviation from linearity becomes more pronounced when compressive stresses are involved. Moreover, the failure modes depend on the combination of stress components.

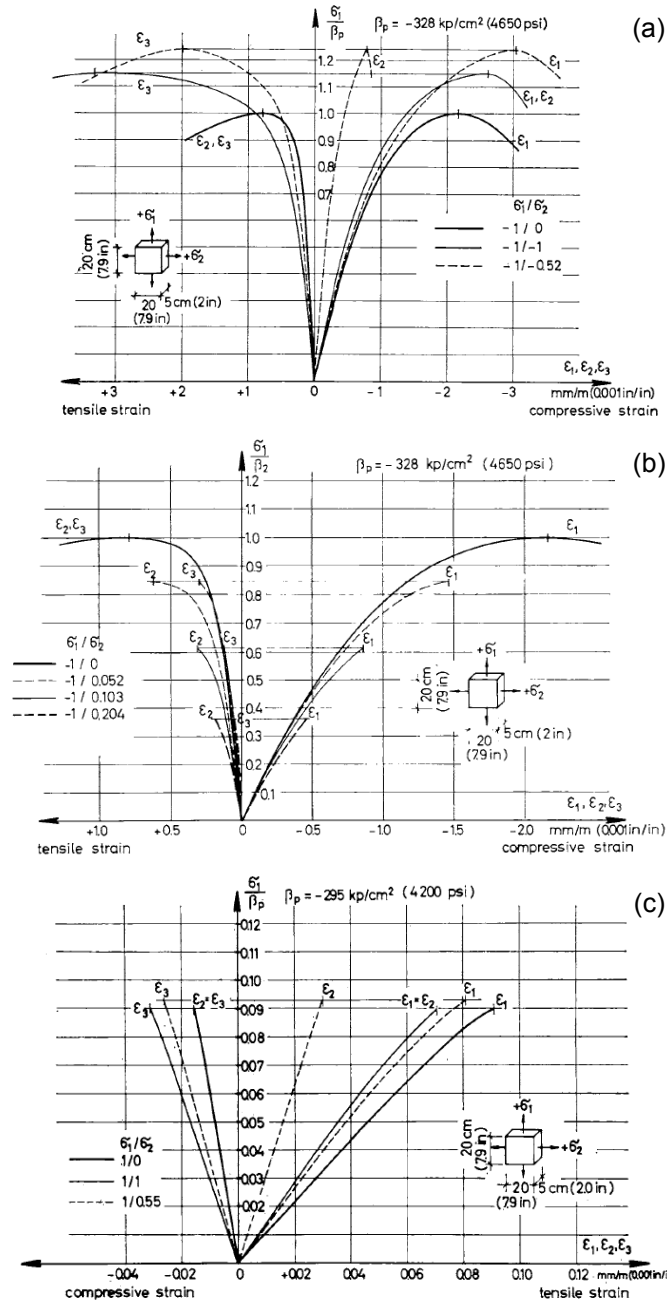


Figure 2.2 Experimental stress-strain curves for concrete under a biaxial state of stress, relative to (a) compression-compression, (b) tension-compression and (c) tension-tension region [50]

One of the most important experimental program aimed to investigate the concrete behavior under biaxial loading is the one carried out by Kupfer et al. in 1969 [50] on concrete panels. As a matter of fact, the most important failure criteria and stress-strain relationships for concrete under a biaxial stress state developed in the subsequent years were calibrated on the basis of the results provided in [50]. Some of the experimental evidences obtained by Kupfer et al. [50] are reported in Figure 2.2 for (a) compression-compression (b) tension-compression and (c) tension-tension region. Similar results were also be found in the same years by other Authors, see e.g. [95,96].

Under biaxial compression, concrete, due to the beneficial effect of confinement, increases its strength, obtaining for example an improvement of about 16% when $\sigma_{1c} = \sigma_{2c}$ and 27% for $\sigma_{1c} = 0.5 \sigma_{2c}$. When a tensile stress is involved, even small, the compressive strength in the perpendicular direction drops rapidly. On the contrary in the tension-tension region the influence of the orthogonal tension can be neglected; thus, concrete exhibits almost the same strength as under uniaxial tension.

Moreover, it can be observed that under biaxial compression, not only the ultimate strength but also the slope of the curve changes when considering different stress states. Ductility for biaxial compressive-compressive failure is higher than uniaxial compression and the higher value of the peak strain is obtained for about $\sigma_{1c} = 0.5 \sigma_{2c}$. Also in the tension-compression region, ductility is markedly influenced by the current state of stress: the value of peak strain in the direction of the compressive stress decreases the more the value of the tensile stress increases.

Moreover, also the Poisson coefficient is influenced by a biaxial state of stress, as depicted in Figure 2.3. The dilatancy is reduced in case of biaxial compression, since cracks get compressed.

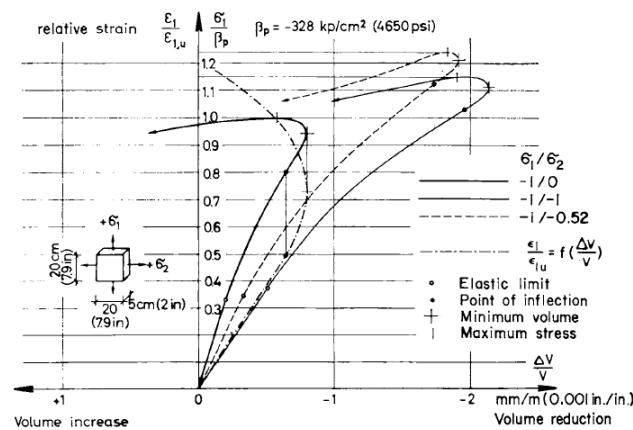


Figure 2.3 Experimental volumetric strain - stress relations for concrete under biaxial compression [50]

2.2.3 Failure criteria

To accurately simulate concrete behavior under a general biaxial state of stress, the strength of concrete should be correctly defined. Hence, before considering the stress-strain models it is convenient to investigate concrete strength. As a matter of fact only the adoption of an accurate failure criterion allows to correctly simulate concrete behavior.

As already said, the experimental results by Kupfer et al. [50], have provided a reference point for the calibration of the analytical failure criteria nowadays more utilized for the modeling of concrete contribution.

The development of an analytical formulation able to generalize experimental results into relationships for the study of structures of engineering interest has always been of primary interest. Because of concrete complex behavior, a lot of approaches for the definition of a failure criterion have been developed over the years, characterized by different complexity and accuracy. For example some of these failure criteria are of immediate applicability; whereas others require numerous experimental tests for the calibration of the model.

One of the first model applied to concrete was the one parameter Rankine criterion. The yield condition of Rankine criterion is indeed only related to maximum principal stress and a brittle failure is assumed when the maximum principal stress exceeds the tensile strength. For plane strain and axisymmetric stress situations, the Rankine yield condition is complemented with a tension cut-off criterion in the out-of-plane direction. As far as one parameter models are concerned, other well-known failure models are the Tresca and Von Mises shear stress criteria. One parameter criteria are very simple but they are nowadays rarely applied for concrete modeling due to their moderate accuracy in describing the experimental results.

The most known two parameter models applied to concrete are the Mohr-Coulomb and the Drucker-Prager criteria. The Mohr-Coulomb criterion represents the linear envelope that is obtained by combining the shear strength of the material with the applied normal stress. Nowadays it is rarely utilized in current concrete modeling partly because of the discontinuity of the yield surface that hinders numerical implementation. Moreover, comparisons with experimental tests have shown that the Mohr-Coulomb criterion only moderately fits experimental evidences. The Drucker-Prager criterion is one of the oldest criterion that represents moderately well the response of concrete subjected to biaxial compression and provides a smooth yield surface, being the first derivative continuous. However, comparisons with experimental data show that the model over-estimates the capacity of concrete subjected to compression-tension or tension-tension states of stress.

Subsequently more refined models have been developed for concrete, among others [51,72,73,97,98], which will be briefly described in the following.

The Bresler-Pister criterion [97], which represents an extension of the Drucker-Prager yield surface, uses three parameters (i.e. tensile strength and compressive strength relative to uniaxial and equally biaxial compression, respectively) and it is characterized by a parabolic dependence of the meridians.

Comparisons with experimental evidences show quite good agreement in the compression-compression region, but poor results in the tension-tension quadrant.

The Willam and Warnke [98] criterion was originally conceived as a three parameters model: the uniaxial compressive strength, the uniaxial tensile strength as well as the strength exhibited by concrete under equal biaxial compressive stresses should be chosen to define the failure surface, as for the Bresler-Pister criterion [97]. This model was later modified by the same Authors in a more general form that requires five experimental points. It includes all the stress invariants and it is periodic in the deviatoric plane with three-fold symmetry. Moreover, it has parabolic meridians with intersection in the deviatoric plane consisting of elliptical sections. It is worth noting that it can contain, for different parameter choices, different models (such von Mises, Drucker-Prager or the three parameter Willam-Warnke formulation). The Willam and Warnke five parameters model is now widely used for concrete modeling, since it is valid for all stress combinations and it gives in all the regions good agreement with experimental data.

Also the criterion proposed by Ottosen [72] is nowadays wide applied for the analysis of concrete behavior under a general state of stress. This model, which is related to all the three invariants, requires the definition of four parameters and it is general and well suited for the representation of concrete behavior. The use of invariants makes superfluous the determination of the principal stresses and the surface is convex with the only exception of the vertex. The meridians are parabolic and the trace in the deviatoric plane changes from nearly triangular to circular shape with increasing hydrostatic pressure. Acting on the parameters, this model can contain other criteria, such as von Mises or Drucker-Prager. Moreover, it well fits experimental data: analyzing for example the evidences provided in [50] well agreements can be stated, with only a slight overestimation of concrete strength in the compression-compression region.

The failure criterion developed by Men  try and Willam [73], derived from the rock mechanics, combines the traditional criterion of Rankine in the tension region with the shear resistance criterion of Mohr-Coulomb. It is characterized by three parameters: the compressive strength, the tensile strength and the eccentricity, whose definition can be found in [73], to which reference is made. This failure criterion provides a correct representation of the tensile strength (cohesive strength) of cementitious materials, as well as a sensible description of the shear friction strength. Since concrete is a composite material, the formulation of the failure criterion must indeed combine the cohesive strength of the cement paste with the adhesion by friction due to the interaction of the aggregates. The cohesive and the friction parameters are decoupled, allowing a direct control on the hardening and softening branch. The failure criterion for concrete should include on the one hand the sensitivity to the external applied pressure and on the other hand the imposed limit by the tensile strength. It is formulated according to three independent invariants of tension, known as Haigh-Westergaard co-ordinates and its geometric representation in principal stress co-ordinates is convex and regular. Moreover, it is characterized by two parabolic

meridians and a deviatoric section that varies from a triangular shape to a circular one as a function of the degree of confinement. The model is general and contains other criteria, such as Rankine, Mohr-Coulomb, Drucker-Prager, as special cases.

In the following the attention will be focused on the criterion proposed by Kufer and Gerstle [51], together with its slight modification by Barzegar and Schnobrich [99], since it represents the biaxial failure envelope adopted in this work.

2.2.3.1 Failure criterion by Kupfer and Gerstle

The biaxial failure envelope formulated by Kupfer and Gerstle in 1973 [51], (see Figure 2.4), was developed on the basis of the results of the experimental programs on concrete panels, undertaken by the same Authors. Analytical formulations are given, in order provide direct interpolations to test data, for all the three regions of the domain (i.e. compression-compression, tension-compression and tension-tension) and so subdividing the failure envelope in order identify the stress combination $(\sigma_{1max}, \sigma_{2max})$ that causes failure. These regions are identified on the basis of the ratio α of the maximum σ_{1c} and the minimum σ_{2c} principal stress, which is expressed in terms of the angle α^* .

$$\alpha = \frac{\sigma_{1c}}{\sigma_{2c}} = \tan \alpha^* . \quad (2.1)$$

where α^* represents the slope of the straight lines through the origin of the axes in the failure envelop diagram.

For biaxial compression ($0 < \alpha \leq 1$) the following equation is provided:

$$\left(\frac{\sigma_{1max}}{f_c} + \frac{\sigma_{2max}}{f_c} \right)^2 - \frac{\sigma_{2max}}{f_c} - 3.65 \frac{\sigma_{1max}}{f_c} = 0 \quad (2.2)$$

that can be rewritten to obtain the value of the stresses defining the failure surface $(\sigma_{1max}, \sigma_{2max})$:

$$\begin{aligned} \sigma_{1max} &= \alpha \sigma_{2max} \\ \sigma_{2max} &= \frac{1 + 3.65\alpha}{(1 + \alpha)^2} f_c . \end{aligned} \quad (2.3)$$

It is worth noticing that σ_{2max} in case of biaxial compression results always higher than the uniaxial compressive strength f_c , as proven by experimental evidences.

In the tension-compression region ($\alpha \leq 0$) a linear reduction in the tensile strength with increasing compressive stress is suggested, as follows:

$$\frac{\sigma_{1max}}{f_{ct}} = \left(1 - 0.8 \frac{\sigma_{2max}}{f_c} \right) . \quad (2.4)$$

This equation provides a good interpolation to experimental data but creates a discontinuity when the principal tensile stress approaches zero.

For tension-tension region ($\alpha \geq 1$) a constant tensile strength, equal to the one exhibited by concrete under uniaxial loading, is assumed:

$$\sigma_{1max} = f_{ct} . \quad (2.5)$$

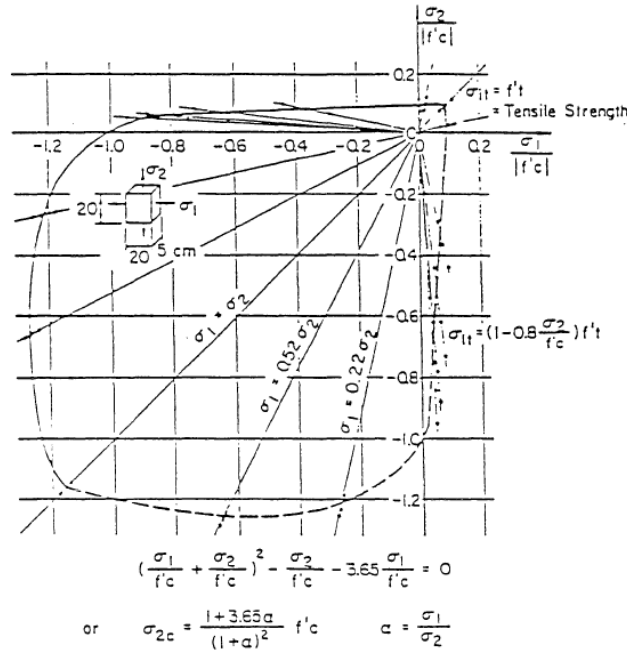


Figure 2.4 Biaxial failure envelope by Kupfer and Gerstle [51]

Modification in the tension-compression region proposed by Barzegar and Schnobrich

As already said, to overcome the discontinuity in the failure envelope when the principal tensile stress approaches zero, Barzegar and Schnobrich [99] developed an empirical law to better describe the states of stress characterized by one tension and one compression ($\alpha \leq 0$), to be used in conjunction with the failure envelope proposed by Kuper and Gerstle [51] in the compression-compression and tension-tension regions. The relationship is a function of the ratio between principal stresses α ($\alpha = \sigma_{1c}/\sigma_{2c}$) and the ratio between the uniaxial

compressive and tensile strength k ($k = f_{ct}/f_c$). The region is subdivided in two portions: when tension or compression are prevalent respectively. The transition between tension-compression and compression-tension regions (where failure respectively occurs for concrete cracking or crushing) is assumed to take place for a ratio between the principal stresses, α , equal to $0.73 f_{ct}/f_c$. Thus, the transition takes place for a principal compressive stress σ_{2c} equal to $0.75 f_c$, while the corresponding principal tensile stress σ_{1c} is varied depending on the ratio f_{ct}/f_c . As this ratio increases, the value of the tensile stress at the transition point, where the principal compressive stress is equal to $0.75 f_c$, also increases. This modification, which is consistent with the experimental data obtained by Kupfer et al. [50], allows to avoid the presence of any discontinuity at the transition point.

When tension is prevalent the following equation is provided:

$$\sigma_{2c} = \frac{f_{ct}}{\alpha - 0.6 \cdot k} \quad \text{if } \alpha \leq -0.73 k \quad (2.6)$$

whereas, when compression is prevalent:

$$\sigma_{2c} = \frac{f_c}{12.8 \cdot k} \left[9 \cdot k + \alpha + \sqrt{(9 \cdot k + \alpha)^2 - 66.56 \cdot k^2} \right] \quad \text{if } -0.73 k < \alpha \leq 0 \quad (2.7)$$

2.2.4 Stress-strain models

After having discussed the strength of concrete under a general biaxial state of stress, the related stress-strain relations will be briefly discussed.

An effective stress-strain model should describe the different phases recognizable in concrete experimental behavior, such as the linear elastic stage, the strain hardening before failure as well as the strain softening that occurs after failure. In particular, an accurate description of the post-failure response is mandatory to correctly predict the load capacity of some types of structures.

Because of concrete complex behavior, a lot of models are available in technical literature. An overview of the most important formulations, here summarized, was presented in [71].

Initially, plasticity models based on linear elastic-ideal plastic behavior applying the failure surface as yield surface were developed (among others e.g. [98,100,101]). Subsequently, a different formulation was presented in [102], which applies different stress transfer strategies when stresses exceed the failure criterion, instead of a standard flow rule. It is indeed mandatory for a model to be able to provide different post-failure behaviors. Subsequently, formulations based on hardening plasticity were proposed to consider the highly nonlinear behavior of concrete before failure, e.g. [103,104]. However, these plasticity models based on Drucker stability criterion could not consider the real response after failure, characterized by strain softening. More recently a lot of plasticity models able to well describe concrete behavior have been developed; however, they are quite

complex and, hence, because of their simplicity, the nonlinear elastic formulations are probably the most applied.

Among the several formulations to describe the concrete behavior, it is also worth of mention the endochronic model developed by Bažant and Bhat [105]. This formulation, which can be applied to general stress states, is able to provide accurate failure stresses and it takes into account dilatation and softening phenomena. However, in [106] the stability and uniqueness of the endochronic equations have been questioned. Moreover, concrete behavior is simulated by only considering the uniaxial compressive strength and this latter does not seem a reasonable approximation, since even under uniaxial compressive loading, different concrete characterized by the same uniaxial compressive strength can exhibit different stiffness and failure strain.

As regards the wide-spread nonlinear elastic models, they can be subdivided into incremental and secant formulations. Referring to the incremental nonlinear elastic approaches, anisotropic (among others, e.g. [47,107]) and isotropic formulations (among others, e.g. [108,109]) can be mentioned. The isotropic formulations neglect the stress-induced anisotropy and, since the tangential value of Young modulus and the Poisson coefficient cannot become negative and higher than 0.5 respectively, softening and dilatation are usually disregarded. Secant formulations, which can easily handle these issues, have been developed; among others, [51,81,91,110] can be mentioned. Generally speaking, one of the main problems related to the nonlinear elastic models is associated to independence, which applies in most formulations, between the stress-strain relations proposed and the failure criterion adopted, which in turn may result in a non-smooth transition from the response before and after failure. Moreover, a lot of these models have little generality, being studied for a particular type of concrete. Hence, the calibration for other types of concrete is complex, since data from biaxial and triaxial tests (in addition to evidences from uniaxial ones) are required for each concrete object of study.

In this work, a secant formulation based on nonlinear elasticity is applied. In more detail, the stress-strain relation adopted in this thesis is a 2D form by Barzegar and Schnobrich [99] of the 3D nonlinear elastic model proposed by Ottosen [70,71]. Moreover, some slight modifications, to improve the smoothness of the curves and to obtain closer correspondence with experimental data especially in the post-crushing phase, will be herein provided. Moreover, this formulation, originally conceived for plain concretes, will be also applied to SFRC elements. The adopted procedure will be described in detail in §2.3; in this section some of the main features of the model proposed by Ottosen [71] will be briefly outlined.

The model belongs to the secant formulations; in particular, the secant values of the elastic modulus and of the Poisson coefficient are properly updated during the analysis to consider the nonlinear behavior of concrete. This is performed by adopting a proper variable (the nonlinearity index) that provides a convenient measure of the current state of stress in relation to the adopted failure envelope, so as to account for the current nonlinearity of the material. The model considers the effect of all three stress invariant and it is easy to calibrate, by

requiring data from uniaxial test alone. It is quite simple and well suits numerical routines, but at the same provides very accurate results for all the stress combinations, including those where tensile stresses are present. It worth noting that any failure criterion available in technical literature can be employed in conjunction with this model, without requiring any change. Moreover, this formulation provides smooth stress-strain curves and an accurate description of concrete behavior both before failure, at failure itself and in post-crushing stage, considering also the dilatation that occurs for high compressive level. Different post-failure behaviors in compression can be simulated, by only properly changing a convenient post-peak parameter; the model is indeed very general; for example, referring to uniaxial compression, the stress-strain relation proposed by Hognestad [85], by Desayi and Krishnan [87] and by Saenz [88] can be obtained as special cases.

2.3 A different approach to model concrete contribution into 2D-PARC

As already said, the goal of the work is to improve the modeling of concrete resistant contribution into 2D-PARC model in order to enhance its computational efficiency, while maintaining the same accuracy in describing concrete behavior both before and after cracking development. As a matter of fact, the approach for the evaluation of concrete stiffness matrix $[D_c]$, detailed described in §1.3.3.2, followed in the original formulation of 2D-PARC model, complicates the general algorithm, considerably lengthening in some cases the calculational time. Moreover, the post-peak behavior of concrete in compression cannot be handled. Hence, a revision of the concrete contribution is required. Thanks to the modular structure of 2D-PARC, this operation can be performed by only changing the part of the algorithm related to concrete behavior (i.e. the evaluation of concrete stiffness matrix in the uncracked and cracked stages), while leaving unchanged the rest, so without interfering with the global structure of the model or with the other contributions. Similarly, this modified formulation can be applied for the analysis of both traditionally reinforced and fiber reinforced concrete elements, since the only difference between these two cases lies in the constitutive laws adopted for the modeling of rebar/fiber contributions, which are automatically selected among those incorporated into 2D-PARC.

Detailed explanations of the original formulation of 2D-PARC are reported in Chapter 1; in this Section, the revised formulation adopted for the evaluation of concrete resistant contribution both in the uncracked and in the cracked stage is provided as well as its implementation into the nonlinear algorithm.

The evaluation of concrete contribution is properly revised, as already said, by implementing into 2D-PARC an isotropic formulation for concrete, which represents a specialized 2D form, according to Barzegar and Schnobrich [99] of the 3D nonlinear elastic model originally proposed by Ottosen [70,71]. Among the several formulation available in technical literature, this one is selected, due to its

peculiar features. In more detail, as already mentioned in §2.2.4, it permits to accurately take into account the short-term high nonlinearity of concrete, even under multi-axial loading and when tensile stresses are present, in a very simple way by modifying a unique parameter, the so called “nonlinearity index”. The adopted constitutive law can indeed describe the concrete strain hardening up to failure and also the post-crushing phase can be modeled. Thanks to its structure, it is numerically feasible and smooth stress-strain curves are provided so ensuring good convergence. Moreover it is flexible, since it can be applied in conjunction with any failure criterion, and it is easy to calibrate, only requiring experimental data obtained from standard uniaxial tests.

The concrete stiffness matrix $[D_c]$ is then reformulated into 2D-PARC model by applying the abovementioned isotropic model: matrix $[D_c]$ can be expressed as a function of only two parameters, i.e. the secant values of the Young modulus E_c and of the Poisson coefficient ν , and can be written directly in the in the global x-y co-ordinate system in the following form:

$$[D_c] = \frac{E_c}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1 - \nu}{2} \end{bmatrix}, \quad (2.8)$$

so having a simpler structure with respect to the previous formulation implemented into 2D-PARC model. Consequently, its evaluation requires a lower computational effort, since the calculation of the two equivalent uniaxial strains is no longer necessary.

The secant values of the Young modulus E_c and of the Poisson coefficient ν , are properly modified during the analysis to account for material nonlinearity, through the introduction of the so-called nonlinearity index β , whose value is strictly related to the failure strength envelope chosen. It is worth noting that the secant values of the Young modulus E_c and of the Poisson coefficient ν are evaluated, for any combination of biaxial stresses, and both in the pre and post-crushing region, by applying always the same procedure (with slight modifications), so making the method very simple and numerically feasible. Moreover, the definition of the whole procedure requires only six input data: five determined on the basis of the results of standard compressive uniaxial tests - the initial value of the Young modulus E_{ci} and of the Poisson coefficient ν_i , the compressive strength f_c with the related peak strain ε_{c0} as well as the tensile strength f_{ct} , - and a last one, the post-peak parameter B , that must be chosen (or calibrated) to define the post-failure behavior.

In the following, the procedure required for the evaluation of the secant values of the Young modulus E_c and of the Poisson coefficient ν will be detailed outlined. A first, it is necessary to define the nonlinearity index and to choose a proper biaxial failure envelope, then the secant values of the Young modulus E_c and of the Poisson coefficient ν can be obtained for both the uncracked and the cracked stage and finally they are inserted into concrete matrix $[D_c]$.

2.3.1 Evaluation of the nonlinearity index

At first it is necessary to define the nonlinearity index β . As already said, it is a measure of the actual loading in relation to the failure envelope and is consequently related to the amount of nonlinearity in the stress-strain curves [70,71,99].

In case of biaxial compression this parameter can be evaluated as:

$$\beta = \frac{\sigma_{2c}}{\sigma_{2fin}} \quad , \quad (2.9)$$

where σ_{2c} is the most compressive principal stress and σ_{2fin} represents its corresponding value on the failure envelope, determined by keeping fixed the other principal compressive stress σ_{1c} (Figure 2.5). It is assumed $\sigma_{1c} \geq \sigma_{2c}$, being the compressive stresses negative.

Thus, $\beta < 1$, $\beta = 1$ and $\beta > 1$ respectively correspond to stress states located inside, on, and outside the considered failure curve.

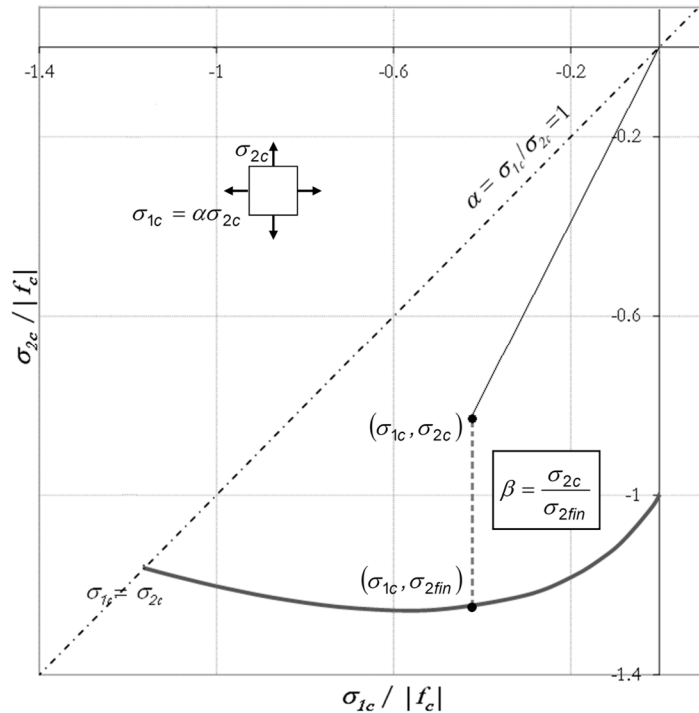


Figure 2.5 Determination of the nonlinearity index β in case of biaxial compression

When tensile stresses are present, the nonlinearity index is instead computed in terms of effective stresses. To this aim, the actual state of stress $(\sigma_{1c}, \sigma_{2c})$, where at least σ_{1c} is a tensile stress, is properly turned into an “equivalent compressive case”, by superposing a hydrostatic pressure $-\sigma_{1c}$ to the existing stress field. In this way, a new state of stress $(\sigma'_{1c}, \sigma'_{2c}) = (0, \sigma_{2c} - \sigma_{1c})$ is obtained and the nonlinearity index is evaluated as:

$$\beta = \frac{\sigma'_{2c}}{\sigma'_{2fin}} = \frac{\sigma_{2c} - \sigma_{1c}}{f_c} \quad , \quad (2.10)$$

where f_c is the uniaxial compressive strength of concrete. By following this procedure, the value of β is properly reduced when tensile stresses occur and $\beta < 1$ always holds in this case. This reduction of the nonlinearity index is consistent with the fact that the more the stress state involves tensile stresses, the more the concrete behavior becomes linear.

2.3.2 Adoption of a proper biaxial strength envelope

The introduction of the nonlinearity index β , which relates the actual state of stress to the failure surface, allows applying this constitutive model without changes in connection with any failure criterion. Consequently, it is precisely the choice of an accurate failure criterion by itself that assures the prediction of realistic stress-strain curves.

On this point, it can be observed that the same envelope already adopted in the original formulation of 2D-PARC (see §1.3.3.2) and based on the work of Kupfer and Gerste [51] is applied. However, its analytical expression is herein modified, in the region corresponding to tension-compression as already proposed by Barzegar and Schnobrich et al. [99], in order to avoid any discontinuity involved when the principal tensile stress approaches zero. As already mentioned in §2.2.3.1, the transition between tension-compression and compression-tension regions (where failure respectively occurs for concrete cracking or crushing) is assumed to take place for a ratio between the principal stresses α equal to $0.73 f_{ct}/f_c$, being f_c and f_{ct} the uniaxial compressive and tensile strength of concrete, respectively. Thus, the transition takes place for a principal compressive stress σ_{2c} equal to $0.75 f_c$, while the corresponding principal tensile stress σ_{1c} is varied depending on the ratio f_{ct}/f_c . As this ratio increases the value of the tensile stress at the transition point, where the principal compressive stress is equal to $0.75 f_c$, also increases. This modification, which is consistent with the experimental data obtained by Kupfer et al. [50], allows to avoid the presence of any discontinuity at the transition point.

The adopted failure envelope, as well as the analytical expressions describing each considered region (tension-tension, tension-compression and compression-compression) are shown in Figure 2.6; the bold line indicates the part of the curve effectively implemented into the revised 2D-PARC model, according to its conventions (that is $\sigma_{1c} \geq \sigma_{2c}$).

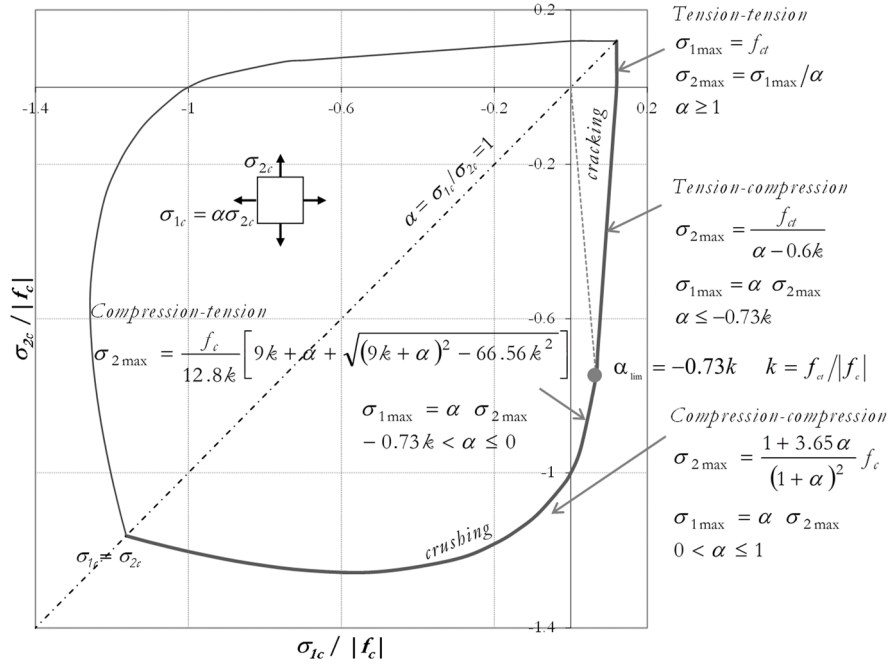


Figure 2.6 Adopted failure envelope

2.3.3 Evaluation of the secant value of the Young modulus in the uncracked stage

Concrete secant elastic modulus E_c , which accounts for biaxial state of stresses, is obtained by applying the relation proposed by Ottosen [70,71].

At first, the case of uniaxial compressive loading is analyzed by approximating the stress-strain relation suggested by Sargin [89]:

$$-\frac{\sigma}{\sigma_c} = \frac{-A \cdot \frac{\varepsilon}{\varepsilon_c} + (B-1) \cdot \left(\frac{\varepsilon}{\varepsilon_c} \right)^2}{1 - (A-2) \cdot \frac{\varepsilon}{\varepsilon_c} + B \cdot \left(\frac{\varepsilon}{\varepsilon_c} \right)^2}, \quad (2.11)$$

where A is a parameter related to the type of concrete, defined as $A = E_{ci}/E_{cf}$. Moreover, E_{ci} represents the initial value of concrete Young modulus, whereas E_{cf} , defined as $E_{cf} = f_c/\varepsilon_{c0}$, (being ε_{c0} the strain value corresponding to peak stress f_c), is the secant value at failure. B is a compressive post-peak nonlinearity parameter that determines the degree of strain softening when concrete crushing

occurs; the adopted values together with further speculations will be reported in §2.3.3.1.

By inverting Equation (2.11) and generalizing it to a biaxial state of stress, by substituting the ratio σ/σ_c with the nonlinearity index β , the expression of the concrete secant elastic modulus E_c can be written as:

$$E_c = \frac{E_{ci}}{2} - \beta \left(\frac{E_{ci}}{2} - E'_{cf} \right) + \sqrt{\left[\frac{E_{ci}}{2} - \beta \left(\frac{E_{ci}}{2} - E'_{cf} \right) \right]^2 + E'^2_{cf} \beta [B(1-\beta) - 1]} \quad (2.12)$$

where E'_{cf} represents the secant modulus corresponding to peak stress. When a tensile stress is present, E'_{cf} is simply evaluated as in case of uniaxial compression, i.e. $E'_{cf} = E_{cf} = f_{cl}/\varepsilon_{c0}$. For biaxial compression the following relation is instead adopted:

$$E'_{cf} = \frac{E_{cf}}{1 + 4(A-1)x} \quad , \quad (2.13)$$

where term x takes into account the dependence on the actual loading and can be evaluated as:

$$x = \left(\frac{\sqrt{J_2}}{f_c} \right)_f - \frac{1}{\sqrt{3}} \quad . \quad (2.14)$$

The first addend of Equation (2.14), $\left(\sqrt{J_2}/f_c \right)_f$, represents the failure value of the invariant $\sqrt{J_2}/f_c$, where J_2 is expressed in the following form:

$$J_2 = \frac{1}{3} \left(\sigma_{1c}^2 + \sigma_{2fin}^2 - \sigma_{1c} \sigma_{2fin} \right) \quad . \quad (2.15)$$

Hence, $1/\sqrt{3}$ is the value of $\left(\sqrt{J_2}/f_c \right)_f$ for uniaxial compressive loading. It is worth noting that $x > 0$ always holds, because Equation (2.13) is only used for compressive stress states (so restituting $E'_{cf} > E_{cf}$), and $x = 0$ holds for uniaxial loading (with $E'_{cf} = E_{cf}$).

By applying Equation (2.12) reliable stress-strain curves for a general biaxial state of stress are gained; so providing the exact initial slope and a zero slope at failure. The correct failure stresses relative to the actual state of stress are obtained by employing the nonlinearity index β , related to the strength envelope chosen. It is worth noting that this formulation describes accurately also stress states that involve tensile stresses, so catching the more linear behavior exhibits by concrete in this cases, by applying the proper expression for the nonlinearity index as reported in Equation (2.10).

Finally, it should be observed that all the above described procedure is applied until either cracking or crushing occurs. Cracking takes place when the current stress state reaches the failure envelope in the cracking region, i.e. $\sigma_{1c} \geq \sigma_{1max}$ in tension-compression or simply $\sigma_{1c} \geq f_{ct}$ in tension-tension region, and it is handled separately according to §2.3.5. On the contrary, crushing occurs when the stress state reaches the failure envelope in the biaxial compression or in the compression-tension (with $\alpha > \alpha_{lim}$, see Figure 2.6) regions and it is handled in the following subsection.

2.3.3.1 Proper modifications in case of crushing

Differently from the previous formulation adopted in 2D-PARC for modeling concrete behavior (see §1.3.3.2), the revised approach allows to take into account also the post-crushing regime during the analysis. In more detail, the procedure suggested in [70,71,99] is followed.

In case of uniaxial or biaxial compression, crushing occurs when $\sigma_{2c} \leq \sigma_{2max}$, with a corresponding value of the nonlinearity index β equal or slightly greater than 1. The post-peak behavior is then governed by the following Equation:

$$E_c = \frac{E_{ci}}{2} - \beta \left(\frac{E_{ci}}{2} - E'_{cf} \right) - \sqrt{\left[\frac{E_{ci}}{2} - \beta \left(\frac{E_{ci}}{2} - E'_{cf} \right) \right]^2 + E'_{cf}{}^2 \beta [B(1-\beta) - 1]}, \quad (2.16)$$

which is formally identical to Equation (2.12), but contains a negative sign before the term under square root.

A proper choice of the post-peak nonlinearity parameter B in Equation (2.16) allows to simulate different post-failure behaviors, while it only very slightly affects the pre-peak response. The calibration of B based on experimental data is quite difficult, since the precise form of the post-peak branch of the stress-strain curves is often not known. Hence, the choice of B is done so to obtain convenient stress-strain curves characterized by increasing values (without inflexion points) until the attainment of the peak and characterized by then decreasing values (with at most one inflexion point), reaching zero values for adequately high strains. To meet these requirements the following limitations are assumed for B , as suggested in [70,71]:

$$\begin{aligned} \left(1 - \frac{A}{2}\right)^2 < B \leq 1 + A \cdot (A - 2) \quad \text{for } A \leq 2 \quad \left(1 - \frac{A}{2}\right)^2 < B \leq 1 + A \cdot (A - 2) \\ 0 < B \leq 1 \quad \text{for } A > 2 \end{aligned} \quad (2.17)$$

where $A \leq 4/3$, being this latter parameter defined as $A = E_{ci}/E_{cf}$. It is worth noting that the aforementioned limitation on the value of A is significant only for high strength concrete.

By varying the post-peak parameter different stress strain curves can be obtained, as for example the Sargin law when $B=0$. It is worth noting that the

range of variation for parameter B that can be found in [70,71,99] is referred to ordinary concrete while no information is provided for fiber reinforced concrete. In this case, parameter B should be calibrated so as to take into account the effect of fibers, which stiffen the material response in compression. To this aim, a possible procedure (that is followed in this work) is that of calibrating parameter B on the basis of the post-peak compressive response obtained from tests on SFRC cylinders. A best-fitting technique can be undertaken and the model can be optimized by minimizing the scatter between experimental results and numerical analyses. When experimental data are not available (or in case of predictive analyses), parameter B can be chosen so to obtain a post-peak branch almost superimposed to that provided by one of the general analytical uniaxial compressive stress-strain relations suggested in current Design Codes or available in technical literature.

Obviously, the suggested procedure can be adopted also in case of ordinary RC elements, by simply verifying that the value of B obtained from numerical calibration is included in the range of variation reported in [70,71], which mainly depends on concrete type and strength.

It should be pointed out that Equation (2.16) is still valid also in the region corresponding to biaxial compression-tension. In this case, crushing still occurs when $\sigma_{2c} \leq \sigma_{2max}$, but the nonlinearity index β remains lower than unity. According to Figure 2.7, the post-peak curve AB is obtained by translating part MN of the original descending branch, parallel to the horizontal axis. Concrete secant elastic modulus E_c is then consequently evaluated as suggested in [70,71]:

$$E_c = \frac{\beta E_{MN} E_A E_M}{\beta E_A E_M + \beta_f E_{MN} (E_M - E_A)} \quad (2.18)$$

where E_{MN} represents the secant value along the original post-peak curve. In more detail, it depends on the current value of the nonlinearity index β and it is determined through Equation (2.16). Moreover, β_f represents the value of nonlinearity index at failure, while E_A and E_M are the secant values corresponding to β_f (see Figure 2.7) in the pre-peak and in the original post-peak branches of the curve, calculated according to Equations (2.12) and (2.16), respectively.

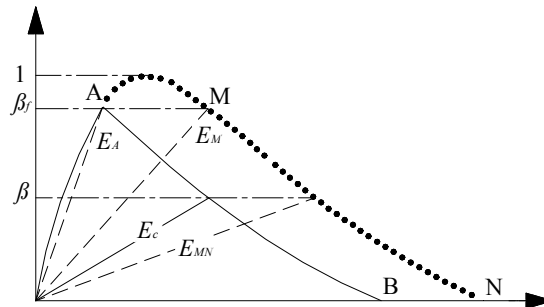


Figure 2.7 Post-peak curve in compression-tension region, adapted from [70]

2.3.4 Evaluation of the secant value of the Poisson coefficient in the uncracked stage

It has been experimentally observed (see e.g. [111]) that, in presence of compressive stresses, concrete tends first to compact and subsequently it tends to expand after the appearance of micro-cracks. To reproduce this behavior, the Poisson ν coefficient is expressed as a function of the nonlinearity index β and properly adjusted during the analysis, according to [70,71,99]. In more detail, ν is kept fixed until β reaches a limit value β_a equal to 0.8, and afterwards it is updated by applying the following relation, representing a quarter of an ellipse:

$$\nu = \nu_f - (\nu_f - \nu_i) \sqrt{1 - \left(\frac{\beta - \beta_a}{1 - \beta_a} \right)^2}, \quad (2.19)$$

where ν_i indicates the initial value of Poisson coefficient (assumed equal to 0.2), while ν_f represents its secant value at peak (approximately equal to 0.36).

Also in this case Equation (2.19) suits the concrete behavior up to cracking or crushing.

2.3.4.1 Proper modifications in case of crushing

In the post-crushing regime only little information are available in the literature relatively to the amount of the increase of Poisson coefficient ν ; however, experimental evidences highlight that the phenomenon of expansion continues also in this region. In order to take into account this aspect, Ottosen [70,71] and Barzegar and Schnobrich [99] suggest to decrease the secant elastic modulus E_c by steps of 5% in the post-crushing region. Dilatation is then ensured by simply posing $\nu = 1.005 \nu^*$, being ν^* the secant value of Poisson coefficient associated to E_c , so that the corresponding secant bulk modulus $K_f = E_c / 3(1 - 2\nu^*)$ remains unchanged. However, in this work a different procedure is proposed, which allows obtaining more regular stress-strain curves in the post-peak region. To this aim, the Poisson coefficient is varied along the second quarter of the same ellipse of Equation (2.19), with vertical axis $\nu = \nu_f$, by using the following relation:

$$\nu = \nu_f + (\nu_f - \nu_i) \sqrt{1 - \left(\frac{\beta - \beta_a}{1 - \beta_a} \right)^2}, \quad (2.20)$$

which is formally identical to Equation (2.19), but has a positive sign between the two addends. When $\beta \leq \beta_a$ a fixed value ν should be assumed, equal to $2\nu_f - \nu_i$. However, this represents only a theoretical case, since an upper bound $\nu \leq 0.45$ is set to eliminate problems associated with Poisson ratio ν approaching to 0.5 in accordance with the finding of Huang [112].

For the biaxial compression-tension region in this work the following procedure is proposed for the evaluation of Poisson coefficient ν . With reference to Figure 2.8 the post-peak curve AB is obtained by translating part MN of the original descending branch parallel to the horizontal axis, so obtaining:

$$\begin{aligned} \nu &= \nu_{MN} - (\nu_M - \nu_A) & \text{for } \beta > \beta_a \\ \nu &= \nu_f + (\nu_f - \nu_i) - (\nu_M - \nu_A) & \text{for } \beta \leq \beta_a \end{aligned} \quad (2.21)$$

ν_{MN} represents the secant value along the original post-peak curve; it depends on the current value of the nonlinearity index β and it is determined through Equation (2.20); while ν_A and ν_M are the secant values corresponding to β_f (the nonlinearity index at failure), in the pre-peak and in the original post-peak branches of the curve, calculated according to Equations (2.19) and (2.20), respectively.

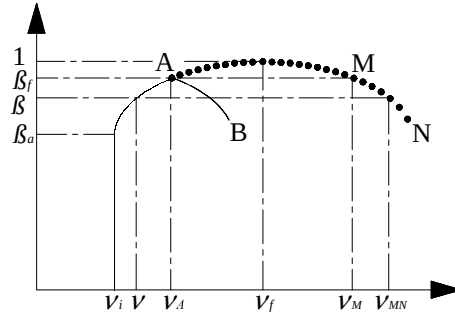


Figure 2.8 Variation of the secant value of Poisson ratio ν in compression-tension region (prevalent compression)

2.3.5 Adaptation of the formulation to the cracked stage

In case of cracking, the fixed-smeared crack approach is applied (see Chapter 1 for further details). Thanks to the abovementioned modular structure of 2D-PARC model, the modeling of concrete contribution can be easily revised also in the cracked stage. In more detail, only the modeling of concrete contribution referred to intact (even if damaged) material between two adjacent cracks is properly modified, while leaving unchanged the rest of the algorithm. In particular the contribution related to steel and to all the mechanisms of the fracture zone are evaluated as in the original formulation, see §1.3.4 for further details.

Therefore, the same formulation for concrete modeling described in §2.3.1-2.3.4, with slight modifications, is applied for the evaluation of the stiffness matrix of concrete between cracks, $[D_c]$ in the cracked stage. These adjustments are required to include concrete damaging due to the presence of cracks; a proper reduction in concrete compressive strength and stiffness is operated.

To this aim, the biaxial strength envelope shown in Figure 2.6 is reduced according to the relation suggested in [113], see also Figure 2.9, by adopting the following relation:

$$f_c^* = f_c \left[1 - 0.2 \left(\frac{\varepsilon_{max}}{\varepsilon_{c0}} \right) \right], \quad f_c \geq f_c^* \geq 0.8f_c \quad (2.22)$$

where ε_{max} is the current maximum tensile strain in cracked concrete and f_c^* is the modified value of the uniaxial compressive strength at peak.

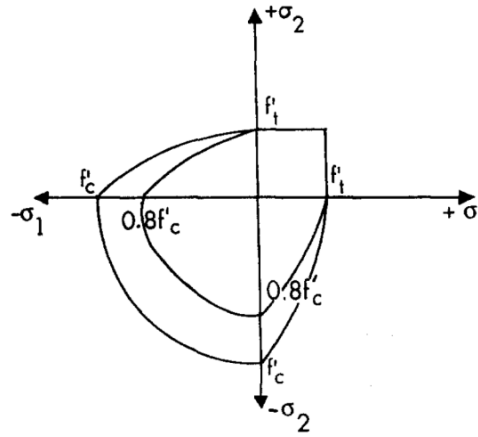


Figure 2.9 Modified failure envelope in the cracked stage [113]

The corresponding modified peak strain ε_{co}^* , necessary for the evaluation of the Young modulus E_c – Equation (2.12), can be in turn calculated as:

$$\varepsilon_{co}^* = \varepsilon_{co} \left[1 + 0.1 \left(\frac{\varepsilon_{max}}{\varepsilon_{co}} \right) \right], \quad (2.23)$$

as a function of the initial value of the strain ε_{c0} corresponding to peak stress in uniaxial compression.

The value of secant elastic modulus E_c and the Poisson coefficient ν relative to the current state of stress are then inserted in matrix $[D_c]$, relative to the intact, even if damaged, concrete between two adjacent cracks. Finally the global cracked stiffness matrix $[D]$ can be found by combining the matrices relative to the uncracked reinforced concrete ($[D_c]$ and $[D_s]$) with the contributions of the fracture zone (namely matrix $[D_{cr}]$), see §1.3.4 for further details.

2.3.6 Implementation of revised concrete model into 2D-PARC

2D-PARC model is implemented into a commercial FE code (ABAQUS) by means of a “user material” subroutine written in FORTRAN language. In correspondence of each loading increment (or iteration within a fixed loading increment), the global updated strain and stress vectors are provided by ABAQUS and read by the user subroutine. Starting from these strains, the stiffness matrix and the total stress vector are evaluated in the subroutine according to the current condition (uncracked or cracked) of each integration point in the FE model, and then passed back to ABAQUS. For further details see §1.3.5.

2D-PARC user subroutine is in turn structured so as to recall different routines that separately evaluate the involved resistant mechanisms. In this way, the implementation of the revised concrete constitutive law simply requires the modification of a single part of the program, by leaving unchanged the rest of the algorithm.

For sake of brevity, the adopted procedure is here briefly illustrated with reference to the uncracked stage only. This latter is managed through a proper subroutine recalled by the main program and named “UNCRACKED STAGE”, whose flow chart is reported in Figure 2.10. The goal of this subroutine is to provide uncracked stiffness matrix $[D]$, that is in turned obtained by summing up the concrete $[D_c]$ and the steel $[D_s]$ stiffness matrices.

In the following the attention will be focused on the evaluation of the concrete matrix and therefore on the determination of the secant values of the Young modulus E_c and the Poisson coefficient ν .

The actual state of stress in concrete $\{\sigma_{cxy}\}$ is evaluated by subtracting from the total stress vector $\{\sigma_{xy}\}$ the amount of stress absorbed by reinforcement $\{\sigma_{sxy}\}$. The total stress vector $\{\sigma_{xy}\}$, which is passed as an input from the main program, is evaluated as the stress provided by ABAQUS; hence, referring to the previous increment. This approximation is not significant at all, since in the following a convergence on stress is performed to obtain the correct value of the elastic modulus E_c and the Poisson coefficient ν . The steel stress vector $\{\sigma_{sxy}\}$ is evaluated as the product between steel stiffness matrix and strains; these latter are set equal to the current global strains provided by ABAQUS, according to the hypothesis of perfect bond, that applies in the uncracked stage.

Subsequently, the subroutine computes the corresponding principal strains and stresses in concrete, which are in turn passed to the subroutine “CONCRETE STIFFNESS” for the determination of the secant values of concrete elastic modulus E_c and Poisson coefficient ν , where the trial and error procedure described in the following is applied. In this work, subroutine “CONCRETE STIFFNESS” is indeed properly modified with respect to the original formulation of 2D-PARC so as to incorporate the revised concrete modeling, according to the flow chart reported in Figure 2.11. A similar scheme is also followed in the cracked stage.

Once the correct values of E_c and ν are obtained; they are incorporated into the concrete stiffness matrix $[D_c]$, Equation (2.8). The uncracked RC stiffness matrix in global co-ordinate system (x,y) is then evaluated by simply summing up concrete stiffness matrix with that relative to steel reinforcement $[D_s]$.

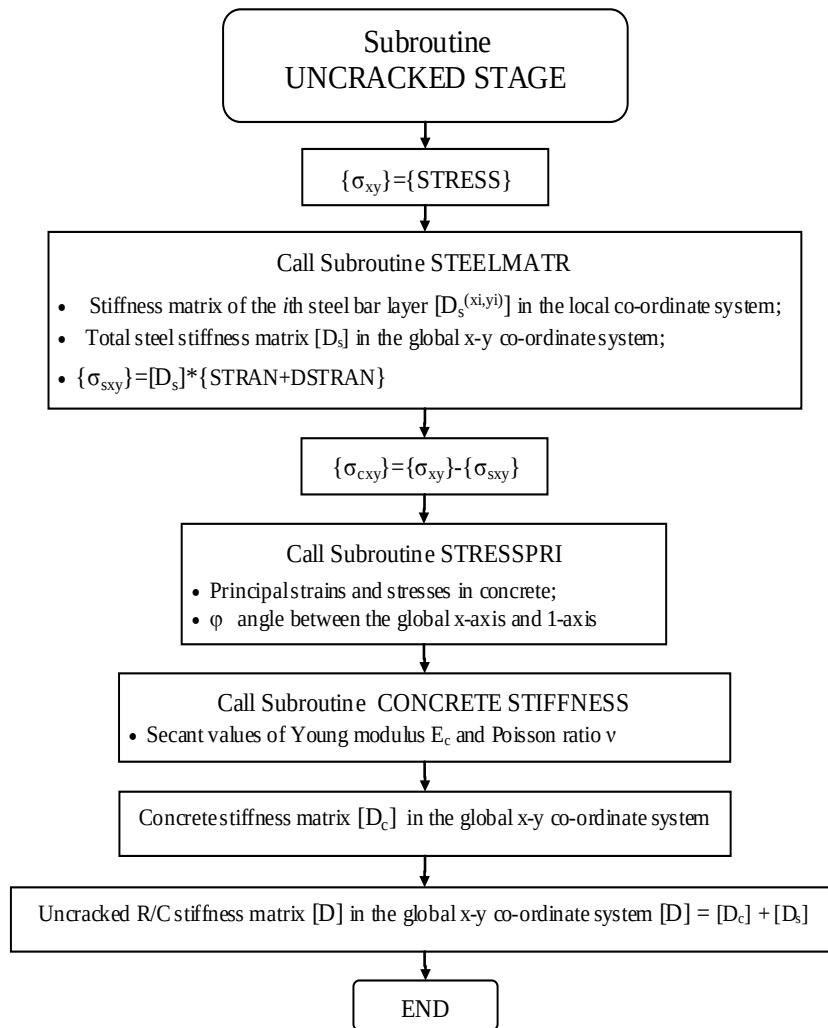


Figure 2.10 Flow chart relative to uncracked stage modeling

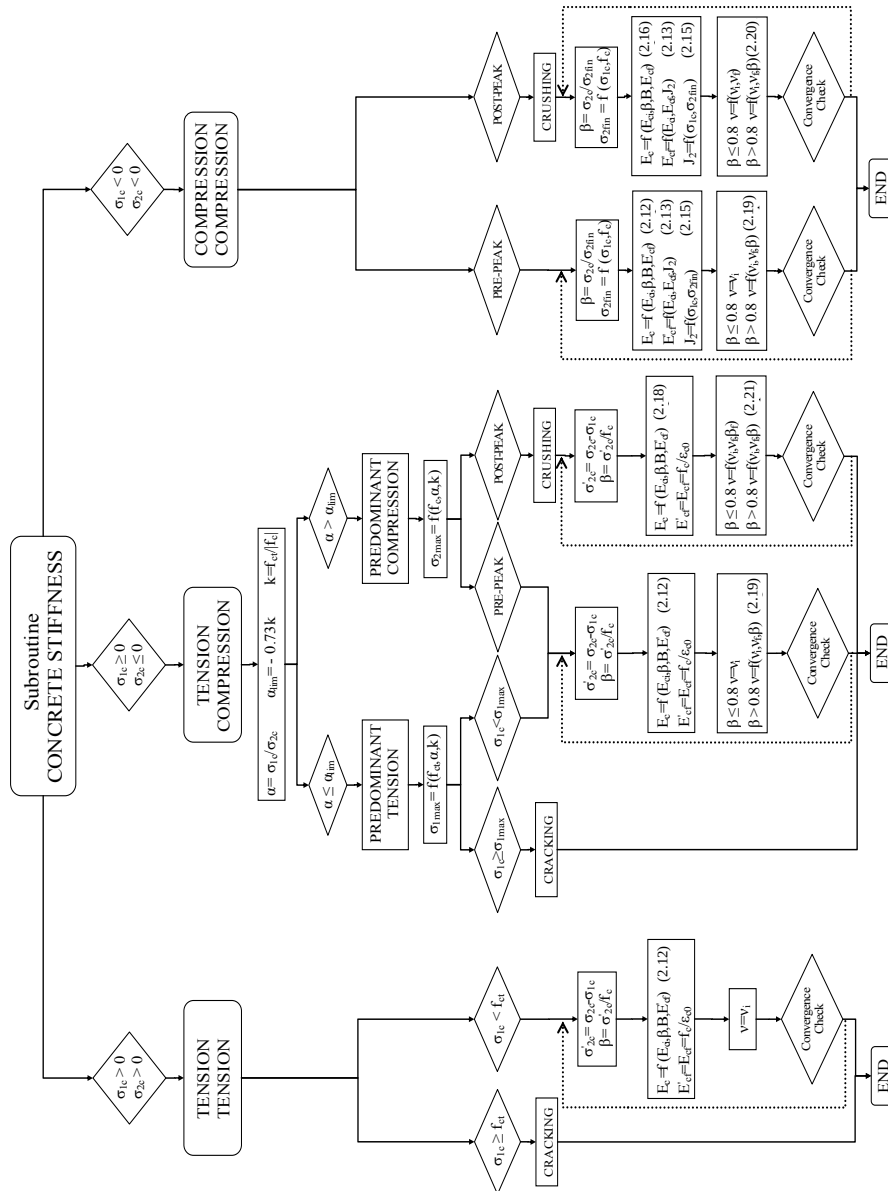


Figure 2.11 Flow chart relative to evaluation of concrete secant elastic values E_c and ν_c .

2.3.6.1 Convergence of stress-strain relation for concrete for the evaluation of the Young modulus and the Poisson coefficient

To obtain the secant value of the Young modulus E_c and of the Poisson coefficient ν related to the current state of stress an iterative procedure is required.

At first the original convergence method suggested by Ottosen [71] and the more refined by Barzegar and Schnobrich [99] have been implemented. According to this latter, starting from the calculated stress and by applying the constitutive concrete model, a new Young modulus is found. Then a convergence on the elastic modulus is performed: when the difference between this latter and the Young modulus related to the calculated stresses is less of a predefined tolerance, the stress-strain curve is assumed to be converged; oppositely, the Young modulus is updated on the basis of both its value at the previous iterations and the tolerance. A similar procedure is applied for ν . These methods are not very convenient, thus a new convergence method is proposed in this thesis, that has proven to more precise, more stable and faster than the ones suggested by Ottosen [71] and by Barzegar and Schnobrich [99], especially approaching failure and in the post-crushing stage.

In this work, for each integration point, convergence is performed on stress, by applying the bisection method. The same results have been however obtained by applying the secant method, omitted for sake of brevity.

Starting from the strain ε_{ic} provided by "UNCRACKED STAGE" subroutine, σ_{ic} is updated in order to get a converged solution; a predefined stress interval is progressively narrowed until the following condition is achieved:

$$\sigma_{ic} - \frac{E_c}{1-\nu^2} (\nu \varepsilon_{jc} + \varepsilon_{ic}) = tol \quad , \quad (2.24)$$

where subscript i and j are respectively equal to 1 and 2 in case of biaxial tension and conversely to 2 and 1 in case of tension-compression or biaxial compression and tol represents the chosen tolerance value, adequately small. Actually the convergence check can be always performed indistinctly on both principal stresses, owing to the isotropic nature of the adopted constitutive model for concrete - see (2.8); it can be indeed posed $E_1 = E_2 = E_c$ and $\nu_1 = \nu_2 = \nu$. The value of the Young modulus E_c and the Poisson coefficient ν are calculated by applying the adopted constitutive model (see Figure 2.11) on the basis of the stress attempt value.

When the condition reported in Equation (2.24) is satisfied, convergence is assumed to be reached on stresses. Consequently, the corresponding elastic parameters E_c and ν represent the updated values that can be used for the construction of concrete stiffness matrix $[D_c]$ of the analyzed integration point.

In case of compression-compression or compression-tension (when compression is prevalent), the evaluation of Equation (2.24) requires to preliminarily identify if the current strain state belongs to pre- or post-peak response, since different expressions should be used for determining E_c and ν .

This operation is performed herein by comparing the current minimum principal strain ε_{2c} at each integration point with the corresponding peak strain value. This latter depends on the maximum (compressive) stress $\sigma_{2\max}$ obtained from the failure envelope of Figure 2.6 on the basis of the acting stress state (in terms of $\alpha = \sigma_{1c} / \sigma_{2c}$) and on the relative secant Young modulus (at peak) evaluated by applying the concrete constitutive relation.

2.4 Comparison between numerical and experimental results

The effectiveness of the proposed procedure and its correct implementation are verified through comparisons with experimental data relative to different structural typologies.

Experimental programs available in technical literature on elements whose structural behavior is mainly governed by concrete performances are selected. For some elements, indeed, such as specimens made of plain concrete or reinforced in a single direction, the adoption of an effective nonlinear stress-strain law for concrete in conjunction with an accurate failure criterion is essential in order to correctly simulate the global and local behavior. As a matter of fact the behavior of concrete under a general multi-axial state of stress cannot be reproduced into a material model by directly using uniaxial stress-strain curves or a simple linear-elastic matrix; but an accurate concrete constitutive model is mandatory for a realistic prediction of the element stiffness, strength and ductility.

First, the attention is focused on the simulation of plain concrete panels tested by Kupfer et al. [50], subjected to different stress states. These tests can be seen as a sort of benchmark case study, which allows to easily check the effectiveness of the adopted model in correctly describing concrete behavior under different loading combinations. Moreover, by modeling the abovementioned concrete panels [50], the additional acquired ability of 2D-PARC model in representing the post-peak behavior of concrete in compression can be assessed, as well as its effectiveness in describing dilatation that occurs for high compressive stresses.

Subsequently, are analyzed RC beams without shear reinforcement, since their numerical response is strongly affected by the effectiveness of the adopted concrete modeling. As known, these elements represent a quite difficult challenge in modeling, especially when a two-dimensional membrane analysis is adopted, with many finite element formulations failing in providing accurate results. The accuracy of the proposed model in describing the global behavior up to failure as well as crack pattern evolution is verified against the results of two experimental programs, available in the technical literature ([74,75]). Among several research projects, the one carried out by Vecchio and Shim [74] is selected owing to its effectiveness, being a duplicate of the classical Bresler and Scordelis beam tests [114]. The choice of the experimental program undertaken by Podgorniak-Stanik [75] is instead related to the availability of several

experimental data monitored during test execution, mainly concerning the crack pattern evolution with increasing loads.

Finally also the SFRC beams without shear reinforcement tested by Cucchiara et al. [76] are modeled, so as to prove the validity of the proposed procedure in case fibrous reinforcement.

2.4.1 Concrete panels subjected to biaxial stresses

Plain concrete panels tested by Kupfer et al. [50] had dimensions equal to 200 x 200 x 50 mm, whereas concrete mechanical properties adopted for the analysis are summarized in Table 2.1. As can be observed, even if panels were subjected to a biaxial state of stress, all the parameters to be inserted in the model simply derive from standard uniaxial tests, except for B , whose value is chosen equal to 0.1 so to obtain a realistic post-crushing behavior.

By following the same procedure suggested in §2.3.3.1 for SFRC, parameter B is chosen so to fit experimental post-peak curve in uniaxial compression, verifying at the same time that the obtained value is included in the range suggested in [70,71,99] for ordinary concrete so as to obtain convenient and numerically feasible curves.

Sample	f_c [MPa]	f_{ct} [MPa]	E_{ci} [MPa]	ν_i (-)	ε_{c0} (-)	B (-)
Compression–Compression	32.8	3.0	32700	0.20	0.0022	0.1
Compression–Tension	32.8	3.0	32700	0.20	0.0022	0.1
Tension–Tension	29.5	2.7	32000	0.18	0.0022	0.1

Table 2.1 Concrete mechanical parameters adopted for the simulation of tests on panels

Figure 2.12 shows a comparison between numerical and experimental results for uniaxial and biaxial tensile loadings. As expected, concrete exhibits a nearly linear response; in fact, the more the stress state involves tensile stresses, the more concrete behavior becomes less nonlinear. This is consistent with the reduction of the nonlinearity index β in the model as tensile stresses increase. In these cases, concrete behavior is governed by cracking and consequently a completely brittle failure takes place.

The stress-strain curves for the region of combined compression and tension are reported in Figure 2.13. As can be seen, failure stresses and strains increase in magnitude as the tensile stress decreases. Based on the limit value α_{lim} dividing the cracking and crushing regions in the adopted failure envelope (see Figure 2.6), the curves reported in Figure 2.13a-b can be referred to the case of predominant tension and are indeed characterized by a brittle failure. When a smaller tensile stress is instead applied, predominant compression takes place and crushing of concrete occurs (see Figure 2.13c).

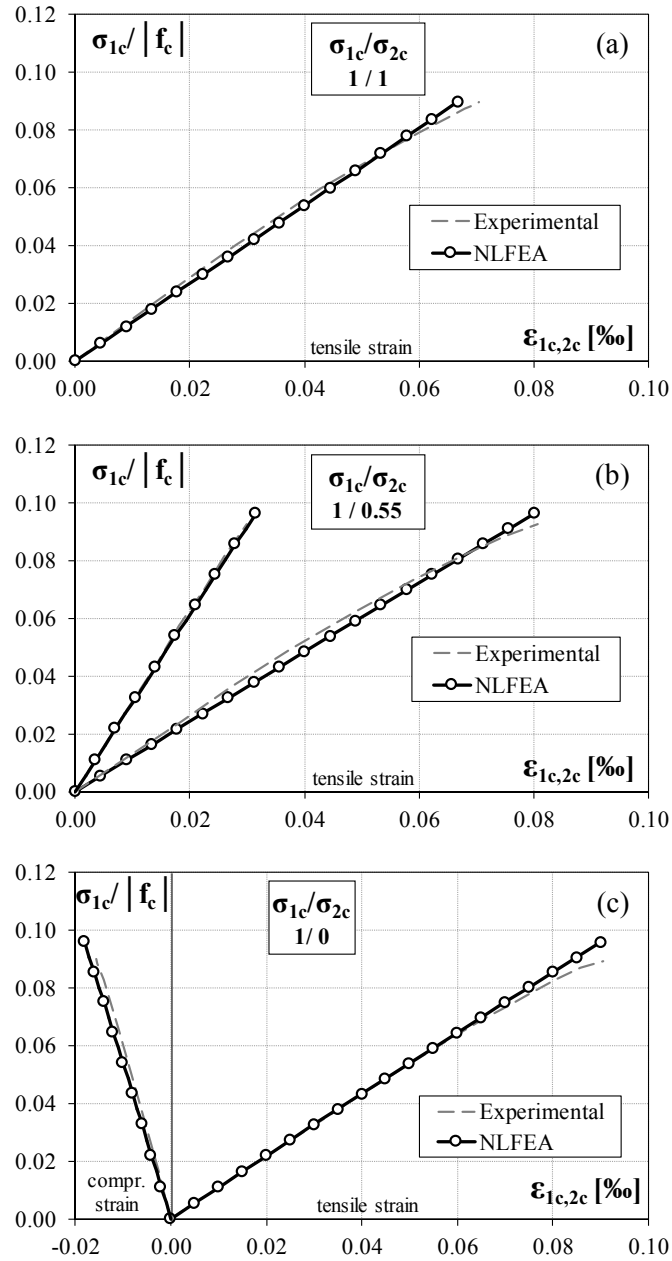


Figure 2.12 Comparison between numerical and experimental [50] results for concrete panels subjected to: (a)-(b) biaxial and (c) uniaxial tension

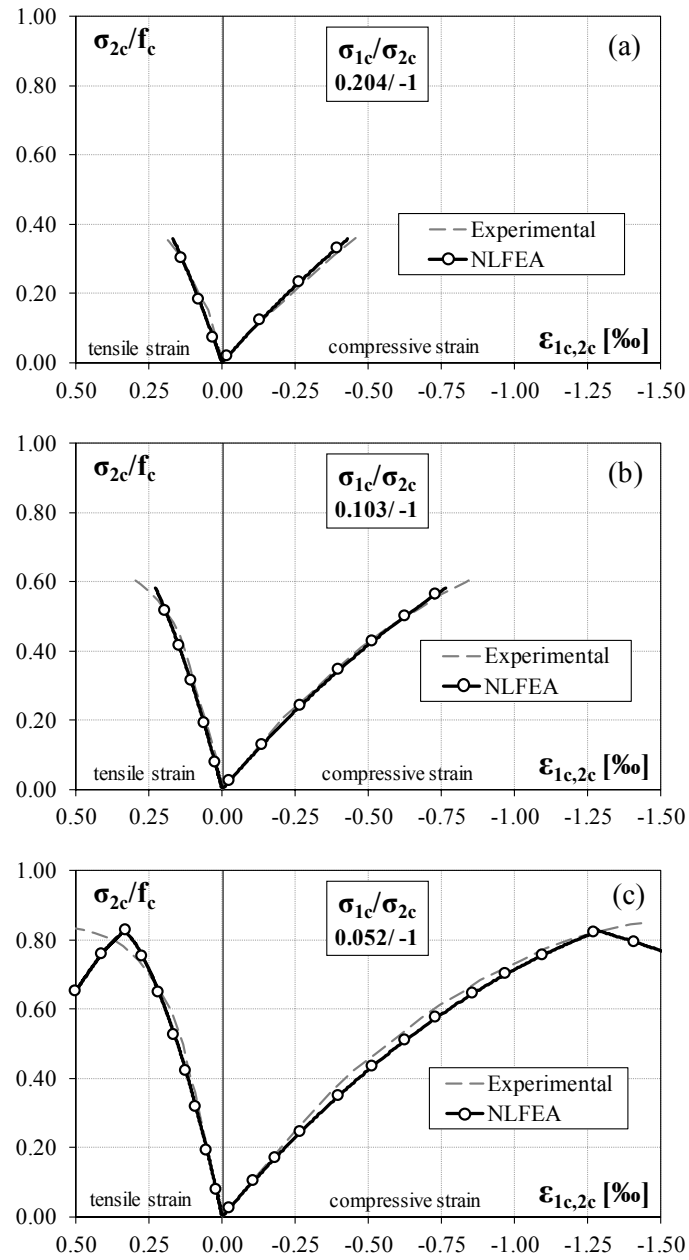


Figure 2.13 Comparison between numerical and experimental [50] results for concrete panels subjected to: (a)-(b) tension-compression and (c) compression-tension

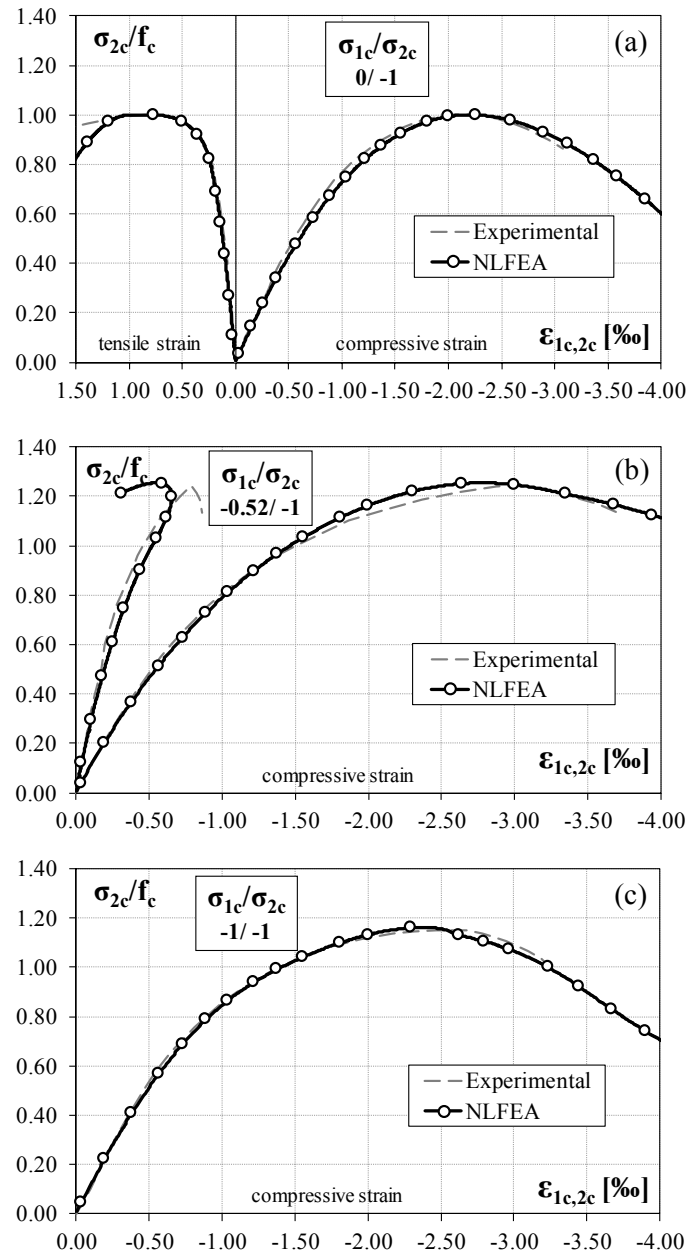


Figure 2.14 Comparison between numerical and experimental [50] results for concrete panels subjected to: (a) uniaxial and (b)-(c) biaxial tension

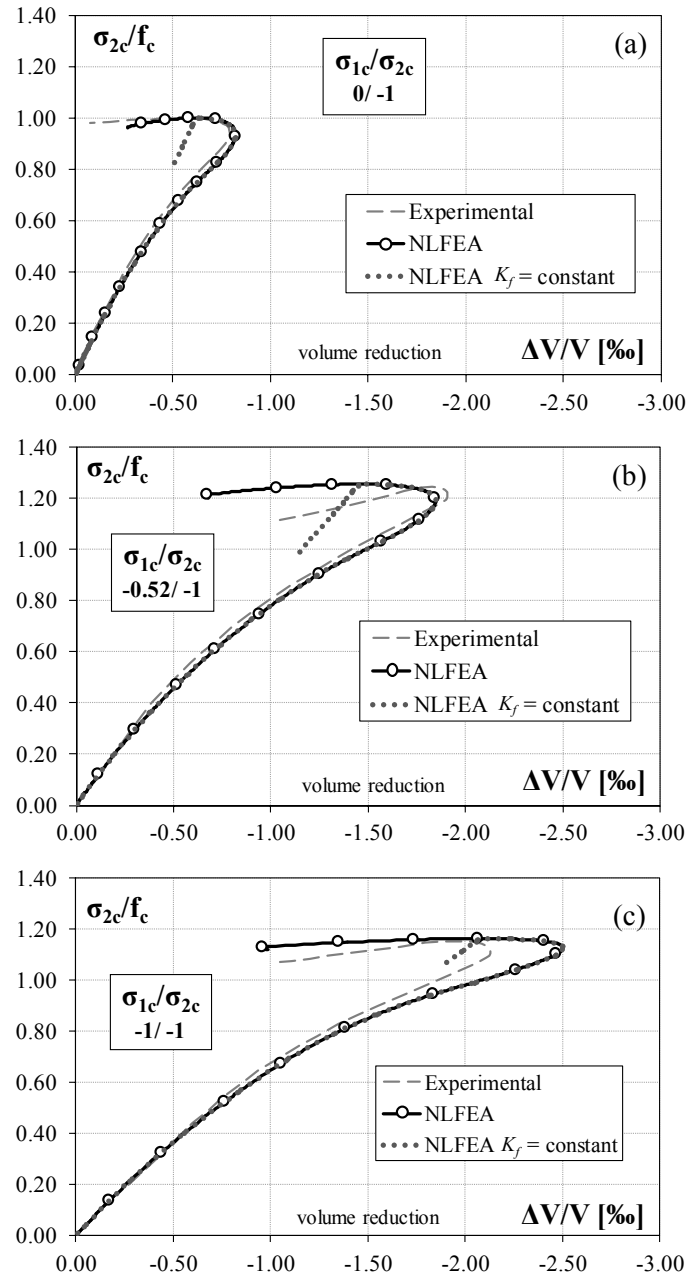


Figure 2.15 Comparison between numerical and experimental [50] results in terms of evolution of volumetric strains with loading for concrete panels subjected to: (a) uniaxial and (b)-(c) biaxial tension

Finally, Figure 2.14 shows panel stress-strain response for uniaxial and biaxial compressive loading. In this case, the adopted model is able to provide an accurate prediction of the experimental response both in pre- and post-peak regions, correctly catching crushing failure.

Moreover, the evolution of volumetric strains $\Delta V/V = \varepsilon_{1c} + \varepsilon_{2c} + \varepsilon_{3c}$, reported in Figure 2.15, proves the effectiveness of the followed approach in representing the volume reduction and then the dilatation typical of concrete loaded in compression. These last graphs show two distinct numerical curves, which are respectively obtained by following a different procedure for the evaluation of Poisson coefficient ν in the post-peak region. At first (dotted line, Figure 2.15), ν is determined by keeping fixed the secant bulk modulus K_f for a given change of the secant elastic modulus E_c , as suggested in [71,99]. Subsequently (continuous line, Figure 2.15) ν is calculated by following the approach proposed by in this thesis, by applying Equation (2.20). It can be observed that this latter approach provides more regular curves, which better fit the available experimental data.

2.4.2 RC beams without shear reinforcement

To prove the effectiveness of the proposed approach in the analysis of more complex structures, two experimental programs ([74,75]) on ordinary RC shear-critical beams subjected to three-point bending are modeled.

2.4.2.1 RC shear critical beams tested by Vecchio and Shim

Three beams without stirrups, named OA1, OA2, OA3, tested by Vecchio and Shim [74] are analyzed. These specimens had the same rectangular cross section - 305 mm wide and 552 mm deep - and a net span respectively equal to 3660 mm, 4570 mm and 6400 mm, corresponding to an increasing amount of tension reinforcement A_s , heavy enough to make the beam shear critical. The main geometrical details of the considered specimens and the amount of steel reinforcement are summarized in Figure 2.16 and Table 2.2.

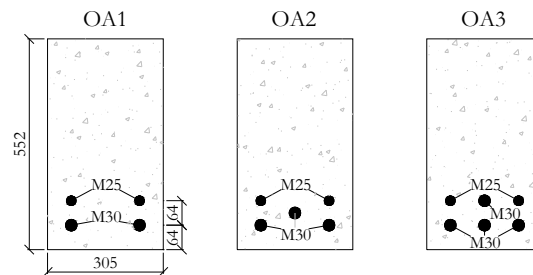


Figure 2.16 Geometric dimensions and reinforcement arrangement of the considered beams, adapted from [74]

The three beams were characterized by a progressively increasing concrete compressive strength f_c . This value, together with the other concrete mechanical properties (the peak strain ε_{c0} , the initial elastic value of the Young modulus E_c , the tensile strength f_{ct}) necessary to define the adopted constitutive relation are reported in Table 2.3.

Sample	b [mm]	h [mm]	d [mm]	L_{span} [mm]	L_{tot} [mm]	Longitudinal Reinforcement	A_s/bd (%)
OA1	305	552	457	3660	4100	2 M25 ^b , 2 M30	1.72
OA2	305	552	457	4570	5010	2 M25 ^a , 3 M30	2.22
OA3	305	552	457	6400	6840	2 M25 ^b , 4 M30	2.73

Table 2.2 Geometrical details of the analyzed beams

As the initial value of Poisson coefficient ν was not experimentally measured it is assumed equal to 0.2. The compressive post-peak nonlinearity parameter B is set equal to 0.1. Moreover, as only the concrete split cylinder strength f_{sp} was available, the value of f_{ct} reported in Table 2.3 is calculated according to Eurocode 2 [115].

Sample	f_c [MPa]	f_{ct} [MPa]	E_{ci} [MPa]	ε_{c0} (-)
OA1	22.6	2.131	36500	0.0016
OA2	25.9	3.031	32600	0.0021
OA3	43.5	2.821	34300	0.0019

Table 2.3 Concrete mechanical properties applied in simulations

The main characteristics of the reinforcement, both in terms of geometrical details (bar diameter d_b and area A_b) and steel properties (elastic modulus E_s and yield f_y and ultimate f_u strength), can be found in Table 2.4.

Bar size	d_b [mm]	A_b [mm ²]	f_y [MPa]	f_u [MPa]	E_s [MPa]
M25 ^a	25.2	500	440	615	210000
M25 ^b	25.2	500	445	680	220000
M30	29.9	700	436	700	200000

Table 2.4 Geometrical details and mechanical properties of the reinforcement

All the tests were performed under loading control, with a central point load, until the approaching of the ultimate stage, when the procedure was switched to displacement control so to allow the evaluation of the post-peak behavior. As already mentioned, the purpose of this experimental program was to recreate, as much as possible, the Bresler and Scordelis tests [114], in terms of geometrical dimensions, reinforcement details, material strengths and loading. Compared to these latter, the beams tested by Vecchio and Shim [74] exhibited indeed a very similar behavior, with only few minor differences; as a consequence, only the specimens described in [74] are considered in the FE analyses reported herein, but in the following graphs, also the results obtained by Bresler and Scordelis [114] on nominally identical beams are reported.

Taking advantage of the symmetry of the problem, only one half of each beam is simulated, by adopting a FE mesh constituted by quadratic, isoparametric 8-node membrane elements with reduced integration (4 Gauss integration points). Numerical analyses are performed under displacement control, by applying an increasingly displacement at the loading point, in order to achieve a better numerical convergence and evaluate also the post-peak behavior.

Numerical and experimental results are first compared by considering the global response, in terms of applied load P vs midspan deflection δ , as can be seen from Figure 2.17. On the same graphs, the experimental results obtained by Bresler and Scordelis [114] on nominally identical beams are reported too.

The graphs highlight the high accuracy of the proposed model in representing the global behavior both at serviceability (cracking load) and at ultimate limit state. The peak load is accurately predicted, as well as the brittle shear failure characterized by no ductility.

For comparison, design Code provisions are also considered. In more details, both the relations suggested in Eurocode 2 [115] and Model Code 2010 [57] (EC2 and MC2010 in the following) are analyzed. Table 2.5 compares the experimental ultimate load capacity $P_{u,exp}$ to the numerical value $P_{u,num}$ and to the maximum load to failure provided by Design Codes: $P_{u,MC2010}$ and $P_{u,EC2}$, by applying respectively MC2010 and EC2 relations. The maximum load is obtained as the double the ultimate resistant shear, depurated by the self-weight.

Sample	$P_{u,exp}$ [kN]	$P_{u,num}$ [kN]	$P_{u,MC2010}$ [kN]	$P_{u,EC2}$ [kN]	$\frac{P_{u,num}}{P_{u,exp}}$	$\frac{P_{u,MC2010}}{P_{u,exp}}$	$\frac{P_{u,EC2}}{P_{u,exp}}$
OA1	331	297	212	265	0.90	0.64	0.80
OA2	320	359	225	290	1.12	0.70	0.91
OA3	385	370	254	341	0.96	0.66	0.88

Table 2.5 Comparison between experimental [74] , numerical and Design Code results in terms of failure load and their relative ratio

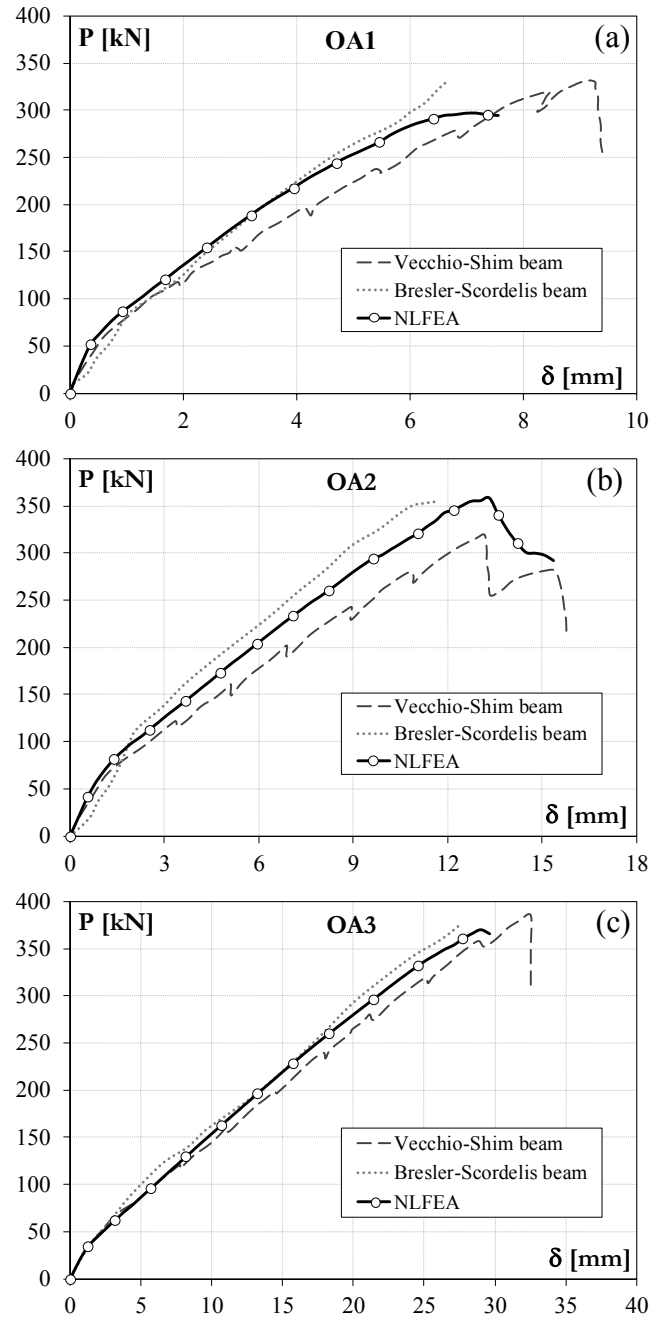


Figure 2.17 Comparison between numerical and experimental [74] results in terms of applied load P vs. midspan deflection δ

With reference to MC2010 [57], the ultimate resistant shear force is computed for the Level II Approximation, by applying the following Equation:

$$V_{Rd,c} = \frac{0.4}{1+1500 \varepsilon_x} \cdot \frac{1300}{1000+k_{dg}z} \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} z b_w , \quad (2.25)$$

being z the internal lever arm, b_w the minimum width of the cross-section and k_{dg} a parameter related to the maximum aggregate size (equal to 20 mm for these beams). ε_x represents the longitudinal strain calculated at the mid-depth of the effective shear depth and it is in turn calculated according to MC2010 [57]:

$$\varepsilon_x = \frac{1}{2E_s A_s} \left(\frac{M_{Ed}}{z} + V_{Ed} + N_{Ed} \left(\frac{1}{2} \mp \frac{\Delta e}{z} \right) \right) , \quad (2.26)$$

where Δe is the load eccentricity and N_{ed} , V_{ed} , M_{ed} are the sectional forces. The applied shear V_{ed} in Equation (2.26) is posed equal to the ultimate resistant shear force of the element - Equation (2.25); thus an iterative procedure is required.

The ultimate resistant shear force according to EC2, [115], can be instead calculated as:

$$V_{Rd,c} = \left[C_{Rd,c} k (100 \rho_l f_{ck})^{1/3} + k_1 \sigma_{cp} \right] b_w d \geq V_{Rd,c \min} , \quad (2.27)$$

where

$$V_{Rd,c \min} = \left(0.035 k^{3/2} f_{ck}^{1/2} + k_1 \sigma_{cp} \right) b_w d , \quad (2.28)$$

being d the effective depth of the cross-section, ρ_l the geometrical ratio (defined as $A_s/b_w d$), σ_{cp} the axial stress due to loading or pre-stressing. The parameters $C_{Rd,c}$, k and k_1 are respectively set equal to $0.18/\gamma_c$, $1+\sqrt{(200/d)}$ and 0.15. Further details can be found in EC2, [115], to which reference is made.

Referring to experimental tests, in Equations (2.25) and (2.27) the characteristic value of compressive strength f_{ck} is substituted by its mean value f_c and the partial safety factor for concrete (γ_c) is posed equal to 1.

As can be observed in Table 2.5, the NLFEA ultimate load and the experimental value are very close to each other, also varying the span length, while MC2010 and EC2 relations provides values further from experimental observations, even if the latter provides better estimations. However, MC2010 and EC2 values can be considered quite satisfactory, considering the high scatter typical of shear tests and bearing in mind that they should provide results on the safe side.

Further comparisons between numerical and experimental results are also provided in terms of cracking development and crack widths, as depicted in Figure 2.18, which shows the crack pattern at failure for the three specimens tested by Vecchio and Shim [74]. It should be noticed that the reported numerical crack pattern corresponds to numerical failure load, which is very close to the experimental one for all the considered beams, as observed from previous comparisons. As can be seen, the model exhibits a fine capability for reproducing the diagonal-tension crack experimentally observed for these beams, so correctly describing the very brittle and sudden failure, typical of beams containing no shear reinforcement. Furthermore, maximum crack widths, which represent one of the most difficult parameters to predict in numerical analyses, are substantially comparable to experimental ones.

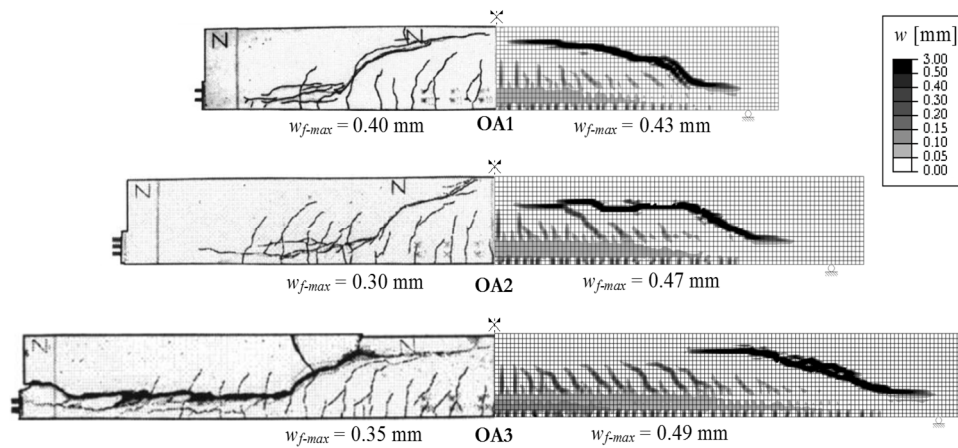


Figure 2.18 Experimental (left side, [74]) vs numerical (right side) crack patterns and crack widths at failure

2.4.2.2 RC shear critical beams tested by Pogdorniak-Stanik

Four ordinary RC shear-critical beams – named BN25, BN25D, BN50, BN50D – tested by Podgorniak-Stanik [75] are then analyzed. Series 25 and 50 differed from each other in terms of transverse cross-section dimensions, respectively equal to 300 mm x 250 mm and 300 mm x 500 mm, and in terms of net span, which was nearly doubled for the second one. Moreover, the two series were characterized by almost the same tension reinforcement ratio, which was equal on average to 0.85% for beams BN25 and BN50, and to 1.21% for beams BN25D and BN50D. These two last specimens contained indeed small longitudinal bars distributed along the web in addition to the main flexural bottom rebars. More details about the geometric dimensions and reinforcement arrangement of the examined beams are illustrated in Table 2.6 and Figure 2.19.

Sample	b [mm]	h [mm]	d [mm]	L_{Span} [mm]	L_{tot} [mm]	Longitudinal Reinforcement	A_s/bd (%)
BN25	300	250	225	1352	1502	3 M15	0.89
BN25D	300	250	225	1352	1502	3 M15, 10 #3	1.31
BN50	300	500	450	2700	3000	2 M20, 1 M25	0.81
BN50D	300	500	450	2700	3000	2 M20, 1 M25, 10M10	1.11

Table 2.6 Geometrical details of the analyzed beams

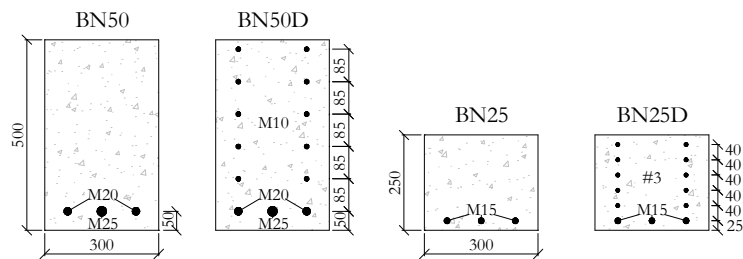


Figure 2.19 Geometric dimensions and reinforcement arrangement

Bar size	d_b [mm]	A_b [mm ²]	f_y [MPa]	f_u [MPa]	E_s [MPa]
#3	9.5	71.3	508	778	200000
M10	11.3	100	458	692	200000
M15	16	200	437	643	200000
M20	19.5	300	438	667	200000
M25	25.2	500	490	689	200000

Table 2.7 Properties of the reinforcing bars

Properties of reinforcing steel are listed in Table 2.7; the elastic modulus E_s is evaluated according to Eurocode 2 [115], since the experimental value was not measured. As regards concrete properties, only the cylindrical compressive strength - $f_c = 37$ MPa - is provided in [75]; as a consequence, the other parameters required for the definition of the adopted constitutive relation are

evaluated through the correlations suggested in [115]. Even in this case, the compressive post-peak nonlinearity parameter B is assumed equal to 0.1.

All the tests were performed under loading control, by applying a central point load in several steps. At the end of each step, the load was lowered to approximately 90% of its current peak value and then increased again. During all test execution, crack pattern evolution and crack width were monitored in detail by noting them at each load stage.

Numerical analyses are performed by following the same modeling choices already described in §2.4.2.1 for the shear critical beams tested by Vecchio and Shim [74].

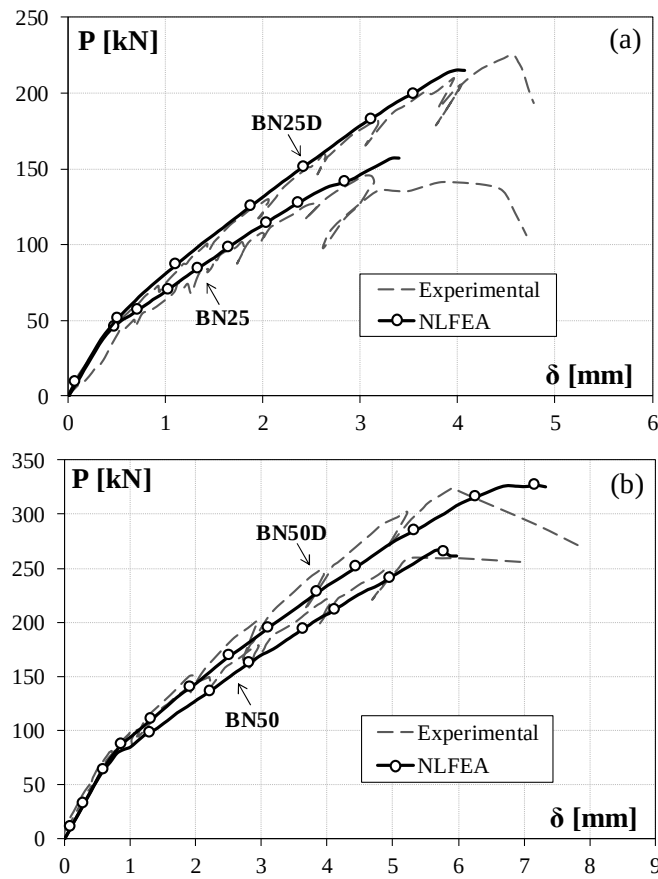


Figure 2.20 Comparison between numerical and experimental [75] results in terms of applied load P vs. midspan deflection δ

Figure 2.20 shows a comparison between numerical and experimental results in terms of applied load P vs. midspan deflection δ . Experimental evidence is well met by numerical predictions both as regards stiffness and failure load and mode. Numerical analyses are also able to predict the variation of bearing capacity related to a different amount and arrangement of longitudinal reinforcement in the cross-section: beams BN25D and BN50D which contain additional distributed reinforcements on element sides fail at a higher shear stress than the corresponding specimens BN25 and BN50, as also evidenced by experimental tests. Therefore, the satisfactory agreement between experimental and numerical results for both the experimental program carried out by Vecchio and Shim [74] (see §2.4.2.1) and by Podgorniak-Stanik [75] proves that the proposed model is able to reliably describe the behavior up to failure (in terms of strength, stiffness, ductility), for different specimen geometries, as well as different reinforcement arrangements.

Comparisons with Code provisions are then provided. The ultimate load is computed as already described in §2.4.2.1, by applying the relations suggested in MC2010 [57] and EC2 [115]; see respectively Equations (2.25)-(2.26) and (2.27)-(2.28).

Sample	$P_{u,exp}$ [kN]	$P_{u,num}$ [kN]	$P_{u,MC2010}$ [kN]	$P_{u,EC2}$ [kN]	$\frac{P_{u,num}}{P_{u,exp}}$	$\frac{P_{u,MC2010}}{P_{u,exp}}$	$\frac{P_{u,EC2}}{P_{u,exp}}$
BN25	145	157	115	148	1.08	0.80	1.02
BN25D	244	215	115	148	0.88	0.47	0.61
BN50	260	266	193	241	1.02	0.74	0.93
BN50D	322	327	193	241	1.02	0.60	0.75

Table 2.8 Comparison between experimental [75], numerical and Design Code results in terms of failure load and their relative ratio

The results presented in Table 2.8 indicate that the failure load is predicted with accuracy by numerical simulations for all the examined specimens. As regards beams BN25 and BN50, also EC2 relation provides precise peak values, while the approach suggested in MC2010 [57] tends to underestimates them. On the contrary the Design Codes fail in providing the correct value for the beam with the designation D, both EC2 and MC2010 relations indeed do not account for the small longitudinal bars distributed along the web of these elements, so providing the same value for the two series of beams.

Moreover, the model provides accurate results not only in terms of global load-deformation response but also in terms of crack widths and crack pattern evolution up to failure, as depicted in Figure 2.21 and Figure 2.22.

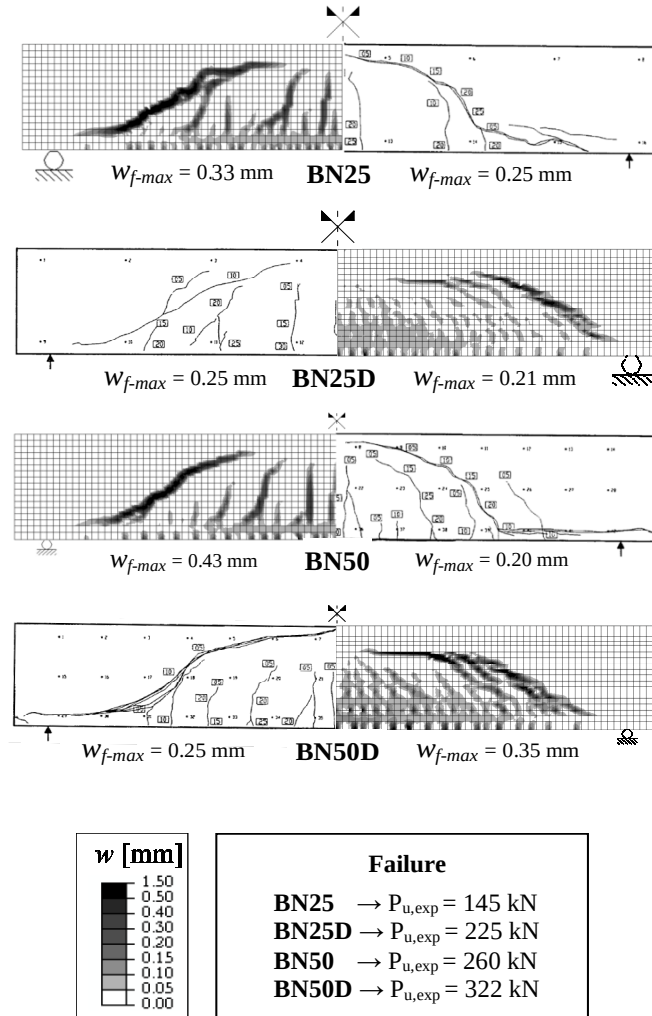


Figure 2.21 Numerical vs. experimental [75] crack pattern and crack width at failure

Figure 2.21 shows the numerical and experimental [75] crack pattern at failure for the four investigated specimens. As previously described for the beams tested by Vecchio and Shim [74], the numerical crack pattern refers to the failure load obtained by simulations, which is almost superimposed to the experimental one for all the considered specimens, as shown in Table 2.8. The model is able to reproduce the brittle experimental failure characterized by the presence of a critical diagonal-tension crack.

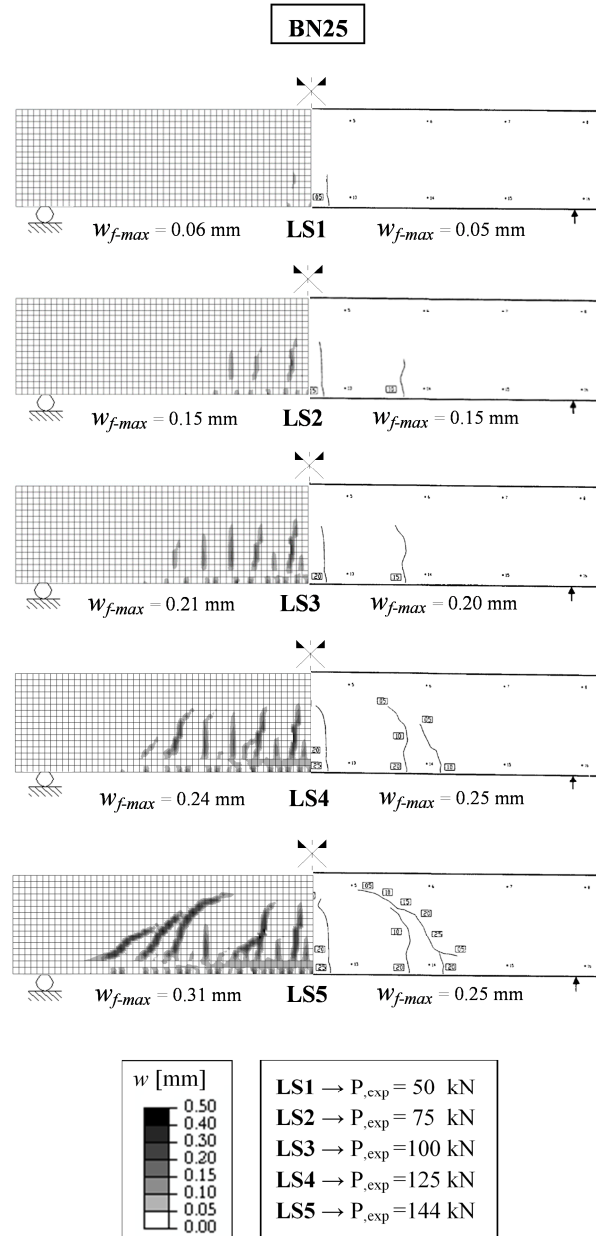


Figure 2.22 Numerical (left side) vs. experimental (right side, [75]) crack patterns and crack widths at different loading stages for specimen BN25

Crack pattern evolution for increasing loads, in terms of crack distribution and width, is instead represented in Figure 2.22 for beam BN25. Similar results are obtained also for the other specimens, but herein omitted for sake of brevity.

As regards the first loading stage (LS1) depicted in Figure 2.22, the comparison highlights that the first numerical flexural crack occurs at approximately the same loading level registered during the experimental test. As loading increases, other flexural cracks develop, until the attainment of loading condition LS5, which is very close to failure load. At this point a significant shear diagonal crack starts to form as extension of existing cracks; this shear crack is well visible at failure, as depicted in Figure 2.21. Crack patterns and widths are reasonably well described during the entire loading history; hence, thanks to its fine capability for predicting crack pattern evolution, the proposed model can represent a valuable design tool also for serviceability verifications, where crack control represents one of the fundamental issues to be checked.

2.4.3 SFRC beams without shear reinforcement

The proposed model is subsequently applied to the analysis of beams without shear reinforcement characterized by the addition of fibrous reinforcement in concrete admixture, in order to verify the applicability of the proposed procedure to structural elements realized with different types of concrete/reinforcement.

The presence of fibers is taken into account in 2D-PARC model in the uncracked stage as described in §2.3.3.1, by only appropriately calibrating the post-peak parameter B in order to account for the beneficial effect provided by fibers in compression increasing the ductility of the concrete.

In the cracked stage, as regards the concrete between cracks, its different behavior in compression (especially in the post-crushing phase) due to the presence of fibers is still taken into account as in the uncracked stage by properly setting the parameter B . Moreover, as for plain concrete, a proper reduction of the failure envelope is adopted in the cracked stage (see §2.3.5). On the contrary, the contribution of fibers to the resistant mechanisms of the fracture zone, is evaluated as in the original formulation of the model. In more detail, the structural improvement due to fibers is taken into account through a modification of the crack stiffness matrix. The resistant mechanisms related to the fibrous reinforcement are modeled by means of proper constitutive laws already implemented into 2D-PARC model, as discussed in § 1.3.4.5: an additional term due to fiber bridging across the crack is considered and the bond-slip law is properly modified so as to correctly represent the confinement action exerted by fibers. In particular the bond-slip law proposed by Harajli et al. [61,62] and the tension softening law by Li et al. [63,64] are applied.

The attention is herein focused on four shear critical beams, tested by Cucchiara et al. [76] and subjected to four-point bending test. These beams were characterized by the same rectangular cross section - 150 mm wide and 250 mm deep – and by the same length (equal to 2500 mm), but had different shear spans a . All the specimens did not contain stirrups and were characterized by the same longitudinal reinforcement ($2\phi 20$), characterized by a Young modulus $E_s=232000$ MPa and a yield strength $f_y=610$ MPa. The main geometrical details and the amount of steel reinforcement of the analyzed beams are illustrated in Figure 2.23 e Table 2.9.

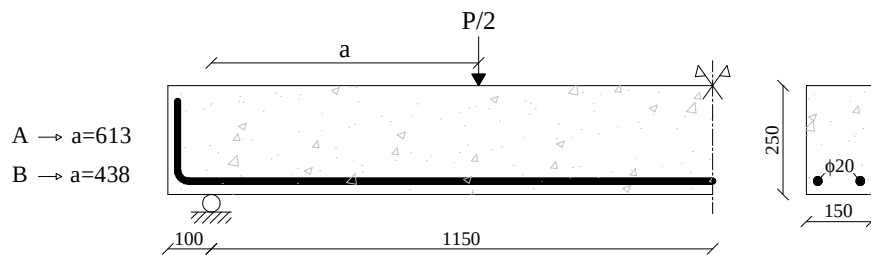


Figure 2.23 Geometric dimensions (in mm) and reinforcement arrangement of the examined beams

Two beams were reinforced by hooked-ended steel fibers, with length of 30 mm and equivalent diameter of 0.5 mm. For comparison, also two plain concrete beams having the same geometric characteristics as the SFRC ones are considered. All the examined specimens presented about the same compressive strength but, as expected, the fibrous concrete was characterized by more ductility and minor loss of strength in the post-peak phase. The fiber amount and the main concrete properties assumed in the simulations are provided in Table 2.10.

Sample	b [mm]	h [mm]	d [mm]	a [mm]	L_{tot} [mm]	Longitudinal Reinforcement	A_s/bd (%)
A00	150	250	219	613	2500	2 $\phi 20$	1.91
A10	150	250	219	613	2500	2 $\phi 20$	1.91
B00	150	250	219	438	2500	2 $\phi 20$	1.91
B10	150	250	219	438	2500	2 $\phi 20$	1.91

Table 2.9 Geometrical details of the analyzed beams

Sample	V_f (%)	f_c [MPa]	f_{ct} [MPa]	E_{ci} [MPa]	ε_{c0} (-)
A00	0.0	41.20	1.84	26094	0.0025
A10	1.0	40.85	1.84	26236	0.0028
B00	0.0	41.20	1.84	26094	0.0025
B10	1.0	40.85	1.84	26236	0.0028

Table 2.10 Amount of fibers and concrete mechanical properties applied in simulations

The adopted nomenclature is the same used by Cucchiara et al. [76] and is formed by a capital letter followed by two digits. Letters A and B respectively denote specimens with a shear span-to-depth ratio (a/d) equal to 2.8 and 2.0; the first digit refers to the amount of fibers (0 for plain concrete and 1 for $V_f = 1\%$) and the second one to the amount shear reinforcement, which is equal to 0 for all the considered beams, since they did not contain stirrups.

All tests were carried out with monotonically increasing displacements.

Numerical analyses are then performed on the four selected beams: the same modeling choices already described in §2.4.2 for ordinary RC beams are applied.

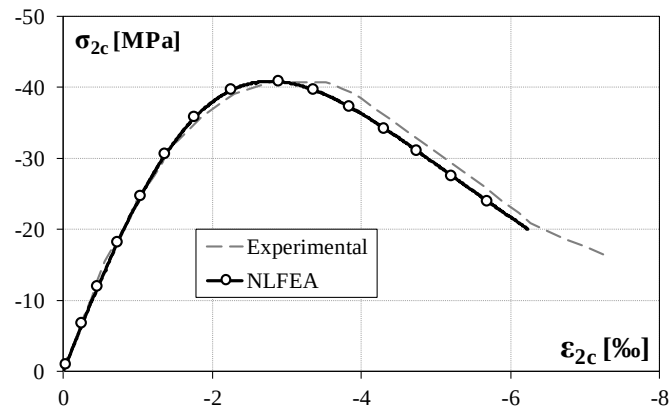


Figure 2.24 Comparison between numerical and experimental [76] response of SFRC cylinder specimen subjected to compression

It is worth noting that the model proposed by Ottosen [70,72] and implemented with slight modifications in 2D-PARC constitutive law, was originally conceived only for plain concrete. To prove the consistence of the model also for fibrous concrete, a cylindrical specimen, realized with the same concrete mixture of SFRC beams and subjected to compression, is first modeled (Figure 2.24). As can be seen, numerical and experimental responses are almost superimposed

not only in the pre-peak branch, where the behavior of fibrous concrete is almost the same of concrete without fibers [76], but also in the descending part of the curves. This result can be easily obtained through a proper calibration of parameter B , which is set equal to 0.5 for the samples reinforced with fibers; while it is maintained equal to 0.1 for the plain concrete ones.

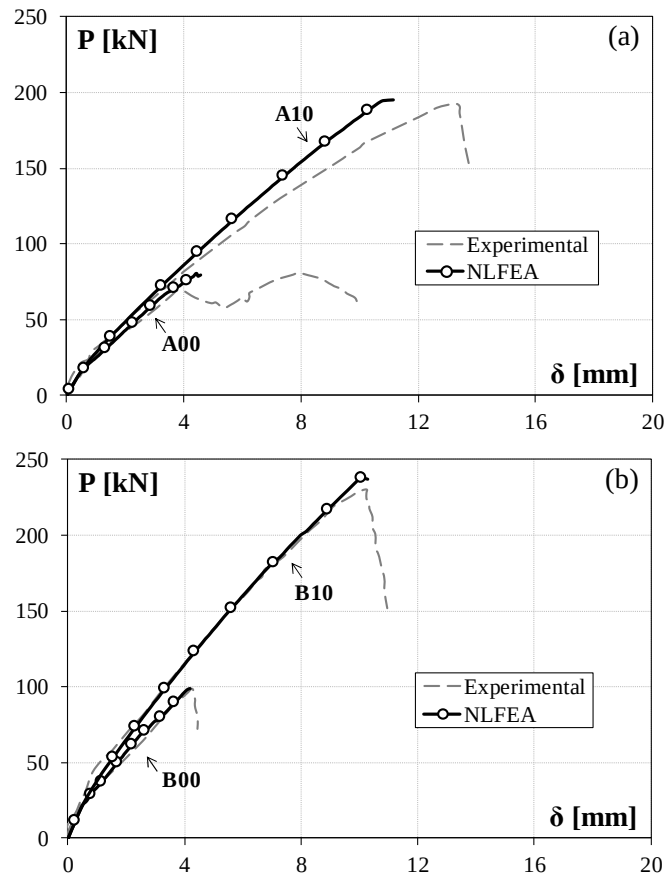


Figure 2.25 Comparison between numerical and experimental [76] results for RC (A00, B00) and SFRC (A10, B10) beams in terms of applied load P vs. midspan deflection δ

The global response of the RC and SFRC beams is first analyzed. Figure 2.25 shows the comparison between numerical and experimental [76] results in terms of total load P vs midspan deflection δ for both the considered series of beams. For immediate comparison each graph reports the curves relative to the specimens characterized by the same shear span (A or B), with and without fiber addition (1 or 0). Experimental behavior for both the beams with greater (Figure

2.25a) and smaller (Figure 2.25b) span shear length is predicted closely. Moreover, the improvement in terms of bearing load in the post-cracking stage due to the presence of fibers is correctly caught, so proving the accuracy of the proposed formulation also in case of SFRC elements.

Design Code provisions are then considered for the evaluation of the failure load. In the following only a theoretical explanation is provided, since the considered Code relations for SFRC specimens require the knowledge of the residual tensile strength f_{Ftuk} , which was not experimentally measured.

In more details EC2 [115] does not suggest any specific relation for specimens with fibrous reinforcement without stirrups; whereas MC2010 provides two formulas.

The first MC2010 relation, which has been validated and so it is considered reliable [57] is given by:

$$V_{Rd,F} = \left\{ \frac{0.18}{\gamma_c} k \left[100 \rho_l \left(1 + 7.5 \frac{f_{Ftuk}}{f_{ctk}} \right) f_{ck} \right]^{\frac{1}{3}} + 0.15 \sigma_{cp} \right\} b_w d \geq V_{Rd,F \min} \quad , \quad (2.29)$$

where

$$V_{Rd,F \min} = \left(0.035 k^{\frac{3}{2}} f_{ck}^{\frac{1}{2}} + 0.15 \sigma_{cp} \right) b_w d \quad , \quad (2.30)$$

being d the effective depth of the cross-section, b_w the minimum width of the cross-section, ρ_l the geometrical ratio (defined as $A_s/b_w d$) and σ_{cp} the average axial stress due to loading or pre-stressing, while k represents a parameter that takes into account the size effect and it is set equal to $1 + \sqrt{(200/d)}$. Further details can be found in MC2010 [57], to which reference is made. This approach is not consistent with the one adopted by MC2010 for RC beams reported herein in Equations (2.25)-(2.26), but it appears aligned to the one suggested by EC2, see Equations (2.27)-(2.28). The difference between EC2 relation for traditional RC elements - Equation (2.27) - and the above described formula - Equation (2.29) - lays indeed only in the addition of the term $7.5 f_{Ftuk}/f_{ctk}$ in the round brackets, which is equal to zero for traditional RC beams.

The latter MC2010 relation is expressed as:

$$V_{Rd,F} = \frac{1}{\gamma_F} \left(\frac{0.4}{1 + 1500 \varepsilon_x} \cdot \frac{1300}{1000 + k_{dg} z} \sqrt{f_{ck}} + k_f f_{Ftuk} \cot \theta \right) \cdot z b_w \quad , \quad (2.31)$$

being z the internal lever arm, θ the angle of the compressive stress field, k_{dg} a parameter related to the maximum aggregate size. These parameters are in turn calculated according to MC2010, while k_f is posed equal to 0.8. ε_x is the longitudinal strain calculated at the mid-depth of the effective shear depth by

Equation (2.26). This approach, more extensively explained in *fib* Bulletin 57 “Shear and punching shear in RC and FRC elements. Workshop proceedings.” [116], even if not fully validated [57], is coherent with MC2010 formulation for RC elements, reported herein in Equations (2.25)-(2.26).

At last, the experimental crack distribution at failure is compared to the final numerical crack pattern in Figure 2.26: the brittle shear failure is well described in all cases, with the development of localized diagonal cracks running along an inclined line, from the support towards the loading point; thus proving once again the effectiveness of the proposed approach.

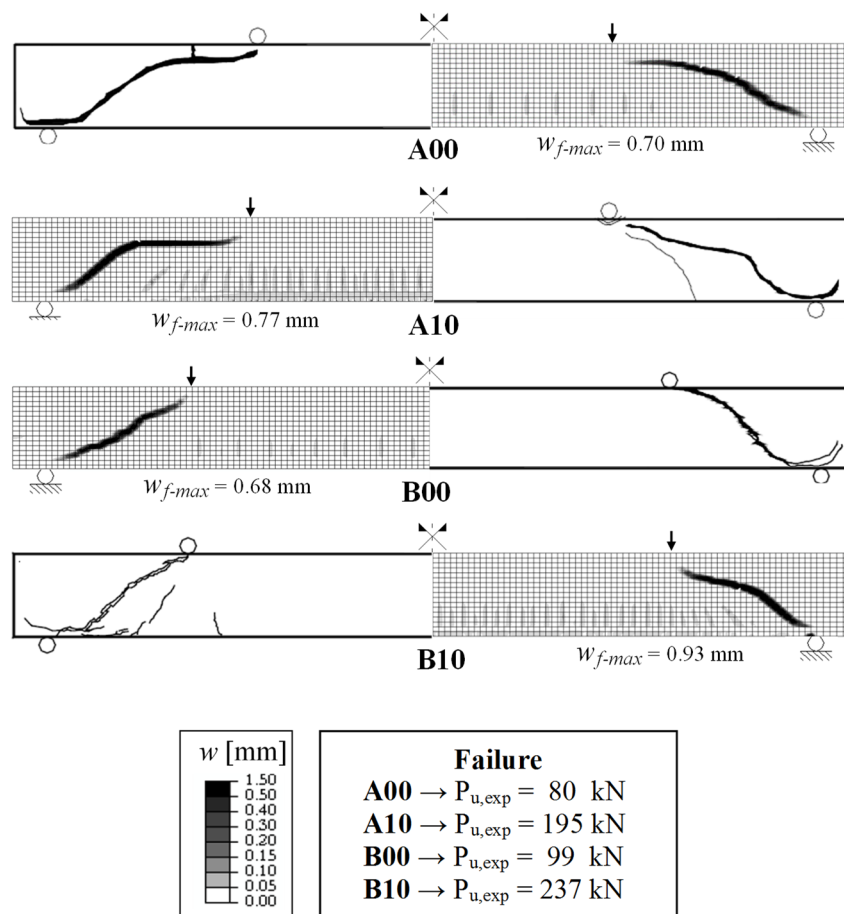


Figure 2.26 Comparison between numerical and experimental [76] crack patterns at failure for RC/SFRC specimens.

2.5 Concluding remarks

In this Chapter, the nonlinear constitutive model for the analysis up to failure of RC and SFRC elements, named 2D-PARC, is properly revised in order to reduce the computational effort, as well as to incorporate the effects of crushing and dilatation of concrete in compression. To this aim, the constitutive relation originally adopted for concrete behavior is properly substituted both before and after cracking. Thanks to the modular structure of 2D-PARC model, this operation can be easily performed by only changing the part of the general algorithm related to concrete contribution (i.e. the evaluation of concrete stiffness matrix), while leaving unchanged the rest.

In more detail, the formulation originally implemented in the model, quite complex, which treats concrete (both in the uncracked and cracked stage) as an orthotropic material, by turning the current biaxial state of stress into two uniaxial states of stress, is then substituted, by following an alternative approach, based on isotropic nonlinear elasticity.

In particular, the evaluation of concrete resistant contribution is performed by applying the formulation proposed by Ottosen [70,72]. In this way, the concrete stiffness matrix can be expressed, at each integration point, as a function of only two parameters, the secant values of the Young modulus and the Poisson coefficient, which are properly updated during the analysis to take into account the current biaxial state of stress.

This formulation, very numerically feasible, enables to improve the computational efficiency of 2D-PARC model, while providing at the same a sophisticated representation of concrete behavior, thus maintaining its capability for providing accurate predictions. Moreover, the post-crushing behavior can be satisfactorily handled, by also considering the dilatation that occurs when concrete is loaded in compression.

It is worth noting that the formulation proposed by Ottosen [70,72] is herein slightly modified in order to obtain better predictions in the post-crushing phase and to increase the rate of convergence; in addition, a more convenient convergence method based on stresses instead of on the Elastic modulus is herein proposed. Moreover, the formulation adopted for concrete, originally conceived for plain and ordinary reinforced concrete structures, is applied also to SFRC elements by only changing a proper post-peak parameter and by exploiting the versatile nature of 2D-PARC.

Comparisons between numerical results and experimental data available in technical literature validate the proposed procedure and its numerical implementation. Different structural typologies are analyzed: plain concrete panels subjected to a biaxial stress state [51], as well as RC [74,75] and SFRC [76] beams without shear reinforcement; all elements whose structural response is mainly governed by concrete performances. The selected case studies point out the flexibility of the proposed approach, as well as its accuracy in describing concrete behavior under different loading combinations, also in the post-crushing region. Based on the obtained results, it can be stated that global load-

deformation response of RC/SFRC elements up to failure can be closely simulated: accurate prediction in terms of stiffness, failure load and ductility are indeed provided. Moreover, also crack pattern evolution, both in terms of crack distribution and crack width, can be satisfactorily evaluated; so demonstrating the effectiveness of the proposed approach and that 2D-PARC model, revised in the concrete contribution, could be a powerful tool for both research and design purposes.

Chapter 3

Inclusion of shrinkage strain

3.1 Introduction

In this chapter, 2D-PARC constitutive model is extended to include the effects of shrinkage occurring prior to loading.

It is indeed generally recognized that for reinforced concrete (RC) or steel fiber reinforced (SFRC) elements, shrinkage markedly influences the structural behavior even under short-term loading [117–120]. In current design practice, shrinkage and creep effects are usually taken into account in the assessment of long-term response of RC structures, as well as in the evaluation of prestress losses. However, it has been demonstrated that also shrinkage occurring prior to loading can significantly influence their behavior. Restraints to concrete shrinkage (due to the presence of bonded reinforcement and/or to external boundaries) are indeed responsible for a not negligible reduction of the cracking load of RC/SFRC elements and exert a significant influence on member deformations even under short-term loading. It should be remembered that shrinkage may also induce tensile stresses in concrete high enough to lead to crack formation before the application of the load, often giving rise to serviceability and durability problems [121,122].

Moreover, shrinkage does not have a uniform distribution within the member, being greater in correspondence of the exposed surfaces, directly subjected to drying, and progressively decreasing towards the core; the so-called differential shrinkage. This phenomenon is related to the moisture loss to environment: the drying process starts indeed at the exposed surface and then penetrates into the inner parts. Due to humidity gradient, non uniform drying shrinkage strains develop in the element, causing restraints to volume change, and thus a stress gradient. Internal restraint due differential shrinkage results indeed in higher tensile stresses near the drying surfaces, which are self-balanced by internal compression and may lead to superficial cracking [123–125].

For all these reasons, it would be advisable to consider shrinkage effects in a rigorous manner in numerical simulations, so as to obtain reliable results. This requires the simulation of the moisture field in concrete in order to obtain the related shrinkage strains, as well as the adoption of an efficient constitutive

model able to catch both shrinkage-induced and load-induced cracking in the structural element.

In this work shrinkage pre-strain are inserted into an existing bi-dimensional numerical model, 2D-PARC, that is able to describe the behavior up to failure of RC/SFRC elements subjected to a general biaxial state of stress. The original formulation of the model [1], which is based on a fixed, smeared crack approach, does indeed not account for any possible concrete initial strain. For further details see Chapter 1, where a comprehensive description of the model can be found.

In the following, 2D-PARC, already revised in the modeling of concrete contribution as described in Chapter 2, will be further improved to consider shrinkage prestrains. In more detail, both in the uncracked and in the cracked stages, shrinkage can be treated as a prescribed deformation, which is properly added to concrete strain, so consequently modifying the state of stress in concrete and in the reinforcing steel. It is worth noting that the peculiar formulation of 2D-PARC model, that smears both the reinforcement and cracking within the element and at the same time follows a strain decomposition procedure in the cracked stage, requires the development of a different strategy for inserting shrinkage effects with respect to all the other constitutive models available in the literature. A new set of equilibrium and compatibility equations is written and the material secant stiffness matrix is consequently rearranged and implemented into the finite element code adopted.

It is worth noting that this modified formulation including shrinkage effects can be applied for the analysis of both traditionally reinforced and fiber reinforced concrete elements, thanks to the modular structure of the model: the only difference between these two cases lies indeed in the constitutive laws adopted for the modeling of rebar/fiber contributions, which are automatically selected among those incorporated into 2D-PARC.

The value of shrinkage strain associated to each integration point is evaluated by following two different approaches characterized by different refinement and complexity. For sake of simplicity, the algorithm is first modified by considering shrinkage strains as uniformly distributed within the element. Subsequently, the procedure is refined, so as to take into account the dependency of shrinkage on moisture gradient. This latter is in turn obtained by performing an “equivalent” thermal analysis, by exploiting the analogy between the equations governing the moisture and the thermal problem, with a simple substitution of the corresponding parameters. In this way, more reliable results can be obtained.

Moreover, in both cases, since shrinkage-induced stresses develop gradually with time, creep effects are inserted in the algorithm.

This chapter is subdivided in three main part: at the beginning the state of the art about shrinkage will be presented, then the revised formulation of 2D-PARC model to include initial prestrains will be outlined and finally, extensive comparisons with data from literature will be provided in order to validate the proposed numerical strategy over a wide range of conditions. Different structural typologies will be considered: plain concrete specimens [126,127] RC and SFRC tension members [128,129], as well as RC beams [130–132], so as to prove the

effectiveness of the proposed procedure in affording serviceability problems (i.e. deformations and cracking), where concrete shrinkage is one of the key parameter to be considered for the performance assessment of RC elements.

3.2 State of the art

Shrinkage and creep greatly influence the response of reinforced concrete (RC) elements. As it is well known, they affect the long-term performances of structures, by causing, under sustained load, a stress redistribution. This leads to an increase of deformations and crack widths over time as well as the appearance of additional primary cracks. However, it has been demonstrated that shrinkage, and the related creep effects, play a major role also on short-term behavior, significantly affecting both cracking resistance and deformations of RC structures. Moreover, because of shrinkage, also cracking before the application of any load may occur. This leads to possible aesthetic, durability and serviceability problems. Cracking results in a less stiff behavior of the element and increased deformations, by progressively limiting, as the cracks open, the ability of the concrete to transfer stresses across the cracks themselves. Moreover, for RC structures, the presence of unexpected cracks can greatly increase the corrosion risk of the steel rebars, especially in case of aggressive environment, large crack widths and prestressing steel.

In the following a brief description of shrinkage phenomenon and its effects on short-term response of RC structures will be provided, whereas long-term effects will be disregarded, being the assessment of long-term behavior out of the scope of this work.

Shrinkage influences the stress history of a concrete element from its early age. The chemical and physical processes involved soon after being cast and moist cured, related to the cement hydration reaction and to the interactions with the surrounding environment, cause indeed a volume change, namely shrinkage. Since the element is almost never free to shrink (due to internal or external restraint), a eigen-stress state takes place. In more detail, considering an unrestrained plain concrete specimen, differential tendencies to shrink take place within the element, leading to a stress build-up. The drying process begins indeed at the exposed surfaces and gradually penetrates into the core; thus leading to an uneven distribution of shrinkage strains, since, while the exposed surfaces tend to shrink a lot, the inner layers have no tendency to shrink. To restore compatibility, the core is compressed while the external fibers are subjected to tension, that may cause premature cracking. The problem becomes even more complex when the reinforcement is embedded in concrete, due to the additional restraint to shrinkage provided by the rebar. Moreover, it should be pointed out that, since shrinkage strains develop gradually with times, also creep plays a major role.

Hence, shrinkage, and the related creep effects, are significant issues even under short-term loading and must be considered in numerical simulations as

well as during the interpretation of test results. As a matter of fact, incorrect design predictions of structural performances under serviceability conditions can be obtained when these phenomena are not properly included into calculations and/or numerical simulations. Moreover, the same experimental results can be misinterpreted – especially in terms of apparent reduction of tension stiffening with increasing amount of reinforcement - when high concrete shrinkage occurs before test execution, but it is not suitably taken into account in the elaboration of test data.

The necessity of studying shrinkage and of assessing its influence of RC behavior has been indeed recognized since the middle twentieth century [133–137], but due to the complexity of the problem this subject is still relevant in academic research, as confirmed by the great number of experimental and theoretical/numerical recent studies on this topic, e.g. among others [118–122, 130–132, 138–143].

In the following the attention will be first focused on the physical description of shrinkage phenomenon. Subsequently the effects of shrinkage on short-term behavior of RC elements will be outlined; tension members and beams will be discussed, being the typologies of elements analyzed in the present work. Finally, some considerations about how shrinkage can be inserted in numerical models will be provided.

3.2.1 Description of the physical phenomenon of shrinkage

Shrinkage is a spontaneous phenomenon for concrete that occurs throughout the service life of concrete structures. It is associated to the cement paste, since aggregate does not experience a volume contraction, but they influence the phenomenon. Shrinkage is indeed mainly related to evaporation of the absorbed water; thus, to the porosity of the cement paste, which consists of air voids, capillary pores and gel pores. The loss of water results in a contraction of concrete, usually termed as shrinkage. It can indeed be defined as the time-dependent change in volume during an unstressed state at constant environmental temperature [10]. Without restraints, concrete shrinkage will only result in a reduction of the concrete volume, otherwise internal stresses arise, which may lead to cracking. However, the condition of zero internal stresses is almost impossible, since for concrete elements with thickness greater than 3 mm, a moisture gradient takes place [144]; thus leading to the appearance of eigenstresses. Shrinkage occurs even in the absence of stresses and develops over time, with a higher rate in the period immediately after casting, while it tends to reach an asymptotic value after very long periods of time.

According to the cause, shrinkage may be divided into five different types, namely plastic, thermal, autogenous, carbonatation and drying shrinkage, that will be discussed in the following subsections. A more detailed discussion will be provided for autogenous and drying shrinkage, since the shrinkage strain is usually evaluated as the sum of only these two components, being the most relevant phenomena for the short-term behavior of RC structures under serviceability conditions.

3.2.1.1 Plastic shrinkage

Plastic shrinkage is associated with the rapid water evaporation from the concrete surfaces into the surrounding environment when it is still in the plastic phase. This phenomenon happens in the freshly poured concrete, thus at early-age, being prominent during the setting period. It favors the onset of cracking on the free surface, with consequent drop in the element durability. In more detail, plastic cracks are likely to appear when moisture loss from concrete surfaces is larger than bleed water. Therefore, this danger can be limited by reducing the evaporation rate or increasing the rate of water bleeding. The first is related to relative humidity, air movement velocity, ambient and concrete temperatures; whereas the latter depends on cement, water and entrained air contents.

3.2.1.2 Thermal shrinkage

Thermal shrinkage is given by the contraction of the material due to the non-adiabatic conditions during the hydration of the cement. The chemical reactions develop heat, with a consequent increase in temperature and expansion of the concrete. Such reactions are progressively slowed down during the setting, and the temperature is lowered up to a value close to that of the external environment, because of heat dissipation through the formwork, so causing the concrete contraction. Thermal shrinkage is usually neglected; it can become important only during setting and for mass concrete structures, where a massive amount of internal heat is generated during the hydration process.

3.2.1.3 Autogenous shrinkage

Autogenous shrinkage, also called chemical shrinkage, is related to the water consumption from the capillary pores due to the chemical reactions of the hydration process of cement. This results in a bulk volume reduction, which happens under isothermal condition and in the absence of moisture exchange with the external ambient. In particular during the hydration process water reacts with the cement particles to create the cement hydration products: capillary pore water, and if it is not enough also the intracrystalline water, are absorbed as the hydration process continues. This process is known as self-desiccation: a negative pressure develops in the pores and it causes the volume reduction.

Autogenous shrinkage can be considered to develop isotropically within the mass of the material, but it is not uniform. Higher autogenous shrinkage develops in the inner parts of the element since chemical reactions are indeed usually stronger in the core of element, because of the development of higher heat of hydration. It can be considered as an intrinsic characteristic of concrete and almost independent of the size of the specimen. The phenomenon evolves in time with similar trend to development of mechanical strength: its increase is very fast in the first days, reaching at 28 days the 60 - 90% of its final value.

Autogenous shrinkage is relatively small in conventional normal strength concrete (about 5%-10% of the maximum drying shrinkage), but can reach high value in case of very low water/cement ratio. The consumption of pore water in

the hydration process is indeed more rapid in case of low water/cement ratio. Therefore, autogenous shrinkage cannot be neglected for high strength concrete. Moreover, also the type of the cement and the additives inserted in the admixture influence chemical shrinkage. For example, the presence of alumina cement, high early-strength cement or fine-grained cement tend to increase this phenomenon as well as the addition of blast-furnace slag, silica fume, and expansive agent.

3.2.1.4 Carbonation shrinkage

Carbonation shrinkage is another kind of chemical shrinkage, since it is related to the chemical reaction between calcium hydroxide, produced by the hydration of the cement, and carbon dioxide in the air, to create calcium carbonate. Carbonation shrinkage begins at the surfaces and gradually penetrates into the core; however, carbon dioxide rarely penetrates through concrete surface for more than a few millimeters; therefore carbonation shrinkage can be usually neglected.

3.2.1.5 Drying shrinkage

Drying shrinkage is the most important type of shrinkage and it is often the single rate considered when referring to shrinkage. It is defined as the volumetric reduction due to the evaporation of water to the surrounding environment, with a constant ambient temperature and relative humidity. The process of drying shrinkage is not fully understood, nevertheless it is thought to begin as soon as the absorbed water is lost to the environment. At first, free water evaporates, even if this loss is found to cause insignificant value of shrinkage. Afterwards, the adsorbed water in the capillary pores starts to evaporate, since the internal humidity tends to equilibrate the inferior environmental one. Also intracrystalline water can be involved in the process after water in the capillary pores is all evaporated but concrete is still exposed to drying. A negative capillary pressure develops and compressive forces are induced on the rigid concrete skeleton, resulting in a volume contraction.

Relating with a unique law drying shrinkage to the water loss is very difficult. It depends on the specimen size and it is far more complicated for real structures, characterized by a size and shape non-uniform throughout. Moreover, it is not isotropic being higher on the external superficies and lower in the core. Drying shrinkage increases with time and it continues for long period: it usually ends months or years after casting. However, the rate of volume reduction decreases with time. As already said, the member size plays a major role on drying shrinkage: thin members can shrink for months or years, while for the core of a larger member, the drying process may continue throughout its lifetime. In particular the volume-to-surface area ratio of the element influences the development of shrinkage strains; lower volume-to-surface area ratio permits indeed a greater moisture loss.

Drying shrinkage is affected, a part from the shape and the size of the specimens, by all the mechanisms related to the drying process. They can be

grouped into two main groups: the first is related to the environmental conditions, such as relative humidity, ambient temperature and wind velocity, whereas the latter is related to the intrinsic characteristic of the concrete material, such as the content and type of aggregates, the water and cement content as well as the presence of additives.

As far as environmental conditions are concerned, the major role is played by the relative humidity; a low ambient humidity produces larger gradients near the drying surfaces, thus increasing the drying rate.

As regards the intrinsic characteristic of the concrete mix, aggregates influence a lot drying shrinkage. They are inert, due to their low permeability, so they restrict the overall deformations of the material, providing restraining actions to the cement paste that undergoes drying shrinkage. In particular higher aggregate concentration and higher aggregate stiffness result in lower shrinkage strains. Greater water and cement concentrations produce instead higher shrinkage deformations. In case of high water content drying shrinkage increases due to larger amount of evaporable water; whereas higher cement content results in a larger fraction of cement paste in concrete which represents the shrinking part of the material. Considering the water to cement ratio, reducing this value produces a reduced porosity of the cement paste, leading in turn to lower shrinkage strains. Moreover, also the inclusion of additives in the concrete admixture influences the drying shrinkage potential, since they modify the microstructure of the cement paste, as well as the pore structure.

3.2.2 Effects of shrinkage on the behavior of structural elements

At first the effects of shrinkage on unreinforced members will be discussed, subsequently RC tension members and beams will be analyzed. Only the short-term behavior will be discussed, being the long-term one out of the scope of this thesis.

3.2.2.1 Unreinforced elements

In general shrinkage would only results in a concrete volume reduction if the element was totally free to deform; thus when no hindrances opposed to free shrinkage. However, in physical reality this condition applies only for unrestrained and unreinforced elements characterized by very a limited thickness. In particular Bissonnette et al. [144] reported that only concrete sample with thickness in the range of 1 mm to 3 mm can be considered characterized by totally free shrinkage, because in this case the internal moisture gradient approaches to zero.

Issues concerning shrinkage arise when any kind of hindrance prevents the concrete from freely deforming. In this case, self-equilibration internal stresses are induced within the element, with the appearance of tensile stresses which may lead to cracking before the application of any load.

Considering an unloaded, unrestrained and unreinforced element (with thickness greater than 3 mm), it exhibits different amount shrinkage at different locations, depending on the distance to the external drying surfaces. Shrinkage is

indeed related to the moisture gradient that is not uniform within the member. The drying front migrates indeed gradually from the exposed surface to the interior of concrete elements. It follows that the tendency to shrink is largest on the external surfaces due to rapid moisture loss and lower in the inner parts of the element, furthest from the drying surface. Since the higher shrinkage strains on the external surfaces are restrained by the lower ones of the inner points, a non uniform strain field arises within the member, termed as differential shrinkage. This phenomenon gives rise to the development of an internal state of stress to restore strain compatibility, so as to ensure that sections remain plane. The induced eigen-stresses consist of tensile stresses near the external surfaces, while the inner core is compressed, as can be seen in Figure 3.1.

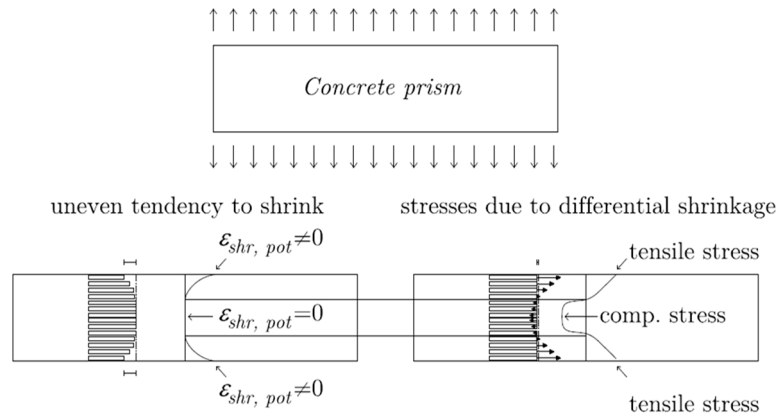


Figure 3.1 Differential drying shrinkage and induced stresses within a cross-section [124]

It is worth noting that also creep is involved in the process; since the internal stresses induced by shrinkage develop gradually with time, they are relieved by creep. Because of the high magnitude of external shrinkage strains the relief caused by creep is not often enough to prevent cracking of concrete. Tensile stresses near the drying surface may indeed exceed concrete tensile strength, leading to surface cracking before the application of any load and giving rise to possible aesthetic, durability and serviceability problems. In particular, the degree of restraint to shrinkage, the extensibility and strength of concrete in tension, as well as creep effects play a major role.

It must be underlined that differential shrinkage due to unsymmetrical drying may even cause warping in the concrete member. Moreover, in case of significant external restraints provided to shrinkage deformation, high tensile stresses develop and wide cracks can be often observed.

3.2.2.2 Reinforced elements

Structural concrete elements are usually reinforced with steel bars. They provide restraint to concrete shrinkage due to the bond action.

Considering a member reinforced with a single centered bar (see Figure 3.2), it can be observed that as the concrete shrinks, the reinforcement becomes compressed and imposes an equal and opposite tensile force to concrete. Due to this constraint to concrete shrinkage, the average shortening of the member is reduced, assuming the compatibilized value ε_{mi} . By following the formulation proposed in [119] and with reference to an uncracked tension member supposed characterized by a linear elastic behavior and subjected to a uniform shrinkage strain ε_{sh} , the average member strain ε_{mi} due to shrinkage can be evaluated as:

$$\varepsilon_{mi} = \varepsilon_{sh} \frac{1}{(1+n\rho)} \quad , \quad (3.1)$$

being n the modular ratio (E_s/E_c) and ρ the reinforcing steel ratio (A_s/A_c). It is worth noting that shrinkage strain ε_{sh} is assumed as negative.

Equation (3.1) is calculated for the condition of zero axial load (i.e. $N=0$) by by substituting the compatibility equations:

$$\varepsilon_{mi} = \varepsilon_c + \varepsilon_{sh} \quad (3.2)$$

$$\varepsilon_{mi} = \varepsilon_s$$

into the equilibrium equation:

$$N = N_c + N_s = A_c E_c \varepsilon_c + A_s E_s \varepsilon_s \quad (3.3)$$

being ε_c and ε_s respectively the concrete and the steel strains caused by stresses.

Moreover, the appearance of tensile stress in concrete equal to $\sigma_c = -(E_c \varepsilon_{sh} n \rho)/(1+n\rho)$ leads to a reduction in the load required to crack the member. It results:

$$N_{cr,sh} = N_{cr} \left[1 + \frac{E_c}{f_{ct}} \frac{\varepsilon_{sh} n \rho}{(1+n\rho)} \right] \quad (3.4)$$

being $N_{cr,sh}$ and N_{cr} the cracking load respectively for a shrunk and a non-shrunk member.

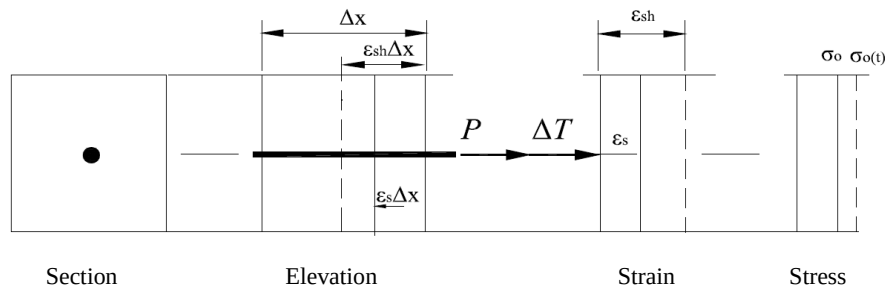


Figure 3.2 Strains and stresses on an uncracked section of a uniaxial tension member, due to an hyopotized uniform shrinkage strain ε_{sh} [132]

In case of high shrinkage strain the member may crack prior to loading. If the element cracks the initial shortening caused by shrinkage, will be reduced compared the value derived from Equation (3.1) and more refined approaches should be applied.

It is worth noting that the reported equations represent only a simplified formulation, reported to better highlight the phenomenon. However, this approach is not exhaustive. The concrete is indeed characterized by a nonlinear behavior and by differential shrinkage, due to the uneven tendency to shrink of points at different distances from the drying surface, as explained in §3.2.2.1. Moreover, since shrinkage induced stresses developed gradually with time, also creep effects influence the development of the stress fields. It follows that creep cannot be neglected in calculation.

Passing now to the description of the behavior of RC elements subjected to shrinkage with reinforcement not symmetrically placed in the depth of the section, it can be stated that a shrinkage-induced curvature takes place (see Figure 3.3).

Shrinkage, in an unsymmetrically reinforced concrete member, like a RC beam, produces indeed deflections, also of significant magnitude, before the application of any external load. While concrete shrinks, it compresses the bonded rebars that impose equal and opposite tensile forces on concrete at the level of the reinforcements. Differently from an element symmetrically reinforced, in this case the resulting tensile force acts at some eccentricity to the centroid of the concrete cross-section, resulting in a gradual warping of the member.

The shrinkage induce curvature (in sign and magnitude) depends on the amount and position of the reinforcements. Considering for example a RC beam if the amount of bottom (tensile) reinforcement is greater than the top (compressive) one a positive deflection takes place, on the contrary for heavy top reinforcements a negative (hence, upward) deflection appears.

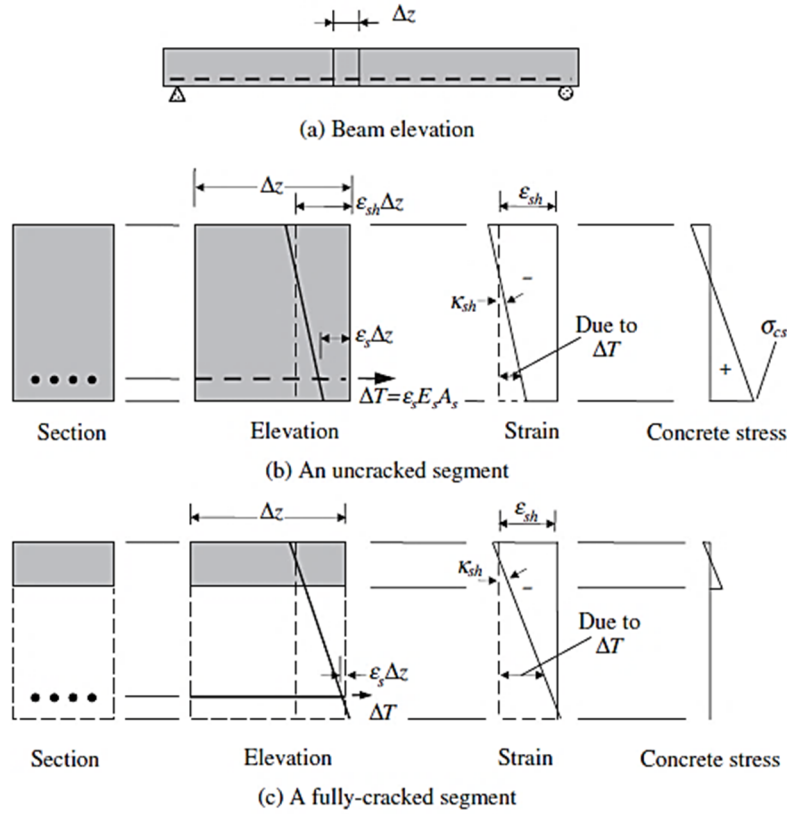


Figure 3.3 Strains and stresses in a singly reinforced beam, due to an hypothetical uniform shrinkage strain ϵ_{sh} [117]

As already observed for symmetrically reinforced concrete elements, cracking may occur prior to loading if the induced tensile stresses overcome the tensile strength of concrete. It is worth noting that shrinkage warping may significantly increase in case of cracked elements. Moreover, as in case of symmetrically reinforced members, creep effects and differential shrinkage should be always considered in simulations.

The importance of considering shrinkage becomes even more pronounced in case of externally restrained RC members. External restraint to shrinkage is very common in RC structure, due to the presence of supports of the structural elements or to the connection of the elements to other part of the structure. These restraints prevent the element to shorten or warp, causing internal stresses and deformations that, in a statically indeterminate structure, result in additional internal actions. As a matter of fact, the reactions of a restrained indeterminate member change because of shrinkage, leading to a possible significant redistribution of moment and shear, as well as to the appearance of tensile stresses that may cause premature cracking.

3.2.3 Numerical modeling of shrinkage effects

From a structural point of view, shrinkage can be taken into account in numerical modeling by following two alternative approaches, that is to say in indirect or direct way [143].

In the first case, material laws that couple tension stiffening with shrinkage are generally adopted. However, the use of “coupled” laws not produces accurate results, especially in presence of a not symmetric reinforcement, which causes a not uniform stress-strain distribution within the height of the section due to restrained shrinkage. This formulation does not consider indeed the actual stress field induced by shrinkage.

Shrinkage can be then better considered by including it explicitly in the modeling as a prescribed deformation or as a fictitious force (e.g., among others [34,53,140,142,145]). In this work shrinkage is inserted as a prescribed deformation applied to concrete.

In the following the two main approaches for evaluation of the effects induced by shrinkage will be discussed. It can be distinguished in models that consider the approximate overall (average) shrinkage strain, and models that apply true constitutive equations, considering the infinitesimal shrinkage of each part of the specimen [138], relating it to the moisture field .

According to the first approach, the mean drying effect on the cross section of the structural element is considered, so applying a uniform shrinkage strain for the overall cross section; hence the environmental humidity must be known, while the actual moisture distribution in the pores within the element is neglected. Obviously, such a formulation is not accurate in describing what happens at the cross-sectional level in a concrete element. The derived macroscopic shrinkage strain represents indeed the compatibilized value of the several infinitesimal shrinkages of the different layers of concrete, which are instead disregarded. The influence of the size and the shape of the sample on the kinetics of drying is computed in an approximated way, by applying semi-empirical coefficients.

This simplified modeling is very useful in design practice; however, it can lead to inaccurate predictions, since it does not allow to evaluate the actual non-uniform distribution of shrinkage strains throughout the cross section and the related self-equilibrated stresses induced by shrinkage to restore compatibility. As a matter of fact, Bazant and Baweja [146] stated that a really effective model for the prediction of the average shrinkage properties in the cross section under general loading and environmental conditions will never be possible and significant errors, increased complexity and a greater number of empirical parameters must be accepted when one decide to characterize the behavior of the cross section as a whole by its average properties. Nevertheless, formulations that adopt a sectional approach, are still widespread, especially in design practice; since such an approach allows to simply determine the effects of shrinkage in terms of deformation and critical load, neglecting the temporal and spatial evolution of the phenomenon, starting only from the average mechanical properties that characterize the material and from the environmental conditions, without passing through the solution of the thermo-hygrometric problem. As a

matter of fact this approach is that currently adopted in Design Codes, e.g. [115,147].

On the contrary, the second approach provides a more rigorous description of physical reality. In this case, true constitutive equations, which describe the behavior of a representative volume of the element, are applied and the value of shrinkage strain at each location of the element is individually evaluated, starting from the moisture field. The evolution of the moisture distribution in the pores is taken into account through the resolution of the moisture diffusion problem. The ambient humidity is a boundary condition and the cross-sectional dimensions and shape are implicitly considered in the spatial resolution of the drying differential equation. A detailed description of the moisture diffusion problem together with some considerations on boundary conditions will be provided in §3.3.2.

Once the moisture distribution within the element is known, it is possible to estimate its influence on the element behavior through a physical law that correlates the shrinkage strain distribution with the moisture gradient. Establishing a relation, at a local level (i.e. at the material level), between the water loss (or change in the relative humidity) and the resulting volumetric shrinkage strains is not so easy. As a matter of fact, in this field there are still many uncertainties, despite a lot of efforts dedicated over the years to this issue. This is mainly related to the difficulties related to the experimental measurements of totally unrestrained shrinkage strains. Experimental measurements of shrinkage strains are indeed affected by internal mechanical restraint of the samples, which causes an alteration of the strain field. Therefore, the difficulties in measuring totally unrestrained shrinkage strains prevent a clear extrapolation of the material (local) behavior from the structural (overall) one. Among the several relations proposed in technical literature over the years to relate the moisture potential to the local unrestrained shrinkage, in this thesis the widespread relation proposed by Bazant and Xi [148] is applied. Further details can be found in §3.3.2.1.

The application of this second refined approach is desirable in the case of very sensitive structures (such as record-span bridges, nuclear containments and vessels, large off-shore structures, large cooling towers, record-span thin roof shells and slender arch bridges [146]), where it is important to know the precise value of shrinkage deformation and the related induced stress field or in case of more accurate levels of analysis. In case of refined finite element modeling, such as the one applied in this thesis, the accurate determination of the local shrinkage strain starting from the moisture gradient, ought to be always used, since it is not convenient to input incorrect material properties into a very accurate computer program.

Therefore, in this thesis a step-by-step computer solution based on a general constitutive law, sequentially coupled with the solution of the differential equations for the diffusion problem, will be provided.

3.3 Inclusion of shrinkage effects into 2D-PARC model

In this section the main concepts concerning the extension of 2D-PARC model, in order to include in the algorithm the effects of concrete shrinkage that occurs prior to loading, will be detailed presented.

It should be reminded that 2D-PARC, [1], was originally developed for the analysis up to failure of RC or SFRC elements subjected to a general biaxial state of stress, but neglects any possible concrete initial prestrain. A comprehensive explanation of its original formulation can be found in Chapter 1, to which reference is made.

In the following, 2D-PARC, already revised in the modeling of concrete contribution as described in Chapter 2, will be further improved to consider shrinkage prestrains, by modify its fundamental equations.

Two revised formulations will be presented: the only difference between them lies in the evaluation of shrinkage prestrains at each integration point, while the new set of equilibrium and compatibility equations written is the same. In the first model, named in the following Model A, shrinkage prestrains are considered uniform within the element, whereas in the latter, called Model B, the actual shrinkage strain field related to the current moisture gradient is taken into account.

3.3.1 Model A

3.3.1.1 Uncracked stage

In the uncracked stage, concrete and steel are schematized like two materials working in parallel, by assuming perfect bond between them. Hence, with reference to the global co-ordinate system, concrete and steel strains, respectively denoted as $\{\epsilon_c\}$ and $\{\epsilon_s\}$, are considered equal to each other and to the total strain vector $\{\epsilon\}$, obtaining:

$$\{\epsilon\} = \{\epsilon_c\} = \{\epsilon_s\} \quad . \quad (3.5)$$

For equilibrium, the total stress $\{\sigma\}$ can be calculated as the sum of the stress in concrete $\{\sigma_c\}$ and in steel reinforcement $\{\sigma_s\}$.

To take into account shrinkage that occurs prior to loading, concrete stress $\{\sigma_c\}$ must be related only to net concrete strain $(\{\epsilon_c\} - \{\epsilon_{sh}\})$, being $\{\epsilon_{sh}\}$ the free shrinkage strain, assumed as negative according to 2D-PARC conventions. It is worth noting that shrinkage strain is independent from the internal state of stress of the considered element and the shear component is assumed equal to zero. The other terms of vector $\{\epsilon_{sh}\}$ are set equal to the average shrinkage values computed analytically as time-dependent variables adopting one of the several formulations reported in the Design Code or in technical literature [115,146,147]. These values, if desired, can be also set equal to the ones measured during experimental tests, if available.

The equilibrium condition for the uncracked RC element assumes the following expression:

$$\{\sigma\} = \{\sigma_c\} + \{\sigma_s\} = [D_c] \left(\{\varepsilon_c\} - \{\varepsilon_{sh}\} \right) + [D_s] \{\varepsilon_s\} . \quad (3.6)$$

being $[D_s]$ the steel stiffness matrix, which is evaluated by summing up all the contributions of the n reinforcement layers (see Chapter 1 and [1]), and $[D_c]$ the concrete stiffness matrix. This latter is herein evaluated by following the improved formulation for the description of concrete behavior under biaxial state of stress, presented in Chapter 2.

3.3.1.2 Cracked stage

The transition from uncracked to cracked stage takes place when the current state of stress violates the concrete failure envelope in the cracking region (see §2.3) and then a smeared fixed crack approach is followed. Crack pattern is assumed to develop at right angle with respect to principal tensile stress direction, with a constant crack spacing a_{m1} , and crack orientation is kept fixed throughout the loading process. As already observed, in case of shrunk elements, crack formation takes place at a lower loading level with respect to non-shrunk elements, due to the presence of tensile stresses in concrete even before the application of any external load.

After crack formation, a strain decomposition procedure is adopted. In this way, all the specific contributions related to what happen at crack location are individually analyzed and modeled; thus, avoiding the main disadvantages typical of classic smeared formulations, see §1.2.2.2 and §1.3.4. The total strain $\{\varepsilon\}$ is subdivided into two components, respectively related to intact material, even if damaged, between cracks, $\{\varepsilon_c\}$, and to all the phenomena taking place at crack surfaces, $\{\varepsilon_{cr1}\}$, so obtaining:

$$\{\varepsilon\} = \{\varepsilon_c\} + \{\varepsilon_{cr1}\} , \quad (3.7)$$

where the crack strain $\{\varepsilon_{cr1}\}$ is first evaluated in the crack local co-ordinate system (i.e. perpendicular and parallel to crack direction) as a function of crack width w_1 and sliding v_1 , and then transferred into the global one.

It should be pointed out that strain decomposition procedure is applied into 2D-PARC model in a different manner compared to the other constitutive model available in technical literature (for further details see §1.3.4); therefore, shrinkage effects must be inserted with a different strategy. In particular since the strain decomposition in 2D-PARC model is applied to the whole RC material (instead of referring only to concrete) close attention must be paid to applied the prescribed prestrain due to shrinkage only to concrete.

Following the equilibrium condition, the stress field in RC between adjacent cracks and that in the crack are assumed coincident to each other, being in equilibrium with the external applied stresses. The stress field in the crack, $\{\sigma_{cr1}\}$,

can be determined as in case of non-shrunk members, i.e. as the product between the crack stiffness matrix $[D_{cr1}]$ and the strain vector of the fracture zone $\{\varepsilon_{cr1}\}$, so obtaining:

$$\{\sigma\} = \{\sigma_{cr1}\} = \{\sigma_{c,cr1}\} + \{\sigma_{s,cr1}\} = \left([D_{c,cr1}] + [D_{s,cr1}] \right) \{\varepsilon_{cr1}\} = [D_{cr1}] \{\varepsilon_{cr1}\} . \quad (3.8)$$

Matrix $[D_{cr1}]$ is calculated by separately considering the resistant contributions due to concrete $[D_{c,cr1}]$, (i.e. aggregate bridging and interlock) and due to steel $[D_{s,cr1}]$, (i.e. tension stiffening and dowel action). The complete expression of $[D_{cr1}]$, which is, once again, not modified in presence of shrinkage, can be still found in §1.3.4.3 and §1.3.4.4, together with a more detailed description of the constitutive laws adopted for the modeling of each resistant mechanism. It is worth noting that shrinkage occurring before the application of external loads does not cause bond deterioration [132] and consequently the bond-slip relation adopted in the model is the same used for non-shrunk elements (see §1.3.4.4).

The evaluation of the stress field in RC material between two adjacent cracks, which can be expressed as the sum of the stress in concrete and steel, requires to modify the equilibrium equation in order to take into account shrinkage influence, so leading to the same expression already derived for the uncracked staged – see Equation (3.6). The steel and the concrete stiffness matrices, $[D_s]$ and $[D_c]$, are determined from the corresponding ones of the uncracked stage, with slight modifications, see respectively §1.3.4.2 and §2.3.5. It should be pointed out that in the cracked stage the hypothesis of perfect bond is no longer valid; consequently, the two strain vectors relative to concrete $\{\varepsilon_c\}$ and to steel $\{\varepsilon_s\}$ between two adjacent cracks cannot be assumed coincident with each other. For sake of simplicity, the steel strain $\{\varepsilon_s\}$ is assumed coincident with the total average strain $\{\varepsilon\}$, due to the limited difference between them. The concrete strain between two contiguous cracks $\{\varepsilon_c\}$ and the crack strain $\{\varepsilon_{cr}\}$ are then obtained by inverting the equilibrium conditions respectively in the uncracked RC between two adjacent cracks - Equation (3.6) - and at crack locations- Equation (3.8):

$$\{\varepsilon_c\} = [D_c]^{-1} \left(\{\sigma\} - [D_s] \{\varepsilon\} \right) + \{\varepsilon_{sh}\} , \quad (3.9)$$

$$\{\varepsilon_{cr1}\} = [D_{cr1}]^{-1} \{\sigma\} , \quad (3.10)$$

which, substituted into compatibility Equation (3.7), lead to:

$$\begin{aligned} \{\varepsilon\} &= [D_c]^{-1} \left(\{\sigma\} - [D_s] \{\varepsilon\} \right) + \{\varepsilon_{sh}\} + [D_{cr1}]^{-1} \{\sigma\} \\ &= \left([D_c]^{-1} + [D_{cr1}]^{-1} \right) \{\sigma\} - [D_c]^{-1} [D_s] \{\varepsilon\} + \{\varepsilon_{sh}\} . \end{aligned} \quad (3.11)$$

The total stress vector $\{\sigma\}$ in the global co-ordinate system x-y can be then expressed as:

$$\begin{aligned} \{\sigma\} &= \left([D_c]^{-1} + [D_{cr1}]^{-1} \right)^{-1} \left([I] + [D_c]^{-1} [D_s] \right) \{\varepsilon\} - \{\varepsilon_{sh}\} \\ &= [D] \{\varepsilon\} - \left([D_c]^{-1} + [D_{cr1}]^{-1} \right)^{-1} \{\varepsilon_{sh}\}, \end{aligned} \quad (3.12)$$

being $[I]$ the identity matrix and $[D]$ the global RC stiffness matrix for non-shrunk elements.

3.3.1.3 Fiber contribution

The formulation described in §3.3.1.1 and §3.3.1.2 for ordinary reinforced concrete elements can be also applied to fiber reinforced members, since the only difference between them lies in the adopted constitutive laws. The global algorithm of 2D-PARC model, thanks to its modular framework, is indeed kept unchanged. Concrete shrinkage is inserted also in this case as a prescribed deformation and the presence of fibers does not modify the aforementioned procedure; therefore, the same rules described in §3.3.1.1 and §3.3.1.2 are followed.

In more detail, in the uncracked stage the formulation in presence of fibrous reinforcement remains almost the same adopted for RC elements, due to the limited influence that fibers exert until cracking occurs. As known their presence only modifies the post-peak branch of concrete law in compression: this stiffening contribution is simply taken into account as for non-shrunk members by modifying the compressive post-peak parameter B , as explained in §2.3.3.1.

Also in the cracked stage fiber influence is considered as for not-shrunk members. In more detail, as far as SFRC between two adjacent cracks is concerned, only matrix $[D_c]$ is modified, so as to take into account fiber inclusion, by adopting a proper value for B , as already done for the uncracked stage. As regards the kinematics of the fracture zone, crack stiffness matrix $[D_{cr1}]$ is modified to account for the additional resistant mechanisms offered by fibers across crack surfaces, by means of proper constitutive laws already implemented into 2D-PARC model, as discussed in § 1.3.4.5. An additional term due to fiber bridging is inserted into crack stiffness matrix relative to concrete $[D_{c,cr1}]$, while the bond-slip law necessary to define crack stiffness matrix relative to steel $[D_{s,cr1}]$ is properly modified so as to correctly represent the confinement action exerted by fibers. In particular the bond-slip law proposed by Harajli et al. [61,62] and the tension softening law by Li et al. [63,64] are applied.

3.3.1.4 Model implementation and convergence checks in the cracked stage

As already explained in §1.3.5, 2D-PARC model is implemented into the adopted finite element code, ABAQUS, in the form of a so-called user-material (UMAT) subroutine. In correspondence of each loading increment (or iteration within a fixed loading increment), the updated strain vector $\{\varepsilon\}$ in the global co-ordinate system is provided by ABAQUS and read by the user subroutine. Starting from this strain, the stiffness matrix and the total stress vector are evaluated in the subroutine according to the current condition (uncracked or cracked) of each integration point in the FE model, and then passed back to ABAQUS.

Since the adopted approach requires a strain decomposition in the cracked stage, an iterative procedure is followed in the user subroutine to achieve a converged solution, which should be properly corrected when shrinkage strains are included in the model. The adopted technique is schematically shown in the flow-chart reported in Figure 3.4.

For reading convenience, all the quantities are directly expressed in the global co-ordinate system, without reporting the necessary transfer operations (each variable is indeed first determined in its own local co-ordinate system).

For the k^{th} iteration within a fixed loading increment, $\{\varepsilon\}^k$ is the strain vector provided by the FE code. In the user subroutine, a first attempt value is assigned to the strain of the uncracked concrete between cracks $\{\varepsilon_c\}^j$, which is assumed equal to the one calculated during the previous internal iteration. The crack strain $\{\varepsilon_{cr1}\}^j$ is calculated as the difference between the external total strain provided by ABAQUS and that in concrete between cracks from previous iteration. Starting from these values, the stiffness matrices relative to uncracked concrete $[D_c]^j$, to steel between cracks $[D_s]^j$ and to the crack $[D_{cr1}]^j$ are first calculated and subsequently the global stiffness matrix $[D]^j$ is determined, accordingly to the procedure already described. Finally, the new stress vector $\{\sigma\}^j$ is first obtained by taking into account shrinkage influence – Equation (3.12), and subsequently the strain values in uncracked concrete and in the crack are properly updated. Also in this case, shrinkage strain should be properly included in the evaluation of vector $\{\varepsilon_c\}^j$. The update strain vectors $\{\varepsilon_c\}^j$ and $\{\varepsilon_{cr1}\}^j$ are then summed up to obtain the total strain $\{\varepsilon\}^j$. Subsequently convergence on strains is performed by verifying that the difference between each updated value and that calculated in the previous internal iteration ($j-1$) is within a predefined tolerance and by comparing the total strain $\{\varepsilon\}^j$ to the current strain vector $\{\varepsilon\}^k$ provided by ABAQUS. If all the convergence checks are overcome, the last calculated stiffness matrix and the total stress vector are passed from the user subroutine to the FE code; otherwise an iterative procedure starts until convergence on strains is reached.

It should be remarked that the inclusion of shrinkage strains in the model also influences the internal convergence procedures respectively required for the evaluation of the concrete stiffness matrix $[D_c]$, and for the calculation of tension stiffening contribution in matrix $[D_{s,cr1}]$.

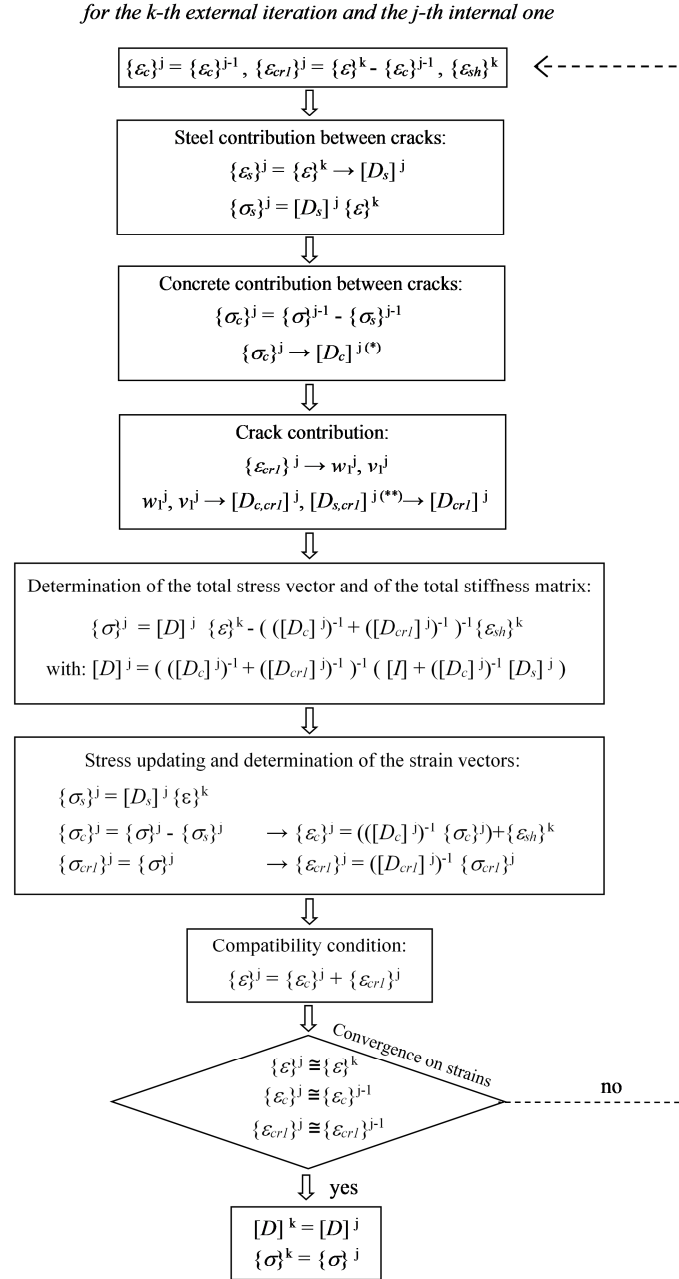


Figure 3.4 Flow chart of the internal iterative procedure adopted in the cracked stage

As better discussed in Chapter 2, matrix $[D_c]$ is expressed as a function of the secant values of concrete Young modulus E_c and Poisson coefficient ν_c , which are properly updated during the analysis to account for material nonlinearity. These two parameters are evaluated through an iterative procedure based on the bisection method, by progressively narrowing a predefined stress interval until the achievement of a convergence check on stresses. In presence of shrinkage, the expression of this convergence check is properly modified by relating the concrete stresses evaluated in the principal directions 1-2 only to the corresponding concrete net strains, so obtaining:

$$\sigma_{ic} - \frac{E_c}{1 - \nu^2} [\nu (\varepsilon_{jc} - \varepsilon_{shj}) + (\varepsilon_{ic} - \varepsilon_{shi})] = tol \quad . \quad (3.13)$$

where subscript i and j are respectively equal to 1 and 2 in case of biaxial tension and conversely to 2 and 1 in case of tension-compression or biaxial compression and tol represents the chosen tolerance value, adequately small. Actually the convergence check can be always performed indistinctly on both principal stresses, owing to the isotropic nature of the adopted constitutive model for concrete (see §2.3). When the condition reported in Equation (3.13) is satisfied, convergence is assumed to be reached on stresses. Consequently, the corresponding elastic parameters E_c and ν represent the updated values that can be used for the construction of concrete stiffness matrix $[D_c]$ of the analyzed integration point.

As regards tension stiffening contribution, it should be reminded that the non-uniform distribution of the strains in the reinforcement is locally evaluated in 2D-PARC model by solving – at each integration point – the governing equations of the problem (i.e. equilibrium conditions for the whole section, for concrete and for steel, as well as compatibility condition) through the finite difference method, [60]. Shrinkage effects should be then properly included in the evaluation of stress and strain distribution in concrete. Moreover, it is necessary to ensure global compatibility by imposing that the mean value of steel strains computed between two adjacent cracks from the tension stiffening formulation is equal to the total strain evaluated in the global procedure, which is in turned modified in presence of shrinkage according to the flow chart reported in Figure 3.4.

3.3.2 Model B

Concrete shrinkage is inserted in the algorithm as a prescribed deformation also in this case; thus, the formulation adopted for Model A, described in §3.3.1, is kept unchanged: the fundamental equations governing the model should be rewritten in the same manner, so leading to the expressions already derived for Model A. The stress field can indeed be expressed by Equations (3.6) and (3.12) respectively for the uncracked and cracked stage. Moreover, the same formulation applied also in case of fibrous reinforcement. The difference between Model A and Model B lies instead in the evaluation of the shrinkage strain vector.

Model B evaluates the actual value of the shrinkage strain associated to each integration point as a function of the moisture field in the element. As explained in the following, this latter is in turn obtained by performing an “equivalent” thermal analysis by exploiting the analogy between the equations governing the moisture and the thermal problem, with a simple substitution of the corresponding parameters. Finally, moisture field is related to shrinkage strains and the stress-strain field in the element is computed by performing the structural analysis. Following this procedure, the differential shrinkage strains (and the related eigen-stresses) between the faces exposed to drying and the core are taken into account in the analysis, so leading to more realistic results.

Simulations are performed by considering two separate mechanisms, namely drying shrinkage and autogenous shrinkage, both related to the decrease of internal relative humidity.

As already explained in §3.2.1, concrete elements subjected to ambient conditions lose indeed capillary water through diffusion and evaporation until the internal relative humidity of concrete is in balance with that of the surrounding environment. A volumetric change of the element takes place due to drying, so leading to the called drying shrinkage. This phenomenon is the most important and often is the only one considered in simulations (for further details, e.g. [149–152]). In this thesis, also autogenous shrinkage is considered, even if this latter effect gains relevance especially at early-age and for concrete characterized by low water-cement ration (typically $w/c < 0.39$) - among others, [153,154]. This phenomenon is related to the process of hydration of the cement paste that causes an additional decrease in moisture content, the so called self-desiccation.

Hence, concrete shrinkage can be successfully predicted only by relating it to the moisture state and its variation in space and time. In more detail, the moisture distribution in a concrete element is non-uniform. As a matter of fact, as already mentioned in §3.2.1, neither autogenous nor drying shrinkage occur uniformly: the rate of hydration is indeed higher in the core because of the presence of stronger chemical reactions and also the drying process proceeds with different speed depending on the distance of each point of the element to the drying surface, leading to a non-uniform field as well. An uneven tendency to shrink occurs: while the exposed surfaces tend to shrink a lot the inner fibers have no tendency to shrink. Differential shrinkage occurs, leading in turn to self-equilibrated stresses: to restore compatibility, the external fibers are subjected to tension while the core is compressed.

Since shrinkage at each point of the element is related to the moisture distribution within the member and its evolution with time, the first step of the numerical analysis is related to the determination of the moisture field in concrete.

There is a general agreement in recognizing that moisture flow in concrete subjected to drying is governed by diffusion principles (among others, [134,155–158]) especially when the average moisture content decreases below 70 to 80% of the initial saturation, [159].

Mass transfer equation for an isotropic material subjected to isothermal condition can be written as ([124,158,160]):

$$\frac{\partial W_e}{\partial t} = \text{div} (D \text{ grad } W_e) - \frac{\partial W_n}{\partial t} \quad (3.14)$$

where D is the diffusion coefficient and W_e and W_n stands respectively for evaporable and non-evaporable water. Thus, the last term of Equation (3.14) is related to the consumption of water due to chemical reactions.

Alternatively, the nonlinear diffusion equation can be written as a function of the internal pore humidity h instead of the water concentration W as the driving potential, so obtaining (see e.g. [57,126,155,158]):

$$\frac{\partial h}{\partial t} = \left(\frac{\partial W}{\partial t} \right)^{-1} \text{div} (D_h \text{ grad } h) + \frac{\partial h_s}{\partial t} \quad (3.15)$$

being D_h the moisture diffusion coefficient and h_s the self-desiccation humidity. Thus, the first term at the right-hand side accounts for the moisture change due to drying, while the latter for humidity variation caused by water consumed in chemical reactions.

This transformation allows to easily consider the boundary conditions ([161,162]) and to better link moisture field with a subsequent shrinkage analysis ([124,155]). Moreover, internal profiling of moisture distribution becomes more feasible utilizing h instead of W ([126,158,163–166]). Thus, in this work the formulation in terms of internal pore humidity h is applied.

To pass from Equation (3.14) to Equation (3.15) the factor $(\partial W / \partial t)^{-1}$ that represents the reciprocal of the slope of the moisture sorption isotherm curve (also called moisture capacity) must be added. However, considering that the moisture capacity of concrete at usual environmental conditions ($h > 50\%$) can be assumed reasonably constant ([162]), Equation (3.15) can be rewritten as:

$$\frac{\partial h}{\partial t} = \text{div} (D^* \text{ grad } h) + \frac{\partial h_s}{\partial t} \quad (3.16)$$

by assembling terms $(\partial W / \partial t)^{-1}$ and D_h to obtain a single coefficient D^* .

This latter, for isothermal conditions, can be expressed as a function of the pore relative humidity ($0 < h < 1$) as:

$$D^*(h) = D_1 \left(\alpha + \frac{1 - \alpha}{1 + [(1 - h)/(1 - h_c)]^p} \right) \quad (3.17)$$

where D_1 is the maximum of $D^*(h)$ for $h = 1$, h_c is the pore relative humidity at $D^*(h) = 0.5 D_1$, α is the ratio between D_0 and D_1 , being D_0 the minimum of $D^*(h)$ for $h = 0$, while p is an exponent. According to Model Code 2010 [57], $\alpha = 0.05$,

$h_c = 0.80$, $p = 15$ and $D_1 = D_{1,0} / (f_{cm} - 8)$ are commonly assumed, $D_{1,0}$ being equal to $1 \cdot 10^{-8} \text{ m}^2/\text{s}$ and f_{cm} being the mean concrete compressive strength.

As regards the boundary conditions to be applied to the moisture field, two main different approaches can be used. A possible solution is to impose the value of environmental humidity to the concrete surfaces, so-called Dirichlet boundary condition (see e.g. [167]). Alternatively, Neumann boundary condition can be set (see e.g. [124,127,158]). This latter better represents reality and is the most widespread strategy, consequently it is also adopted in this work. It consists in the introduction of a proportionality factor $f_{boundary}$ between the exposed surface flux and the difference between the environmental humidity h_{en} and the relative humidity of the exposed surface h_{surf} , so obtaining:

$$D^* \left(\frac{\partial h}{\partial n} \right)_{surf} = f_{boundary} (h_{en} - h_{surf}) \quad (3.18)$$

where n is the normal axis of the boundary surface.

3.3.2.1 Adopted modeling strategy

Moisture distribution within plain and reinforced concrete elements is herein evaluated by exploiting the formal analogy between the moisture and thermal problems, through a simple substitution of the parameters governing the corresponding diffusion equations. There are indeed striking resemblances between Equation (3.16) with its boundary condition – Equation (3.18) – describing the diffusion problem and the well-known heat equations.

This choice is related to the fact that the adopted commercial FE Code (ABAQUS) has only the heat transfer option and not the moisture diffusion one. Moreover, temperature belongs to those field variables which are passed from ABAQUS to the user-material subroutine, where 2D-PARC is implemented, that describes the constitutive behavior of the material.

As a consequence, the mutual correspondence between thermal and moisture quantities is exploited. In more detail, temperature is considered in spite of moisture content and thermal conductivity in spite of moisture diffusivity. Furthermore, density and specific heat are set equal to unity in thermal analyses, so as to avoid the presence of extra-multipliers that are necessary in the heat transfer equation, but not in the moisture diffusion one. As regards boundary conditions, the moisture emissivity coefficient is substituted by convection heat transfer one. Moreover, the self-desiccation is taken into account by considering the internal heat generation in place.

A sequentially-coupled thermo-mechanical analysis is then performed; in this way, temperature (i.e. moisture) values in each node of the FE mesh are read by 2D-PARC subroutine and used for the evaluation of the free shrinkage strain vector $\{\epsilon_{sh}\}$. The assumption that micro-cracking due to shrinkage does not influence the moisture transport has been highlighted by several research ([168–170]), since the usual width of these cracks is remarkably lower than the limit value, according to [171], which begins to influence the drying rates.

Several numerical formulations relate the moisture potential to the local unrestrained shrinkage, among them the well-known relation suggested in [148] is applied in this work. In more detail, the free shrinkage strain is calculated as a function of pore relative humidity h through the following relation:

$$\dot{\varepsilon}_{sh} = k_{sh} \dot{h} \quad (3.19)$$

where the shrinkage coefficient k_{sh} is defined as the product between the ultimate drying shrinkage at complete drying (ε_s^0) and the ratio of elastic modulus with time ($E_c(t_0) / E_c(t)$). The same approach has been applied also in [127,172–176]

Since shrinkage-induced stresses develop gradually with time, creep effects should be also considered in the analyses. To this aim, the compliance function is evaluated in 2D-PARC model by following the procedure described in [177], to which reference is made. The same approach is also adopted for model A; its description is not discussed herein for brevity, being out of the scope of this work.

Once the shrinkage strain vector is evaluated, strain and stress fields in the member are computed by applying the same formulation presented in §3.3.1.

3.4 Comparison between numerical and experimental results

In order to verify the effectiveness of the proposed models, the results obtained from numerical analyses are compared with some experimental data from technical literature.

At first Model A is considered, so as to validate the inclusion of shrinkage pre-strains in 2D-PARC model, as well as the implementation of the revised formulation into the commercial FE code adopted (ABAQUS).

Subsequently the results of more refined Model B are analyzed, in order to validate the effectiveness of the proposed approach in representing moisture distribution and the relative differential shrinkage within the element; thus, leading to more reliable strain and stress fields.

3.4.1 Model A

To validate the proposed procedure over a wide range of conditions, Model A is verified through comparison between numerical and experimental results relative to different structural typologies and concrete mix (i.e. with the addition of fibrous reinforcement or not).

At first, Model A is verified through the simulation of tension members. These types of elements are chosen because their behavior is significantly affected by shrinkage and they represent the simpler RC elements to be study. The extensive experimental program carried out by Deluce [128] on both RC and SFRC tension members is selected, so as to validate the model also in case of fibrous reinforcement.

Then the model is applied to more complex specimens, where 2D-PARC can express all its potentialities; in more detail, RC beams in bending [130,131] are selected for comparison. These elements are characterized by a stress and strain gradient. Moreover, shrinkage causes not only the reduction of the cracking load but also the warping of the member since the reinforcement is not symmetrically placed in the depth of the section, as detailed explained in §3.2.2.2. Two experimental programs are considered [130,131]. The first tests, carried out by Sato et al. [130], are simulated to compare the flexural behavior of beams subjected or not to shrinkage. The experimental program undertaken by Gribniak [131] is instead considered to highlight the effects of concrete shrinkage on deflections and cracking moment in case of different amount and distribution of reinforcement.

3.4.1.1 RC and SFRC tension members tested by Deluce

The proposed model is first applied to the analysis of tension ties with and without the addition of fibrous reinforcement in the concrete mix. Among the extensive program carried out by Deluce [128], the attention is herein on focused twelve tension ties.

All the samples were characterized by a length of 1000 mm and by a square section with three different sides (50, 80, 100 mm, for specimens H50, H80 and H100, respectively). The amount of reinforcement was the same for H50 and H80 - constituted by a central steel rebar of 11.3 mm diameter - while for specimens H100 the central steel bar was 19.5 mm diameter. For each specimen, four different concrete mixes were adopted: one non-fibrous RC series and three series containing hooked-end high-carbon steel fibers, having a length of 30 mm and a diameter of 0.38 mm. The fiber volume fractions were 0%, 0.5%, 1.0% and 1.5%, for the elements with designation PC, FRC1, FRC2, FRC3 respectively.

Tests were performed by means of hydraulic servo-controlled (closed-loop) testing machine.

The main mechanical properties of steel and concrete are provided in Table 3.1, together with the average free shrinkage strain values ε_{sh} measured at the date of testing on three prismatic specimens for each concrete batch. Further details can be found in [128], to which reference is made. The experimental value of ε_{sh} is used to compose the shrinkage strain vector that must be defined in the numerical model, as described in §3.3.1.

Since shrinkage-induced stresses develop gradually with time, the relief caused by creep should be included in numerical simulations, as suggested by many Authors in the literature (e.g., [121,142]). The procedure suggested in [177] is herein adopted.

Sample	Concrete					Steel	
	V_f (%)	f_c [MPa]	f_{ct} [MPa]	E_c [MPa]	ϵ_{sh} (10^{-6})	f_{sy} [MPa]	E_s [MPa]
H50/10 PC	0.0	91.70	4.93	38500	-310		
H50/10 FRC1	0.5	85.25	3.50	39200	-550	442	199000
H50/10 FRC2	1.0	57.80	3.41	26200	-630		
H50/10 FRC3	1.5	52.30	3.12	21200	-830		
H80/10 PC	0.0	91.70	4.93	38500	-310		
H80/10 FRC1	0.5	79.15	3.62	35500	-600	442	199000
H80/10 FRC2	1.0	57.50	3.41	26200	-630		
H80/10 FRC3	1.5	52.90	3.12	21200	-830		
H100/20 PC	0.0	92.20	4.93	38500	-310		
H100/20 FRC1	0.5	91.40	3.28	35000	-540	456	194000
H100/20 FRC2	1.0	58.10	3.41	26200	-630		
H100/20 FRC3	1.5	62.00	3.40	28000	-780		

Table 3.1 Material properties of RC/SFRC tension members tested by Deluce [128]

Taking advantage of the symmetry of the problem, only one half of each tensile member is modeled, by adopting a mesh constituted by quadratic, isoparametric 8-node membrane elements with reduced integration (4 Gauss integration points).

In the performed analyses, a uniform displacement is applied to all the nodes belonging to the terminal section of the specimen, in order to achieve a better numerical convergence.

In the following, for sake of brevity, only the most significant results are presented, the others, being similar, are herein omitted.

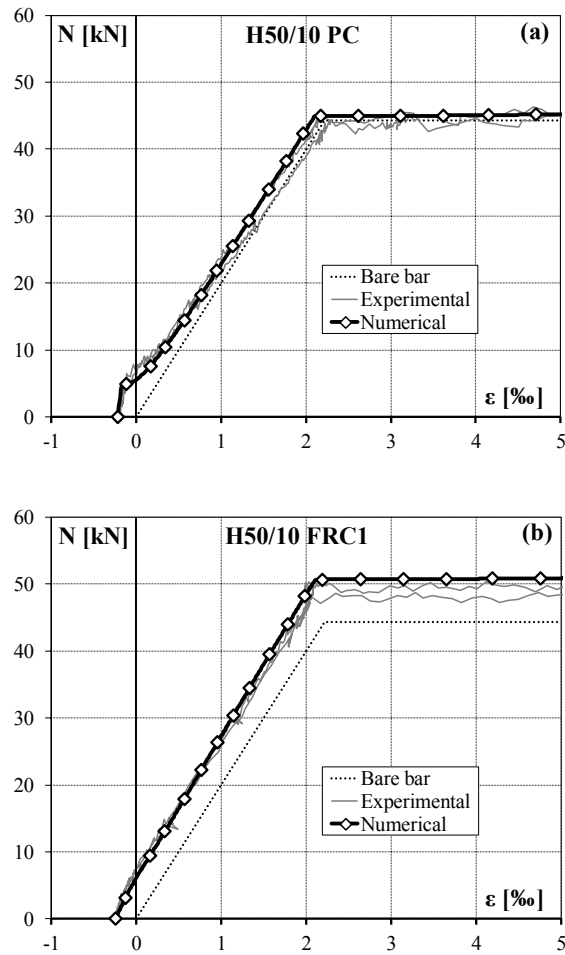


Figure 3.5 Comparison between numerical and experimental [128] results in terms of axial load N vs. average axial strain ε for specimens: (a) H50/10 PC and (b) H50/10 FRC1

Numerical results are compared to experimental evidences in terms of axial load N vs. average axial strain ε . Since the initial shortening due to shrinkage was not experimentally measured, the corresponding value obtained from FE analyses is used to shift experimental curves backwards along the horizontal axis, so as to make easier the comparison with the numerical outcomes. The same procedure was adopted also by Deluce [128].

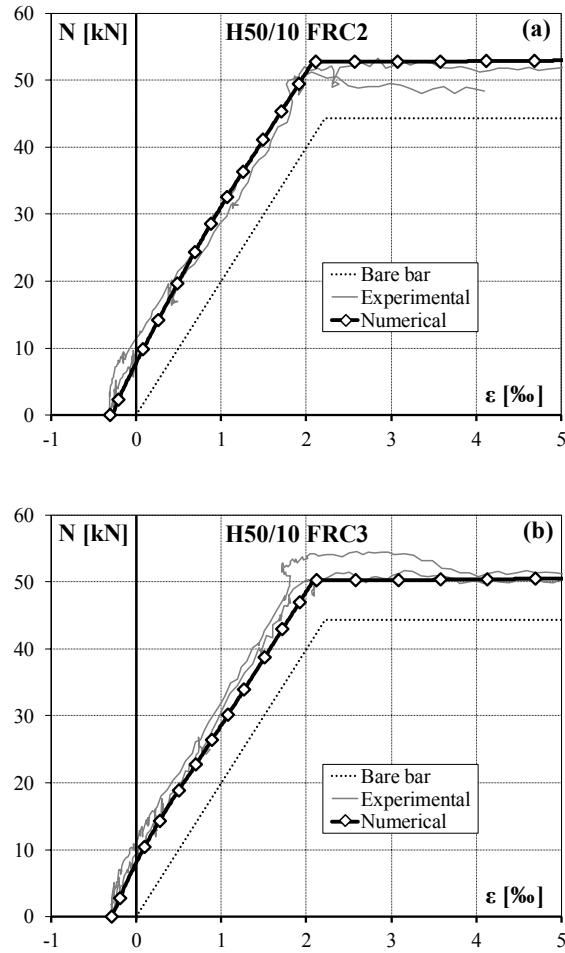


Figure 3.6 Comparison between numerical and experimental [128] results in terms of axial load N vs. average axial strain ε for specimens: (a) H50/10 FRC2 and (b) H50/10 FRC3

As can be seen from Figure 3.5-Figure 3.8, the model is able to well describe the global behavior of tension members subjected to shrinkage prior to loading. Satisfactory results are provided for all the analyzed tension ties, independently of their dimensions, reinforcement ratio and fiber content, so proving the versatile nature of the proposed approach.

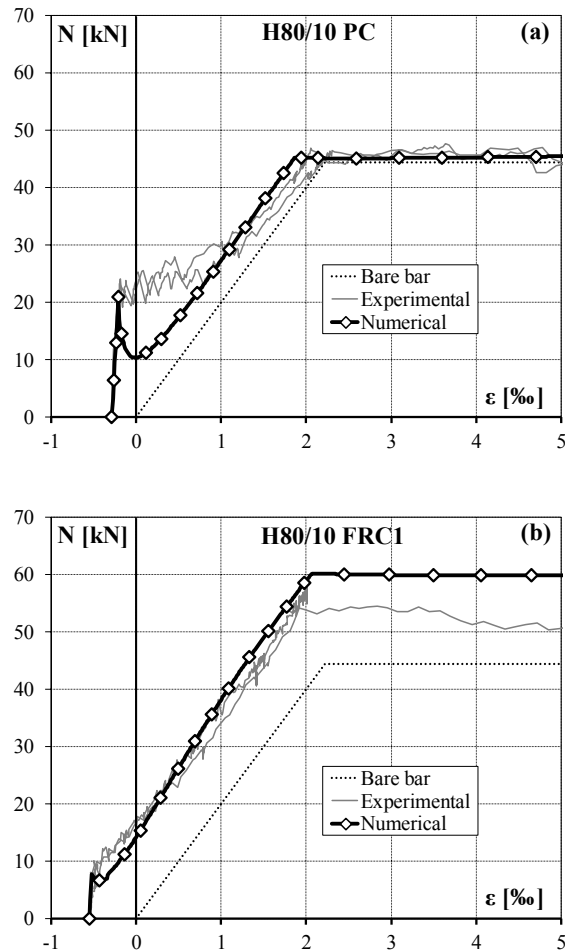


Figure 3.7 Comparison between numerical and experimental [128] results in terms of axial load N vs. average axial strain ε for specimens: (a) H80/10 PC and (b) H80/10 FRC1

The cracking load (that is reduced due to shrinkage prestrains that cause tensile stress in the concrete balanced by compression in the rebar) is satisfactory caught, as well as the initial shortening of the element. Moreover, both the initial stage of crack formation as well as the stabilized cracking stage, when the tension stiffening contribution slightly decreases due to bond degradation, are well described.

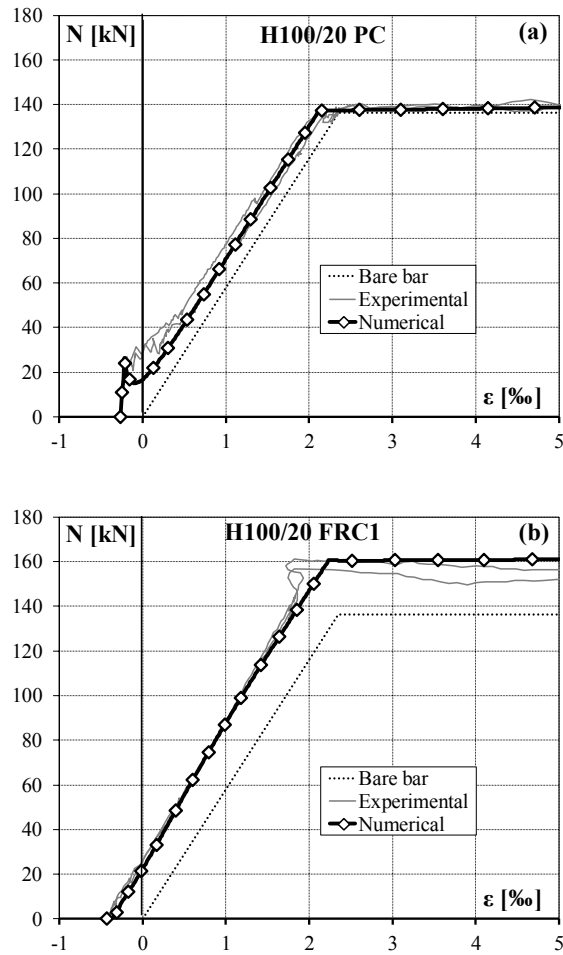


Figure 3.8 Comparison between numerical and experimental [128] results in terms of axial load N vs. average axial strain ε for specimens: (a) H100/20 PC and (b) H100/20 FRC1

It is worth noting that the specimens were characterized by a significant initial shortening before testing and in some cases this led to cracking before the application of the load. In this case the model enters the cracking stage at the beginning of the analysis and it calculates the total strain, by dividing it into two components respectively referring to concrete between two adjacent cracks and to the fracture zone, by adopting the iterative procedure described in §3.3.1.4, while assuring that the total applied force is equal to zero. Therefore, the model is verified through a wide range of conditions, including the case of pre-cracked specimens and also in presence of fibrous reinforcement.

3.4.1.2 RC beams tested by Sato et al.

Two RC beams tested by Sato et al. [130], named V-01-13WB and V-01-13DB, and subjected to four-point bending are analyzed.

The selected specimens were characterized by the same geometrical details and amount of reinforcement, having a size of 150 x 200 x 2800 mm (with a net span of 2200 mm) and two D13 bars as tensile reinforcement.

They belonged to the same concrete batch, but were subjected to different curing conditions. After demolding, beam V-01-13WB was sealed at room temperature with saturated paper ("Wet curing") until the time of testing, whereas beam V-01-13DB was first subjected to wet curing for one week and subsequently exposed to room atmosphere for 114 days ("Drying condition"). The main mechanical properties of steel and concrete, together with the average free shrinkage strain ε_{sh} measured at the date of testing, are summarized in Table 3.2. Also in this case ε_{sh} is used to composed the shrinkage strain vector that must be defined in the model (see §3.3.1).

Sample	Steel				Concrete		
	$A_{s,bottom}$ [mm ²]	E_s [GPa]	f_{sy} [MPa]	f_c [MPa]	$f_{ct,sp}^*$ [MPa]	E_c [GPa]	ε_{sh} (10 ⁻⁶)
V-01-13WB	253.4	193.2	353	30.6	2.9	27.5	-
V-01-13DB	253.4	193.2	353	32.5	3.0	28.5	-425

*: f_{ct} adopted in the analyses is calculated from $f_{ct,sp}$ according to [115]

Table 3.2 Material properties of RC beams tested by Sato et al. [130]

The two beams were subjected to four-point bending test, with the two concentrated load symmetrically placed at 400 mm from the center section.

Numerical analyses are performed under displacement control; moreover, taking advantage of the symmetry of the problem, only one half of each beam is simulated, by adopting a FE mesh constituted by quadratic, isoparametric 8-node membrane elements with reduced integration (4 Gauss integration points).

Figure 3.9 shows the comparison between numerical and experimental [130] results in terms of bending moment M vs midspan deflection δ . Since initial deflections due to shrinkage were not experimentally measured, numerical curves are shifted to the origin of the horizontal axis, so as to match experimental data.

For both the considered beams good agreement between numerical and experimental results can be stated during the entire loading history.

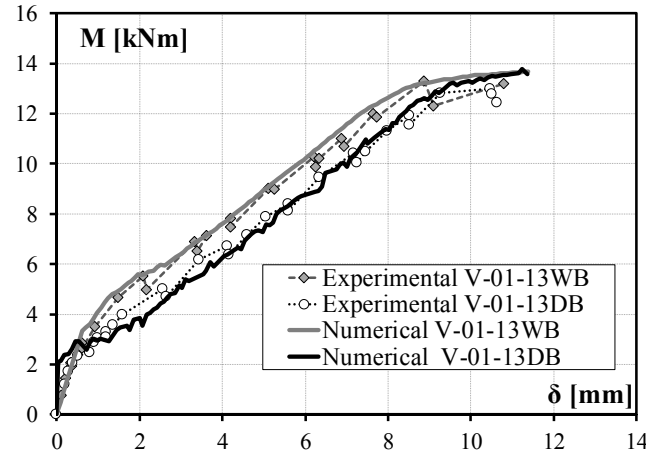


Figure 3.9 Comparison between numerical and experimental [130] results in terms of in terms of bending moment M vs. midspan deflection δ

As can be seen from Figure 3.9, the shrunk specimen (V-01-13DB) is characterized by a reduced cracking load and a larger deflection with respect to sample V-01-13WB, due to the presence of shrinkage-induced tensile stresses in concrete before loading. It is also worth noting that in the cracked stage the curves still remain almost parallel to each other, since all the involved resistant contributions remain the same.

Sample	M_{cr} [kNm]		δ_s^{**} [mm]		$w_{s,max}^{**}$ [mm]		$w_{s,av}^{**}$ [mm]	
	Exp.	Num.	Exp.	Num.	Exp.	Num.	Exp.	Num.
V-01-13WB	3.6	3.2	3.7	3.7	0.12	0.10	0.07	0.05
V-01-13DB	2.1	2.1	4.6	4.7	0.12	0.15	0.08	0.06

******: Values at $M=7.2$ kNm

Table 3.3 Numerical vs. experimental [130] results under serviceability conditions

Additional comparisons are provided in Table 3.3, where numerical and experimental values of cracking moment M_{cr} as well as deflection δ_s , average and maximum crack width ($w_{s,av}$ and $w_{s,max}$) under serviceability conditions are listed, showing good agreements.

Finally, a satisfactory result is also found in terms of crack pattern at failure, as proved by Figure 3.10, which reports numerical and experimental results for the shrunk specimen V-01-13DB.

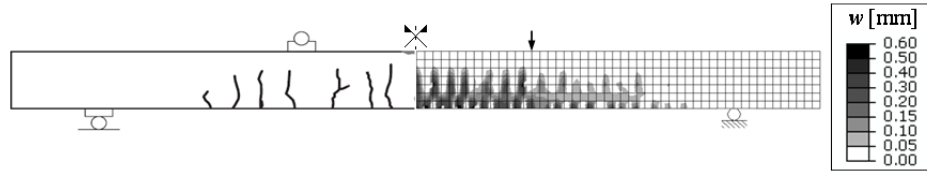


Figure 3.10 Numerical and experimental [130] crack pattern at failure for specimen V-01-13DB

3.4.1.3 RC beams tested by Gribniak

Four RC beams tested by Gribniak [131] – respectively named S1, S1R, S2, S2R – and subjected to four-point bending, are selected for further comparisons.

The considered specimens were characterized by the same geometry with a rectangular cross section (300 deep and 280 wide) and a total length equal to 3280 mm (with a net span of 3000 mm). Series 1 and 2 were obtained from different concrete batches, showing slightly different compressive strengths f_c , as reported in Table 3.4.

Sample	Steel		Concrete			
	$A_{s,bottom}$ [mm ²]	$A_{s,top}$ [mm ²]	E_s [GPa]	f_{sy} [MPa]	f_c [MPa]	ϵ_{sh} (10 ⁻⁶)
S1	309.0	56.6	212	566	47.3	-194.6
S1R	309.0	749	212	566	47.3	-188.2
S2	309.0	56.6	212	566	48.7	-152.6
S2R	309.0	749	212	566	48.2	-155.7

Table 3.4 Material properties of RC beams tested by Gribniak [131]

All the specimens contained the same amount of lower tensile reinforcement (denoted as $A_{s,bottom}$ in Table 3.4, corresponding to 4 ϕ 10 mm bars), as well as of transverse reinforcement (ϕ 6 mm / 100 mm spaced). The two specimens with designation “R” had an higher amount of top reinforcement ($A_{s,top}$, Table 3.4), constituted by 3 ϕ 18 mm bars, instead of 2 ϕ 6 mm bars. The main mechanical properties of steel reinforcement are still provided in Table 3.4, together with concrete compressive strength f_c and shrinkage strains ϵ_{sh} measured at the test date. As regards concrete properties, only f_c is provided in [131]; as a consequence, the other parameters required to perform the simulations are evaluated through the correlations suggested in [115].

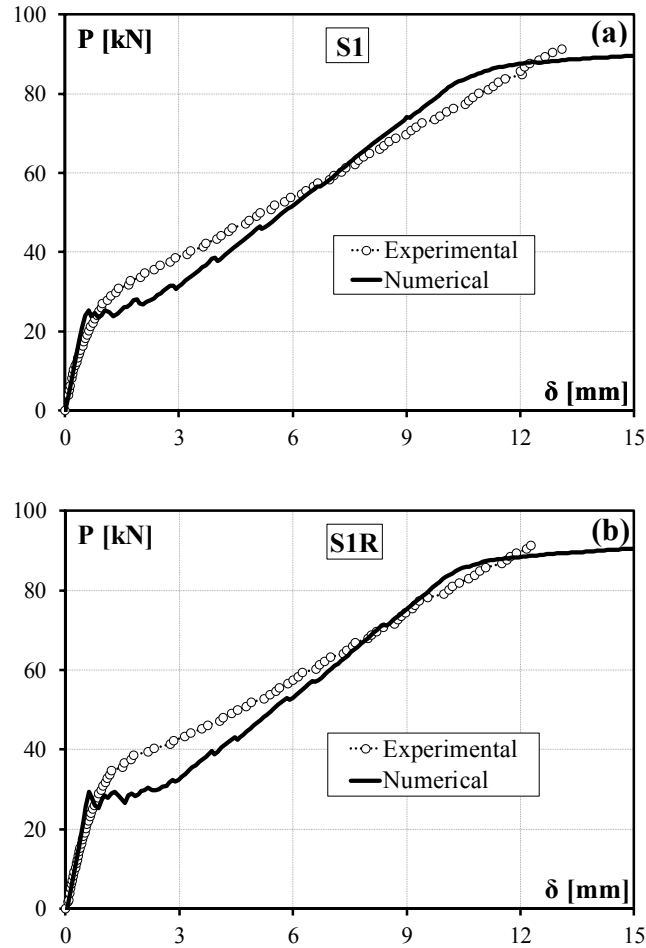


Figure 3.11 Comparison between numerical and experimental [131] results in terms of applied load P vs. midspan deflection δ for beams: (a) S1 and (b) S1R

All tests were carried out under loading control. On the contrary, numerical analyses are performed under displacement control, in order to achieve a better numerical convergence. Moreover, they are performed by following the same modeling choices already described in §3.4.1.2.

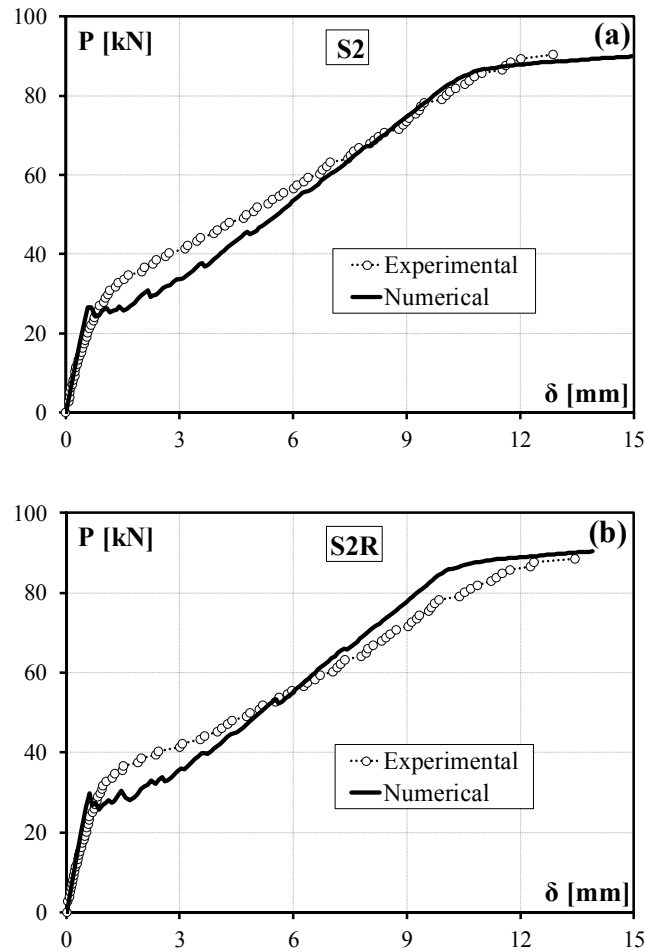


Figure 3.12 Comparison between numerical and experimental [131] results in terms of applied load P vs. midspan deflection δ for beams: (a) S2 and (b) S2R

Numerical and experimental results are first compared in terms of applied load P vs midspan deflection δ . Since initial deflection due to shrinkage was not experimentally measured, also in this case numerical curves are shifted to the origin. As shown in Figure 3.11 and Figure 3.12, an accurate simulation of the experimental behavior is obtained for all the considered specimens.

Further comparisons are made for beams S-1 and S-1R in terms of bending moment M – concrete strains ε within the constant moment zone at different level of the section (1,2,3,4) - see Figure 3.13. Once again, satisfactory agreements are archived.

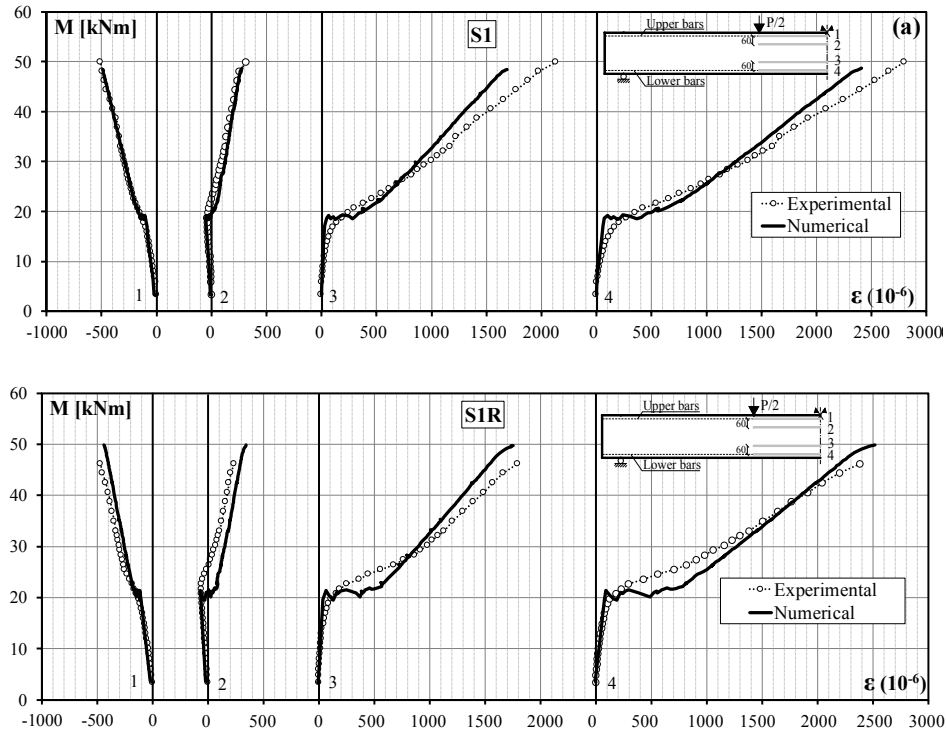


Figure 3.13 Comparison between numerical and experimental [131] results in terms of bending M vs concrete strains ε for beams: S1 and S1R

Moreover, experimental and numerical cracking moments M_{cr} together with the initial numerical deflections δ_i (thus, calculated before any external load is applied) are listed in Table 3.5. Beams with a larger amount of top reinforcement (designation “R”, Figure 3.12) are characterized by an initial negative deflection δ_i due to shrinkage. Moreover, they show a higher cracking resistance with respect to their twin specimens (Figure 3.11). This can be attributable to the increment of moment of inertia due to the presence of a heavier top reinforcement, but mainly to the difference of tensile stresses caused by shrinkage in the extreme bottom fiber of the cross-section. To better clarify this last aspect, the numerical variation within the depth of the section of the total strain ε , as well as of the stresses in concrete σ_c and in steel σ_s just before loading are reported in Figure 3.14 for the twin beams S1 and S1R.

Sample	M_{cr} [kNm]		δ_i [mm]
	Experimental	Numerical	Numerical
S1	16.8	14.9	0.13
S1R	19.8	17.5	-0.16
S2	15.9	15.4	0.09
S2R	17.9	17.6	-0.13

Table 3.5 Experimental [131] and numerical cracking moments M_{cr} , numerical initial deflections δ_i

The restraint to shrinkage provided by the embedded reinforcements, which are not symmetrically placed in the beams cross-section, is not uniform. This causes in turn the element curvature and the appearance of a stress gradient, which is, of course, very different in S1 and S1R beams. Heavy compressive reinforcement (S1R specimen) provides a larger restraint to the top of the beam, so leading to a reversed curvature and to a great reduction of tensile shrinkage stresses in the extreme bottom fiber. For this reason, a greater applied moment is required to crack the member.

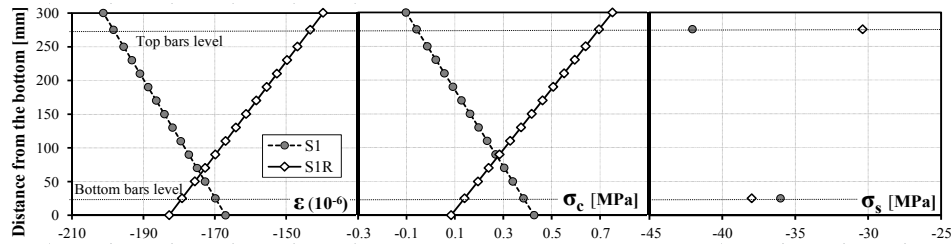


Figure 3.14 Numerical values of total strain ε , concrete σ_c and steel σ_s stresses within the depth of the section at midspan just before loading for beams S1 and S1R

3.4.2 Model B

First the attention is focused on the capability of the proposed procedure to correctly represent humidity distribution and differential shrinkage within prismatic concrete specimens, through the simulation of the tests carried out by Kim and Lee [126,127].

Subsequently, the model is applied to the analysis of structural elements like RC tension members [129] and RC beams in bending [132], that are restrained not only internally, by compatibility arise from differential shrinkage, but also by the reinforcing bar.

Finally, some more comparisons between numerical - relative to both model A and B - and experimental results are provided, with reference to RC/SFRC tension members [128], to highlight the advantages of taking into account differential shrinkage.

3.4.2.1 Prismatic concrete specimens tested by Kim and Lee

At first, the effectiveness of Model B in representing moisture distribution is considered. To this aim, numerical analyses are performed on prismatic concrete specimens, characterized by a 100 mm x 100 mm transversal cross-section and a total depth of 200 mm, tested by Kim and Lee [126].

During the tests, three different water/cement ratios (corresponding to different compressive strengths) were considered, respectively equal to 0.28 (specimen H), 0.40 (specimen M) and 0.68 (specimen L). At the age of 1 day, the specimens were removed from their molds and moist-cured in water for 28 days; subsequently, they were placed in a climatic chamber with $h_{en} = 50 \pm 2\%$ and $T = 20 \pm 1^\circ\text{C}$. A uniaxial moisture diffusion was obtained during tests by sealing five sides of each specimens with paraffin wax and allowing evaporation only through the exposed square surface, having a side of 100 mm. Humidity sensors were placed at three different depths from the exposed surface, respectively equal to 30 mm, 70 mm and 120 mm, as can be seen in Figure 3.15a; for further details see [126]. Humidity decrease due to self-desiccation was also measured on totally sealed cubic specimens, with an edge length of 100 mm.

Numerical simulations are performed according to the approach discussed in §3.3.2, by adopting the values of maximum moisture diffusion coefficient D_1 suggested by Kim and Lee in [126] for the three specimens; however, only the results relative to sample M are discussed herein for sake of brevity.

Humidity distribution in the specimen is obtained numerically by performing the “equivalent” thermal analysis described in §3.3.2.1. Neumann boundary condition is applied on the exposed surface, by using a proportionality factor $f_{boundary}$ similar to that deduced by Oliveira et al. in [158], while no moisture interaction with the surrounding environment is assumed for the sealed surfaces; thus assuming null flux.

Moreover, self-desiccation effects are taken into account through a proper calibration of the body heat flux, based on the experimental measurements reported in [126].

As can be seen from Figure 3.15b, numerical results are in satisfactory agreement with the experimental data for all the considered depths of the specimen, so proving the effectiveness of the proposed strategy that exploits the analogy between heat transfer and moisture diffusion.

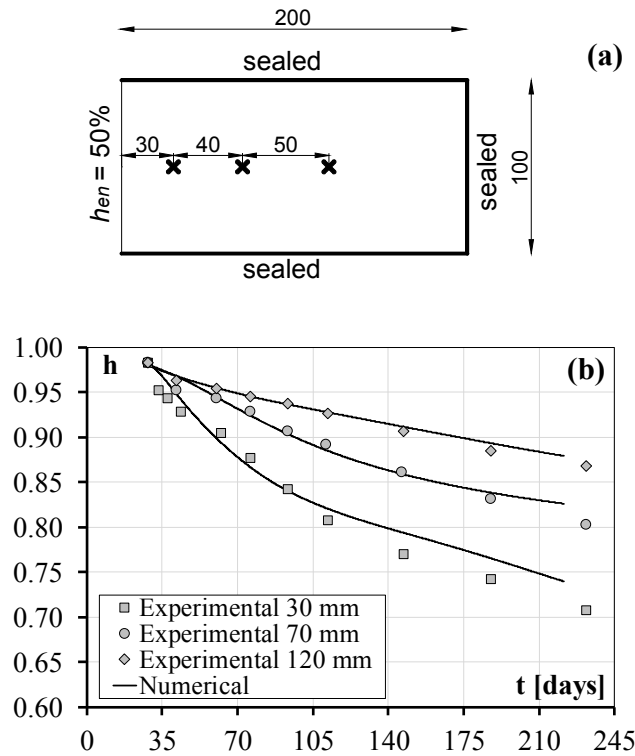


Figure 3.15 (a) Geometry and instrumentation of the prismatic sample; (b) relative humidity distribution in concrete due to moisture diffusion and self-desiccation in sample M ($t_0 = 28$ days).

The second step in the validation of the Model B concerns the correct simulation of differential shrinkage induced by moisture variations. To this aim, the tests performed by Kim and Lee [127] on prismatic concrete specimens, characterized by a 300 mm x 300 mm transversal cross-section and a total depth of 150 mm, are analyzed.

During the tests, two different water/cement ratios were considered, respectively equal to 0.65 (Mix I) and 0.40 (Mix II). After moist-curing for 7 days, the specimens were placed in a climatic chamber with $h_{en} = 68 \pm 2\%$ and $T = 20 \pm 1$ °C. Also in this case, a uniaxial moisture diffusion was obtained during tests by sealing five sides of each specimens with paraffin wax and allowing evaporation only through the exposed surface, with 300 mm side. The differential drying shrinkage was measured by means of embedded strain gauges, which were placed at four different depths from the exposed surface, respectively equal to 20 mm, 50 mm, 80 mm and 120 mm (Figure 3.16a; for further details see [127]).

Also in this case, numerical analyses are carried out by adopting the values of maximum moisture diffusion coefficient D_1 indicated by Kim and Lee in [127], while the proportionality factor $f_{boundary}$ to be used in the Neumann boundary condition is chosen within the typical range suggested in technical literature for concretes with similar properties.

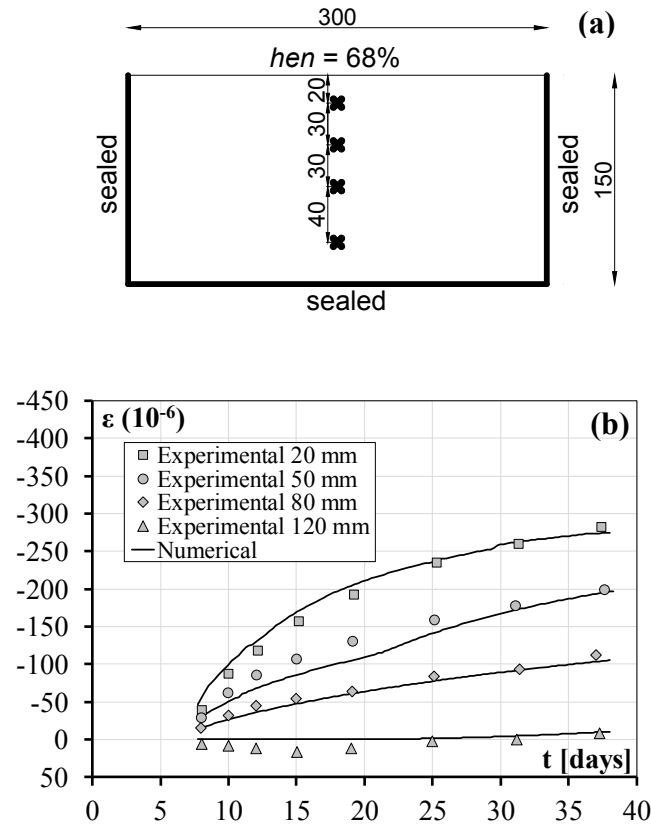


Figure 3.16 (a) Geometry and instrumentation of the prismatic sample; (b) comparison between numerical and experimental results of internal drying shrinkage for Mix I ($w/c = 0.65$).

The free shrinkage vector $\{\epsilon_{sh}\}$ in each integration point of the FE mesh is calculated in 2D-PARC user-material subroutine, starting from the moisture field obtained in the “equivalent” thermal analysis (see §3.3.2.1); in this way, each point of the element is subjected to its own infinitesimal shrinkage. By applying the same equations proposed in §3.3.1, the actual strain and stress fields in specimen are then provided. These fields account for compatibilization arising from differential shrinkage that leads to internal self-equilibrated stresses.

Figure 3.16b shows the comparison between numerical and experimental results for Mix I in terms of strain evolution with time, at different depths from the exposed surface. As can be seen, both shrinkage strain and its increase in the early stages of drying are markedly affected by the distance from the exposed surface, being greater in the external part of the specimen and significantly lower in the inner core, due to the moisture gradient.

The good agreement between numerical and experimental curves for all the considered depths confirms also in this case the effectiveness of the proposed modeling strategy, which can be then applied to the analysis of more complex structural elements.

3.4.2.2 RC tension members tested by Wu and Gilbert

As regards the application of Model B to reinforced elements, at first four RC tension members tested by Wu and Gilbert [129] are analyzed.

All the specimens, named STN12, STS12, STN16, STS16 were 1100 mm long and characterized by a square cross-section with 100 mm side. Moreover, they all contained one single central reinforcing bar. The adopted nomenclature is the same used by the Authors [129] and is formed by three capital letters followed by two digits. The first two letters “ST” stand for short-term tests, the third letter refers to specimen condition at the beginning of tests (“N” = non-shrunk, “S” = shrunk), while digits 12 and 16 indicate the reinforcing bar diameter (in mm). Samples STN12 and STN16 were tested soon after the end of moist curing and so exhibited little shrinkage, whereas STS12 and STS16 were allowed to dry for 25 days at room temperature and humidity levels.

Mechanical properties of concrete and steel are summarized in Table 3.6, which also provides the average shrinkage strains ε_{sh} . This value was experimentally measured before test execution by using two identical unrestrained plain concrete samples, having a length of 600 mm and a square cross-section with 100 mm side.

Sample	Concrete				Steel	
	f_c [MPa]	$f_{ct,sp}^*$ [MPa]	E_{ci} [MPa]	ε_{sh} (10^{-6})	f_{sy} [MPa]	E_s [MPa]
STN12 STN16	21.56	2.04	22400	-28	500	200000 204000
STS12 STS16	24.73	2.15	21600	-249	500	200000 204000

* : f_{ct} adopted in the analyses is calculated from $f_{ct,sp}$ according to [115]

Table 3.6 Concrete and steel mechanical properties of RC ties [129]

Tests on tension members were carried out under displacement control, by applying a monotonically increasing axial elongation at the ends of the steel bar.

Unrestrained plain concrete specimens are first selected for comparison so as to provide a further confirmation of the effectiveness of the proposed procedure in catching shrinkage effects related to moisture gradient. Once the thermal (i.e. moisture) analysis is completed, the static one is performed. Taking advantage of the symmetry of the problem, only one-half of the member is simulated, by adopting a mesh constituted by quadratic, isoparametric 8-node membrane elements with reduced integration (4 Gauss integration points).

Figure 3.17 provides a comparison between experimental and numerical results in terms of the development of the average shrinkage strains in time. This value refers to the compatibilized average shrinkage because, even if the specimen is not externally restrained nor rebars are present, mechanical impediment occurs. Thus, the actual shrinkage of the element does not correspond to free shrinkage at the exposed surface or to the zero of the center at the beginning of drying, but it is an intermediate value. Once again, the comparison reported in Figure 3.17 proves that the numerical model is able to correctly describe the development of shrinkage strains after the beginning of the drying process, providing results that are almost superimposed to experimental ones [129].

In addition, Figure 3.18 shows the numerical stress state induced in the specimen by shrinkage 57 days after casting, corresponding to the beginning of tests on shrunk RC ties, STS12 and STS16. The two main directions x and y are considered (tangential stresses are omitted since they are almost null).

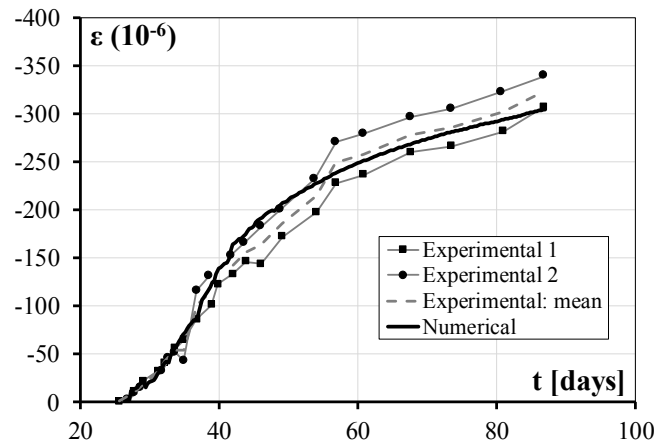


Figure 3.17 Development of shrinkage strains in plain concrete specimens during drying process

It can be seen that self-equilibrated stresses arise, due to differential shrinkage. Evaporation begins at the exposed surfaces that, thus, tend to contract, while the inner part attempt to remain unchanged. Therefore, due to compatibility, there is the appearance of tensile stresses near the drying surfaces; on the contrary, the inner part of the specimen is compressed. The magnitude of tensile stresses near the external surfaces are comparable to concrete tensile strength, so leading to cracking (see Figure 3.19). The moisture gradient is indeed large enough to induce surface cracking, even in absence of external restraint or reinforcement.

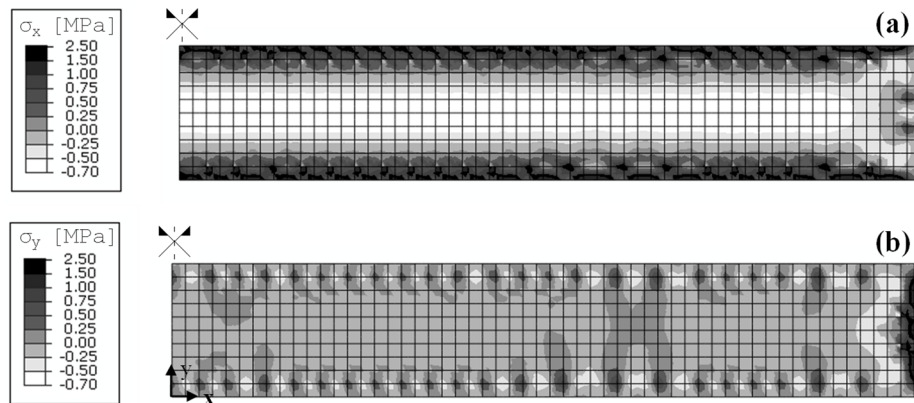


Figure 3.18 Numerical stress field in the specimen due to differential shrinkage, 57 days after casting: (a) in x-direction and (b) in y-direction

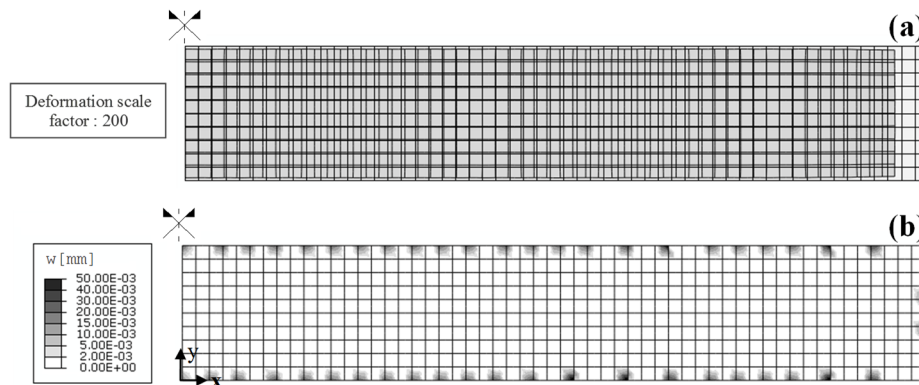


Figure 3.19 (a) Deformed shape and (b) shrinkage-induced cracks in plain concrete specimen, 57 days after casting, as obtained from numerical simulation

Figure 3.19 shows the differential shortening of the specimen after 57 days from casting, together with the corresponding shrinkage-induced orthotropic crack pattern obtained from the numerical simulation. As expected, cracks are characterized by very small opening values and are oriented perpendicularly to the privileged desiccation direction (Figure 3.18), which is in turn coincident with the x -axis.

Finally, the RC tension members are analyzed by performing the “equivalent” thermal analysis, followed by the structural one. Also in this case, taking advantage of the symmetry of the problem, only one-half of the member is simulated, by adopting a mesh constituted by quadratic, isoparametric 8-node membrane elements with reduced integration (4 Gauss integration points).

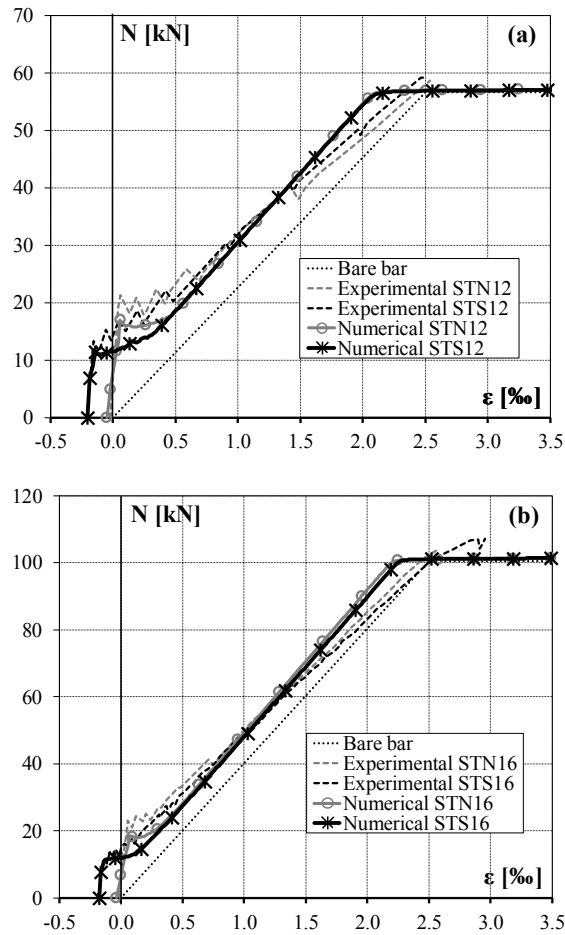


Figure 3.20 Comparison between numerical and experimental [129] results in terms of total load N vs. average axial strain ϵ for specimens: (a) STN12 and STS12; (b) STN16 and STS16

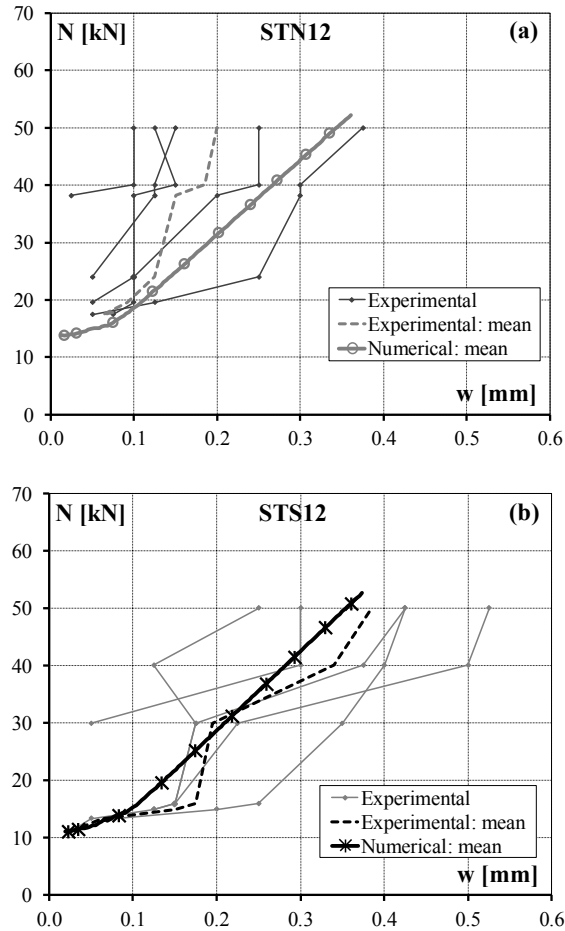


Figure 3.21 Comparison between numerical and experimental [129] results in terms of total load N vs. crack opening w for specimens: (a) STN12 and (b) STS12

A uniform displacement is applied to all the nodes belonging to the terminal section of the specimen to simulate test conditions, while ensuring a better numerical convergence.

A comparison between numerical and experimental results in terms of axial load N vs. average axial strain ε , together with the bare bar response, is presented in Figure 3.20. Each graph reports the curves relative to specimens with the same reinforcement amount, but affected or not by shrinkage; (actually samples STN experienced very little shrinkage).

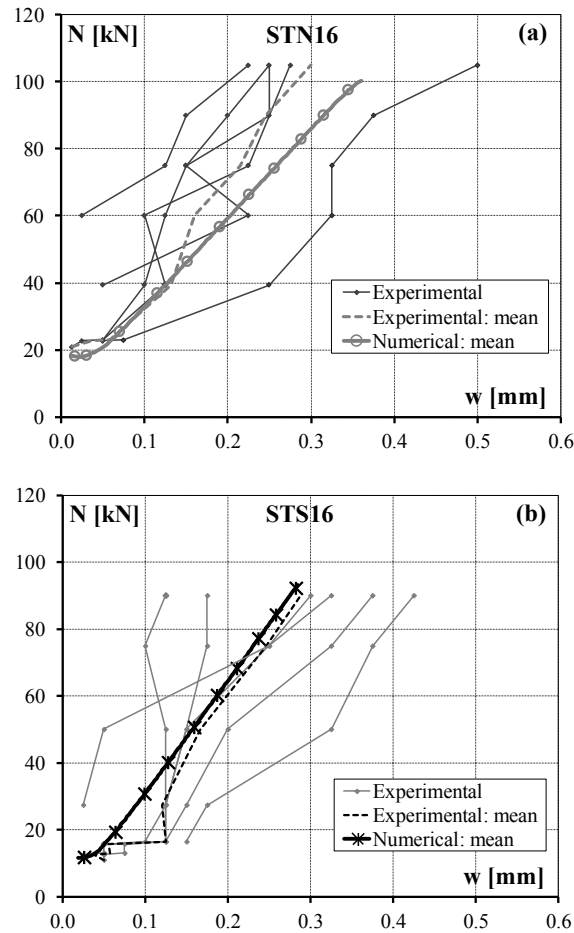


Figure 3.22 Comparison between numerical and experimental [129] results in terms of total load N vs. crack opening w for specimens: (a) STN16 and (b) STS16

The obtained results point out that the model B is able to correctly describe the initial shortening of the element caused by shrinkage before any load is applied, as well as the corresponding reduction of the cracking load (N_{cr}). The initial shortening is related to moisture gradient, as well as to the restrained provided by the reinforcement. Therefore the close correlation between the experimental [129] and numerical results proves the effectiveness of the proposed procedure. Moreover, it should be pointed out that, since shrinkage-induced stresses develop gradually with time, also the relief caused by creep is considered in simulations.

Finally, mean crack width values obtained from numerical analyses are reported in Figure 3.21 and Figure 3.22, together with experimental data for non-shrunk (Figure 3.21a and Figure 3.22a) and shrunk (Figure 3.21b and Figure 3.22b) elements; a good agreement can be stated; so proving once again the effectiveness of the proposed procedure. Moreover, the obtain results underline that the effect of shrinkage on crack width is not very significant, as also observed by other Authors (e.g. [119]).

3.4.2.3 RC beams tested by Wu

Four ordinary RC beams subjected to four-point bending, tested by Wu [132] and named BSTN2-16, BSTS2-16, BSTN3-16, BSTS3-16, are analyzed.

The specimens were characterized by the same geometry, with a rectangular cross-section (200 mm wide and 400 mm deep) and total length equal to 2650 mm, with a net span of 2400 mm. The beams were reinforced with a single reinforcing layer, placed in the bottom part of the cross-section and formed by two or three 16 mm rebars, corresponding to digits “2-16” or “3-16” in the name of each specimen. This latter also includes some capital letters, indicating the type of test (“BST” = short term beam) and the specimen condition at the beginning of tests (“N” = non-shrunk, “S” = shrunk).

Table 3.7 summarizes the main steel and concrete properties, as well as the average shrinkage strains ϵ_{sh} at the time of testing as experimentally determined on prismatic concrete specimens.

Sample	Concrete				Steel		
	f_c [MPa]	$f_{ct,sp}$ [MPa]	$f_{ct,fl}$ [MPa]	E_{ci} [MPa]	ϵ_{sh} (10^{-6})	f_{sy} [MPa]	E_s [MPa]
BSTN2-16 BSTN3-16	26.00	2.20	3.50	25000	0	500	204000
BSTS2-16 BSTS3-16	40.00	2.40	3.90	32000	-220	500	204000

Table 3.7 Concrete and steel mechanical properties of RC beams tested by Wu [132]

Tests on RC beams were carried out under loading control, by monotonically increasing two concentrated loads applied at one-third of specimen net span.

Also in this case, the FE mesh and the adopted modeling strategy are the same already described in §3.4.2.2 for the simulation of tension tests.

Figure 3.23 shows the comparison between numerical and experimental [132] results in terms of bending moment M vs. midspan deflection δ for both the analyzed series of beams. Each graph reports the curves relative to beams reinforced with the same numbers of rebars, subjected or not to initial shrinkage strains.

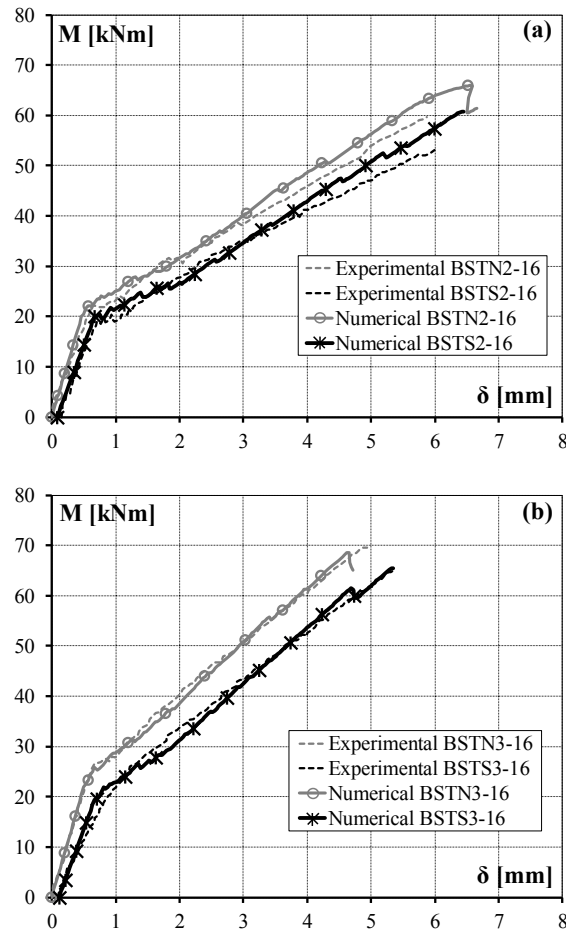


Figure 3.23 Comparison between numerical and experimental [132] results in terms of bending moment M vs. midspan deflection δ for beams: (a) BSTN2-16 and BSTS2-16; (b) BSTN3-16 and BSTS3-16

A good agreement is found for all the considered specimens. Shrunk beams are characterized by lower values of cracking moment, due to the presence of shrinkage-induced tensile stresses in concrete before loading. Moreover, the eccentricity of the reinforcement with respect to the centroid of concrete cross-section causes an initial curvature of the element, which in turn determines an increase in member deflections under applied load (with respect to twin non-shrunk specimens).

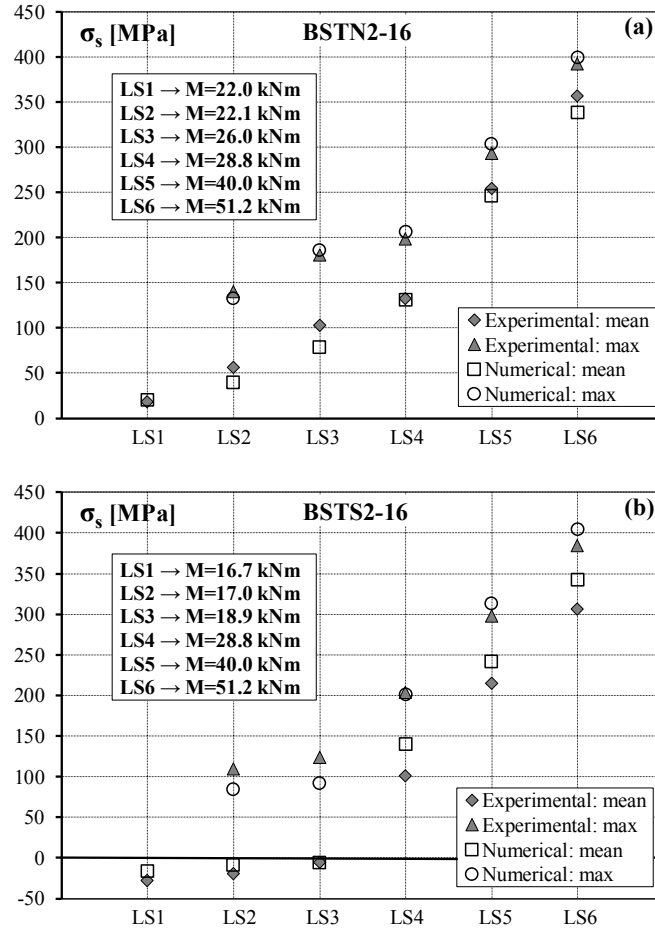


Figure 3.24 Comparison between numerical and experimental [132] results in terms of steel stresses σ_s at different loading stages (LS) for beams: (a) BSTN2-16 and (b) BSTS2-16

Experimental and numerical values of maximum and average stresses in steel over the constant moment region for different loading levels are reported in Figure 3.24 and Figure 3.25, for both shrunk and non-shrunk samples. In the uncracked stage (corresponding to the first point of each graph), steel rebars of specimens BSTS2-16 and BSTS3-16 exert a restraining action towards shrinkage and consequently undergo negative stresses.

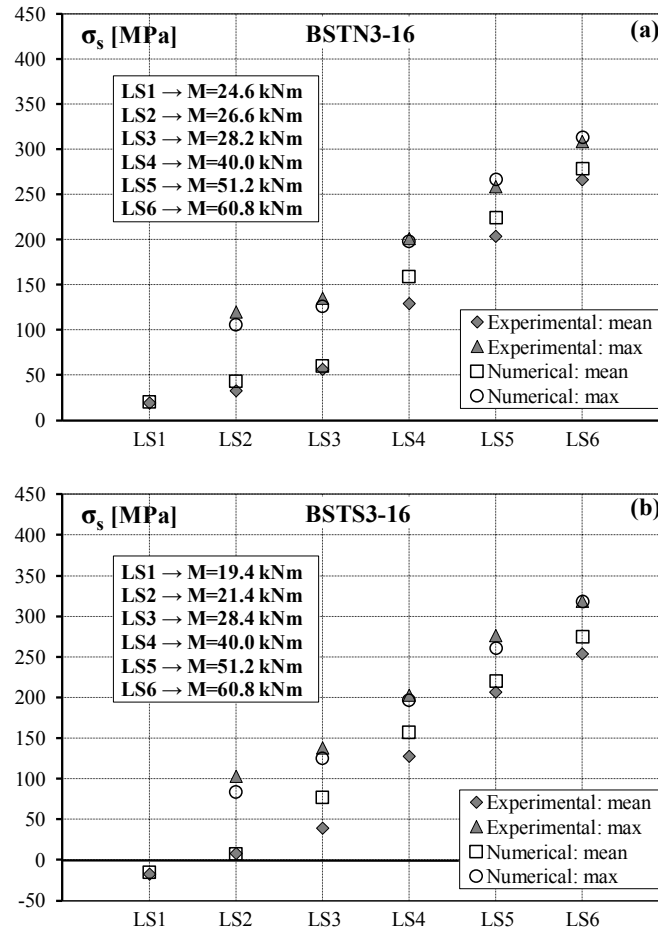


Figure 3.25 Comparison between numerical and experimental [132] results in terms of steel stresses σ_s at different loading stages (LS) for beams: (a) BSTN3-16 and (b) BSTS3-16

From the second loading level onward all the considered specimens are cracked; as a consequence, medium and maximum steel stresses differ from each other. These latter are always positive, being related to a cracked section where the reinforcement carries almost the entire load. The difference between maximum and average stresses in reinforcing bars is higher in shrunk members and tends to decrease for increasing values of the applied moment.

Further comparisons are reported in Figure 3.26 in terms of crack pattern at failure, limited to those members that undergone significant shrinkage (BSTS2-16, BSTS3-16). Crack distribution and widths are indeed similar in case of non-shrunk specimens BSTN2-16 and BSTN3-16, so the corresponding contours are omitted for brevity.

It should be also noticed that the reported numerical crack pattern corresponds to the numerical failure load, which is very close to the experimental one for all the considered beams, as observed from previous comparisons. Both the contours and the experimental evidences highlight that the constant moment region is interested by vertical flexural cracks, which becomes inclined flexure-shear cracks moving to the supports, due to the presence of more significant tangential stresses in the beam web.

It is worth noting that external cracks induced by differential shrinkage cannot be appreciated in the reported contours due to their limited widths with respect to loading-induced cracks at failure.

Finally, it can be remarked that specimen BSTS3-16, characterized by a greater reinforcement ratio ρ , is interested by narrower crack width than BSTS2-16. This aspect can be better appreciated with reference to the results reported in Table 3.8, which compares numerical and experimental values of average crack width within the constant moment region at different loading stages. Once again, experimental behavior is closely predicted by the simulations.

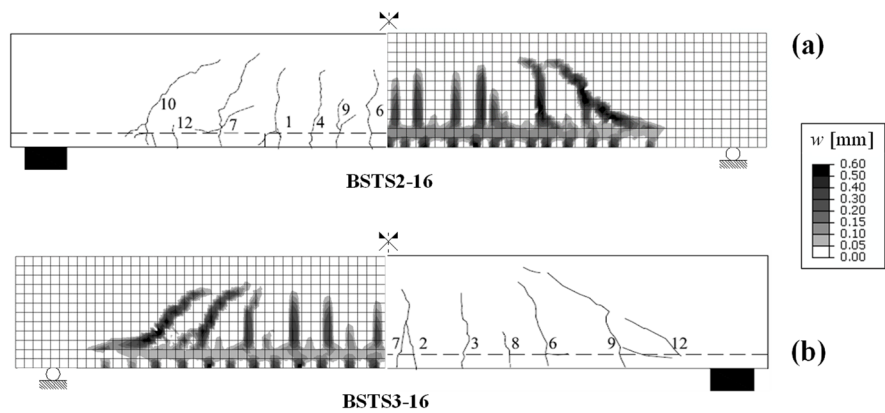


Figure 3.26 Comparison between numerical and experimental [132] crack pattern and widths at failure for RC beams subjected to initial shrinkage: (a) BSTS2-16 and (b) BSTS3-16

Sample BSTS2-16		
Bending Moment [kNm]	Experimental crack width [mm]	Numerical crack width [mm]
20	0.0344	0.0345
30	0.0675	0.0647
40	0.1025	0.0867
48	0.1021	0.1028
Sample BSTS3-16		
Bending Moment [kNm]	Experimental crack width [mm]	Numerical crack width [mm]
30	0.0250	0.0470
40	0.0575	0.0663
48	0.0850	0.0739
60	0.1400	0.1017

Table 3.8 Numerical vs. experimental [132] average crack width at different loading stages for RC beams subjected to initial shrinkage

3.4.2.4 RC and SFRC tension members tested by Deluce

Finally, some RC and SFRC ties among the extensive experimental program carried out by Deluce [128] are modeled for a double purpose: to validate Model B also when fibers are added to the concrete mix and to highlight the differences between Model A and B.

As a matter of fact, the analyzed specimens have already been model in §3.4.1.1, to which reference is made for the descriptions of the geometrical details and the material properties.

For sake of brevity, among the twelve samples simulated, only the results referring to H80/10 PC, H80/10 FRC1 and H100/20 PC are herein provided. These cases are selected since the differences between the results obtained by applying Model A and Model B are more pronounced, because these elements were characterized by a low reinforcement ratio ρ and they were not pre-cracked before the application of the load.

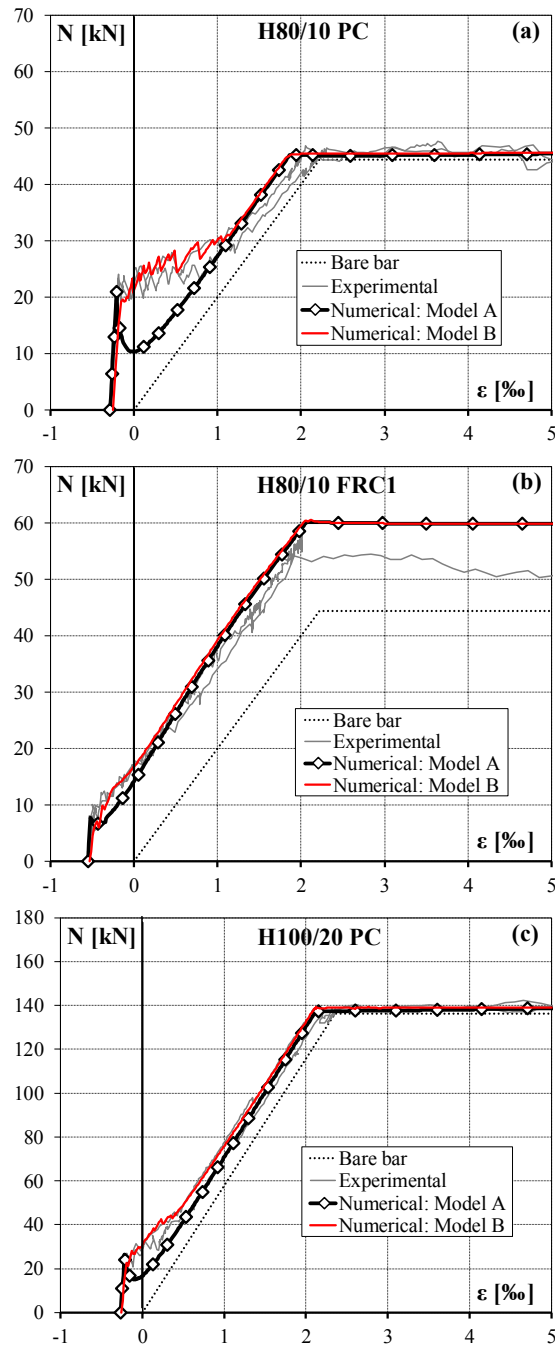


Figure 3.27 Comparison between numerical (Model A - Model B) and experimental results [128] in terms of axial load N vs. average axial strain ϵ

As can be seen in Figure 3.27, Model B, which takes into account differential shrinkage related to moisture gradient, better represents the global behavior of the tension members at the beginning of cracks formation, even in presence of fibers. Therefore, this refined approach, not only provides the actual self-equilibrated stresses due to shrinkage (see e.g. §3.4.2.2, Figure 3.18) that Model A obviously cannot give, but it also shows a closer correspondence to experimental evidences, even after the application of the load. Thus, the necessity of considering shrinkage in an accurate way in simulations is proven. Obviously, this effect is more pronounced the more the behavior of the element is governed by concrete performances, i.e. at the beginning of the loading history and when the concrete cover is grater and fibers are not added in the concrete admixture.

Moreover, while Model A calculates the initial shortening of the members on the basis of the average shrinkage experimentally measured, so obviously fits the experimental evidences, Model B evaluates this value from the moisture analysis, providing the same satisfactory results.

Therefore, it can be concluded that the application of the more refined (and computationally demanding) Model B is convenient for the evaluation not only of the local but also of the global mechanical response, especially for elements, like plain concrete specimens or reinforced tension members, whose behavior is greatly affected by shrinkage. On the contrary, it is worth noting that minor differences arise when analyzing more complex structures, like beams in bending: these results are herein omitted for sake of brevity.

3.5 Concluding remarks

In this Chapter an existing nonlinear constitutive model, 2D-PARC, is extended so as to include shrinkage effects. Shrinkage indeed greatly influences the behavior of RC and SFRC elements, even under short-term loading; thus, its inclusion is essential to obtain reliable results and avoid mistaken outputs in simulations. To this aim, the 2D-PARC model, already revised in the modeling of concrete contribution as described in Chapter 2, is herein further improved to insert shrinkage prestrains.

The general algorithm of the model is revised both in the uncracked and in the cracked stages, by inserting free shrinkage strain as a prescribed deformation. A new set of equilibrium and compatibility equations is written and the material secant stiffness matrix is consequently rearranged and implemented into the adopted finite element code. Moreover, the internal procedures for ensuring convergence are properly updated. It is worth noting that 2D-PARC, which can be included in the framework of smeared-fixed crack formulations, applies the strain decomposition procedure in the cracked stage to the whole RC/SFRC material (differently from the others model available in technical literature that applies this technique only to concrete and treat the bar separately); thus, a different strategy is required to include shrinkage strain in the

algorithm. Moreover, since shrinkage-induced stresses develop gradually with time, creep effects are also included in the model.

Once the fundamental equations of the model are reformulated, the value of shrinkage strain associated to each integration point is evaluated by following two different approaches characterized by different refinement and complexity, termed as Model A and Model B. The first, evaluates the strain and stress fields in the element by considering shrinkage strains as uniformly distributed within the element; whereas the latter, more refined, computes shrinkage as a function of moisture gradient. This latter is in turn obtained by performing an “equivalent” thermal analysis, by exploiting the analogy between the equations governing the moisture and the thermal problem, with a simple substitution of the corresponding parameters. The punctual values of pore relative humidity (i.e. temperature) are then passed to 2D-PARC user subroutine for the evaluation of the free shrinkage strain vector and the consequent strain and stress fields in the RC/SFRC element object of study. In this way, more reliable results can be obtained, since the self-equilibrated stresses, related to the uneven tendency to shrink of each point of the element, can be computed.

Both Model A and Model B are then validated through comparisons with significant experimental data available in technical literature relative to different structural typologies: respectively [130,131] for model A and [126,127,129,132] for model B. Also SFRC elements [128] are considered, so proving the versatile nature of the proposed approach. The obtained results highlight the potentialities of both the proposed approaches for the assessment of serviceability requirements of RC and SFRC structures, where shrinkage is one of the key parameter to be considered. However, Model B that additionally accounts for self-equilibrated stresses arisen from mechanical hindrance due to differential shrinkage, provides more accurate responses in terms of both global and local behavior, especially for elements, like plain concrete specimen or reinforced tension members, whose behavior is greatly affected by shrinkage.

Finally, it is worth noting that the addition of shrinkage strains in 2D-PARC algorithm represents only the first step of a more extensive work, devoted to the inclusion of all concrete prestrains in the model, that can be inserted in the algorithm by following the same procedure described for shrinkage strains.

Chapter 4

NLFEA of concrete tunnel linings subjected to fire

4.1 Introduction

In this chapter a numerical procedure is developed to investigate the behavior of concrete tunnel linings under fire conditions.

During the last decades, several catastrophic tunnel fires have occurred. Only in Europe, from 1990 to date more than ten absolutely disastrous fires have taken place in tunnels, such as English Channel in 1996, Mont Blanc in 1999, Tauern in 1999, Kitzsteinhorn in 2000, St. Gotthard in 2001, Frejus in 2005. These events have obviously increased the interest of technical community in the assessment of the performances of tunnel linings under fire conditions. In most cases, a tunnel fire results indeed in human losses and high financial damages. Moreover, the structural damage of concrete linings may determine prolonged service disruptions after fire development. Long-term effects, like loss of load bearing capacity of the lining and durability problems, may also occur. Therefore, the evaluation of tunnel safety during and after a possible fire is of fundamental importance in limiting human and economical losses and in deciding whether and when a tunnel can be reopened to traffic; for this reason it represents also a very important topic in academic research. In recent past, several studies have been focused on the development of more or less sophisticated approaches, aiming at providing a reliable estimate of the strength and stiffness degradation of the lining system during and after fire exposure (among others, e.g., [178–185]).

Generally speaking, fire design of concrete structures is a very complicated task and the problem becomes even more complex in case of tunnel linings. The response of these elements at high temperatures is indeed influenced by several parameters, which should be properly taken into account during the analysis in order to provide reliable predictions. Some of the most relevant factors are represented by geometrical configuration, loading level, temperature variation both with time and over the lining depth, appearance of temperature-induced stresses and strains, as well as temperature effects on material properties (among others, e.g., [186,187]). On this last point, it is known that the behavior of

concrete under high-temperature is relatively good with respect to other construction materials. Nevertheless, its mechanical properties experience a marked decay, whose extent depends on fire duration and intensity. Besides, the presence of a temperature gradient in the element, together with other restraints to dilatation, may lead to concrete cracking, due to the induced thermal stresses. Moreover, in case of tunnel linings the interaction between the lining itself and the surrounding ground plays a major role and should be accurately considered.

Three main approaches can be applied in the evaluation of tunnel safety, even under fire conditions: analytical solutions, empirical methods and numerical models. Due to the complexity of the considered problem, these latter (and in particular 3D models) should be preferred, since they do not require coarse simplifications and allow to handle more general cases, taking into account all the main involved phenomena. It can be observed that numerical models often require considerable computational efforts, but the increasing computational power over the last decades has partially solved this drawback.

In this work, a 3D numerical finite element (FE) procedure is developed to simulate both the ground-support interaction, with special attention to the excavation/construction phases, and the concrete lining behavior during fire. This last aspect is dealt by performing a sequentially coupled thermo-mechanical finite element analysis: a heat-transfer analysis is linked with a nonlinear structural simulation that considers thermal strains, concrete degradation due to fire and the interaction between tunnel lining and the surrounding ground.

In order to correctly simulate the excavation and lining installation phases, the structural analysis is in turn subdivided into several steps. Firstly, the lithostatic load is applied and subsequently the solid elements corresponding to tunnel opening are progressively deactivated to simulate the excavation process. At the same time, lining elements are activated at a prescribed distance from tunnel face, so as to model the support construction. When lining installation process is completed, fire development is simulated by inserting in the model the temperature field obtained from the previously performed heat-transfer analysis.

Concrete lining behavior, both before and during fire, is modeled through 2D-PARC constitutive relation. This model, which was originally developed by Cerioni et al. in 2008 [1] in the form reported in Chapter 1, has been properly improved to handle this complex problem.

In particular the model has been herein firstly revised in the modeling of concrete contribution, see Chapter 2. As already mentioned, this operation has been performed in order to improve the computational efficiency of the model, as well as to incorporate the effects of crushing and dilatation of concrete in compression. Both these aspects are of primary importance for the simulation of tunnel linings subjected to fire. FE analyses of tunnels, which are high computationally demanding by nature, require indeed particularly efficient numerical algorithms. On the contrary, the procedure adopted for concrete modeling in the original formulation of 2D-PARC was quite complex and considerably lengthened the calculation times; thus a change was essential. Moreover, the correct simulation of crushing and dilatation phenomena in the

analysis of tunnel linings is mandatory, since concrete support may experience high compressive stresses, especially during fire.

Subsequently, 2D-PARC model has been extended to include thermal strains. To this aim, equilibrium and compatibility equations governing the model have been reformulated and the material stiffness matrix has been consequently rearranged, following the same procedure described in Chapter 3 for shrinkage deformations.

Moreover, 2D-PARC model has been further revised to take into account concrete degradation due to fire, by inserting the dependence of the main mechanical properties on temperature in the algorithm.

This chapter is subdivided in four main parts. At the beginning, the state of the art relative to tunnels subjected to fire will be presented. Then, the developed analysis procedure will be outlined in detail. Subsequently, the effectiveness of the proposed model is validated through comparisons with analytical closed-form solutions. In particular the accuracy of the simulation of excavation and lining installation phases is verified by comparing FE results to well-known analytical formulations available in technical literature [188–192]. On the contrary, a simplified analytical model is herein developed on purpose, based on the solutions originally proposed in [179,180,187,193,194], so as to validate the simulation of concrete lining behavior under fire conditions. Finally, a parametric study is performed so as to better highlight the influence exerted by different parameters on the structural response of the lining, especially during fire. In more detail, different ground mechanical properties, tunnel depths and fire curves are considered, to evaluate their effects on the stress and strain fields of tunnel lining.

4.2 State of the art

The study of tunnels under fire conditions is a very relevant and current topic in research, which is often promoted by local governments in several countries, due to the severe consequences caused by fires developed in tunnels, as well as because of the importance of the infrastructure transport systems for the society and the related high investment costs. As a matter of fact, fire scientists from Sweden, Norway, Finland, Netherland, French, Germany, Australia, USA, Japan, Korea and China have all been involved in National projects on tunnel related fire research [195].

The hazard of fire in tunnels is indeed high. Numerous tunnel fires have occurred in the last years, more than ten absolutely disastrous (among others, the English Channel in 1996, Mont Blanc in 1999, Tauern in 1999, Kitzsteinhorn in 2000, St. Gotthard in 2001 and Frejus in 2005). It can be observed that many of them have been developed in road tunnels; however those happened in railway systems are usually more severe and seem to cause a greater number of fatalities. A review of the most important fires developed in road and rail tunnels can be found in [196,197]. An increasing risk is also expected in the future [178],

due to the growing number of tunnels, their greater length, the increment of traffic density (and also of transportations of flammable materials) and the increasing travelling speed. Thus, an accurate representation of tunnel linings under fire conditions is of primary interest not only for academic purposes, but also for design purposes, in order to provide a sound assessment of the structural safety of real tunnels.

The most important task of tunnel fire safety protection is to avoid or diminish the number of incidents and, once they have occurred, to reduce fatalities and injuries, economical losses and prolonged service disruptions.

In this sense, the development of a wide range of detection devices to obtain a rapid suppression of fire, and the study of several active and passive protection systems is mandatory. In the past, a trial-and-error procedure was often applied to test fire safety systems: only the ones that were useful during previous tunnel fire have been subsequently applied in other tunnels [196]. This approach has been quickly abandoned due to the high social, political and economical impact of the fire accidents. Nowadays, the study of fire safety systems is related to a deeper understanding of fire dynamics, so as to guarantee systems that are able to deal with tunnel fires with proven efficacy. Depending on the possible fire loads, the structural characteristics of tunnels and the consequences of fire on structural elements, appropriate fire protection measures are provided, aimed at ensuring life safety, preventing heavy structural damages and enabling the reopening of the tunnel as quick as possible.

In the past, passive protection measures were largely adopted because they were considered sufficient to reduce the fire hazard in tunnels. They consist for example in the use of building materials resistant to high temperatures which help in preserving the most important and susceptible structural elements from the damage caused by fire. Passive protections are still of major importance; nevertheless, during years, fire research has suggested that passive fire protections are often not enough to protect tunnels exposed to severe fires [198]. Nowadays, the addition of active systems to traditional passive ones is indeed widespread. As a matter of fact most of modern tunnels incorporate sophisticated fire detectors, suppression systems and ventilation systems to control and restrict fire spread so providing an adequate level of protection and maintain the risks within acceptable ranges.

On the other hand, the assessment of structural safety in tunnel linings exposed to high temperatures is equally important in order to guarantee a bearing capacity enough to sustain the fire effects without collapsing as well as to provide a reliable estimation of the strength and stiffness degradation after fire exposure.

In this work, the attention is focused on the structural behavior of tunnels supported with concrete linings. In the following, the most important characteristics of fires in tunnel will be provided, highlighting the distinguishing features from fires in traditional buildings. Tunnel fires are indeed characterized by different heat release rates, peak temperatures, boundary conditions and design details compared to above ground buildings. Finally, the effects of fire on concrete itself and on the behavior of the whole tunnel will be briefly outlined.

4.2.1 Characteristics of fires in tunnels

Fire is the manifestation of a chemical reaction, called combustion, which takes place between two different substances, the fuel (i.e. the combustible species involved in the fire) and the combustive agent (usually represented by oxygen), with consequent emission of energy. The reaction takes place when the fuel is exposed to a source of heat or ambient temperature above its ignition point and it is able to sustain a rate of rapid oxidation that produces a chain reaction; then heat, light, flames, gases and smoke are released.

Four main phases can be recognized in fire evolution: ignition, growth, fully development and decay. The first stage begins when heat, oxygen and a fuel source have a chemical reaction resulting in fire. In the area around the fire, the temperature field increases. However, the process is very unstable and depends on the immediate energy balance. This stage is characterized by a fire confined to a small area and sometimes it goes out on its own, before the following phases are reached. When the development of general combustion takes place, the growth stage begins. The energy generated by the fire increases, as well as the reached temperature, with significant emission of heat, smoke and flammable gases. In particular, the fire continues to grow and additional fuel becomes involved. In a confined space, in this stage, the flames reach the ceiling and then start to extend horizontally. The dominant heat transfer mechanism within the fire compartment shifts from convection to radiation. When the fire, from being localized, begins to involve all the exposed surfaces and all the combustible materials, a rapid transition from the growth to fully development stage takes place. This transition, termed as flashover, makes the fire reaching its maximum temperature. In the fully development phase also the energy release is at its greatest. Finally, the decay stage is entered when all the combustible materials are consumed or when not enough oxygen is present.

The rate at which energy is generated by fire, generally termed as “Heat Release Rate” is the most important variable characterizing the behavior of a fire [199], since it represents the strength of the fire. As a matter of fact, it is usually more important to know the rate at which heat is released rather than the total heat released. The achieved temperatures depend indeed on the rate of heat release of the fire and on the rate of heat loss of the system. Moreover, other important parameters, such as ventilation of smoke, flame length, radiation and fire spread are related to the HRR.

Even if a fire is the manifestation of a chemical reaction, its features are mainly influenced by the distribution of the fuel and by the characteristics of the environment, rather than by its chemical nature. A fire in open air is indeed very different from a fire in a compartment and, in particular, fires in tunnels show peculiar characteristics with respect to fires in buildings. However, for many years, fires in tunnel were not adequately studied and only more recently, with the contextual development of important National and European research projects, the interest of the fire scientific community in this field is greatly increased.

Even if, some results deriving from different studies are contradictory, several aspects that characterize fires in tunnels can be recognized. The heat

release rate and the gas temperatures registered for fires in tunnels are usually higher than in traditional buildings; moreover, also fire duration is prolonged. High temperatures and rapid temperature increases are of main concern in the design of tunnels under fire conditions since concrete spalling may appear. A description of this phenomenon will be provided in §4.2.2.

The peculiarity of fire in tunnels is mainly due to the high calorific potential of the involved materials (such as vehicle fuels and vehicles themselves), to the confinement of the heat release and to the interaction of the ventilation system with the growing fire.

The effect of confinement is of great importance, since, even traditionally non-hazardous materials may become dangerous when burned in tunnels, see e.g. flour and margarine during the Mont Blanc fire or tyres in the Gotthard one [200]. Moreover, the same heat release rate of a car fire seem to be significantly larger in tunnels [196]. The HRR is also influenced by the geometry of the tunnel, since both height and width seem to change its value; however results on this topic are contradictory and this represents still an interesting research field.

As already mentioned, also ventilation plays a major role, influencing the fire spread and growth rate, the maximum heat release and the gas temperature as well as the smoke stratification. Generally speaking, the ventilation can be natural or mechanical (forced); in both cases, due to the interaction with fire, aerodynamic disturbances in the air flow are generated throughout the tunnel, causing possible changes in the ventilation pattern. The ventilation system must be then designed and dimensioned to ensure the control of smoke and heat in case of fire. In particular, a sufficient ventilation capacity able to destroy smoke stratification is desirable, because smoke and toxic gases emitted from fire can greatly reduce the breathability of the air and the visibility, thus making harder evacuation and fire-fighting operations. The ventilation velocity should be greater than a minimum value, named "critical velocity", in order to prevent back-layering (that is the reverse flow of hot gases and smoke in the longitudinal direction), so as to obtain a tunnel free of smoke upstream of the fire site [195,201]. As already said, ventilation also influences the spread of fire in tunnels. This phenomenon can be subdivided in five mechanisms, with increasing severity: flame impingement, surface spread, remote ignition, fuel transfer and explosion. A description of these mechanisms can be found in [195], together with the explanation of the influence on them by ventilation.

It is then clear that a complete understanding and the consequent representation of the phenomenon of fire is a complicated matter, that becomes even more complex in case of tunnel fires. The analysis of tunnel fires can be performed by applying the computational fluid dynamic (CFD). This approach is very accurate and provides reliable results, since the most important phenomena are taken into account. The domain is divided into a large number of small control volumes and different formulations are considered: a combustion model to simulate the course of combustion, a turbulence model to consider the turbulent flow and a radiation model to simulate the thermal radiation. In this way the gas temperature, which is of primary interest in structural analyses of tunnels under fire conditions, can be determined accurately. To this end, the main

parameters influencing the temperature level, such as the type of fuel, its distribution and geometry, the tunnel cross-section and length, the ventilation system as well as the combustion efficiency are taken into account in simulations. Moreover, the non-uniform distribution of temperature in tunnel is accurately considered; since, as known, tunnel crown reaches higher temperatures with respect to the benches, due to the direct flame impingement. The greatest drawback is the high computational effort required; to this aim, simplified models have been developed. Among them, the most applied are the so called “zone models.” In this case a limited number of control volumes (usually two) are used to describe the fire; the most applied “zone model” is probably CFAST [196].

It is worth noting that the evaluation of the thermal response (in particular the temperature field in the lining and its rate of increase in time) is very important for the assessment of the structural safety of the tunnel, since thermal and mechanical properties of concrete strongly depend on temperature. However, when the focus of the work is the structural performance rather than the study of the fire itself, standards fire curves are often applied, due to their high simplicity and to the quite satisfactory results provided over the years. This approach is also followed in this work. Time-temperature curves most commonly applied in the analysis of tunnel can be found in §4.3.2.2.

4.2.2 Effects of fire on concrete linings

As already said, this work focuses on the evaluation of structural safety of concrete linings subjected to fire; thus, in the following the behavior of concrete under high temperature will be outlined, whereas the influence of fire on other kind of supports is not considered, being out of the scope of this work.

It is generally recognized that concrete behaves well during fire with respect to other construction materials. It can be indeed considered incombustible compared to wood and it is characterized by a lower thermal diffusivity when compared to steel. Nevertheless its mechanical properties show a marked decay with increasing temperatures and it can experience spalling.

Concrete at high temperatures exhibits a more complex behavior than other materials, mainly due to the presence of different components in the admixture. The behavior of concrete at high temperatures depends indeed on several parameters related to the material itself; among them, the characteristics of the constituent materials (cement and aggregates), the moisture content and the porosity. Obviously its response is also influenced by environmental and boundary conditions; especially the temperature level, the heating rate, the external sealing, the applied loads and the confinement.

As regards thermal properties, they must be evaluated correctly, since their values influence temperature rise and distribution in the concrete element. For this reason, their dependency on temperature and moisture fields must be considered in the analyses. The relations adopted in this work for density, specific heat, conductivity and radiation are reported in §4.3.2.3.

As regards mechanical properties, their degradation is strictly related to the physical and chemical processes taking place inside the material as temperature increases. In more detail, the most important factors are the physical and chemical changes in the cement paste and in the aggregates as well as the thermal incompatibility between them [202]. As an example, Figure 4.1, shows the main processes occurring in Portland cement during heating. As can be seen chemical reactions as well as phase transformations take place, together with an evolution of the pore structure.

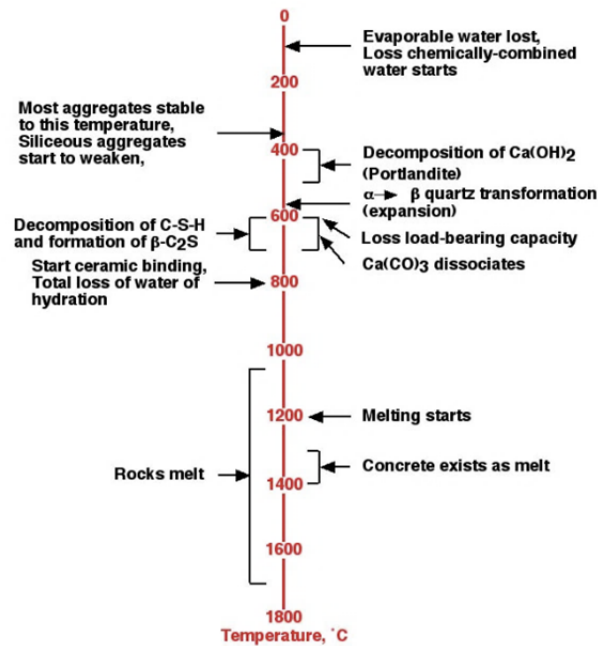


Figure 4.1 Physical and chemical processes in Portland cement as temperature increases [203]

As far as compressive strength is concerned, it is quite difficult to provide a typical strength behavior at high temperature, because this mechanical property is influenced by several material and environmental characteristics, such as type of concrete (normal or high strength), aggregate type, cement blend and moisture content. Aggregate type plays a major role since different aggregate types exhibit very different thermal stability, melting from below 350°C (flint) to above 600°C (gabbro). Moreover, also their thermal expansion, the roughness of the surface and the possible presence of reactive silica influence the results. In particular, aggregates characterized by low thermal expansion improve the strength performance of concrete at high temperatures, since a higher thermal compatibility with the cement past is guaranteed. Rough angular surfaces and the presence of reactive silica increase as well the compressive strength,

because improve respectively the physical and the chemical bond with the cement paste [202].

As can be seen in Figure 4.2, which refers to normal concretes, compressive strength, first drops slightly and then increase a little as temperature increases, whereas the decay is marked above 300°C-400°C [204]; even if a high scatter of experimental data can be recognized, due to the influence of the above described factors.

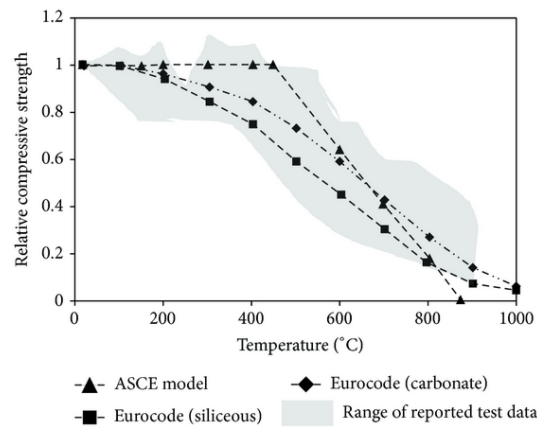


Figure 4.2 Relative compressive strength of concrete under high temperatures [205]

Also the stress-strain curve in compression is markedly influenced by temperature. While compressive strength decreases during heating deformability increases, so resulting in a high decrease of the slope of stress-strain curve. The strain corresponding to peak stress increases indeed a lot as well as ductility, especially above 500°C. However, also in this case, concrete and aggregate types, as well as the environmental conditions can significantly modify the mechanical response, as proved by the general high scatter of experimental data. It is also worth noting that concrete specimens tested at high temperatures seem to exhibit more strength and stiffness than companion samples first heated and subsequently tested at room temperature once they have cooled down [203].

In case of biaxial compressive loading, a beneficial effect takes place [183,202] and then temperature exerts a lower influence on concrete behavior, resulting in a reduced decay of both strength and stiffness. This is related to the compacting effect of compressive loadings that reduce the development of cracks.

Tensile strength decreases as temperature increases. As reported in [205], concrete generally exhibits about 80% of its initial strength at 300°C, then the decay rapidly increases and tensile strength drops down to about 20% of the initial strength at 600°C. As regards fracture energy, some contradictory results can be found in the literature, probably due to the different test methods and

types of specimens adopted; anyhow it is generally suggested that fracture energy does not show clear dependence on temperature [186].

It should be also reminded that concrete tends to expand during heating. The coefficient of thermal expansion mainly depends on aggregate type and temperature, but also cement type, water content and age influence the results. As temperature increases, thermal expansion is characterized by a rapid escalation until the attainment of about 700°C; then it remains almost constant, as can be seen in Figure 4.3.

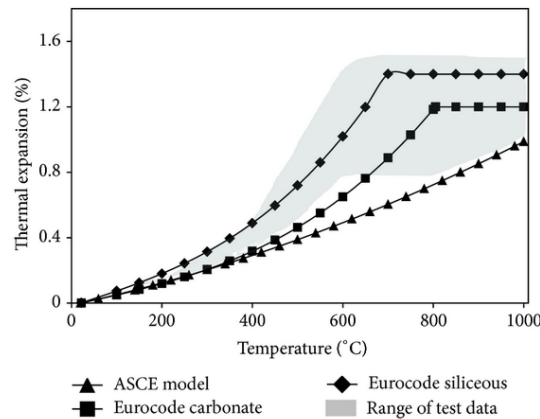


Figure 4.3 Thermal expansion of concrete under high temperatures [205]

The relations adopted in this work for describing the variation of mechanical and deformation properties of concrete with temperature are reported in §4.3.3.3.

In the following, the attention is instead focused on two important aspects that characterize the behavior of concrete under high temperatures: transient creep and spalling.

Transient creep, also known as load-induced thermal strain, develops when a concrete specimen is first loaded (under compression) and then heated, resulting in the appearance of an additional strain compared to concrete loaded at elevated temperature. As a matter of fact, concrete is characterized by different strains during a steady-state test (i.e. when the sample is first heated uniformly and then loaded while keeping the same temperature) and during a transient test (i.e. when it is first loaded and then heated keeping the same load); the difference between these two values represents the load-induced thermal strain [206,207], see Figure 4.4. It develops during the first-time heating and it is irrecoverable [208,209]. It mainly depends on temperature and loading, but due to the complexity of the phenomenon, other factors are involved, such as concrete strength, moisture content and mix proportions [205]. This mechanism should be inserted in any fire analysis involving concrete in compression, because if it is neglected erroneous and unsafe results are found. Nevertheless, the level of accuracy of its inclusion required to perform reliable analyses is still under discussion [210,211], as better discussed in §4.3.3.3.

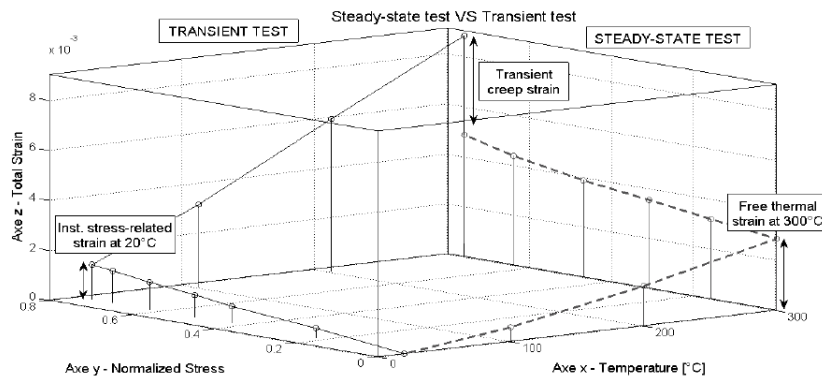


Figure 4.4 Difference between steady-state and transient tests [207]

Spalling is defined as the breaking up of layers or pieces of concrete from a structural element when it is exposed to high and rapidly rising temperatures [202], such as those encountered in tunnel fires. On the basis of its location or its origin, different classifications can be found in the literature. Aggregate, surface and corner spalling can be recognized following the first approach, whereas progressive and explosive spalling are the categories of the latter [201]. The consequences of spalling are severe, since it may lead to early loss of stability and integrity and it influences the temperature distribution inside the member. It exposes indeed deeper layers of concrete to fire temperatures, thereby increasing the rate of heat transmission to the inner layers of the element [205].

Since explosive spalling may appear in tunnel linings, it will be discussed in the following. It appears during the first 20-30 min of fire and it is characterized by the sudden burst-out of concrete pieces, with the related release of energy and loud sounds. The phenomenon is very complex and it is governed by several factors, related to the geometry of the element (section size and shape), to the characteristics of the material itself (moisture content, pore pressure, permeability, aggregate size and type, concrete strength and age, presence and type of fibers), as well as to environmental conditions (heating rate and profile, load level, restraint to thermal expansion) [201,202].

The two main mechanisms underlying explosive spalling are pore pressure and thermal stresses. A high moisture content (more than 2-3%) and a low permeability (such as that of high strength concretes) result in a pore pressure that can overcome the material tensile strength. On the contrary, explosive thermal spalling is related to the elevated thermal stresses, caused by the restrained thermal dilatation. Compressive stresses are indeed induced in the region close to the heated surface, while tensile stresses appear for equilibrium in the inner part.

A lot of measures have been proposed to reduce the risk of spalling; the most effective of them are nowadays represented by thermal barriers and by the addition of PP fibers in the concrete mix [201,202].

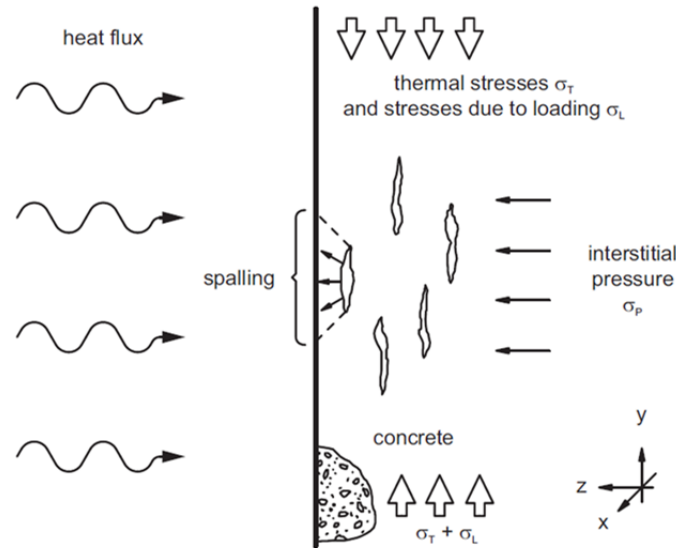


Figure 4.5 Mechanisms underlying concrete spalling [202]

Summing up, it can be stated that concrete linings during heating exhibit a marked decay of strength and stiffness, the appearance of thermal and load-induced thermal strains, as well as the possibility of spalling. Moreover, the appearance of induced stresses related to the confinement provided by the ground (or by the inner and colder region) may lead to cracking.

The behavior of concrete lining depends on all the above described factors, such as the type and quantity of the constituent materials or the moisture content; however, in case of tunnel linings, also tunnel geometry and in situ lithostatic stress play a major role. In particular in circular deep tunnels (that are specifically considered in this thesis) the most important loading is represented by hoop compression [201,212]; the imposed constraints in the circumferential and longitudinal directions result in a compressive stress peak near the exposed surface. In more detail, this phenomenon is due the stress relaxation of the hotter region combined with the lower tendency of expansion of the colder and inner part, which provides a restraint to thermal deformation. The higher compressive stresses in concrete lining during heating may increase the risk of spalling, while the hoop tensile stresses that may appear near the cold extrados, can lead to crack formation.

4.3 A numerical procedure to simulate tunnel lining behavior

This section deals with the numerical procedure developed to simulate tunnel lining behavior. A three-dimensional FE analysis is performed to model concrete lining response before and during fire exposure, accounting for both ground-structure interaction and temperature-induced effects, such as the loss of strength and stiffness in the support system and the appearance of thermal induced strains and stresses. The numerical approach that will be present is general and can be then applied to a large number of realistic cases.

The adoption of a numerical approach allows indeed the study of problems characterized by complex geometries or ground conditions, such as tunnels excavated in rock masses with anisotropic behavior or characterized by a non-homogeneous initial stress distribution. Moreover, with respect to bi-dimensional models, 3D ones are characterized by a larger range of applicability, allowing a more realistic simulation of tunnel behavior in the longitudinal direction. In particular, these models are able to provide a realistic description of the interaction between the rock mass and the support system. The effects of the excavation process are indeed taken into account with accuracy, since tunnel excavation and lining installation phases are simulated step by step. Moreover, possible variations of both tunnel cross-sectional geometry and mechanical properties of the surrounding ground along the longitudinal axis can be handled, as well as the change of mechanical properties of the lining, due to progressive hardening. In this way, the stress field in the lining and its not uniform distribution along the length of the tunnel can be computed with accuracy, as well as the effects exerted by the deformation of the excavation face.

The numerical procedure required to simulate tunnel lining behavior, considering also the development of a possible fire, is organized in several steps. First at all, it is necessary to define the model characteristics and carry out the discretization process. The geometry and the dimensions of the studied region must be then detailed, together with the boundary conditions. Moreover, appropriate element typologies should be chosen and the significant thermal and mechanical properties for the involved materials should be specified. To model the fire conditions, a sequentially coupled thermo-mechanical analysis can be performed. The basic assumption is that the temperature distribution in the studied element is independent from its structural behavior, namely the thermal field affects the stress and displacement variables, but not vice-versa. In this way, the FE simulation can be performed into two subsequent phases, represented by a heat-transfer analysis, which provides the temperature field in each node of the model under an assigned fire scenario, and a mechanical analysis based on the results of the heat-transfer analysis itself.

The mechanical analysis is in turn structured in several steps, corresponding to the application of the initial in-situ state of stress, to the modeling of excavation and lining installation phases, and finally to the simulation of the fire load. To this aim, it is necessary to specify reliable constitutive law for the involved materials;

in this work a very accurate constitutive law is applied to describe concrete lining behavior, that is 2D-PARC, which has been properly improved on purpose

To better explain the adopted numerical procedure, this section is subdivided in three main parts: at first the most important modeling choices will be outlined, subsequently the thermal analysis will be presented and finally the mechanical analysis will be explained in details, also highlighting the improvement made to 2D-PARC model in order to accurately describe concrete behavior also under fire loads.

4.3.1 Modeling choices

The FE model, which is strictly related to the considered problem, represents the study domain, which is in this case represented by a significant portion of the tunnel and the surrounding ground. The model should be indeed realized by taking into account the real extent of the rock mass region involved in the excavation process, since it modifies the stress and strain fields around the opening. So, the FE mesh is chosen wide enough to reach steady-state conditions and to eliminate almost any influence of the outer boundaries. Fixities of stress and displacement are applied at boundaries so as to prevent rigid body movements of the model and to maintain the appropriate boundary conditions during the analysis.

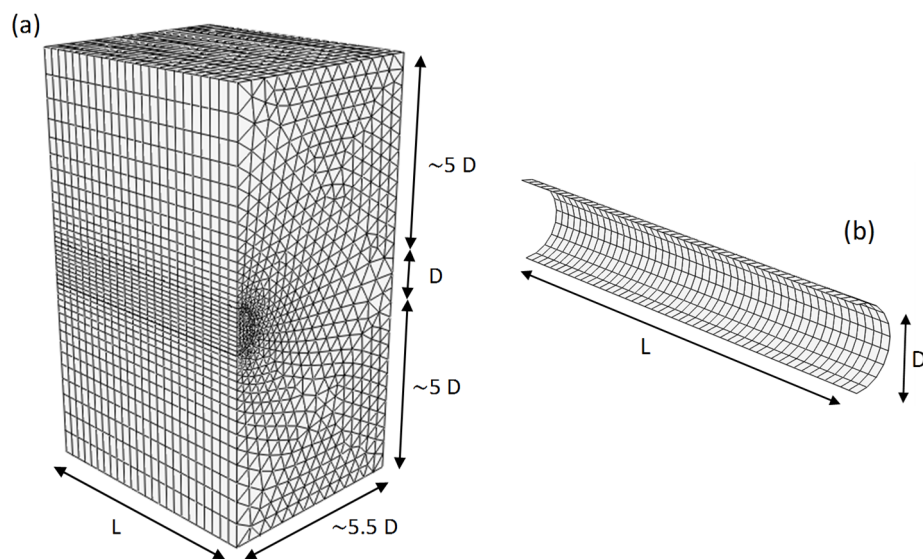


Figure 4.6 Three-dimensional model: (a) geometry and FE mesh of the entire model
 (b) FE mesh of the lining

Generally speaking, the whole tunnel can be modelled; however, transversal symmetric conditions are assumed in this thesis to reduce the computational effort. By exploiting symmetry with respect to the vertical axis passing for the center of the tunnel, only one-half of the tunnel cross-section is modeled. More in detail, the extent of the FE mesh from the vertical axis of symmetry is taken five times the mean tunnel diameter, so that the boundaries have a negligible influence on the results of the analyses.

Moreover, since this work focuses on deep tunnels, also in the vertical direction starting from the center of the tunnel the size FE mesh is limited to maximum five times the mean tunnel diameter, in order, once again to limit the computational time. In case of higher overburden, an external pressure is also applied to the upper surface of the FE mesh, so as to simulate the surcharge due to the overburden not explicitly considered in the mesh.

To further reduce the computational effort only a significant portion of tunnel length is considered, without modelling its whole extension.

It is worth noting, that the same mesh is adopted for both the heat-transfer analysis and the nonlinear structural simulation, by using heat-transfer elements in the first case and stress elements in the second one. In more detail, the rock mass is modelled through 6-node triangular prismatic elements, while 4-node shell elements are applied for concrete lining. These latter are chosen in order to use 2D-PARC constitutive model for the description of concrete lining behavior in the mechanical analysis.

The aspect ratios of all elements are taken less than 5:1 to ensure reliable results and good convergence. In order to correctly catch the temperature gradient and the related stress field in the lining, 19 integration points (the maximum allowed for thermal analyses by the adopted FE code, ABAQUS) are considered in the shell thickness. Moreover, by exploiting the layering of the shell elements, a refinement of the integration points is provided near the surface exposed to fire or where the peak of the compression is expected.

The composite shell elements are connected to the triangular prisms placed on the outer perimeter of the opening by surface-based ties, for both the heat-transfer and the mechanical analyses. Since the default reference surface of the shell elements coincides with their midsurface, an offset is assigned between the shell middle plane and the reference surface, in order to correctly represent the lining thickness and avoid element interpenetrations.

As known, the fineness of the mesh affects the convergence speed and the accuracy of the results; as a consequence, more elements should be used in all those regions characterized by larger stress gradients or where a higher resolution is required. Hence, the discretization is properly refined in correspondence of the lining, as can be seen in Figure 4.6.

4.3.2 Thermal analysis

Thermal analysis is performed in order to obtain the transient temperature field to be applied in the subsequent mechanical simulation. The determination of the correct temperature variation in time and space within the studied element is indeed essential to evaluate the corresponding thermal strains and the degradation of concrete mechanical properties which are mandatory for carrying out a correct mechanical simulation.

To solve the thermal problem the FE software ABAQUS is employed. The adopted mesh consists of 6-node triangular heat-transfer prisms for the ground and 4-node heat-transfer shells for the lining. Static boundaries are not required, while thermal ones must be specified in space and time, as described in the following subsection.

Thermal analysis is performed by taking into account the three mechanisms that rule heat-transfer, which are convection, radiation and conduction.

The thermal input consists in the temperature evolution with time (the so-called fire scenario); in this work, empirical standard fire curves are adopted. For sake of simplicity, in this thesis the fire temperature is uniformly applied along the whole length and section of the tunnel; however, a more realistic fire scenario could be inserted.

In the following some details about the governing equations of the thermal problem, as well as the adopted fire curves and thermal properties of rock and concrete to be defined in the transient temperature analysis will be provided.

4.3.2.1 Mathematical formulation

The governing equation of the heat transfer analysis, which provides the time-dependent distribution of the temperature T within the element, is the Fourier differential law:

$$\lambda \nabla^2 T + q_g = \rho c \frac{\partial T}{\partial t} \quad . \quad (4.1)$$

where t is the time and λ, ρ, c respectively represent the thermal conductivity, the density and the specific heat of the material. Moreover, q_g is the internal generated heat, which is assumed in this case equal to zero. It is worth noting that the values of parameters λ, ρ, c for concrete depend in turn of the current temperature, see §4.3.2.3.

To solve Equation (4.1), boundary conditions in time and space must be specified. The initial temperature (at $t=0$) is assumed equal to 20 °C, while the transient heat flux at the boundaries accounts for both convection and radiation. The heat flux on a surface due to convection q_c is governed by:

$$q_c = -h|T - T_0| \quad , \quad (4.2)$$

where h is the heat transfer coefficient, T is the temperature on the surface and T_0 is the fire or environmental temperature, depending on type of exposure; whereas the heat flux due to radiation q_r is:

$$q_r = \varepsilon \sigma [(T - T_z)^4 - (T_0 - T_z)^4] , \quad (4.3)$$

where ε is the emissivity of the surface, σ the Stefan-Boltzman constant and T_z the value of absolute zero on the temperature scale being used.

In this way, the physical heat transfer phenomenon is correctly modeled. The heat flux, defined by the fire curve, flows to the inner surface of the lining exposed to fire by means of radiation and convection. Then the heat is transferred from the heated surface into the materials by conduction.

The history of the temperature is recorded at each integration point of the model in order to be used as an input for the subsequent mechanical analysis.

4.3.2.2 Fire curves

A fundamental aspect in the study of tunnel fire resistance is the definition of the fire scenario. Traditionally, the heat exposure of the tunnel is based on the use of standardized time–temperature curves [213] and this approach is also followed in this thesis; the thermal input is indeed represented by fire curves, available in technical literature. In the following the most applied fire curves for tunnel analyses will be outlined.

A widely-used fire curve is represented by the standard ISO 834. This curve is based on a cellulosic fire and it is commonly adopted in fire testing of structural elements, being suitable for materials found in typical buildings. Its analytical expression is expressed by:

$$T = 20 + 345 \log_{10}(8t + 1) . \quad (4.4)$$

This curve has been applied for many years also in the analysis of tunnels, even if it is not able to correctly represent highly combustible materials, such as fuels and chemicals, and the actual ventilation conditions that apply for tunnels [201]. Tunnel fires are indeed characterized by the burning of fuel and vehicles with high calorific potential, in combination with the confinement of the released heat, as detailed explained in §4.2.1. For this reason, also other curves are commonly applied in the thermal analysis of tunnels.

The HC (Hydrocarbon) curve approximates the fire evolution of small tanks of flammable liquids and was developed for the petrochemical and off-shore industries. Compared to the ISO 834, this curve is characterized by a higher temperature increase, associated to a faster fire development, typical of petroleum fire. It is expressed by the following relation:

$$T = 20 + 1080 \left(1 - 0.325e^{-0.167t} - 0.675e^{-2.5t} \right) . \quad (4.5)$$

Specific fire curves were also developed in some Countries to better simulate hydrocarbon fire in tunnels: among them the HCM, the RABT/TVZ and the RWS fire curves should be mentioned.

An increased version of the HC fire curve was developed in France, and it is known as HCM (Hydrocarbon modified) or HC_{inc} (Hydrocarbon increased). Its analytical expression is obtained by multiplying the temperatures of the original HC curve by a factor of 1300/1100, with a slight correction for low times so as to provide the same initial temperature ($T=20\text{ }^{\circ}\text{C}$) at $t = 0\text{ min}$, so obtaining:

$$T = 20 + 1280 \left(1 - 0.325e^{-0.167t} - 0.675e^{-2.5t} \right) . \quad (4.6)$$

The HCM curve provides higher temperatures ($1300\text{ }^{\circ}\text{C}$ versus $1100\text{ }^{\circ}\text{C}$) and consequently represents a more severe fire scenario, with a rapid and complete combustion of the flammable fluids compared to HC curve; while the temperature gradient in the first minutes of the fire remains the same.

The RABT/TVZ curves were developed in Germany on the basis of several experimental programs. Two different fire curves are provided for highway and railway tunnels respectively. Both the curves are characterized by a steep linear ascending branch, reaching 1200°C within 5 minutes, followed by a plateau, where the maximum temperature ($T=1200^{\circ}\text{C}$) is maintained for 25 and 55 minutes respectively for car (highway) and train (railway) fires. Afterwards, a linear cooling phase of 110 minutes is applied to both fire curves. The points defining the curves are reported in Table 4.1.

RABT/ZTV (Railways)		RABT/ZTV (Highways)	
Time [min]	Temperature [$^{\circ}\text{C}$]	Time [min]	Temperature [$^{\circ}\text{C}$]
0	15	0	15
5	1200	5	1200
60	1200	30	1200
170	15	140	15

Table 4.1 Tabular values for RABT/TVZ curves

The RWS curve was developed in the Netherlands by the Rijkswaterstaat (Minister of Infrastructures) on the basis of laboratory scale tunnel tests. It represents a very severe hydrocarbon fire characterized by a rapid increase of the temperature until 1200°C , with a subsequent peak at 1350°C . The temperature development of the RWS fire curve is described by the co-ordinates reported in Table 4.2. The RWS fire curve simulates the initial and rapid development of a fire produced by tankers and the subsequent gradual reduction in temperature once the fuel is all burned. It refers to an enclosed space (such as tunnels) where the possibilities of dissipating heat into the surrounding atmosphere are limited or absent.

	Time [min]	0	3	5	10	30	60	90	120	180
RWS	Temperature [°C]	20	890	1140	1200	1300	1350	1300	1200	1200

Table 4.2 Tabular values for RWS curve

In order to apply the appropriate fire curve for tunnel design, the World Road Association (PIARC) [214], together with the International Tunnelling Association (ITA) [215] have provided some instructions [201]. Both the use of the standard ISO 834 and the RWS/HCM curve is proposed, according to the amount and type of traffic (cars/vans vs. trucks/tankers) and to the consequences of a structural failure. Moreover, a fire duration of 120' is proposed. Nevertheless, a final agreement of the scientific community on fire curves to be used in design has not been reached yet [213]. It is indeed worth noting that the maximum temperature provided by RWS/HCM, which represent the most severe fire scenario, have been registered during the well know Runehamar test series [195], but they have not been reached in the recent major tunnel fires (English Channel - 1100°C, Mont Blanc - 1000°C, Tauern - 1100°C , St. Gotthard - 1100°C), [202].

The comparison between the aforementioned fire curves is reported in Figure 4.7.

In this work the ISO 834 standard curve is adopted as reference fire curve in the analyses, due to its large diffusion in structural applications. Moreover also HC and RWS fire curves will be considered, in order to consider an increasing severity of the fire scenario.

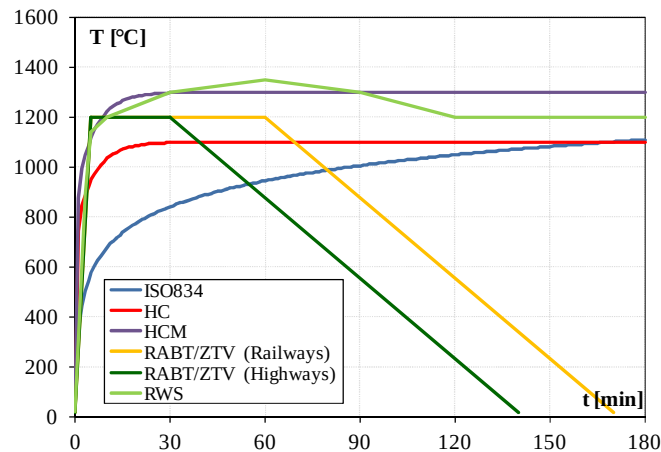


Figure 4.7 Time-temperature fires curves applied for tunnels

4.3.2.3 Thermal properties

The reliability of thermal analysis results is strongly affected by the thermal properties adopted for concrete tunnel lining. They depend on several parameters, such as the mix proportion, the type of aggregates, the age of concrete and the moisture content. Several formulations can be found in technical literature; in this thesis, the empirical laws suggested in Eurocode 2 [216] (EC2 in the following) are assumed.

Concrete specific heat c_c is evaluated as:

$$c_c = \begin{cases} 900 & 20^\circ\text{C} \leq T \leq 100^\circ\text{C} \\ 1470 & 100^\circ\text{C} \leq T \leq 115^\circ\text{C} \\ 1470 - 470/85 \cdot (T - 115) & 115^\circ\text{C} \leq T \leq 200^\circ\text{C} \\ 1100 + (T - 400)/2 & 200^\circ\text{C} \leq T \leq 400^\circ\text{C} \\ 1100 & T \geq 400^\circ\text{C} \end{cases} \quad \left[\frac{\text{J}}{\text{kgK}} \right]. \quad (4.7)$$

Since the moisture content is not considered explicitly in the calculation method, the values assumed for specific heat take into account the heat adsorbed by water. Equation (4.7) refers to a moisture content of 1,5 % of concrete weight.

The density of concrete is subjected to slight variations with temperature, that is:

$$\rho = \begin{cases} \rho(20^\circ\text{C}) & 20^\circ\text{C} \leq T \leq 115^\circ\text{C} \\ \rho(20^\circ\text{C}) \left(1 - \frac{0.02(T - 115)}{85} \right) & 115^\circ\text{C} \leq T \leq 200^\circ\text{C} \\ \rho(20^\circ\text{C}) \left(0.98 - \frac{0.03(T - 200)}{200} \right) & 200^\circ\text{C} \leq T \leq 400^\circ\text{C} \\ \rho(20^\circ\text{C}) \left(0.95 - \frac{0.07(T - 400)}{800} \right) & 400^\circ\text{C} \leq T \leq 1200^\circ\text{C} \end{cases} \quad (4.8)$$

and it can be approximately assumed as constant.

Two equations of thermal conductivity are provided in EC2 [216], defining the upper and lower limits; as suggested in [216], the lower limit is adopted:

$$\lambda_c = 1.36 - 0.136 \frac{T}{1000} + 0.0057 \left(\frac{T}{100} \right)^2 \quad \left[\frac{\text{W}}{\text{mK}} \right]. \quad (4.9)$$

The convection and radiation properties of concrete lining are evaluated once again according to EC2 [216]; hence, the convection factor is assumed equal to 25 W/(m²K), whereas the emissivity of heated concrete is posed equal to 0.7.

The thermal properties of the rock are assumed within the range provided in technical literature.

4.3.3 Structural analysis

As already stated, structural analysis is subdivided into two main stages: the simulation of the excavation and lining installation phases is first performed and then the fire load is applied. Therefore, the stress state caused by construction operations is obtained in the first phase and next the response of the material subjected to fire is evaluated. In more detail, the simulation of tunnel behavior under fire conditions is carried out by imposing the temperature distribution previously obtained from the heat-transfer analysis and consequently evaluating the stress-strain field. In this way, a realistic simulation of both ground-structure interaction and temperature-induced effects is provided.

The nonlinear structural analyses are performed by using the FE code ABAQUS. The soil is modeled through an elasto-plastic law based on the Mohr-Coulomb failure criterion, available in the library of the adopted FE code. On the contrary, concrete lining is modeled by applying 2D-PARC nonlinear model. In more detail, as already explained in §1.3.5, 2D-PARC is incorporated into a User-defined MATerial subroutine (UMAT for short), recalled by the FE program for each integration point at the beginning of each loading increment (or iteration within the loading increment). The model, originally developed by Cerioni et al. in 2008 [1], can be used to study uncracked and cracked concrete elements subjected to plane stresses and reinforced with both steel bars and fibers. A comprehensive description of the model can be found in Chapter 1, to which reference is made. This constitutive law, already revised in the modeling of the concrete contribution as described in Chapter 2, is herein properly extended in order to take into account thermal strains as well as the deterioration of mechanical properties in case of high temperatures; the detailed explanation of the followed procedure will be presented in §4.3.3.3.

The FE mesh is the same applied for the previously made heat-transfer analysis but the adopted finite elements are different; in this case four node stress-displacement shell elements with reduced integration are indeed used for the lining and six node stress-displacement triangular prism for the soil. It is worth highlighting that the adopted shell elements are characterized by 19 integration points in the thickness; moreover, a refinement of the integration points is provided where highest temperature or stress-strain gradients are expected.

In the following some details concerning the two main stages characterizing the structural analysis, that is the excavation and lining installation phases and the fire phase, are provided. Afterwards a detailed explanation of the adopted constitutive model is given, with particular attention to the improvement made to 2D-PARC nonlinear model, in order to handle the studied case.

4.3.3.1 Excavation and lining installation phases

In order to realistically simulate the soil excavation and subsequent lining installation, a step-by-step procedure is followed. At the beginning of the analysis all the elements representing the concrete lining are deactivated and the lithostatic load is applied to the rock mass. In the following steps, the model simulates the different excavation and support installation stages, so to reproduce stresses and strains occurring during the advancement of the excavation face. Tunnel excavation starts from the external cross-section of the model and the prismatic elements corresponding to tunnel opening are progressively deactivated. In each computational step one slice of elements referring to rock is switched off, while a ring of elements referring to lining is switched on to support the previous excavation, hypothesizing that the activation of the lining takes place at a predefined distance from the face. The excavation and lining installation phases are then repeated alternatively until the tunnel is full excavated.

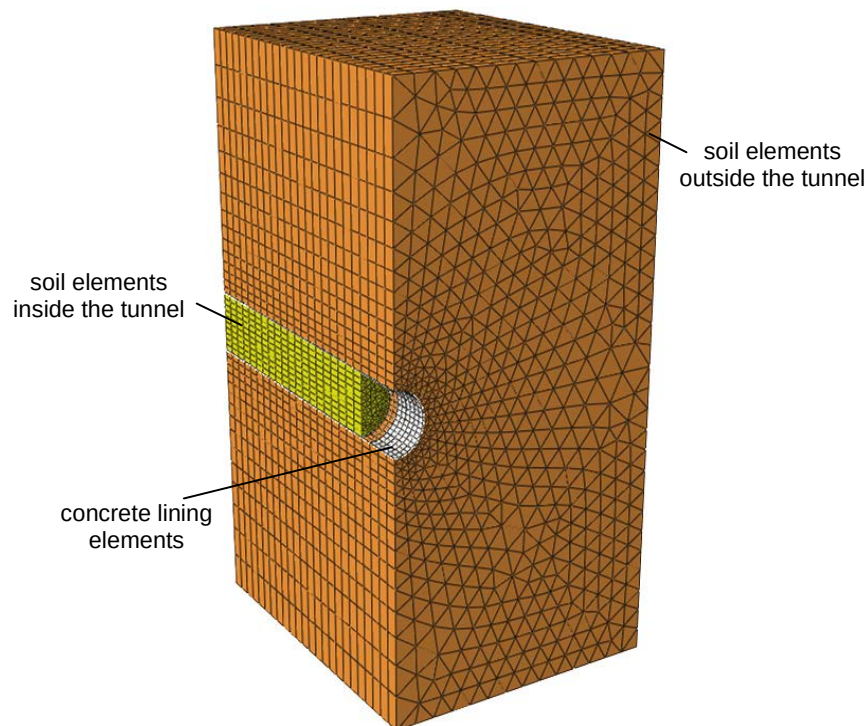


Figure 4.8 Excavation and lining installation phases: three-dimensional model adopted

4.3.3.2 Fire phase

After the completion of the construction phases, the fire load is applied. As already mentioned, a sequentially coupled thermo-mechanical procedure is followed in order to link the heat transfer analysis to a nonlinear mechanical simulation able to provide the stress-strain field in the materials. In particular, the temperature distribution derived from the thermal analysis is read by the structural analysis and used to evaluate thermal strains as well as to provide the decay of material mechanical properties. For concrete this operation is performed within 2D-PARC subroutine, which has been herein properly extended to the purpose. The structural performance can be then calculated based on the modified mechanical properties and taking into account the effect provided by thermal strains.

For sake of simplicity, the phenomenon of concrete spalling is not considered in this work. However, this assumption does not seem to compromise the results in case of normal strength concretes and moisture contents lower than 3%. In any case, spalling can be included in the presented procedure by incorporating a temperature-dependent removal of finite elements, as proposed by several authors [179,182,185,201,217]. In particular, the elements can be deactivated once the temperature has reached a predefined value. Afterwards the heating process continues until the critical temperature is once again reached by the next concrete elements, and so on until the final configuration is attained.

4.3.3.3 Material properties

The effectiveness of a structural analysis strongly depends on the constitutive laws adopted to describe material behavior. In the following, the constitutive models adopted for rock and concrete lining will be presented.

Soil modeling

The rock mass is modeled using an elasto-plastic constitutive model based on the Mohr-Coulomb criterion, available in the library of the adopted FE code, ABAQUS [218]. The input parameters are the elastic modulus E , the Poisson ratio ν , the cohesion c , the friction angle ϕ and the dilatation angle ψ necessary to properly define the yield criterion and the flow rule. Thermal expansion is included in calculations, whereas temperature effects on ground properties are neglected, as also suggested by others Authors (e.g. [178,212]).

The Mohr-Coulomb failure criterion is typically used in the geotechnical field and it represents the linear envelope that is obtained by combining the shear strength τ with the applied normal stress σ (Coulomb friction hypothesis). The Mohr-Coulomb criterion is based on the representation of an envelope of Mohr's circles that result from experimental tests performed up to yield/rupture condition. The envelope defines a straight line tangent to circles that designate the plastic limit. The combination of shear and normal stress that produce yielding is defined, as well as the corresponding angle of the plane in which the failure occurs.

The Mohr-Coulomb failure criterion can be written as:

$$\tau = c - \sigma \tan \phi \quad . \quad (4.10)$$

Beyond the elastic limit a flow rule is applied. A strain rate decomposition is assumed, by subdividing the total strain rate into the elastic and plastic components. The flow potential has a hyperbolic shape in the meridian stress plane and has no corners in the deviatoric stress space; thus, being completely smooth, it provides a unique definition of the direction of the plastic flow.

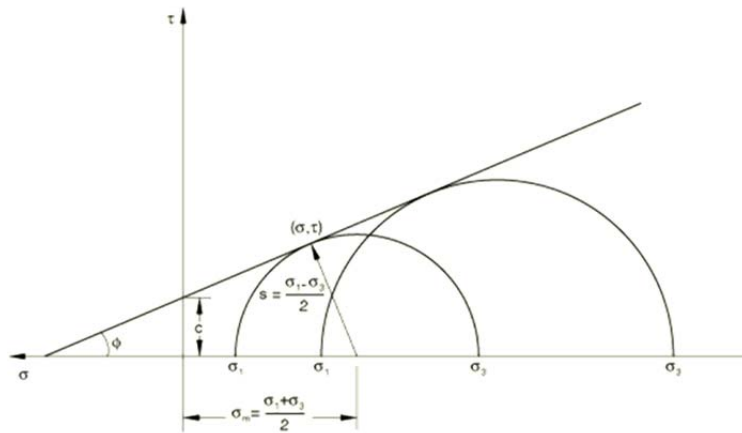


Figure 4.9 Mohr-Coulomb failure criterion [218]

Concrete modeling

As already mentioned, concrete lining is modeled through 2D-PARC model, whose original formulation is described in detail in Chapter 1. In order to be successfully applied to the analysis of concrete linings, 2D-PARC model must be revised to overcome some deficiencies and to include the effect of high temperature.

In particular, it has been firstly improved in the concrete modeling (see Chapter 2). This operation has been performed in order to improve the computational efficiency of the model, as well as to incorporate the effects of crushing and dilatation of concrete in compression. The FE analyses on tunnels are indeed very computationally demanding and hence require a particular efficient numerical algorithm. In addition, the correct simulation of crushing and dilatation phenomena in the analysis of tunnel lining is mandatory, since concrete lining may presents high compressive stresses, especially during fire.

Furthermore, the analysis of concrete lining under fire conditions requires to include thermal strains. To this aim, equilibrium and compatibility equations governing the model are reformulated and the material stiffness matrix is

consequently rearranged, following the same procedure described in Chapter 3 to insert shrinkage deformations (thus representative of all concrete prestrains).

Finally the model is further revised to take into account the loss of strength and stiffness undergone by concrete exposed to fire, by implementing the dependence of the main material mechanical properties on temperature in the algorithm, by adopting the relations proposed in EC2 [216].

In the following the main equations governing the so revised 2D-PARC constitutive law will be presented. It is worth noting that, in this Chapter, 2D-PARC model is extended to represent the behaviour of concrete under fire conditions, whereas the steel reinforcement is not inserted in the calculations. This represents indeed a first step of a more extensive work aimed to apply 2D-PARC model to RC elements exposed to fire.

In the uncracked stage, the stress field $\{\sigma_c\}$ in the concrete lining can be written as:

$$\{\sigma_c\} = [D_c] \left(\{\varepsilon_c\} - \{\varepsilon_{th}\} \right), \quad (4.11)$$

being $\{\varepsilon_c\}$ the total concrete strain and $\{\varepsilon_{th}\}$ the free thermal strain, assumed as positive according to 2D-PARC conventions; $[D_c]$ represents the secant concrete stiffness matrix and it is evaluated by following the improved formulation for the description of concrete behavior presented in Chapter 2. Moreover, in this case the dependence of the main concrete mechanical properties on temperature is inserted in the algorithm.

The transition from uncracked to cracked stage takes place when the current state of stress violates the concrete failure envelope in the cracking region (see §2.3). Crack pattern is then assumed to develop at right angle with respect to principal tensile stress direction, with a constant crack spacing a_{m1} ; moreover, crack orientation is kept fixed throughout the loading process.

After crack formation, a strain decomposition procedure is adopted. The total strain $\{\varepsilon\}$ is subdivided into two components, respectively related to intact, even if damaged, concrete between cracks, $\{\varepsilon_c\}$, and to all the phenomena taking place at crack surfaces, $\{\varepsilon_{cr1}\}$, so obtaining:

$$\{\varepsilon\} = \{\varepsilon_c\} + \{\varepsilon_{cr1}\}, \quad (4.12)$$

where the crack strain $\{\varepsilon_{cr1}\}$ is first evaluated in the crack local co-ordinate system (i.e. perpendicular and parallel to crack direction) as a function of two main variables, namely crack width w_1 and sliding v_1 , and then transferred into the global one.

Following the equilibrium condition, the stress fields related to concrete between adjacent cracks and that in the crack are assumed coincident to each other, being in equilibrium with the external applied stresses.

The stress field in the crack, $\{\sigma_{cr1}\}$, can be determined as the product between the crack stiffness matrix $[D_{cr1}]$ and the strain vector of the fracture zone $\{\varepsilon_{cr1}\}$, so obtaining:

$$\{\sigma\} = \{\sigma_{cr1}\} = [D_{cr1}] \{\varepsilon_{cr1}\} . \quad (4.13)$$

Since steel contribution is not considered in this Chapter, the crack stiffness matrix $[D_{cr1}]$ is assumed coincident to the concrete crack stiffness matrix $[D_{c,cr1}]$. The complete expression of $[D_{c,cr1}]$ can be found in §1.3.4.3, together with a detailed description of the constitutive laws adopted for the modeling of each single resistant mechanism (i.e. aggregate bridging and interlock).

The stress field in the concrete between two adjacent cracks can be evaluated as in the uncracked stage according to Equation (4.11), where the concrete stiffness $[D_c]$ is determined from the corresponding one of the uncracked stage, with slight modifications, see §2.3.5. As in the uncracked stage, the influence of temperature on concrete properties is considered.

Concrete strain between two contiguous cracks $\{\varepsilon_c\}$ and crack strain $\{\varepsilon_{cr}\}$ are then obtained by inverting the equilibrium conditions referred to uncracked concrete between two adjacent cracks – Equation (4.11) – and at crack location - Equation (4.13), respectively. By substituting $\{\varepsilon_c\}$ and $\{\varepsilon_{cr}\}$ into the compatibility Equation (4.12) and inverting the so obtained relation, the total stress vector $\{\sigma\}$ in the global co-ordinate can be written as:

$$\{\sigma\} = \left([D_c]^{-1} + [D_{cr1}]^{-1} \right)^{-1} (\{\varepsilon\} - \{\varepsilon_{th}\}) . \quad (4.14)$$

It is worth noting that the above described procedure can be applied also in case of fiber reinforced concrete. As a matter of fact, their influence, thanks to the modular structure of 2D-PARC, can be easily inserted by only properly modifying the terms of concrete stiffness matrix $[D_c]$ and crack stiffness matrix $[D_{c,cr1}]$, as already explained respectively in §2.3.3.1 and §1.3.4.5.

The free thermal expansion vector $\{\varepsilon_{th}\}$ is assumed as stress independent and isotropic, with shear component equal to zero. The other terms of vector $\{\varepsilon_{th}\}$ are evaluated for each integration point as a function of the temperature T by following the relation suggested in EC2 [216]:

$$\begin{cases} \varepsilon_{th}(T) = -1.8 \cdot 10^{-4} + 9 \cdot 10^{-6} T + 2.3 \cdot 10^{-11} T^3 & 20^\circ\text{C} \leq T \leq 700^\circ\text{C} \\ \varepsilon_{th}(T) = 14 \cdot 10^{-3} & 700^\circ\text{C} < T \leq 1200^\circ\text{C} \end{cases} . \quad (4.15)$$

Since thermal expansion is significantly affected by aggregate type, EC2 [216] proposes two different expressions for calcareous and siliceous aggregates. The equation referring to the latter is adopted, since it provides higher thermal-strain values.

It is worth noting that transient creep strain should be considered in fire analyses. As a matter of fact, when a concrete specimen is first loaded (under

compression) and then heated the resulting thermal strain can differ a lot from the one occurring in an unloaded sample. The difference between these two values represents the load-induced thermal strain (or transient creep), as already explained in §4.2.2.

This strain can be considered implicitly or explicitly in calculations, resulting in a different complexity and degree of accuracy, however the need for considering transient creep explicitly is still a matter of discussion [210,211]. It has been shown that this kind of modeling is preferable when the cooling phase is considered or when the transient creep itself is the main object of the discussion. On the contrary the simplified implicit formulation can be considered reliable when analyzing the heating phase of fire; as demonstrated by the satisfactory results this approach has provided over the years [206,219]. Since in the present study only the heating phase is considered, for sake of simplicity, transient creep strain is taken into account in an implicit manner in the algorithm. Consequently, it is not inserted as a separate strain component, but it is simply considered by adopting EC2 stress-strain relation for concrete at high temperature. As pointed out by several research work [209,220–222] the uniaxial compressive stress–strain relationship provided in EC2 [216] indeed implicitly incorporates the effects of transient creep; in particular the value of the peak strain accounts for the load-induced thermal strain [208].

The EC2 uniaxial compressive stress–strain relation defining the behavior of concrete at elevated temperatures is used herein to calibrate the nonlinear isotropic concrete model implemented in 2D-PARC; a detail explanation of this model as well as of its implementation into 2D-PARC can be found in Chapter 2. In more detail, this isotropic model describes the concrete nonlinear stress-strain relations under general biaxial state of stresses by only properly changing the secant values of Young modulus E_c and Poisson ratio ν on the basis of the actual state of stress. To be properly defined, the model requires six main input data, that are: the initial elastic value of the Young modulus E_{ci} and the Poisson coefficient ν_i , the compressive f_c and the tensile f_{ct} strength, the compressive peak strain ε_{c0} and the post-peak parameter B .

In order to take into account the loss of strength and stiffness undergone by concrete during heating (see Figure 4.10), the dependence of the main variables on temperature is inserted in the model; moreover, a calibration upon the EC2 stress-strain relation for uniaxial compression is performed. This concrete model is indeed very flexible and can match EC2 compressive relation by simply calibrating its parameters. In this work, the biaxial failure envelope proposed by Kupfer and Gerstle [51] with the slight modifications introduced by Barzegar and Schnobrich [99] is adopted and it is kept the same, for sake of simplicity, for all temperatures. However, any failure criterion available in technical literature can be employed in conjunction with this model, without requiring any change to the global algorithm, so making it possible also to consider the lower influence on biaxial compressive strength f_{bc} than on uniaxial compressive strength f_c that temperature seems to exhibit (e.g. $f_{bc}/f_c \approx 1.16$ at 20°C, $f_{bc}/f_c \approx 1.30$ at 300°C, $f_{bc}/f_c \approx 1.70$ at 750°C [183]).

The compressive stress-strain relation suggested by EC2 [216] can be written in the following form:

$$\sigma_c(T) = \frac{3 \cdot \varepsilon / \varepsilon_{c0,T} \cdot f_{c,T}}{2 + (\varepsilon / \varepsilon_{c0,T})^3} \quad \varepsilon \leq \varepsilon_{cu,T} \quad (4.16)$$

The reduction factor for the compressive strength f_c as well as the values of the peak strain ε_{c0} and the ultimate strain ε_{cu} as a function of concrete temperature T can be found in Table 4.3, according to [216] for siliceous aggregates.

During fire, the initial value of the elastic modulus E_{ci} to be provided in the model is then computed as the tangent of Equation (4.16) in the origin of the axes, thus obtaining $E_{ci} = 1.5 f_{c,T} / \varepsilon_{c0,T}$. Moreover, the decay of concrete mechanical properties in compression with temperature is also inserted in the model, according to Table 4.3.

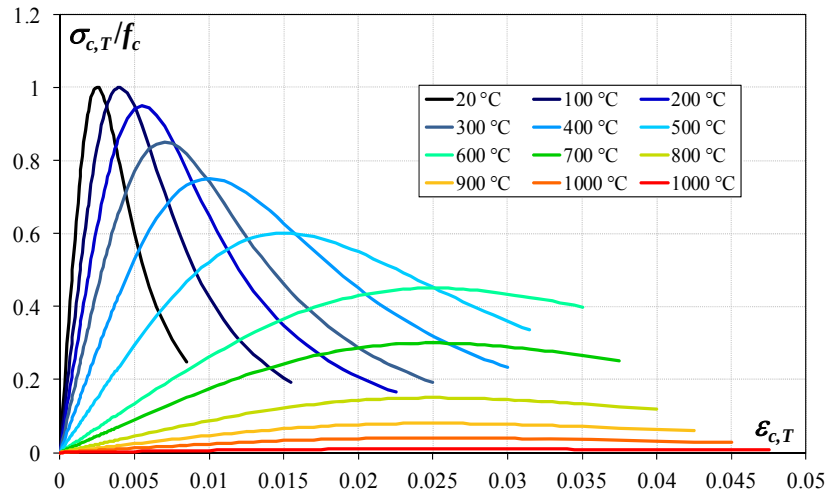


Figure 4.10 Uniaxial compressive stress-strain relation for concrete according to [216] for siliceous aggregates, at different temperature T

Also the decay of concrete tensile strength is inserted in the algorithm, by adopting once again the relation proposed in [216]:

$$f_{c\theta,t} = k_{ct}(T) \cdot f_{ck,t} \quad (4.17)$$

with:

$$k_{ct}(T) = \begin{cases} 1 & 20^{\circ}\text{C} \leq T \leq 100^{\circ}\text{C} \\ 1 - \frac{T - 100}{500} & 100^{\circ}\text{C} \leq T \leq 600^{\circ}\text{C} \\ 0 & T \geq 600^{\circ}\text{C} \end{cases} \quad (4.18)$$

As regards concrete fracture energy, the experimental data suggest that it does not show a clear dependence on temperature [186]; thus, herein it is assumed to be independent of temperature values.

$T [^{\circ}\text{C}]$	$f_{c,T}/f_c$	$\epsilon_{c0,T}$	$\epsilon_{cu,T}$
20	1	0.0025	0.02
100	1	0.004	0.0225
200	0.95	0.0055	0.025
300	0.85	0.007	0.0275
400	0.75	0.01	0.03
500	0.6	0.015	0.0325
600	0.45	0.025	0.035
700	0.3	0.025	0.0375
800	0.15	0.025	0.04
900	0.08	0.025	0.0425
1000	0.04	0.025	0.045
1100	0.01	0.025	0.0475
1200	0	-	-

Table 4.3 Tabular values for the main parameters defining the stress-strain curve as a function of temperature T , according to [216] for siliceous aggregates

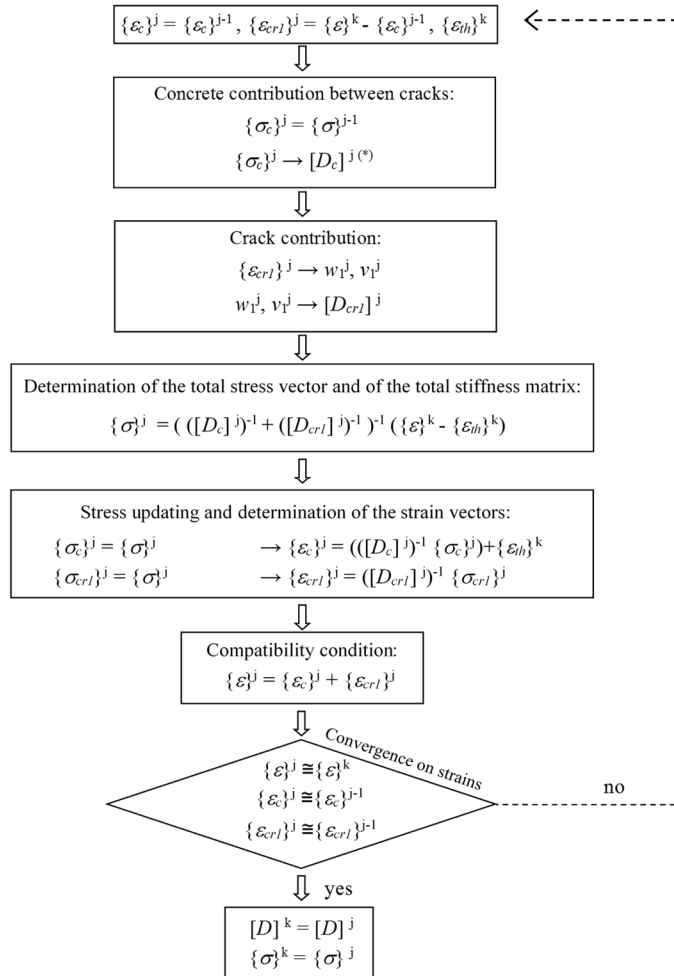
It is worth highlighting that the modular structure of 2D-PARC allows the implementation of other relations available in the literature describing the variation of concrete properties with temperature, similarly different stress-strain relations and failure criteria can be also considered without altering the algorithm structure. A comprehensive database of available relations for the thermal problem can be found, for example in [204,223–225].

It should be also observed that in this work concrete properties are kept constant during the excavation phase, since the focus of the work is the concrete behaviour under fire conditions. Nevertheless, the proposed procedure also allows to simulate the progressive hardening of concrete during the construction period. Concrete properties can be indeed inserted in the algorithm as a function of time and thus updated continuously during the analysis, so accurately accounting for strength and stiffness changes of concrete during tunnel advancement.

Model implementation and convergence checks in the cracked stage

As regards the implementation of the modified algorithm into the FE code, the procedure sketched in Figure 4.11 is followed. Since the problem is nonlinear, the solution strategy followed by ABAQUS is based on an incremental-iterative procedure: for each loading increment, iterations (indicated as “external” in the flow chart of Figure 4.11) are required to achieve convergence.

for the k-th external iteration and the j-th internal one



(*) The evaluation of matrix $[D_c]$ requires internal iterations to calculate the correct values of E_c and v_c ; see §2.3.6 for further details

Figure 4.11 Flow chart of the internal iterative procedure adopted in the cracked stage

In correspondence to each loading increment (or iteration within a fixed loading increment), the updated strain vector $\{\varepsilon\}$ as well as the temperature field obtained by the transient heat transfer analysis are provided by ABAQUS and read by the user subroutine that in turn compute the stress and strain fields in the element. An iterative procedure is also implemented within the user subroutine to achieve the solution in the cracked stage (so-called “internal iterations” in the flow chart of Figure 4.11). As can be seen from Figure 4.11, the followed numerical procedure is very similar to that adopted to include concrete shrinkage strain in the algorithm, which is described in detail in §3.3.1.4.

Moreover, as in case of shrinkage, the inclusion of thermal strains in the model influences the internal convergence procedure required for the evaluation of the concrete stiffness matrix $[D_c]$. To obtain the correct secant values of concrete Young modulus E_c and Poisson coefficient ν_c , which are properly updated during the analysis to account for material nonlinearity, an iterative procedure, based on the bisection method, is indeed required. In particular, when thermal strains are inserted in the algorithm, the expression of the convergence check is properly revised, since only the stress-related strain must be considered. Also in this case the procedure is the same adopted for shrinkage strain, hence for further details see §3.3.1.4

4.4 Validation of the FE model

The effectiveness of the proposed model is validated through comparisons with analytical closed-form solutions. In particular, the accuracy of the simulation of excavation and lining construction phases is verified by comparing FE results to well-known analytical formulations available in technical literature [188–192]. On the contrary a simplified analytical model is herein developed, based on the works [179,180,187,193,194], on purpose to validate the simulation of concrete lining under fire conditions.

4.4.1 Definition of the reference case study

The attention is first focused on a reference case study. It should be noticed that the numerical procedure herein described is general and can be applied to the analysis of realistic cases, characterized by a complex geometry of the opening and complex ground conditions, such as tunnel excavated in soils with anisotropic behavior or characterized by a not homogeneous initial stress distribution, taking also into account the presence of different geomechanical units around the tunnel. However, in order to validate the proposed procedure with analytical formulations, simplified hypothesis are herein adopted.

A deep circular tunnel subjected to a natural isotropic stress state ($k_0=1$) is then considered. The tunnel is hypothesized to be placed at a depth of 100 m and to be characterized by an external radius of 5.15 m. A 300 mm thick concrete lining is applied as support at a distance of 2.5 m from the excavation

face. A similar case was also studied in [187] and, with reference to shallow tunnels, in [179]. The ISO 834 standard fire curve is applied referring to 120' exposure duration.

The thermal properties adopted in the heat transfer analysis are reported in §4.3.2.3. As regards mechanical properties of the materials, a concrete characterized by a characteristic compressive strength f_{ck} equal to 37 MPa ($f_{cm}=45$ MPa) is considered for the lining, while the properties reported in Table 4.4 are considered for the surrounding rock mass.

The constitutive laws applied for soil and concrete are described in detail in §4.3.3.3. On this point, it can be simply recalled that thermal properties of concrete, the decay of its mechanical characteristics and the stress-strain law at elevated temperatures are inserted in the analysis according to EC2 [216] prescriptions, by considering the case of siliceous aggregates in the admixture.

	GSI	γ [kN/m ³]	E [MPa]	ν	ϕ	c [MPa]
Marble	30	25	5200	0.25	50	0.3

Table 4.4 Rock mass mechanical properties

4.4.2 Validation of the thermal analysis

The heat transfer analysis provides the distribution of temperatures in space and time. The predicted temperature evolutions in time at various locations of concrete lining are reported in Figure 4.12.

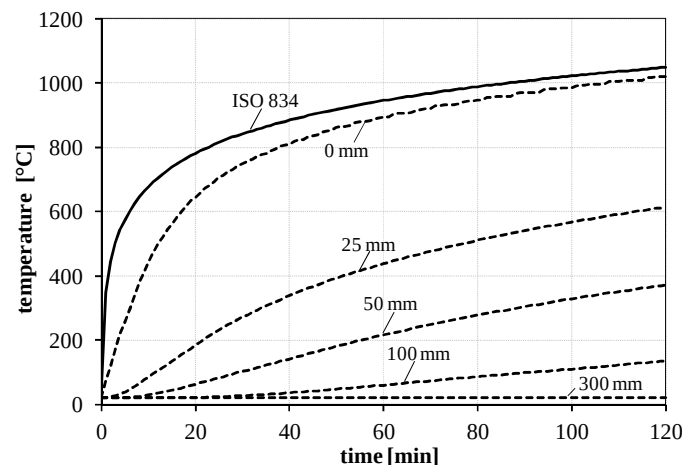


Figure 4.12 Evolution of temperatures in time at different distances from the heated surface

It can be noticed that due to the thickness of lining, only the inner parts of concrete are subjected to temperature increases, whereas the external points as well as the surrounding soil remains at about the initial temperature value ($T=20^{\circ}\text{C}$). The pronounced nonlinear temperature distribution across the lining thickness is shown, for selected time instants, in Figure 4.13. Moreover, Figure 4.13 compares the numerical results to the temperature profiles provided in [216] for slabs, showing a very close agreement.

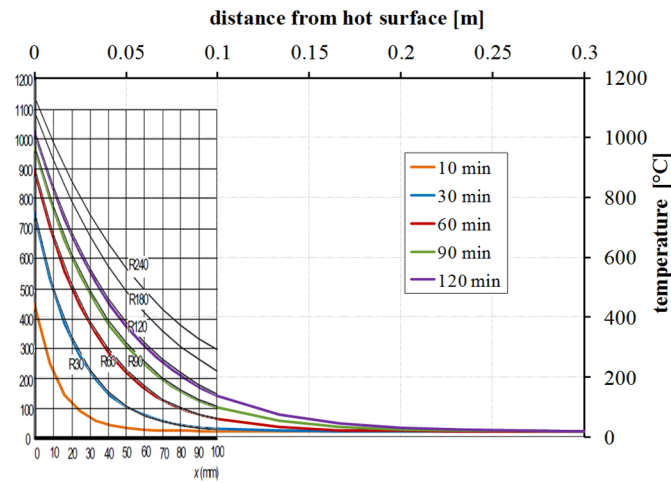


Figure 4.13 Distribution of temperature across concrete lining thickness: comparison between numerical results and EC2 temperature profiles for slabs

4.4.3 Validation of the structural analysis: excavation and lining installation phases

In order to validate the excavation and installation phases comparisons with closed-form analytical formulations are provided.

Preliminary, the simulation of tunnel excavation without applying any support is performed. The obtained results are then compared to the formulations proposed by Lembo-Fazio and Ribacchi [188] and Carranza-Torres and Fairhurst [189–191], referring to a circular deep tunnel with zero internal pressure and to a rock mass characterized by an elastic-plastic behavior.

As can be observed from Figure 4.14 a good agreement between numerical and theoretical results is obtained in term of both principal (radial and hoop) stresses and displacements along a direction radial to the opening. Numerical results refer to a point placed along tunnel perimeter, in correspondence of a cross-section at the middle of tunnel length, at the end of the excavation phase (i.e. when the tunnel is full excavated).

The tunnel excavation determines a stress concentration near the opening. Since the maximum stresses exceed the Mohr-Coulomb failure criterion, soil

plasticization in the region surrounding the bore occurs. Consequently, the stress field shows a discontinuity in correspondence of the plastic radius (which represents the limit between the plastic and the elastic zone of the rock mass), since the plastic strains determine a stress re-distribution. The extension of the plastic zone is correctly described by the model, as well as the values assumed by the radial and the hoop stresses at different distances from the opening.

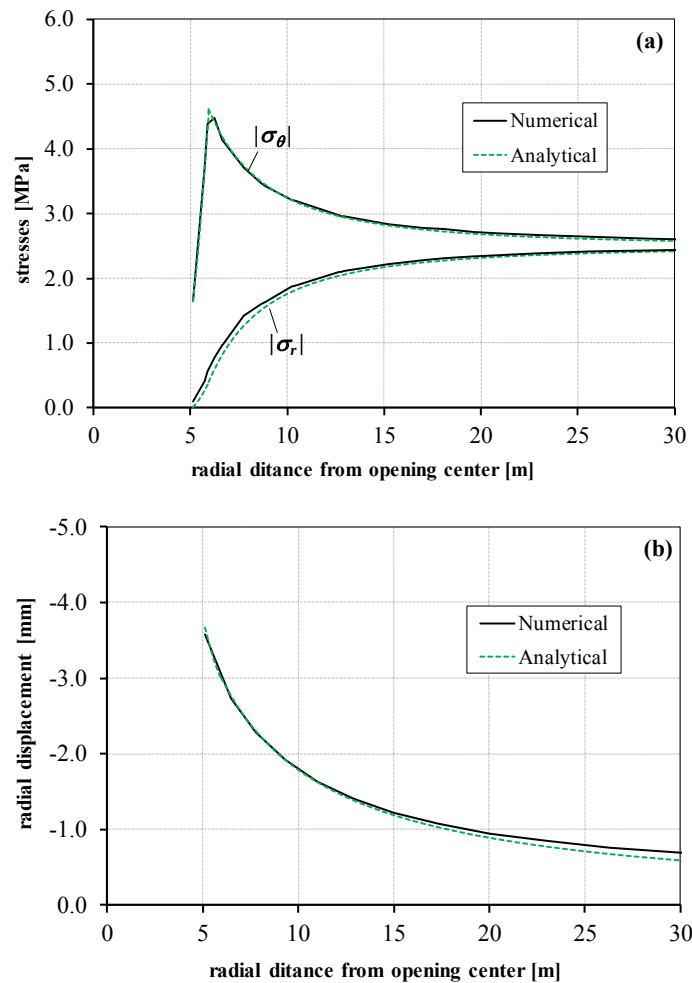


Figure 4.14 Comparison between numerical and theoretical [188–191] results in terms of: (a) radial and hoop stresses, (b) radial convergence displacement along a radial direction to the opening, for the tunnel without lining

Then, the simulation of the tunnel supported with the concrete lining is carried out. Comparisons between numerical and theoretical [188–191] results are provided in Figure 4.15: close agreement is once again achieved. The analytical curves refer to a tunnel characterized by an internal pressure of about 0.75 MPa, corresponding to the one provided by the support (placed at 2.5 m from the excavation face).

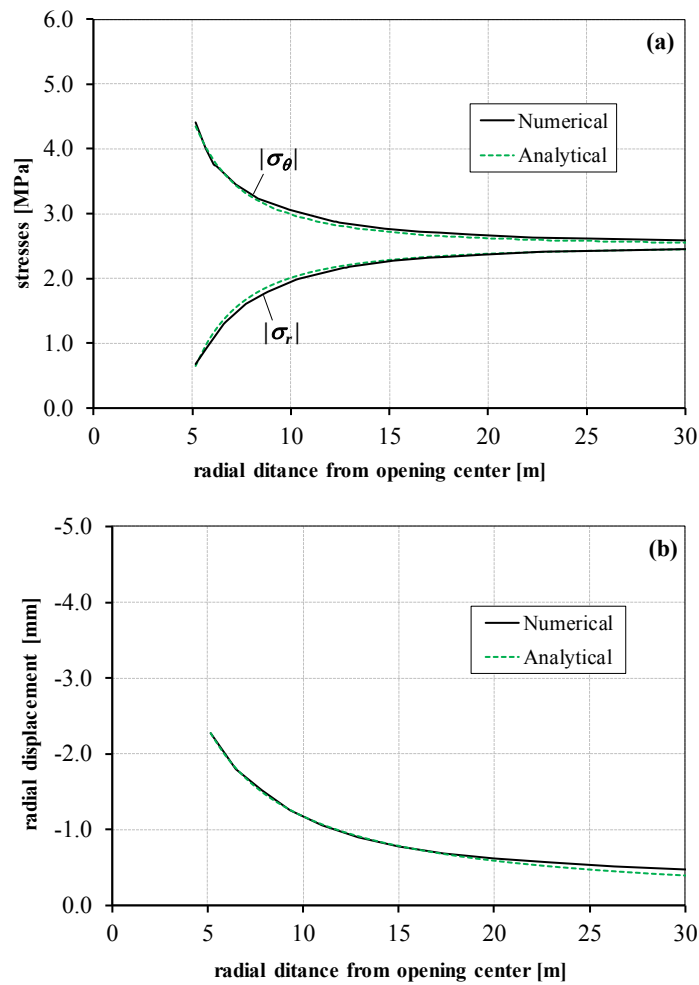


Figure 4.15 Comparison between numerical and theoretical [188–191] results in terms of: (a) radial and hoop stresses, (b) radial convergence displacement along a radial direction to the opening, for the reference case study

As can be inferred by Figure 4.15, the rock remains in this case in the elastic field, since part of load is supported by the concrete lining. The support also provides an effective action in reducing the stress field in the rock as well as the convergence displacements.

The proposed 3D FE procedure also allows to obtain the longitudinal deformation profile, which represents the profile of radial displacements along the tunnel axis. Figure 4.16 reports the comparison between numerical results obtained for the unsupported tunnel and the elasto-plastic relation proposed by Corbetta et al. [192]. The horizontal axis of Figure 4.16 represents the normalized distance to the front, i.e. the ratio between the distance from the excavation face and the tunnel radius; positive values are assumed for the excavated part, whereas negative ones are used for the ahead of the face. The vertical axis represents instead the scaled radial displacement, i.e. the ratio between the convergence displacement and a reference value. This latter is herein evaluated as the maximum value reached by the radial displacement far enough from the front in elastic conditions. Closer correspondence between numerical and analytical [192] results can be stated.

Moreover, on the same graph the numerical longitudinal deformation profile for the supported tunnel is provided. As can be seen, concrete lining highly reduces the convergence displacements behind the face of the tunnel, whereas its influence ahead the face is limited.

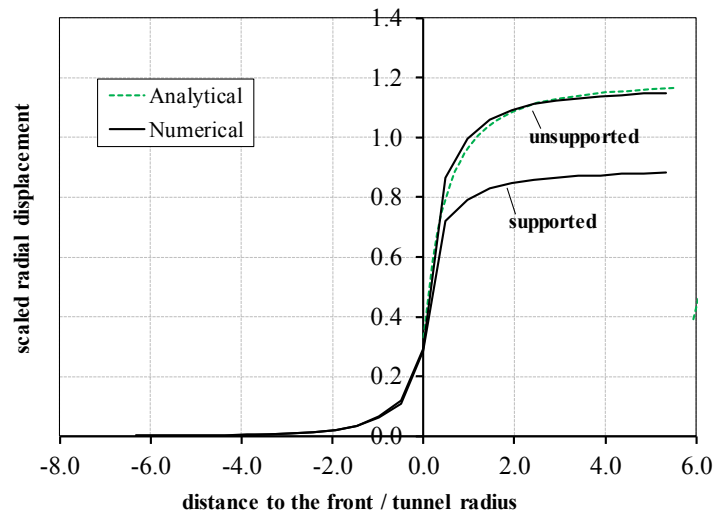


Figure 4.16 Longitudinal deformation profile for unsupported and supported tunnel

4.4.4 Validation of the structural analysis: fire phase

In order to verify the effectiveness of the proposed procedure during the fire phase, a simplified analytical model is herein developed, on the basis of some previous works available in technical literature [179,180,187,193,194].

The state of stress in the lining is calculated by treating the case of the tunnel subjected to fire as an axisymmetric problem, as proposed in [193,194]. The reference case study described in §4.4.1 is considered; in particular the model is based on three main assumptions, that is deep tunnel, circular opening and initial isotropic stress state ($k_0 = 1$). Under the hypothesis of deep tunnel (i.e. the tunnel diameter can be considered negligible with respect to tunnel depth) the state of stress in the lining before the beginning of fire can be indeed considered as uniform. Moreover, the temperature distribution is assumed to be dependent only from the radial co-ordinate r , and not from the angular one θ (see Figure 4.17). As a consequence, each variable involved in the problem only depends on the radial co-ordinate r , thus obtaining an axisymmetric problem.

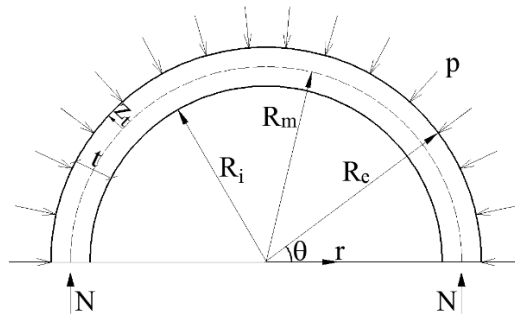


Figure 4.17 Sketch of external loads and internal actions in the lining, with indication of the assumed polar co-ordinate system

Calculations are performed by assuming a uniform distributed load p acting on lining extrados. This load is expressed as the sum of two contributions, which are respectively related to the amount of geostatic load directly carried by the lining, p_g , as well as to the reaction, r^* , opposed by the surrounding ground to lining thermal expansion during fire.

The geostatic pressure, p_g , is evaluated by applying the convergence-confinement method and it represents the pressure transferred from the rock mass to the lining once the excavation face is far enough from the support (i.e. in this case, it represents the equilibrium pressure when the tunnel is fully excavated). In order to compute p_g , the procedure proposed in [190] is herein applied. Once the longitudinal deformation profile (LDP), the ground reaction curve (GRC) and the support characteristic curve (SCC) are determined, the equilibrium condition is found by considering that the lining is applied at a predefined distance from the tunnel face.

The reaction opposed by the ground to lining thermal expansion is evaluated through the so-called “beam-spring” model, which consists in schematizing the rock mass with springs (see also [179]), having a stiffness equal to:

$$K = \frac{E}{(1 + \nu) R^*} \quad , \quad (4.19)$$

being E and ν respectively the Young modulus and Poisson ratio of the rock mass. It is worth noting that R^* represents the external tunnel radius if the rock is still elastic in the equilibrium condition, otherwise it can be assumed as the extent of the plastic region that develops around the tunnel, hypothesizing a Mohr-Coulomb elasto-plastic behavior for the soil. The reaction opposed by the surrounding ground to the thermal expansion of the lining, r^* , is then equal to the product between the rock stiffness K and the radial displacement u undergone by the lining itself during fire.

To solve the problem, Mariotte relation is considered: the axial force N per unit length of the tunnel (see Figure 4.17), related to the stress field in the lining should be equilibrated by the resultant of the external distributed load p , as also suggested in [193,194].

Hoop stresses are computed by dividing the lining thickness into layers and by assigning to them a temperature distribution based on the results of a heat-transfer analysis. Such analysis technique is similar to that used in [180].

At different fire stages, thermal strains ε_{th} in each concrete layer are then calculated according to EC 2 [216]. The combination of these thermal strains with the preexisting strain field in the lining (due to its interaction with the surrounding ground) results in a nonlinear strain distribution along lining thickness. Compatibility is then restored by the appearance of a strain field ε^* , related to internal stresses and defined as:

$$\varepsilon^* = \varepsilon_{tot} - \varepsilon_{th} \quad , \quad (4.20)$$

where ε_{tot} represents the total circumferential strain in the lining. With the assumption that sections remain planes, the total circumferential strain ε_{tot} can be in turn expressed as a linear function within the depth of the lining of the deformation in correspondence of the barycentric axis, ε_0 , through the relation:

$$\varepsilon_{tot} = \varepsilon_0 \left(1 - \frac{z_t}{R_m} \right) \quad , \quad (4.21)$$

where R_m represents the average tunnel radius, while z_t indicates the radial coordinate along lining thickness (see also Figure 4.17). It is worth noting that the barycentric strain ε_0 is not known, representing the only unknown of the problem.

By integrating the stress field produced by strains ε^* along the thickness of each layer and summing up all the contributions, the resultant (per unit length) of the circumferential stresses in the lining can be obtained as:

$$N = \sum_{i=1}^{n_{layers}} \left(\int_{t_i} \sigma(\varepsilon^*) dz_t \right) \quad (4.22)$$

On the basis of the mechanical strains ε^* , the stresses at each layer can be computed from the stress-strain curves adopted for concrete, properly modified to account for the decay of mechanical properties due to fire. The stress-strain laws at elevated temperatures for both compression and tension are evaluated according to EC2 prescriptions. When concrete cracks, the fracture energy is instead assumed to be independent of temperature and the stress-crack opening relation is computed according to Model Code 2010 [57].

Finally, according to Mariotte relation, the force equilibrium can be written as:

$$N = p R_e = (p_g + Ku) R_e = p_g R_e + K (\varepsilon_{tot,ext} - \varepsilon_g) R_e^2 \quad (4.23)$$

where R_e is the external tunnel radius, $\varepsilon_{tot,ext}$ is the total circumferential strain at lining extrados, while ε_g represents the strain contribution related to the hoop stresses in the lining deriving from the interaction with the surrounding ground. It can be observed that, except when a collapse mechanism develops in the lining, no significant changes in the interaction with ground are expected, since the tube outline is little affected by thermal strains [212]. Due to model assumptions, ε_g is constant along the thickness and can be easily estimated based on the amount of geostatic load p_g carried by the lining. By substituting Equations (4.21) and (4.22) into Equation (4.23), the problem can be then solved by using a trial and error method as a function of the only unknown ε_0 .

The obtained results together with the numerical responses are reported for comparison in Figure 4.18 in terms of hoop stress distribution along the radial direction, inside the lining thickness. Five different fire durations (10', 30', 60', 90' and 120') are considered.

A stress peak arises within the lining thickness, due to stress relaxation near the heated surface and to the deformation constraint in the circumferential direction exerted by the cold extrados, as observed by other Authors (e.g. among others, [181,187,212]). An almost biaxial compressive state of stress occurs and a typical bell-shape stress distribution appears both in the circumferential and in the longitudinal direction. On the contrary, the rock mass does not exhibit a significant increase of loading. However these additional results are obviously not provided by the proposed analytical model based on the axisymmetric condition; thus, only circumferential stresses are considered for comparison.

As shown in Figure 4.18, both the proposed analytical approach and numerical procedure are able to correctly describe the typical bell-shape distribution of hoop stresses in the lining thickness, as well as the change in peak location with an increasing duration of fire loading. The location of the peak of the hoop stresses undergoes indeed a shift along lining thickness, moving from the layers closer to the heated surface to the colder and farther ones for longer times of fire exposure, as a consequence of the diffusion of high temperatures in the lining with the related decay of concrete mechanical properties. It can be also observed that, in the considered case study, the maximum hoop stress value remains almost the same. This can be explained by observing that hoop stresses in the lining for different fire durations never reach concrete compressive strength at the corresponding attained temperatures. Moreover, the colder part of the lining is not interested by the appearance of longitudinal cracks, which might cause a reduction of the circumferential confinement [181].

The close correspondence between numerical and analytical results for all the considered fire duration proves the accuracy of the FE procedure.

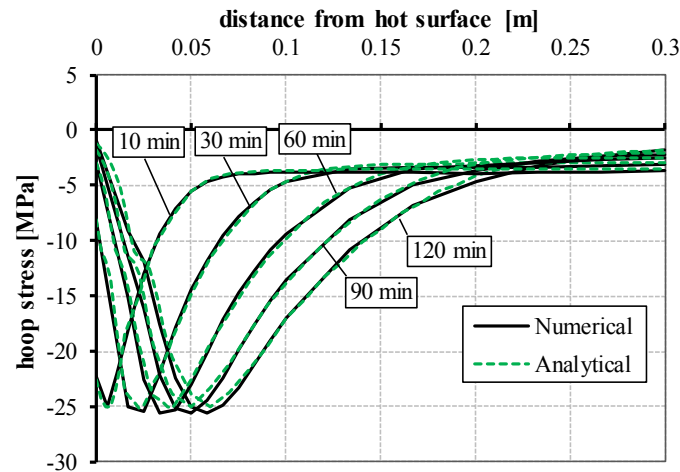


Figure 4.18 Comparison between numerical and analytical results in terms of circumferential stress distribution within the lining thickness at different time instants (10', 30', 60', 90' and 120')

4.5 Parametric study

Once the proposed FE procedure has been validated, the influence exerted by some significant variables on the stress field in concrete linings under fire conditions can be studied. The most influencing parameters are usually represented by rock mass properties, tunnel depth, in situ stress field (in terms of lateral earth pressure coefficient k_0), geometry of the opening, type of support system and its characteristics. Moreover, under fire conditions, also the relations adopted for describing concrete behavior at high temperatures as well as the type of chosen fire play a major role.

In this work, three factors affecting the structural response are selected, that is type of fire, ground mechanical properties and tunnel depth. Sensitivity study to assess the influence of ground parameters are advisable, since the soil is often characterized by high spatial variability and its characteristics are not known with certainty. Moreover, since a final agreement of the scientific community on which fire curve is better suited for tunnel analysis has not been reached yet, the effect of different types of fire on the stress field in the lining is also analyzed.

4.5.1 Type of fire

As explained in §4.3.2.2, several fire curves are proposed in the literature to study tunnels under fire conditions; among them three relations are herein considered: ISO 834, HC and RWS. The ISO 834 standard curve is adopted as reference fire curve, due to its large diffusion in structural applications. However, it is known that tunnel fires differ from typical building fires due to the high calorific values of burning fuel and vehicles, together with the confinement of released heat. To this aim, HC and RWS curves, related to increasing severities of the fire scenario, are considered for comparison.

Figure 4.19 show the results relative to the reference case study subjected to the three different types of fire. In more detail, hoop stress distributions across the lining thickness are reported in Figure 4.19b,d,f. while the corresponding temperature profiles are plotted in Figure 4.19a,c,e. As can be seen, a greater warming of the exposed surface is obtained when adopting the RWS or HC fire curves, with respect to the standard ISO 834 curve. Moreover, the maximum difference among the reached temperature values is more pronounced in the first minutes of fire, since RWS and HC fire curves refer to a faster fire development in the first minutes (see also Figure 4.7). Both RWS and HC fires cause indeed the appearance of temperatures above 800°C in the concrete lining within the first 10' of exposure. This lead to a stress reduction close to the heated lining intrados, due to concrete degradation, with the appearance of a stress peak, which is slightly moved towards the colder part of the lining. On the other hand, the ISO 834 curve cannot produce such high temperatures in a so limited exposure time. For this reason, in case of cellulosic fire, the typical bell-shaped stress distribution is translated in the direction of the heated surface, compared to the other fire curves. It can be also observed that, after 120', both the

temperature profiles and the hoop stress distributions in the lining obtained with the three fire curves are more similar to each other.

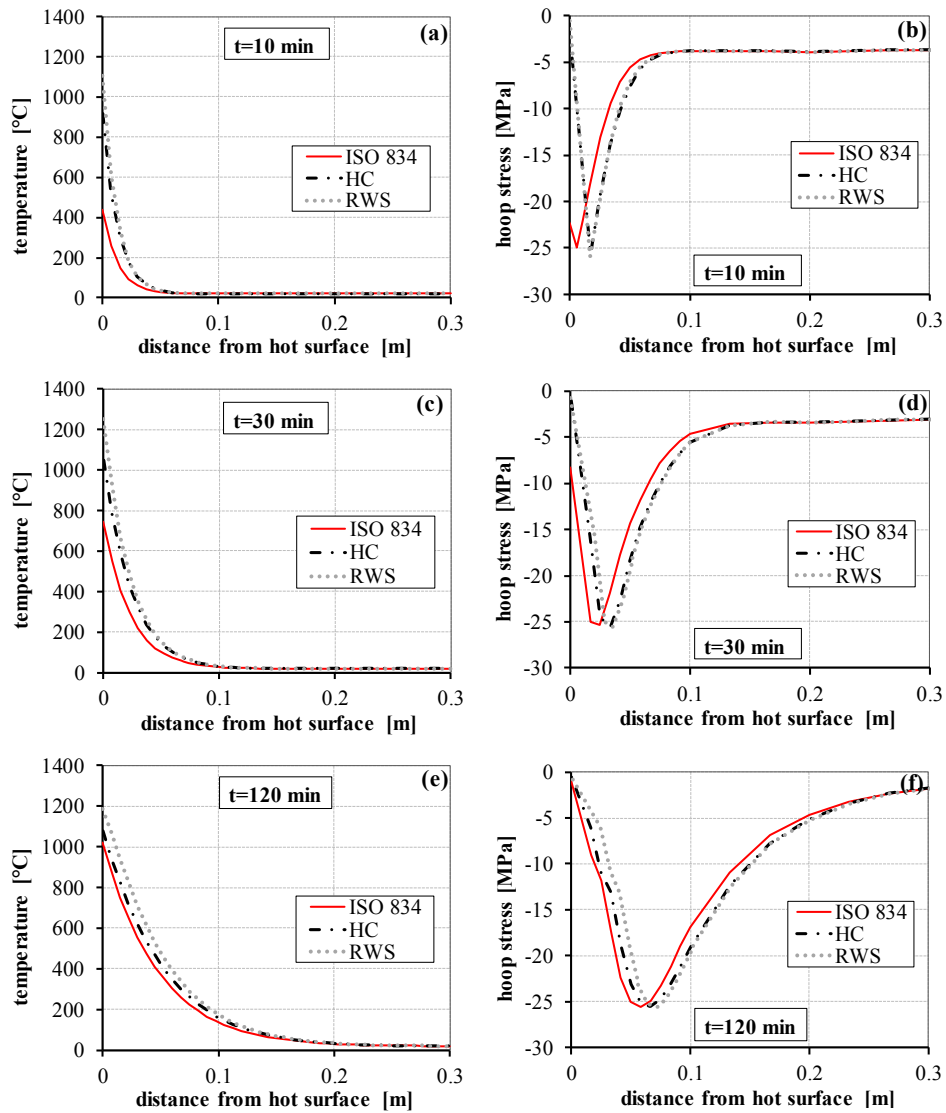


Figure 4.19 (a), (c), (e) Temperature distribution and (b), (d), (f) hoop stress across concrete lining thickness for the reference case study and different fire curves after 10', 30' and 120'.

4.5.2 Ground properties and tunnel depth

Another important aspect influencing the structural behavior of the lining under fire conditions is represented by the surrounding boundary conditions, which depends on the mechanical properties of the ground, as well as on tunnel depth. To this aim, three different rock masses, named 1, 2 and 3, respectively characterized by decreasing quality are considered. The most significant geotechnical parameters assumed in the analyses are reported in Table 4.5. At the same time, tunnel depth is made varying between 50, 75 and 100 m, so corresponding to almost 5, 7.5 and 10 tunnel diameters; thus, having a total of 9 cases. It is worth noting that type 2 rock mass with 10-diameter overburden corresponds to the reference case study described in §4.4.1. The fire curve is instead kept the same in all the analyses (standard ISO 834).

Rock ID	Type	GSI	γ [kN/m ³]	E [MPa]	ν	ϕ	c [MPa]
1	Sandstone	50	25	10800	0.25	60	0.8
2	Marble	30	25	5200	0.25	50	0.3
3	Schist	15	25	1900	0.25	44	0.2

Table 4.5 Rock mass mechanical properties

The influence exerted by rock mass properties and different overburdens on the state of stress in the lining during fire is shown in Figure 4.20 - Figure 4.23.

Comparisons are provided at first for different overburdens keeping the same rock type (see Figure 4.20 and Figure 4.21): on each graph, three curves are plotted, respectively referred to the considered three tunnel depths (50, 75 and 100 m), for a fire duration of 30', 60', 90' and 120'. Generally speaking, it can be inferred that the first part of the bell-shaped stress curve is not influenced by tunnel depth, since both the peak value and its position are almost the same. Different tunnel overburdens determine instead a variation in the post-peak branch, where ground influence is more pronounced. As an example, for a given time of fire exposure, greater overburdens result in increased geostatic stresses, which in turn lead to increased hoop stresses in the colder part of the lining, near the interface with the surrounding rock mass. Moreover, for longer time exposures (120', Figure 4.20d,h and Figure 4.21d), it can be observed in some cases the appearance of tensile stresses near lining extrados, which are more pronounced for a lower overburden.

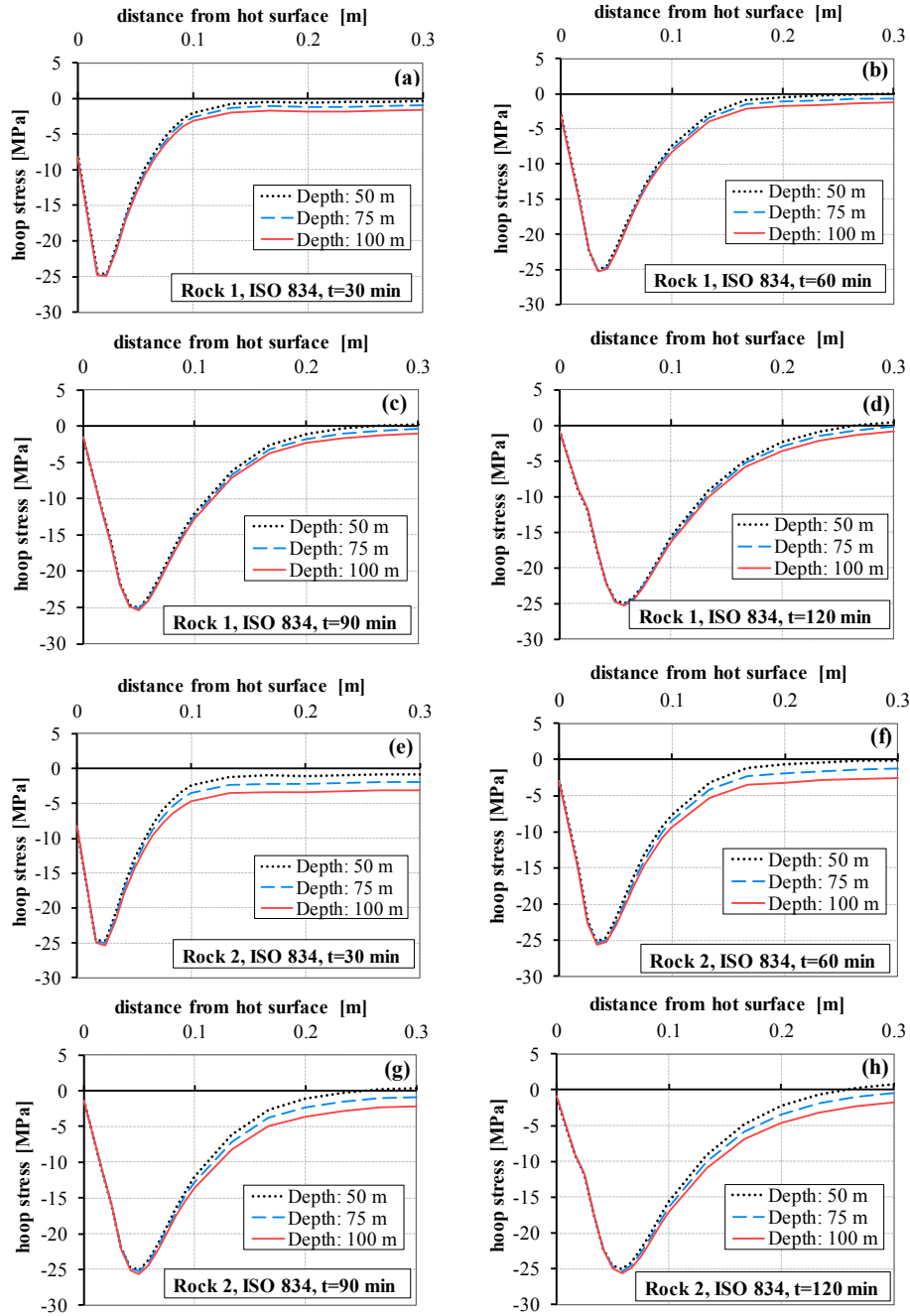


Figure 4.20 Hoop stress distribution across lining thickness after 30', 60', 90' 120' for: (a)-(d) rock type 1; (e)-(h) rock type 2, by considering different tunnel depths

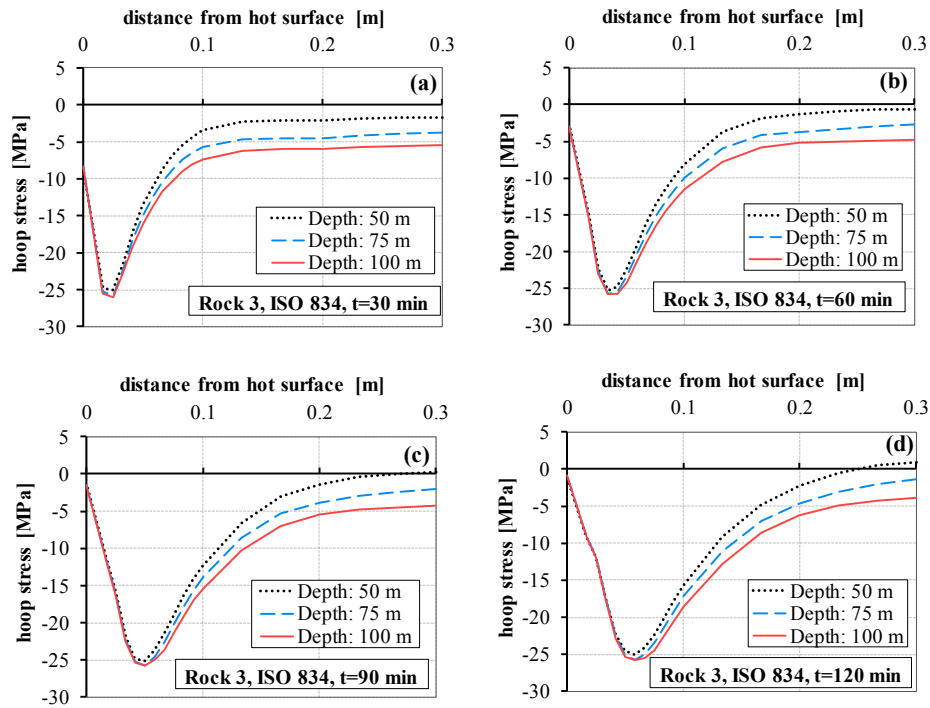


Figure 4.21 Hoop stress distribution across lining thickness after 30', 60', 90' 120' for rock type 3, by considering different tunnel depths

Similarly, the influence exerted by ground properties on the structural response of concrete lining under fire conditions keeping the same overburden is shown Figure 4.22 and Figure 4.23. The effect of rock mass properties is more or less significant depending on tunnel depth and fire exposure. Major variations in the hoop stress entity are indeed registered for higher overburdens and shorter fire durations (e.g. 100 m and 30'). Also in this case, only the post-peak branch of the curve is influenced by a change in ground mechanical properties, while the peak value and its position are almost the same for all the considered cases. As can be expected, higher compressive stresses in the lining appear in presence of poor rock characteristics, due to the larger extension of the plastic zone around the opening and the consequent larger amount of geostatic pressure carried by the lining itself at the time of its installation. This effect is more pronounced for a shorter fire exposure, since the stress field in the lining is disturbed by the temperature gradient only in a limited part near the opening. In some cases, tensile stresses may appear near lining extrados; this effect is more pronounced in case of poor mechanical properties of the ground (rock type 3) and longer fire exposure.

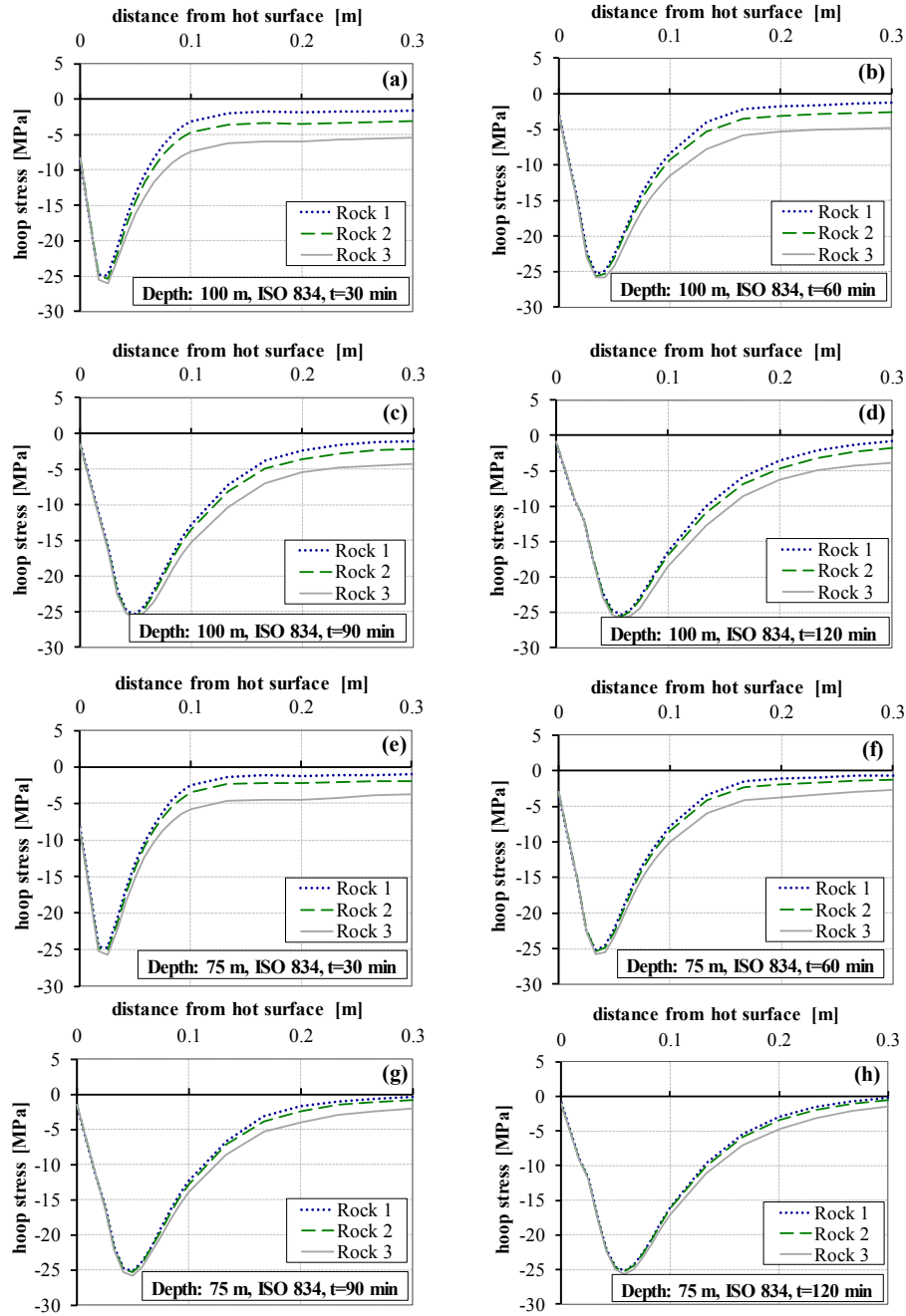


Figure 4.22 Hoop stress distribution across lining thickness after 30', 60', 90' 120' for a tunnel: (a)-(d) 100 m; (e)-(h) 75 m deep, by considering different rock types

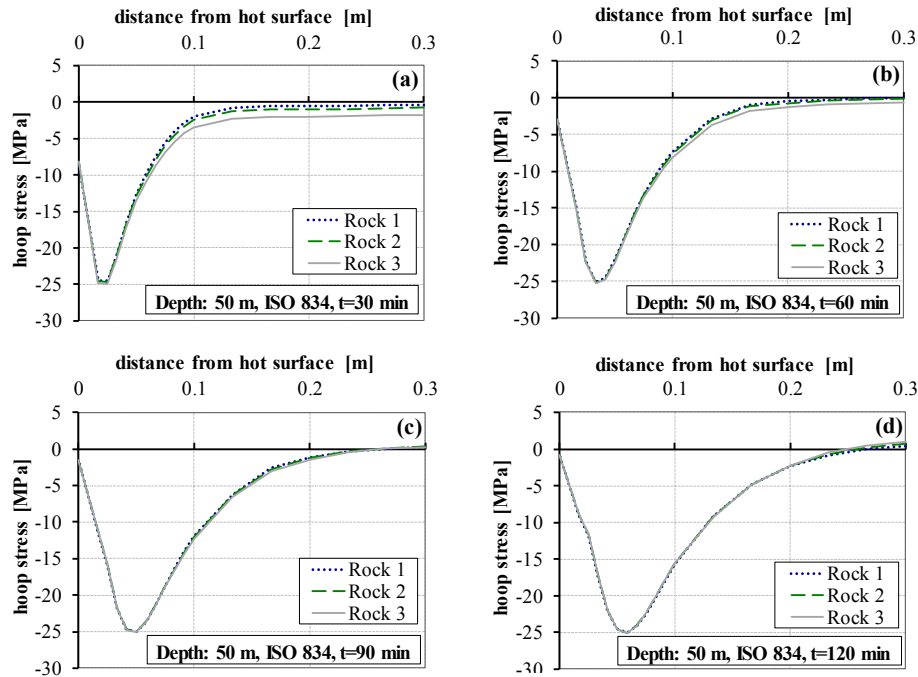


Figure 4.23 Hoop stress distribution across lining thickness after 30', 60', 90' 120' for a tunnel 50 m deep, by considering different rock types

This phenomenon can be explained by considering the heating and the equilibrium conditions in the concrete lining.

As can be observed from the temperature distribution across lining thickness referred to 120' of fire exposure (Figure 4.13), only a layer with depth approximately equal to 15 cm and closer to the exposed surface is characterized by temperatures higher than 100°C. The lining can be then considered composed by two concentric portions: an external cooler part and a hot inner one (see Figure 4.24). The hot inner ring tends to expand more and more with increasing temperatures, i.e. with increasing fire durations; whereas the external layers farther from the exposed surface do not expand at the same manner, being much colder. As a consequence, they tend to oppose to the expansion of the hotter part (see also [181,182]). It results that the cooler part of the lining is subjected to tensile stresses deriving from this confinement action in the circumferential direction, as well as to compressive stresses deriving from the interaction with the surrounding ground. When the stresses in the lining before the application of fire are small or the rock mass does not oppose significant resistance to the lining expansion, that is in case of lower tunnel depth or poor mechanical characteristics of the soil, tensile stresses prevail and may also lead to crack formation in the cooler layers.

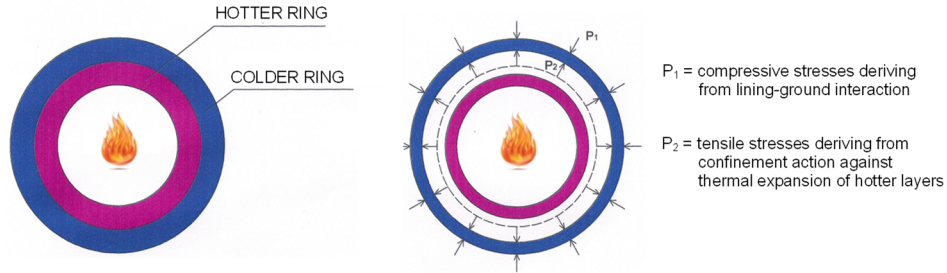


Figure 4.24 Sketch of the hotter and the colder parts of the lining

As a matter of fact, the rock mass does not always behave as a rigid constraint and this behavior is lost especially when the soil is characterized by poor mechanical properties. This effect can be better understood by comparing the stiffness of the ground to the stiffness of the lining, both evaluated with reference to the axisymmetric schematization of the problem presented in §4.4.4 (see also [194]). In particular, the stiffness of the rock mass K_{ground} can be computed as reported in Equation (4.19), whereas the stiffness of the lining K_{lining} can be evaluated by exploiting Mariotte relation as:

$$K_{lining} = \frac{tE_c}{R_e^2}, \quad (4.24)$$

where t represents the lining thickness, R_e the external tunnel radius and E_c the Young modulus of concrete.

For the analyzed cases, it can be observed that K_{ground} and K_{lining} are comparable for rock type 3.

For further verification, the numerical results obtained by performing the parametric study are also compared to the theoretical ones obtained by applying the analytical model proposed in §4.4.4. Close results are obtained for all the considered cases; for sake of brevity, only some of them are reported herein. As can be seen in Figure 4.25, analytical curves are almost superimposed to numerical ones, also when tensile stresses appear near lining extrados.

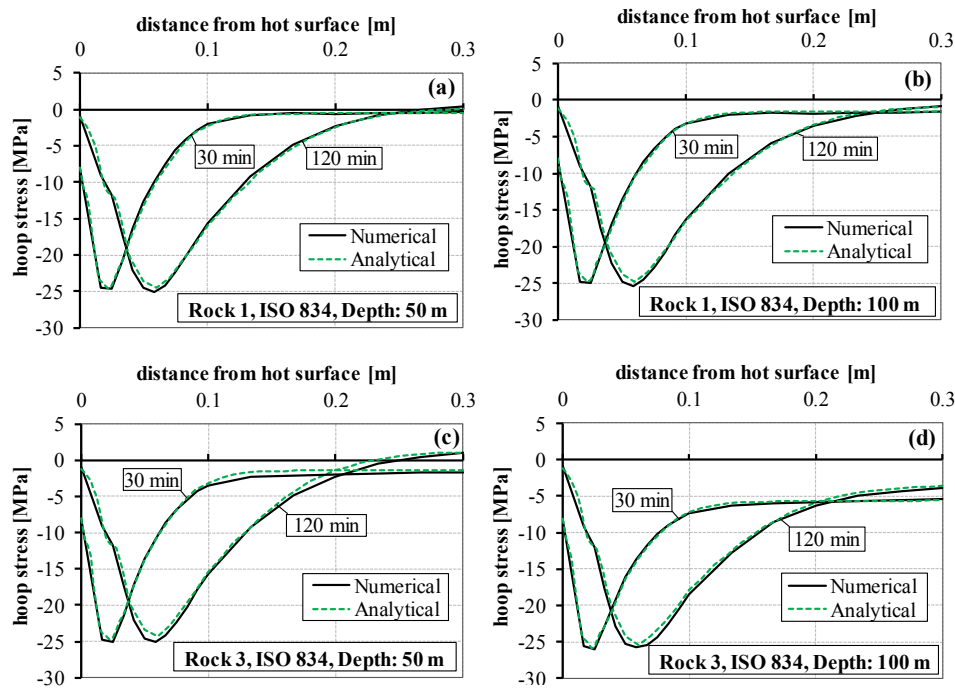


Figure 4.25 Hoop stress distribution across lining thickness after 30' and 120' for different rock types (rock 1 and rock 3) and different tunnel depths (50 and 100 m): comparison between numerical and analytical results

4.6 Concluding remarks

In this chapter a 3D nonlinear finite element procedure is developed in order to investigate concrete lining behavior under fire conditions.

Fire design of concrete structures is a very complicated task and the problem becomes even more complex in case of tunnel linings. The response of these elements at high temperatures is indeed influenced by several parameters, which should be properly taken into account during the analysis in order to provide reliable predictions. In this work attempts are made so as to simulate with accuracy both the ground-support interaction, with special attention to the excavation/construction phases, and the concrete lining behavior during fire, by applying a thermo-mechanical sequentially coupled approach.

In more detail, the FE nonlinear analysis is carried out by firstly performing a heat-transfer analysis, which provides the temperature field in each node of the model under an assigned fire scenario, and then by undertaking a mechanical simulation. The structural analysis is in turn subdivided into two main stages: the

simulation of the excavation and lining installation phases is first performed and then the fire load is applied.

In order to realistically simulate the soil excavation and subsequent lining installation, a step-by-step procedure is followed, by at first applying the lithostatic load and then progressively deactivating solid elements corresponding to tunnel opening while at the same time activating lining elements, at a prescribed distance from tunnel face, so as to model the support construction.

Then, the simulation of tunnel behavior under fire conditions is carried out by imposing the temperature distribution previously obtained from the heat-transfer analysis and consequently evaluating the stress-strain field.

It is worth noting that concrete lining behavior, both before and during fire, is described through a very accurate constitutive law, 2D-PARC, which is herein properly improved and extended on purpose. In more detail this constitutive relation, already revised in the concrete modeling (see Chapter 2) in order to enhance the computational efficiency of the model, as well as to incorporate the effects of crushing and dilatation of concrete in compression, is further extended so as to insert thermal strains in the algorithm and to properly take into account the decay of the mechanical properties under high temperatures.

The effectiveness of the proposed FE procedure is validated through comparisons with analytical closed-form solutions. In more detail, the simulation of excavation phase and lining construction is verified by comparing FE results to well-known analytical formulations available in technical literature [188–192]. On the contrary, a simplified analytical model is developed on purpose, based on the works [179,180,187,193,194], so as to validate the numerical results obtained for concrete linings under fire conditions. The close correlation between analytical and numerical results proves the potentialities of the proposed approach for the assessment of the structural safety of tunnels before and during fire.

Moreover, a parametric study is performed so as to better highlight the influence exerted by different parameters on the structural response of the lining, especially during fire. To this end, different ground mechanical properties, tunnel depths and fire curves are considered to evaluate how these variables can affect the stress-strain field in the tunnel lining. It can be observed that for higher fire exposures, lower overburdens and poor mechanical properties of the ground, tensile stresses may appear near lining extrados also in case of initial isotropic stress-state and circular opening, due to the differential tendency to expand of the inner (and hotter) concrete layers and the external (and cooler) ones.

The proposed numerical approach is herein successfully validated under simplified conditions, since it represents only a first step in the field of the study of tunnels under fire conditions. This work will be further developed in order to treat more general and realistic cases, as well as to take into account crucial aspects in the assessment of tunnel safety under fire condition, disregarded in this phase. As an example, some aspects to be further deepened regards the realistic representation of fire development and propagation in the tunnel, a more refined inclusion of transient creep in the algorithm, the simulation of the phenomenon of concrete spalling and the extension of the model to different kinds of support (e.g. concrete lining reinforced with steel rebars and fibers).

Conclusions

The present work aims at contributing to the ongoing research in the field of modeling of reinforced concrete (RC) structures. An existing nonlinear model, named 2D-PARC, is revised and extended to be finally applied to study of tunnel linings under fire conditions.

The model, which belongs to the approaches that smear both the reinforcement and cracking within the material, is able to describe the behavior from the beginning of the loading history up to failure of reinforced or fiber reinforced (RC/FRC) concrete elements subjected to in-plane stresses. It is formulated in terms of a secant stiffness matrix, to be implemented into a finite element (FE) code. In the uncracked stage, concrete and steel are treated like two materials working in parallel, by assuming perfect bond between them; thus, the stiffness matrix is composed by summing up the concrete and the steel contributions. When the maximum principal stress exceeds the failure envelope in the tensile region, the transition to the cracked stage takes place and the adopted constitutive matrix is properly modified so as to correctly represent the softening effect resulting from the cracking process. Then, the model follows a fixed crack approach and applies a strain decomposition procedure. The total strain is subdivided into two components, respectively related to the intact RC/FRC material, even though damaged, between cracks and to all the resistant mechanisms of the fracture zone (i.e. aggregate bridging and interlock, tension stiffening, dowel action and fiber bridging in case of FRC). Moreover, thanks to the modular framework of the model, these two main contributions can be in turn evaluated by summing up all the mechanisms referring to the two material components (i.e. concrete and steel), so inserting in a transparent manner all the resistant contributions related to crack kinematics. Nevertheless, even if this model is proven to provide reliable results, some limitations are noticeable, with respect to the modeling of concrete contribution and to the impossibility of consider concrete prestrains. The formulation presented in this thesis addresses these deficiencies, in order to apply 2D-PARC model to the analysis of tunnel linings under fire conditions.

As far as the modeling of concrete contribution is concerned, the constitutive relation adopted in 2D-PARC for the representation of concrete behavior, both before and after cracking, is properly modified in order to reduce the required computational effort as well as to incorporate the effects of crushing and dilatation in compression. Thanks to the abovementioned modular structure of

the formulation, this operation is performed by only changing the part of the general algorithm related to concrete behavior (i.e. the evaluation of concrete stiffness matrix in the uncracked and cracked stages), while leaving unchanged the rest. While in the original formulation concrete was modeled as an orthotropic material, by turning the effective biaxial state of stress into two uniaxial states of stress, an isotropic nonlinear elastic formulation is adopted in this work. The concrete stiffness matrix is then expressed as a function of only two parameters, namely the secant values of elastic modulus and Poisson coefficient, which are properly updated during the analysis in order to describe the current biaxial state of stress. By adopting this formulation, the computational efficiency of 2D-PARC is greatly improved. Moreover, the post-crushing behavior can be satisfactorily handled, by also considering dilatation that occurs when concrete is loaded in compression. The versatile nature of 2D-PARC allows also to extend this revised formulation to the case of FRC elements, by simply providing a reliable estimate of the post-peak parameter, which governs the softening regime in compression. Comparisons between numerical results and experimental data from the literature relative to different structural typologies (i.e. plain concrete panels subjected to a biaxial stress state, as well as RC and SFRC beams without shear reinforcement) prove the flexibility of the proposed approach, as well as its accuracy in describing concrete behavior under different loading combinations, also in the post-crushing region; thus confirming the potentialities of the revised formulation of 2D-PARC herein proposed. It allows to closely simulate the global load-deformation response up to failure of RC/SFRC elements, by providing accurate predictions in terms of stiffness, failure load and ductility. Moreover, also crack pattern evolution, both in terms of crack distribution and crack width, can be satisfactorily evaluated.

Subsequently, the so revised model is extended to include initial deformations of concrete, which were not considered in the original formulation of 2D-PARC model. At first, shrinkage strains are inserted in the algorithm, as an initial step of a more extensive work devoted to the inclusion of all concrete prestrains. A new set of equilibrium and compatibility equations is written and the material secant stiffness matrix is consequently rearranged and implemented into the adopted finite element code, by properly updating the internal convergence procedures. Shrinkage strains are inserted both in the uncracked and in the cracked stage as a prescribed deformation. Moreover, since shrinkage-induced stresses develop gradually with time, creep effects are also included (in a simplified way) in the model. It is worth noting that the peculiar formulation of 2D-PARC model requires the development of a different strategy for inserting shrinkage effects with respect to all the other constitutive models available in the literature. The strain decomposition procedure is indeed applied into 2D-PARC in a different manner, referring to the whole RC material (and not of only to concrete); thus, close attention must be paid to apply the prescribed prestrain due to shrinkage only to concrete. For sake of simplicity, the algorithm is first modified by considering shrinkage strains as uniformly distributed within the element. Subsequently, the procedure is refined, so as to take into account the dependency of shrinkage on the moisture gradient. This latter is in turn obtained

by performing an “equivalent” thermal analysis, by exploiting the analogy between the equations governing the moisture and the thermal problem, with a simple substitution of the corresponding parameters. In this way, the differential shrinkage strains (and the related eigen-stresses) between the faces exposed to drying and the core of the element are taken into account in the analysis, so leading to more realistic results. It is worth noting that this extended formulation of 2D-PARC can be applied for the analysis of both traditionally reinforced and fiber reinforced concrete elements, thanks to the modular structure of the model: the only difference between these two cases lies indeed in the constitutive laws adopted for the modeling of rebar/fiber contributions, which are automatically selected among those incorporated into 2D-PARC. The obtained numerical results, validated over a wide range of conditions through several comparisons with experimental evidences available in the literature (relative to plain concrete samples as well as RC and SFRC elements), prove the effectiveness of the proposed formulation, highlighting its potentialities for the assessment of serviceability requirements of RC/SFRC structures, where shrinkage is one of the key parameter to be considered.

Finally, the model is further extended and applied to the structural assessment of concrete tunnel linings under fire conditions. Concrete lining behavior is indeed analyzed by performing 3D FE simulations where mechanical nonlinearity is taken into account through 2D-PARC constitutive law, properly revised in the modeling of concrete contribution and further extended to include thermal effects. An accurate concrete modeling able to account for the effects of crushing and dilatation in compression is mandatory in this case, since concrete linings may present very high compressive stresses during fire, due to the stress relaxation near the heated surface and to the deformation constraint in the circumferential direction exerted by the cold extrados and the ground. Moreover, FE analyses on tunnels, which are high computational demanding by nature, require an extremely efficient numerical algorithm; whereas the original formulation adopted for the simulation of concrete contribution into 2D-PARC model considerably lengthened the computational times. To correctly describe the behavior of concrete during heating, thermal strains are inserted into the algorithm by following the same procedure adopted for shrinkage deformations. The dependence of the main material mechanical properties on temperature is also inserted in the model to take into account the loss of strength and stiffness undergone by concrete exposed to fire. Numerical analyses are then performed so as to simulate with accuracy both the ground-support interaction, with special attention to the excavation/construction phases, and the concrete lining behavior during fire, by applying a thermo-mechanical sequentially coupled approach. To this aim, the temperature field under an assigned fire scenario is obtained by performing a heat-transfer analysis, which is then followed by a nonlinear structural simulation, where both the tunnel construction phases and the application of the fire load are properly taken into account. In more detail, in order to realistically represent the soil excavation phase and the subsequent lining installation, a step-by-step procedure is followed, consisting in the progressive deactivation of slices of FE elements modeling the rock to be excavated and in

the activation of those elements representing the support, hypothesizing that the installation of concrete lining takes place at a certain distance from the excavation face. After the completion of the tunnel construction phases, temperatures deriving from the heat-transfer analysis are passed to the static simulation, and the stress and strain fields in the lining are properly updated by considering the effects of thermal strains as well as the decay of the mechanical properties during heating. In this way a realistic simulations of both ground-structure interaction and temperature-induced effects is provided. The effectiveness of the proposed FE procedure is validated through comparisons with analytical closed-form solutions. In more detail, the accuracy of the proposed approach in simulating the excavation and lining installation phases is verified by comparing FE results to well-known analytical relations available in the literature. A simplified analytical model is instead developed on purpose, so as to validate the simulation of concrete lining under fire conditions. The close correlations between analytical and numerical results prove the potentialities of the proposed approach for the assessment of the structural safety of tunnels before and during fire exposure. Finally, a parametric study is also performed so as to better highlight the influence exerted by different parameters on the structural response of the lining, especially during fire; to this end, different ground mechanical properties, tunnel depths and fire curves are considered. Generally speaking, it can be observed that for high fire exposure, lower overburden and poor mechanical properties of the ground, tensile stresses may appear near lining extrados also in case of initial isotropic stress-state and circular opening, due to the differential tendency to expand of the inner (and hotter) concrete layers and the external (and cooler) ones. It is worth noting that the proposed numerical approach currently provides accurate results under certain simplified conditions, since it represents only a first step in the field of the study of tunnels under fire conditions. This work will be further developed in order to treat more general and realistic cases, as well as to take into account crucial aspects in the assessment of tunnel safety under fire condition, disregarded in this phase. In particular, some of the recommended future developments are outlined below:

1. Providing a more realistic representation of fire development and propagation in the tunnel, considering the real non-uniform distribution of temperature, which is usually characterized by higher values at the ceiling, due to the direct flame impingement, and smaller at the benches.
2. Accounting for concrete spalling, a phenomenon which is particularly significant for concrete linings.
3. Considering different kind of supports, such as concrete lining reinforced with steel rebars and fibers, by properly extending the model in order to account for thermal effects on these materials.
4. Inserting transient creep in a more rigorous manner in the algorithm, by adopting an explicit formulation and considering it as an additional deformation. This aspect is crucial in order to correctly describe the actual unloading stiffness at high temperatures, which is essential in the modeling of the cooling phase.

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List of Publications

In the following, the list of publications by the author on the topic of the thesis.

Papers in Reviewed Scientific Journals

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