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# On Sparse Gain Flattening in Pump-Constrained Submarine Links

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**Abstract**—Having in mind the capacity optimization of power-constrained submarine links, by following the work in [1] we first compare the achievable information rate (AIR) of gain-flattened and un-flattened blocks of  $N_b \leq 12$  spans with span loss 16.5dB and with end-span single-stage co-pumped erbium-doped fiber amplifiers (EDFA) when the transmitted wavelength division multiplexed (WDM) channels all have the same transmitted power. All EDFAs have the same pump power and the same physical parameters. In the flattened case, each EDFA is followed by an ideal gain-flattening filter (GFF) that chops off the EDFA gain exceeding the span loss. No GFFs are used in the un-flattened case. We show that, for block length  $N_b > 7$ , at large-enough input power the AIR of the GFF block exceeds that of the no-GFF block, while for  $N_b \leq 7$  at large input power the AIR is about the same. We next build a long submarine link by concatenating the  $N_b$ -span no-GFF blocks, and placing a GFF at the last EDFA of each block in order to flatten the block gain down to the  $N_b$ -span loss, and calculate the AIR of the resulting sparse-GFF submarine link, accounting also for nonlinear interference. For the 287-span case-study link with span loss 9.5dB used in [5], [9], we show that the best power efficiency is achieved by blocks of size  $N_b = 6$  (i.e., one GFF every 6 spans) when the pump is around 12 mW. When the GFF excess loss is 0.3dB the top-AIR gain over the standard all GFF system is 9.5%, a value that decreases to 4% when the excess loss is zero. Considering that modern submarine-grade GFFs have almost zero excess loss, and that the most efficient pump power is likely too low to operate with, we conclude that sparse-GFF links offer little advantage in practice over the current design.

**Index Terms**—Optical Communications, Optical amplifiers, Submarine transmission, Signal Droop.

## I. INTRODUCTION

The recent advent of rate-adaptive, capacity-approaching transceivers for power-constrained submarine systems has renewed the interest in revisiting the design of the line erbium-doped fiber amplifiers (EDFA). In this context, there were reports based on machine-learning, where better power efficiency (i.e., capacity at EDFA fixed pump power) was achieved by partially removing the usual gain flattening filters (GFF) [1]–[3]. Recent experimental work also used a partial removal of GFFs [4].

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In this paper we wish to explore the settings where the link with partial removal of GFFs is more power efficient than the classical submarine link with GFFs at all amplifiers. For simplicity, we compare the two kinds of links when the transmitted wavelength division multiplexed (WDM) channels all have the same transmitted power, since the performance of optimized submarine links with such a flat input power is known to be quite close to that of the optimal input power profile [5], [6]. We accurately numerically model the transmission line by using the extended Saleh EDFA physical model with amplified stimulated emission (ASE) noise self-saturation [5], [7], [8].

By following the work in [1], we first compare the achievable information rate (AIR) of gain-flattened and un-flattened blocks of  $N_b = 12$  spans with end-span single-stage co-pumped EDFAs at a span loss of 16.5 dB. All block EDFAs have the same pump power and the same physical parameters. In the flattened case, each EDFA is followed by an ideal GFF that chops off the EDFA gain above the span loss. We show that, when the WDM input power is large-enough, the AIR of the unfiltered 12-span block is smaller than that of the block with gain-flattened EDFAs. We also show that, when the flattening filter is imperfect, with a significant residual frequency tilt as in [1], then the unfiltered block is superior at all powers, thus confirming the findings in [1]. Finally, we show that for shorter blocks  $N_b \leq 7$  there is no clear superiority of the GFF block even at large powers.

We next concatenate the unfiltered blocks of  $N_b$  spans to build a long submarine link. We place an ideal GFF at the last EDFA of each block in order to flatten the block gain down to the  $N_b$ -spans attenuation. The obtained link is referred to as sparse-GFF. For various block sizes, we calculate the AIR of the sparse-GFF link for the case-study 287-span submarine link with span loss 9.5dB considered in [5], [9]. EDFA-induced droop [10] is accurately included by the Saleh EDFA model. For such long sparse-GFF links we do consider also the nonlinear interference (NLI) due to fiber propagation, analytically calculated at each span by an extension of the Gaussian Noise model [11] that accounts for the non-uniform WDM channel power in the line fibers [12]. Since the WDM powers used to calculate the generated NLI are those launched into each fiber, including signal, ASE and NLI at each frequency bin, the nonlinear signal-noise and signal-NLI interactions are automatically accounted for.

We show that, for the case-study link, blocks of size 6 and 7 achieve the largest possible power efficiency at an optimal pump power around 12mW, with a theoretical maximum top-

AIR gain over the all-GFF reference link of 9.5% at 0.3dB GFF excess loss (i.e., background and splicing loss), and below 4% at zero excess loss.

The paper is organized as follows. Section II tackles the AIR of short blocks of  $N_b$  spans in isolation, both with and without GFFs. Section III shows AIR results of 287-span sparse-GFF submarine links of various block sizes. Section IV concludes the paper.

This paper is an extension of the work presented in the short conference paper [13].

## II. GFF VERSUS NO-GFF BLOCKS IN ISOLATION

By following [1], we compare the AIR of gain flattened and unflattened blocks of  $N_b$  end-amplified spans. The tacit assumption of this approach taken in [1] is that if the AIR of the block is optimized, then also the AIR of the concatenation of such blocks will be optimum.

The structure of the block is sketched in Fig. 1(top). It consists of  $N_b$  single-mode fiber spans with span loss  $A > 1$  followed by a single-stage co-pumped EDFA. All EDFAs in the block have the same physical parameters and same optical pump power  $P_p$ , but possibly different inversions and thus gain  $G$ . As in [1], we consider a transmitted (TX) WDM signal composed of  $N_c = 40$  channels with bandwidth  $B_c = 50\text{GHz}$ , spaced by  $100\text{GHz}$ , with carrier wavelengths from  $1532.64$  to  $1563.80$  nm, covering about  $4\text{THz}$  in the C band. In the gain-flattened case, each EDFA is followed by an ideal GFF with excess loss  $E > 1$ , that chops off all the EDFA gain in excess of  $A$  on all channels such that  $G/E = A$ , while all remaining channels have loss  $E$  (see Fig. 1(bottom)). Since EDFA inversions in general differ from EDFA to EDFA in the block, the GFFs we consider are ideally tailored to each EDFA in the line, and to each pump and signal level, differently from experimental works where the GFF is tailored to a specific EDFA, pump, and signal power, and applied to all line EDFAs, also at different pump and signal powers [1], [4], [6]. As in [5], the EDFAs are numerically simulated by the homogeneously-broadened Saleh gain model [7], enriched with ASE noise self-saturation [8], i.e., forward and backward ASE generated inside each EDFA over a broad bandwidth from  $1470$  to  $1670$  nm is considered for calculating each EDFA inversion. Then only the WDM signal range is propagated down the line. This way, we realistically model both the correct EDFA gain and noise-figure frequency profiles. Having in mind transmission in a space division multiplexed (SDM) submarine link [1], [9], of which our single-mode link represents one spatial mode, we initially assume that only ASE impairs transmission, so that for a given input WDM power distribution  $[P_1, \dots, P_{N_c}]$  the achievable information rate (AIR) is

$$AIR = 2B_c \sum_{j=1}^{N_c} \log_2(1 + SNR_j) \quad (1)$$

where  $SNR_j$  is the received (RX) signal to noise ratio (SNR) at channel  $j$ , i.e., the ratio of received signal power and cumulated ASE power, which depend on the inversions of the EDFAs in the block and their non-flat noise figures. For some selected values of the EDFAs common pump, we wish

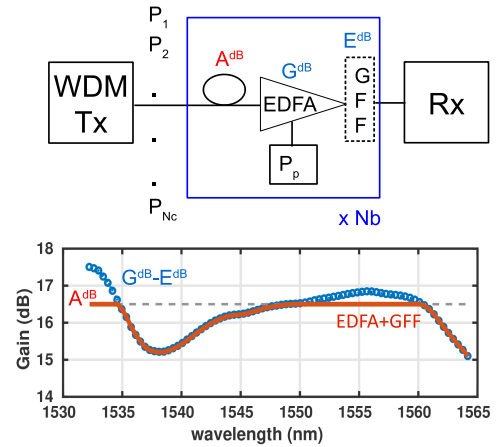


Figure 1. (Top) Single-mode WDM block with  $N_c = 40$  channels and  $N_b$  spans, span attenuation  $A$  and co-pumped single-stage EDFAs with optical pump power  $P_p$ . In the gain-flattened case, every EDFA is followed by a GFF with excess loss  $E > 1$ . (Bottom) Sketch of a typical EDFA unfiltered Gain  $G$  at channel locations (circles), decreased by the GFF excess loss, and amplifier gain after the GFF (solid line). The dashed horizontal line indicates the span loss  $A$ .

to compare the AIR of this block without and with GFFs as we vary the inversion  $x_1$  of the first EDFA, which induces that of the remaining line EDFAs [5]. Throughout this paper, for the purpose of simple AIR comparisons among different systems, we assume a constant input power (CIP) transmission (TX), where all 40 WDM channels have the same TX power  $P_c$ . Although the works in [1], [3] consider also an optimized, non-flat WDM distribution, it was shown in [5] that for GFF submarine links the CIP distribution (around the optimal EDFAs inversion) has AIR very close to Capacity, i.e., the AIR maximum over all possible input WDM distributions subject to the constraint on  $x_1$ . Thus the CIP distribution is more than appropriate for comparing the no-GFF to the GFF systems.

### A. Block Results

For the above 40 WDM CIP signal, we analyze a 12-span block with span loss  $16.5\text{dB}$  similar to the one in [1]. The  $980\text{nm}$  co-pumped single-stage EDFAs we use have the same absorption and emission profiles as in [9, Fig. 7]. In order to reasonably match the WDM power values after  $N_b = 12$  spans in the unfiltered block in [1, Fig. 4b], with input WDM CIP total power  $5\text{dBm}$  ( $P_c = -11\text{dBm}$ ) into the first EDFA, we selected an EDFA length  $\ell = 8.3\text{m}$ , with optical pump levels  $P_p = [25, 80, 170]\text{mW}$  roughly corresponding to the  $[75, 150, 450]\text{mA}$  reference pump currents in [1].

For the lowest pump  $P_p = 25\text{mW}$ , Fig. 2 shows the AIR versus TX power per channel  $P_c$  for the 40 channel WDM CIP input. The dashed curve shows the AIR in absence of GFF. The GFF label shows the AIR when using true flattening filters, both without loss (solid) and with  $0.3\text{dB}$  GFF excess loss (dash-dot). The tilted GFF label shows the AIR for an imperfect GFF such that the flattened gain equals the span loss at the center WDM channel, with a  $2\text{dB}$  linear tilt across the C band, as in [1]. Even here, solid line is for zero excess loss, and dash-dotted for  $0.3\text{dB}$  loss. Note that, in all plots in

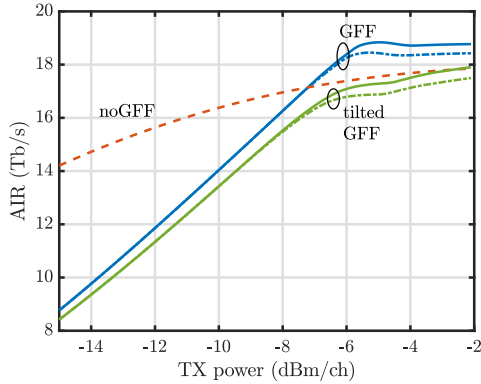


Figure 2. AIR (Tb/s) versus TX power per channel  $P_c$  (dBm) for a block of  $N_b = 12$  spans, span loss  $A = 16.5$  dB, pump power  $P_p = 25$  mW, doped fiber length  $\ell = 8.3$  m, 40 WDM equal-power input channels as in [1]. No GFF: dashed. True GFF: solid (no loss), dash-dot (0.3 dB excess loss). Tilted GFF as in [1]: solid (no loss), dash-dot (0.3 dB excess loss). At this low pump, we do not include NLI.

this paper, at the maximum value of the TX power  $P_c$  axis the total WDM TX power equals the pump power, and it makes little sense to go beyond that. From the figure we note that:

- 1) the tilted GFF AIR is always inferior to the no GFF case, consistently with the findings in [1];
- 2) the no-GFF block has larger AIR than the GFF block at all powers  $0 \leq P_c \leq P_c^*$  up to a cross-point  $P_c^*$  above which the GFF link is superior. This is true even considering a GFF excess loss of 0.3 dB. The top AIR in the GFF case is reached at  $P_c = -5$  dBm at zero excess loss, and 0.3 dB below at 0.3 dB loss.

The reasons of the GFF block superiority at point 2) are explored in Fig. 3, where at both a low-power  $P_c = -15$  dBm and at the GFF-optimal power  $P_c = -5$  dBm we show for both the no GFF and the (truly flat) GFF cases at zero excess loss the EDFAs Gain (EDFA 1, EDFA 12 and the link-average EDFA gain (dashed)) and the RX power after 12 spans versus wavelength.

At low power ( $P_c = -15$  dBm, left box), in the no GFF case EDFA 1 works in the unsaturated small-signal regime with largest gain, but all remaining EDFAs work in deep saturation, with large output power and thus large RX SNR (the SNR can be deduced from the RX power on the ASE-only channels). In the GFF case instead the WDM signal does not saturate the EDFAs since the GFFs chop off all the EDFA gain in excess of the span loss  $A$  (horizontal dashed red line). Hence the EDFAs all work in the small-signal regime, with largest gain and a minimum noise figure. However the RX power is small, only  $A$  times the input power, and thus also the RX SNR is small. This is the regime where GFFs uselessly “waste power in the over-performing channels” [14], so that the no-GFF case has larger AIR.

As power grows ( $P_c = -5$  dBm, right box), in the no-GFF block also EDFA 1 works now in saturation, and the RX SNR slightly increases, mostly because of the improved SNR at the first span. Also the GFF block now has all EDFAs working in saturation and thus large RX power, but the gain-chopping effect of the GFF spreads more evenly the RX SNR among

channels compared to the no-GFF case, so that there is a larger number of WDM channels with a “significant” SNR, and the overall GFF AIR is larger than the one without GFFs. In fact, the AIR in eq. (1) is seen to linearly increase with the number of significant-SNR channels, while the SNR gives only a logarithmic contribution to the AIR. The top AIR in the GFF case is reached at the power such that some channels (in the region around 1538 nm) have EDFA gain that starts sinking below the span loss  $A$ , and the number of significant-SNR channels starts decreasing. If we push the input power above the optimum, more channels have sinking gain below  $A$  in the 1538 nm region even in the GFF block, but the GFF AIR still remains larger than the no-GFF AIR, at least for reasonably small GFF excess losses such as those employed in modern submarine links

While for blocks longer than 12 spans the AIR advantage of the GFF block over the no-GFF block becomes more striking and extends to a larger TX power range [13], Fig. 4 shows the AIR vs.  $P_c$  when considering short blocks of sizes  $N_b = [12, 9, 7, 5, 3]$  at 25 mW pump, both for the no-GFF block (dashed) and the GFF block with 0.3 dB excess loss (solid). We note that the no-GFF AIR dominates the GFF AIR at all powers when  $N_b \leq 7$ . Does this dominance persist when concatenating no-GFF blocks to form a long submarine link? We explore the answer in the next section.

### III. SPARSE-GFF LINKS

In this section, using the same 40 WDM CIP channels across the C band as before, we wish to compare the transmission over a long GFF link with GFFs at all EDFAs, i.e., the reference case in deployed submarine links, against a sparse-GFF link of the same length composed of (see Fig. 5) a concatenation of no-GFF blocks, having a GFF at the last EDFA of each block that flattens the block gain down to the block loss at all channels where the block gain (minus the GFF excess loss) exceeds the block loss, and just attenuates by its excess loss the remaining channels. The EDFA at the GFF has the same pump and physical parameters as all remaining EDFAs. Similarly to [5], [9], we consider an  $M = 287$  span submarine link, formed by a concatenation of  $\lfloor \frac{M}{N_b} \rfloor$  no-GFF blocks of size  $N_b$  with end-block GFF having an excess loss of 0.3 dB, plus the remaining  $M - N_b \lfloor \frac{M}{N_b} \rfloor$  unflattened spans. The reference case indeed corresponds to  $N_b = 1$ . The span loss is  $A = 9.5$  dB. EDFAs have a doped-fiber length  $\ell = 5.3$  m, optimized to the new span loss at a pump  $P_p = 25$  mW.

We now include NLI in the AIR computation as an extra Gaussian additive noise using the Gaussian Noise model [11]. For NLI calculations, we assume a pure silica core fiber (PSCF) with attenuation 0.162 dB/km, nonlinear coefficient  $n_2 = 2.5 \cdot 10^{-20}$  m<sup>2</sup>/W, effective area 130  $\mu$ m<sup>2</sup> (yielding a coefficient  $\gamma = 0.78$  W<sup>-1</sup>km<sup>-1</sup>) and dispersion 21 ps/nm/km. The span loss  $A = 9.5$  dB includes 1.25 dB of margin (as in [9]), hence the fiber span length is  $L = 50.9$  km. In this highly-dispersive line we calculate the NLI variance by including only

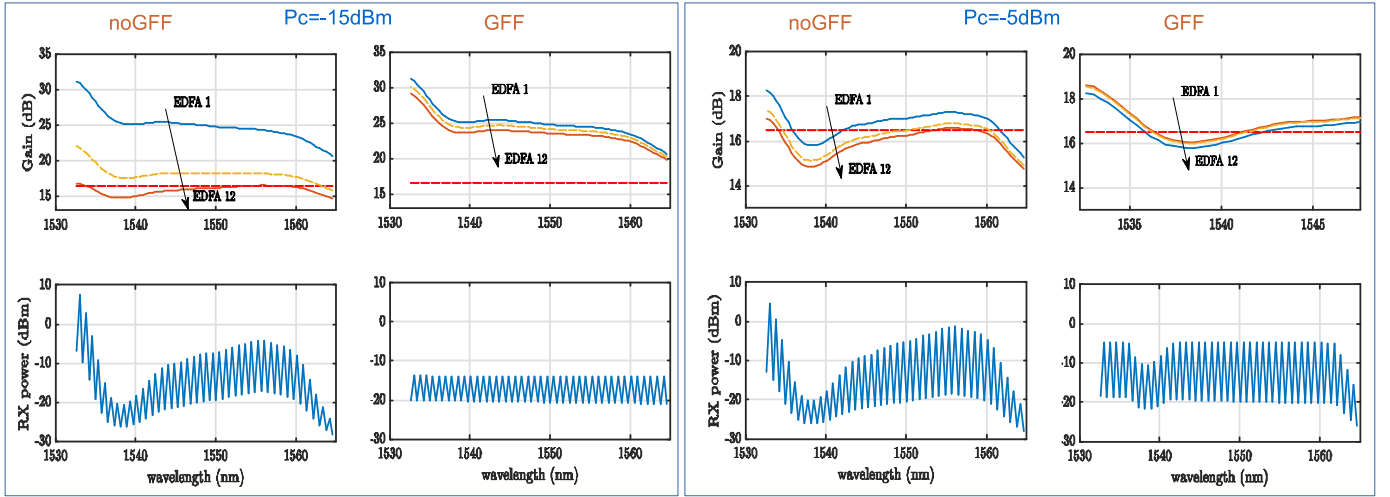


Figure 3. (a) EDFAs 1, 12 (solid) and block-average (dashed) Gain (top row) and RX power (bottom row) vs. wavelength for a block of  $N_b = 25$  spans, span loss  $A = 16.5\text{dB}$  (horizontal dashed line), doped fiber length  $\ell = 8.3\text{m}$ , 40 WDM channels (bandwidth 50GHz, spacing 100GHz), at pump powers  $P_p = 25\text{mW}$ . Both the GFF (zero excess loss) and the no-GFF cases are shown, at both a small input power ( $P_c = -15\text{dBm}$ ) and the GFF-top-AIR power ( $P_c = -5\text{dBm}$ ).

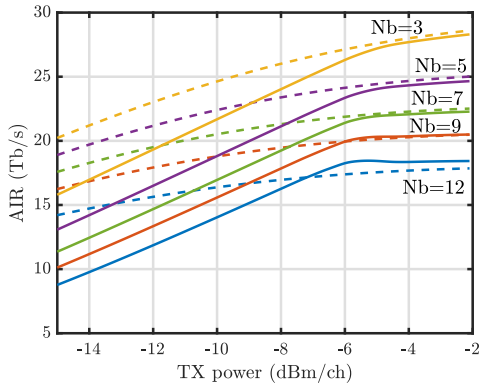


Figure 4. AIR (Tb/s) vs.  $P_c$  (dBm) for blocks of length [3, 5, 7, 9, 12] spans, 40 WDM CIP channels, pump  $P_p = 25\text{mW}$ , span loss  $A = 16.5\text{dB}$ , EDFA length  $\ell = 8.3\text{m}$ . no NLI. Solid: GFF with excess loss 0.3dB; dashed: no-GFF.

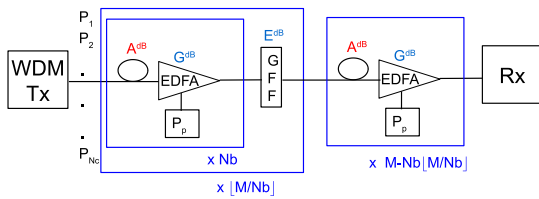


Figure 5. Block diagram of a sparse-GFF link, obtained by concatenating no-GFF blocks of size  $N_b$ , with GFFs in between them, up to a total  $M$  span link. All fiber spans have the same loss  $A$ . All EDFAs in the link have the same physical parameters and pump.

single- and cross-channel interference, using [12, eqs. (124)-(126)].

For the reference submarine link with GFFs at every EDFA ( $N_b = 1$ ), Fig. 6 shows (a) AIR versus TX power per channel and (b) AIR versus EDFA 1 inversion  $x_1$ , at the three pump values 25, 80 and 170 mW considered in [1]. Dashed lines are without NLI, solid lines include NLI.

When NLI is neglected, we see that the top AIR occurs at an optimal inversion around  $x_1 \cong 0.68$  at all pumps [5], which corresponds to a common optimal gain-versus-wavelength profile, such that the gain at 1538nm is just slightly smaller than the span loss  $A$ , similarly to the  $P_c = -5\text{dBm}$  box (GFF case) in Fig. 3. The corresponding optimal TX CIP power can be found in closed form as a function of  $x_1$  by solving the Saleh equation [5, eq. (14)].

Note also that the dashed no-NLI AIR curves in Fig. 6(a) coincide at low power at all pump values, which indicates the presence of a signal-independent noise figure, i.e., the GFF line has EDFAs working in their small-signal regime, up to a little before the top AIR. The AIR maximum comes from the best compromise between the EDFA saturation-induced decrease of the number of WDM channels with a significant SNR and their SNR increase with TX power. The AIR decrease after the maximum is mostly due to the power fading of several channels with a gain below the attenuation  $A$ .

If instead NLI is included in the calculations (solid lines), we see from Fig. 6(a) that the AIR curve at pump 25mW is little affected, while the top AIR for pumps 80 and 170mW is set by nonlinearity, which clamps the optimum TX power  $P_c$  to the NLI optimal power predicted by the GN model, here around  $-1\text{dBm/ch}$ . We also note from Fig. 6(b) that the larger the pump, the larger the inversion of the EDFAs at the top-AIR working point.

As a sanity check of the above results with NLI, Fig. 6(c) shows a split-step Fourier method (SSFM) estimation of the AIR versus TX power at the largest pumps 170 and 80 mW, for a shorter  $M = 100$  span link with GFFs at all EDFAs, where SSFM simulations were feasible in a reasonable time. We used the same 40 WDM channel allocation as in the theory, modulated at 50Gbaud per channel with root-raised-cosine supporting pulses with roll-off 0.01 and complex Gaussian symbols (to be consistent with the used GN model [12]). On

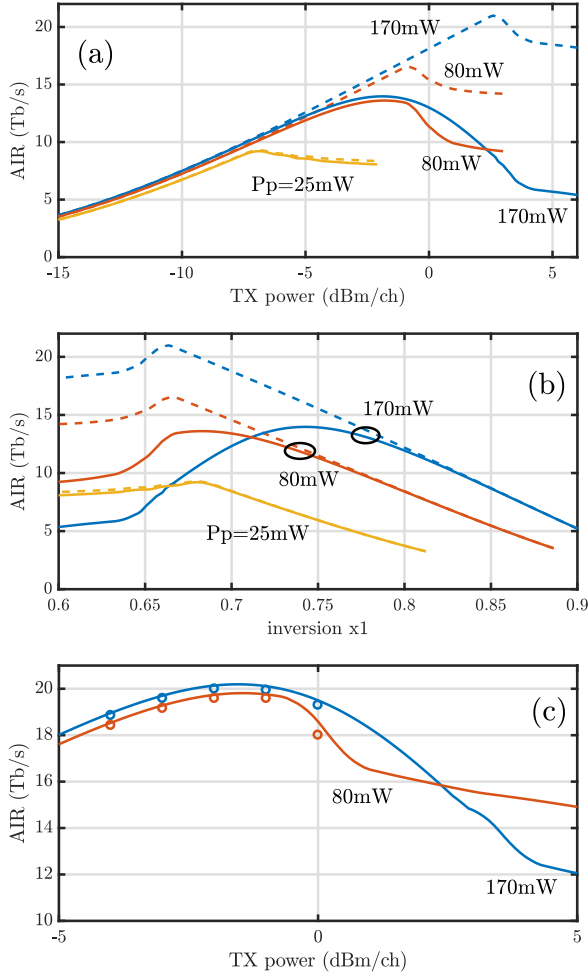


Figure 6. (a) AIR vs.  $P_c$  and (b) AIR vs. EDFA-1 inversion  $x_1$ , for a reference GFF link ( $N_b = 1$ ) with 287 spans, span loss  $A = 9.5$ dB, span length 50.9km, EDFA length  $\ell = 5.3$ m, 40 WDM channels, at pump powers  $P_p = [25, 80, 170]$ mW. Dashed: no NLI; Solid: with NLI on a PSCF fiber, attenuation 0.162 dB/km, nonlinear coefficient  $\gamma = 0.78\text{W}^{-1}\text{km}^{-1}$  and dispersion 21 ps/nm/km; span connectors losses 1.25dB. GFF excess loss 0.3dB. (c) AIR vs.  $P_c$  at 100 spans and same data as above, at  $P_p = [80, 170]$ mW, with NLI. Solid: theory. Circles: split-step simulations.

each channel we used a modulating sequence of  $2^{17}$  symbols. The SSFM step size was chosen as in [16]. The inversion of each EDFA was obtained by solving the Saleh equation [5, eq. (14)] in which the input fluxes were derived from the Fast-Fourier Transform spectral lines of the EDFA input field. We see from Fig. 6(c) that our theory is in reasonable agreement with the simulated AIR.

At the 25mW pump, in a range of interest for SDM submarine links [4], [6], Fig. 7(a) shows for the above 287 span link the AIR versus TX power  $P_c$  for various sparse-GFF block sizes  $N_b = [1 : 7]$ , with block-connecting GFF excess loss 0.3dB. We included NLI in the simulations, although its effect is sizable only at the largest powers. We observe that the top-AIR for block sizes smaller than 7 exceeds the reference  $N_b = 1$  top-AIR, with decreasing optimal power as the block size increases. The fact that the  $N_b = 7$  case has top-AIR below that of the reference  $N_b = 1$  all-GFF line shows that comparing the AIR of the blocks in isolation as done in Fig.

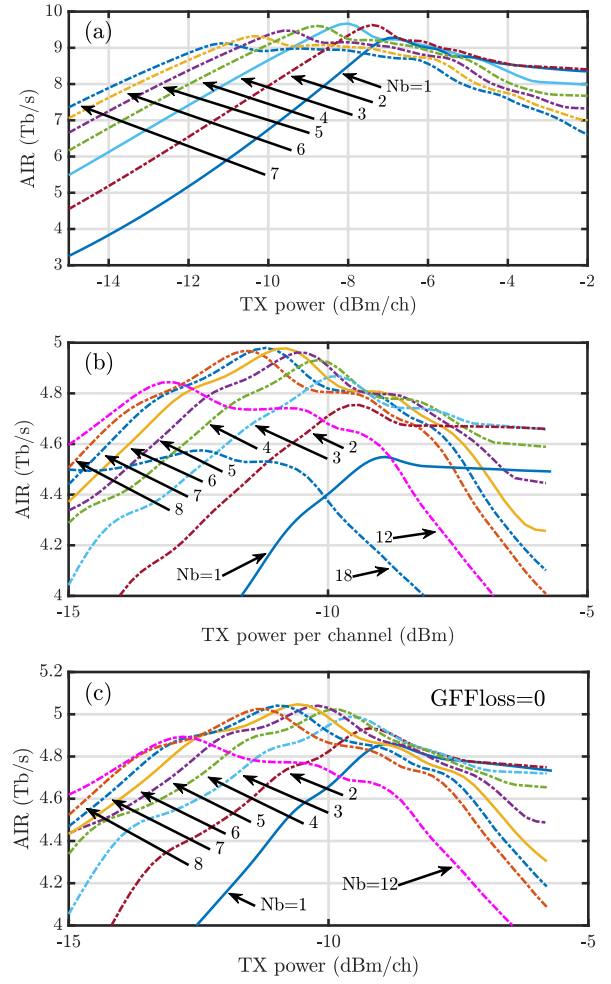


Figure 7. AIR vs.  $P_c$  for transmission of 40 WDM CIP channels over 287 spans, span loss  $A = 9.5$ dB, EDFA length  $\ell = 5.3$ m, at various sparse-GFF block sizes  $N_b$ . (a)  $P_p = 25$ mW, GFF loss 0.3dB (b)  $P_p = 11$ mW, GFF loss 0.3dB; (c)  $P_p = 11$ mW, zero GFF loss. NLI included.

4 is a reasonable, but not perfect practice. The most important message is that, at 0.3dB GFF excess loss, the  $N_b = 3$  block size at  $P_b = 25$ mW is the optimal one, and it offers a 4% top-AIR increase w.r.t. the reference  $N_b = 1$  case. We verified that at zero GFF excess loss,  $N_b = 3$  is still the best size, but its top-AIR gain reduces to a negligible 0.6%.

Fig. 7(b) shows the AIR and PE vs. TX power curves when the pump is reduced down to  $P_p = 11$ mW, i.e., the most power-efficient pump value as we will see later. We observe that now the block size values that maximize the top-AIR are  $N_b = 6$  and 7 with a gain over the reference  $N_b = 1$  case that reaches its theoretical maximum of 9.5%, and for block sizes up to 18 the top AIR is larger than that of the reference  $N_b = 1$  case. Hence the best block size depends on the pump value. Fig. 7(c) shows that  $N_b = 6$  and 7 are still optimal at zero GFF excess loss, but their theoretical gain over the reference  $N_b = 1$  case reduces to 4%.

For our 287 span system and block sizes  $N_b = [1, 3]$ , Fig. 8 shows (a) top-AIR and (b) power efficiency  $PE \triangleq (\text{top-AIR})/P_p$  versus EDFA pump power  $P_p$  for EDFA length 5.3m. Dashed lines are without NLI, solid lines include NLI. From

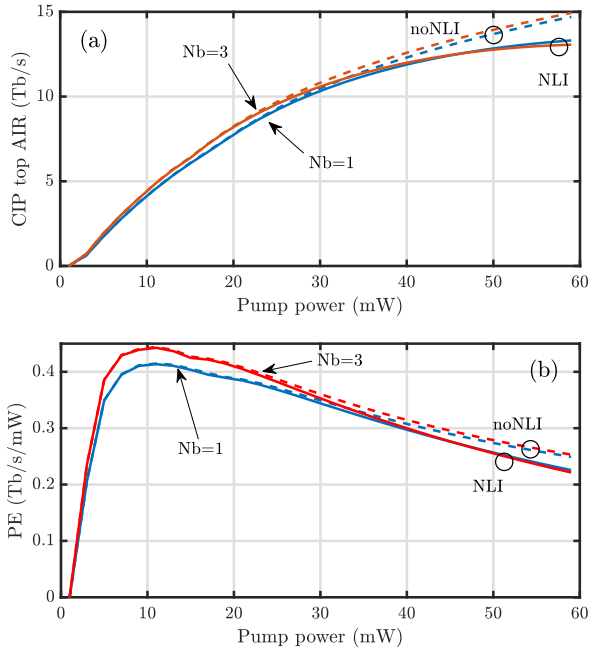


Figure 8. (a) top-AIR for our 40 WDM CIP channels vs. EDFA pump power  $P_p$  at optimal EDFA length 5.3m, both for reference system ( $N_b = 1$ ) and optimal block size  $N_b = 3$ . Dashed: no NLI, solid: NLI. 287 spans, span loss  $A = 9.5$ dB, span length 50.9km, PSCF fiber. (b) Power efficiency  $PE = \text{top-AIR}/P_p$  versus  $P_p$ .

Fig. 8(a) we see that for our 40 WDM CIP signal the effect of NLI becomes visible above  $P_p = 30$  mW<sup>1</sup>. We also note that in the real case with NLI the  $N_b = 3$  block ceases to dominate for pumps above 45mW. The reason is that the larger output powers in the  $N_b = 3$  block cause more nonlinear effects. Therefore it is *only at the very low pump powers envisaged for SDM submarine links that removing some of the GFFs may become advantageous*, as recently demonstrated in [4]. Fig. 8(b) reveals that the pump power at largest power efficiency is 11mW for both block sizes, and in line with the value extrapolated from the top-AIR versus pump power in [5, Fig. 6]. We also verified that if the number of channels is doubled to 80 by decreasing the channel spacing to 50GHz, the top power efficiency is obtained at a similar value 13mW. The larger pump values at top power-efficiency reported in recent system experiments [4], [6] are probably mostly due to the fixed GFFs used at all pump power values in the experiments [6].

To highlight the importance of selecting the correct EDFA length, Fig. 9 shows, for the case including NLI, and for block sizes  $N_b = [1, 3]$ , the top-AIR and PE versus  $P_p$  curves at both the optimized  $\ell = 5.3$ m (same as in Fig. 8) and those for a sub-optimal length  $\ell = 6.3$ m. We see that at 6.3m the  $N_b = 3$  block is practically never superior to the reference all-GFF system. Also, the top power efficiency occurs at even lower pumps when the length is sub-optimal.

<sup>1</sup>This is about half of the value found in [9, Fig.4a], [15] and is due to the 50% spectral occupancy of our WDM system. We verified that when populating also the ASE-only bins, for a total of 80 WDM channels with spacing 50GHz, the effect of NLI becomes visible above  $P_p = 60$  mW.

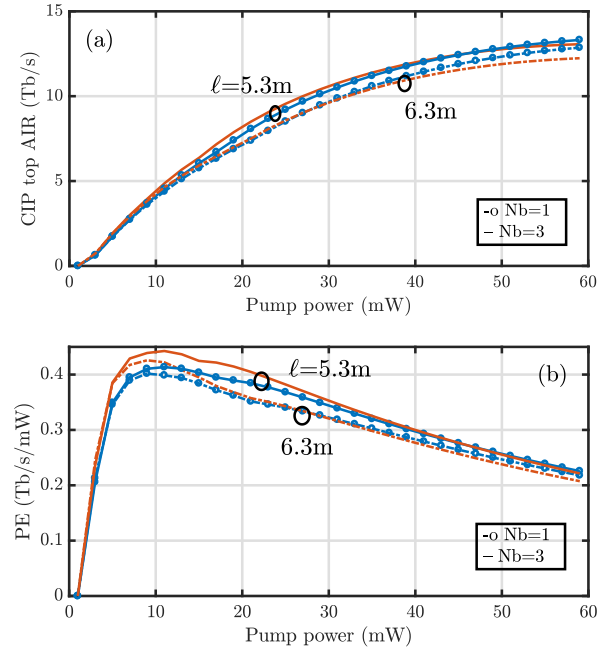


Figure 9. (a) top-AIR for our 40 WDM CIP channels vs. EDFA pump power  $P_p$  at both optimal EDFA length 5.3m (solid) and sub-optimal length 6.3m (dash-dot), both for reference system ( $N_b = 1$ , circles) and optimal block size  $N_b = 3$ .  $M = 287$  spans, span loss  $A = 9.5$ dB, span length 50.9km, including NLI from PSCF propagation. (b) power efficiency  $PE = \text{top-AIR}/P_p$  versus pump power.

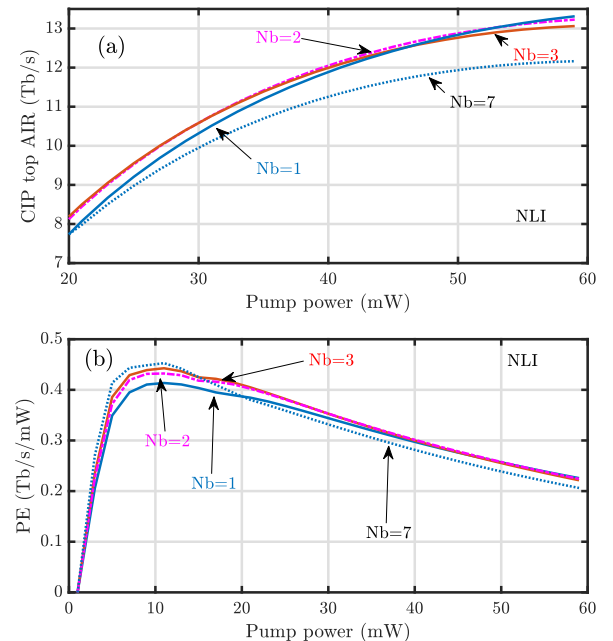


Figure 10. Same as Fig. 8 (only NLI case) but we added the  $N_b = [2, 7]$  cases. GFF excess loss 0.3dB.

Fig. 10 finally shows the same top-AIR and PE curves as in Fig. 8 for  $N_b = [1, 3]$  (only NLI case) but now also including the block sizes  $N_b = [2, 7]$ . We see that in our case-study link the AIR of the  $N_b = 2$  system is always very close but slightly inferior to the  $N_b = 3$  system up a pump of 33mW. From 33 to 53mW,  $N_b = 2$  is the dominant system. Above 53mW the  $N_b = 1$  reference system is the dominant one, which confirms that for traditional submarine systems, which are operated at larger pumps, the all-GFF system is the best possible. We also see that at pumps above 20mW the  $N_b = 7$  case is worse than the reference case since more affected by NLI, while it dominates (and hence provides the best power efficiency) at the best pump power 12mW.

#### IV. CONCLUSIONS

In this paper we critically verified the comparison presented in [1] between a block of  $N_b = 12$  EDFA amplified spans without GFF, and 12 spans of gain-flattened EDFAs, when the span loss is 16.5dB and the pump power is 25mW. We confirmed that the no-GFF block has superior AIR at all input powers when the flattening filters are imperfect, such as those in [1]. However, for truly flattening filters, the 12-span GFF block has superior AIR at large-enough input power, and this remains true for block lengths  $N_b$  down to 7. For shorter blocks there is no clear advantage of using the flattening filters. However, the comparison of such short blocks in isolation is not sufficient to prove superiority of long sparse-GFF submarine links, i.e., having one GFF every  $N_b$  spans. We thus studied such sparse-GFF links and found that in the 287 span, 9.5dB span loss case-study link tackled in [5], [9], for a 25mW pump power the most power-efficient choice is having one GFF every 3 spans, for GFF excess loss below 0.3dB. However, at the most power-efficient pump power around 12mW, the top-AIR and PE are achieved by block sizes  $N_b = 6$  and 7, i.e., one GFF every 6-7 amplifiers. The top-AIR gain with respect to the standard all-GFF case is 9.5% at 0.3dB GFF excess loss, and decreases to 4% at zero excess loss. The no-GFF block could have been better optimized by allowing the last EDFA of the block which precedes the GFF to have length and pump power possibly larger than the remaining EDFAs, akin to the extra EDFA at the GFF considered in [3]. However, we find that preceding the GFF with an EDFA identical to the others is always a very reasonable choice, almost close to optimal. Although the comparisons are presented for a power-flat WDM input signal, the qualitative conclusions are not expected to change if optimized input allocations are used.

The final system conclusion we draw from this study is the following. Since today's submarine-grade thin-film GFFs can have excess losses well below 0.1dB, sparse-GFF links offer in practice less than 4% potential top-AIR gain over the traditional all-GFF links at the most efficient pump power 13 mW, which is likely too low to operate with. With higher, more practical pump values, sparse-GFF links have negligible or no gain at all in top-AIR. If one also considers the construction simplicity of the traditional all-identical amplifier design, then sparse-GFF links seem to offer very minor advantages over traditional ones.

#### V. ACKNOWLEDGMENTS

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