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# Power scaling of normal-dispersion continuum generation using higher-order modes in microstructured optical fibers

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The use of a higher-order HE<sub>12</sub>-like mode to produce weak normal dispersion over a substantial wavelength range in a microstructured optical fiber is investigated numerically. It is shown that the effective area, and thereby the pulse energy, can in this way be scaled by an order of magnitude compared to using the fundamental mode in a single mode fiber. Multimode nonlinear simulations indicate that nonlinear mode coupling will not disturb single-mode operation in the HE<sub>12</sub> mode at least up to the threshold where polarization modulational instability sets in. © 2021 Optical Society of America

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Supercontinuum generation (SCG) from femtosecond (fs) pump pulses by self-phase modulation and wave breaking in fibers with weak all-normal dispersion (commonly denoted ANDi fibers) has in recent years become an object of intense research [1–3]. The deterministic nature of the nonlinear processes results in low noise and excellent shot-to-shot coherence up to a power threshold set by the appearance of Raman and possibly polarization modulational instability [4, 5]. Compared to picosecond-pumped SCG in fibers with anomalous dispersion at the pump wavelength [6], this is a distinct advantage, since the latter approach relies on pulse breakup by modulational instabilities seeded by quantum noise, resulting in chaotic shot-to-shot variations of spectra and pulse profiles [7], and a high degree of complexity in both spectral and temporal domains. Disadvantages of ANDi-SCG are the need for fs pump pulses, somewhat narrower spectra (although bandwidths of several hundred nm are routinely generated), and severe limitations in power scaling. The present theoretical study aims to resolve the latter issue through SCG in higher-order modes (HOMs).

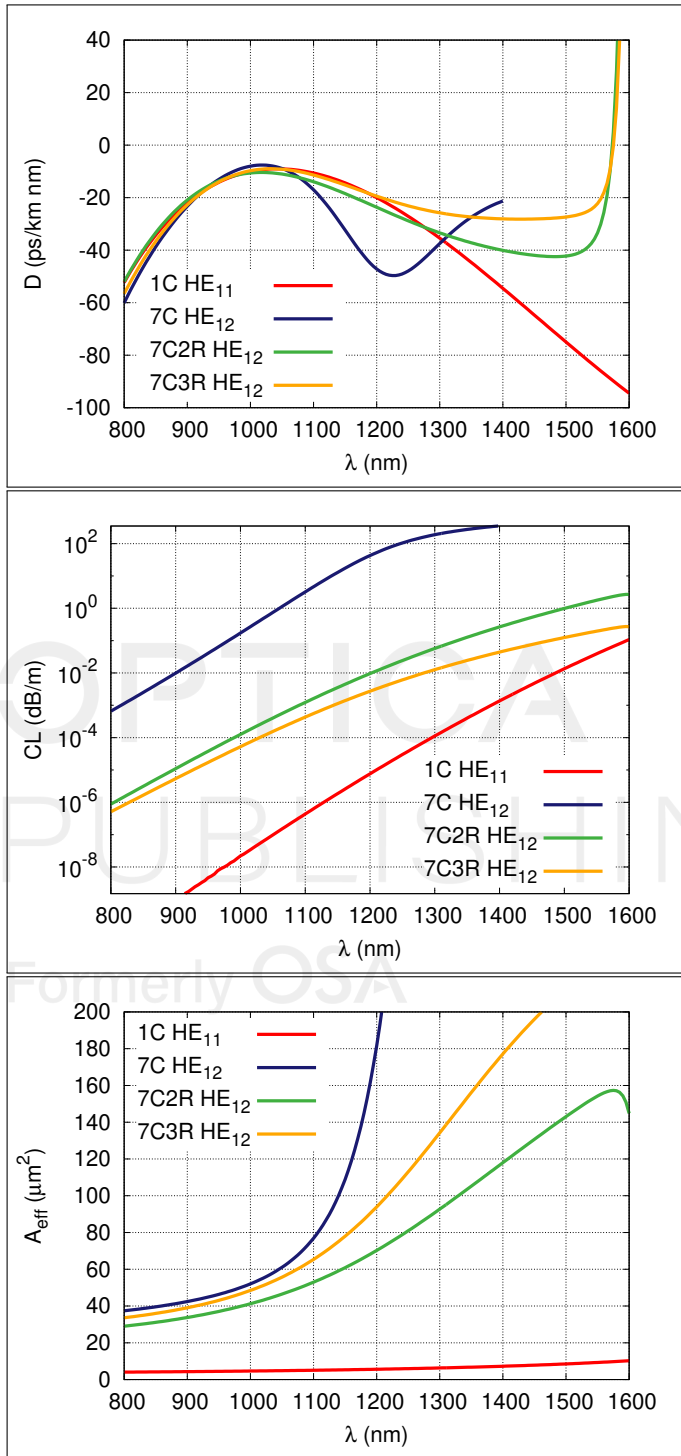
The key technological enabler for ANDi-SCG was the realization that flat dispersion profiles with a small negative value of the dispersion coefficient  $D$  could be obtained in microstructured optical fibers with relatively small airholes, and very small

core diameters on the order of 2.5  $\mu\text{m}$  for operation in the Yb gain band just above 1  $\mu\text{m}$  [1]. The need for a small core size obviously impedes power scaling, and typical pump pulse energies are in the few-nJ or even sub-nJ range. A similar limitation is encountered when designing for anomalous dispersion around 1  $\mu\text{m}$ , although the core diameters are in this case typically around 3–5  $\mu\text{m}$ . It is by now well appreciated that the use of HOMs can enable anomalous modal dispersion at wavelengths where the material dispersion is normal, even in very large cores, thus providing a route to power scaling of anomalous-dispersion SCG and related processes such as dispersive-wave generation or four-wave mixing [8–12]. A pertinent question is then whether a similar potential exists for engineering of large-core ANDi fibers.

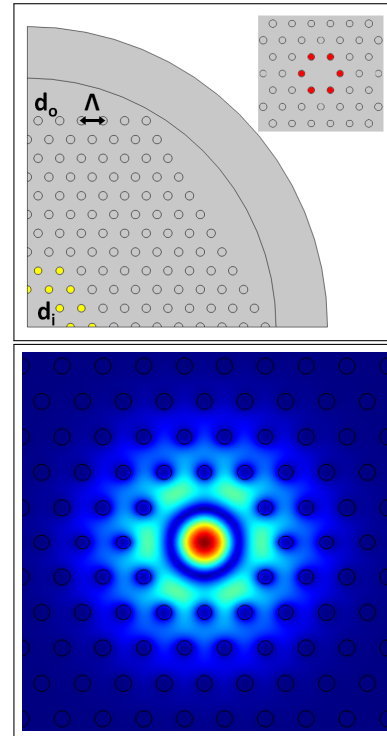
The conventional ANDi design consists of a single-mode microstructured fiber with a triangular array of airholes, where the solid single-mode core is defined by a single missing air-hole, here denoted a 1C design [1]. We have found that similar dispersion properties to those of the 1C HE<sub>11</sub>-like mode can be achieved in the HE<sub>12</sub>-like mode of an enlarged core defined by seven missing airholes, which we will denote a 7C-design.

In Fig. 1 the dispersion coefficient ( $D$ , top panel), confinement loss (CL, middle panel) and effective area ( $A_{\text{eff}}$ , bottom panel) are shown for the 1C HE<sub>11</sub> mode and the 7C HE<sub>12</sub> modes. These parameters, as well as the mode field distributions, have been evaluated with a full-vector modal solver based on the finite-element method [13, 14], taking into account the Sellmeier equation to calculate the silica refractive index [15]. All designs are characterized by the center-to-center distance between neighbouring airholes,  $\Lambda$ , and the ratio  $d/\Lambda$  where  $d$  is the airhole diameter. The fibers have been designed to have similar dispersion around 1  $\mu\text{m}$ , but the HE<sub>12</sub> mode in the 7C design with  $d/\Lambda=0.35$  has a steeper curvature, especially towards longer wavelengths. In addition, this mode deconfines completely at 1.4  $\mu\text{m}$  and already shows dB/cm loss around 1.25  $\mu\text{m}$ . Since the 1C design allows continuum generation out to wavelengths around 1.5  $\mu\text{m}$  (see below), this is a significant drawback of the HOM approach.

The situation may be remedied by enlarging the outer airholes in the 7C design to improve confinement, while retaining  $d/\Lambda=0.35$  for the holes closest to the core to control dispersion. A similar approach was earlier demonstrated to be experimen-



**Fig. 1.** Dispersion coefficient  $D$  (top), confinement loss  $CL$  (middle) and effective area  $A_{eff}$  (bottom) for the 1C, 7C, 7C2R and 7C3R designs, with  $\Lambda=1.5 \mu\text{m}$ ,  $2.5 \mu\text{m}$ ,  $2.1 \mu\text{m}$  and  $2.3 \mu\text{m}$  respectively.



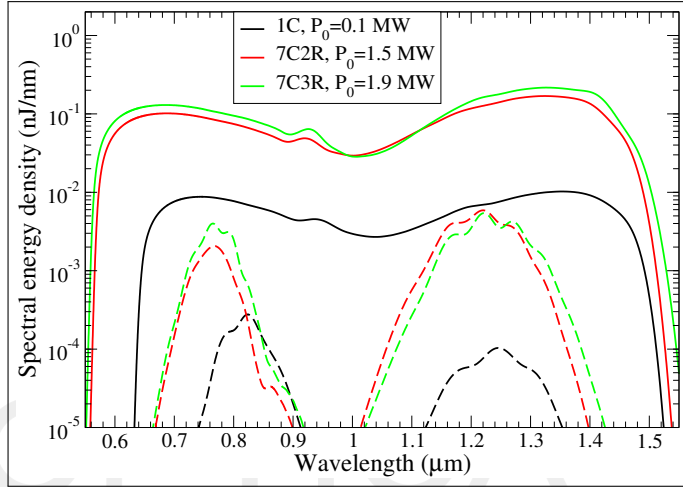
**Fig. 2.** Top: Schematic depiction of the 7C2R structure with inner hole diameter  $d_i$  (yellow holes), outer hole diameter  $d_o$  and pitch  $\Lambda$ . Inset shows 1C core, the red holes are removed to form the 7C core. Bottom: Electric-field modulus at  $1 \mu\text{m}$  for the HE<sub>12</sub> mode at  $1 \mu\text{m}$  in the 7C2R structure.

tally feasible in 1C fibers [16]. In the designs denoted 7C2R and 7C3R the airholes in the two or three innermost ‘rings’ around the core, respectively, have been kept at  $d_i/\Lambda=0.35$ , whereas the outer airholes are enlarged to  $d_o/\Lambda=0.39$ . A schematic depiction of the 7C2R design is shown in the top panel of Fig. 2. The result is a dramatic improvement of CL, making the HE<sub>12</sub> mode useful up to at least  $1.6 \mu\text{m}$ . In addition, the dispersion curve is flattened and comparable to the 1C HE<sub>11</sub> dispersion at intermediate wavelengths, and even flatter beyond  $1.3 \mu\text{m}$ . The HE<sub>12</sub> mode in these designs is confined to the core for wavelengths around  $1 \mu\text{m}$  and shorter, then gradually expands into the ‘inner cladding’ with  $d/\Lambda=0.35$ , leading to an increased slope of the  $A_{eff}$  curve, without deconfinement setting in. The electric-field modulus of the HE<sub>12</sub> mode in the 7C2R design at a wavelength of  $1 \mu\text{m}$  is shown in the bottom panel of Fig. 2. The  $A_{eff}$ -curves in Fig. 1 show the 7C3R area expanding faster than in the 7C2R structure due to the larger area of the inner cladding.

The HE<sub>12</sub>  $A_{eff}$  at  $1.064 \mu\text{m}$  is  $48 \mu\text{m}^2$  in the 7C2R design, and  $58 \mu\text{m}^2$  in the 7C3R fiber, whereas the HE<sub>11</sub>  $A_{eff}$  in the 1C design is  $4.9 \mu\text{m}^2$ . Given the similar dispersion properties and a negligible CL for propagation lengths of  $\sim 10$  cm, these results suggest that an order-of-magnitude scaling of pulse energy for a given nonlinear process should be within reach of the HOM approach.

Nonlinear simulations are performed by two approaches: A conventional two-mode approach, considering two polarization states of the same (HE<sub>12</sub>-like) eigenmode using the effective-area approximation [17, 18] is used to quickly explore the power levels needed to obtain efficient SCG within the limits set by

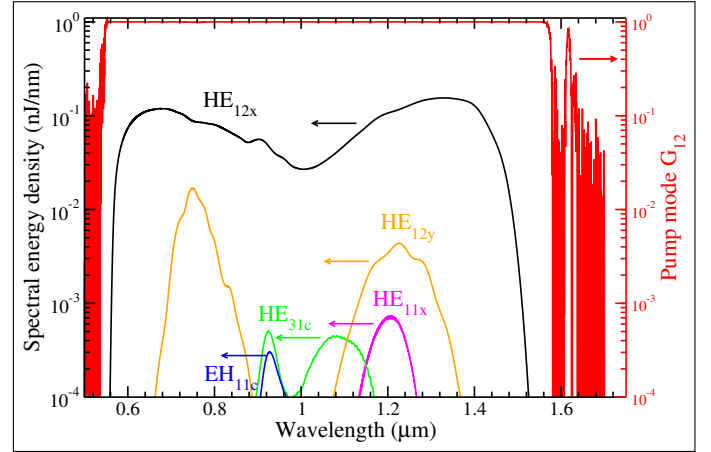
polarization modulational instability [19]. In addition, multi-mode (MM) simulations incorporating all 14 guided modes are carried out for the 7C2R design using a recently developed real-space gaussian quadrature approach to evaluate the nonlinear operator [20, 21]. The simulation results shown below were all obtained with linearly polarized gaussian-shaped input pulses having 50 fs full width at half maximum, and propagating in 10 cm of fiber. To study the impact of polarization modulational instability and possibly other intermodal couplings, quantum noise in the form of one random-phase photon per frequency mode was added to the input pulse.



**Fig. 3.** Output spectra after 10 cm propagation in 1C, 7C and 7C2R fibers for a 50 fs pump pulse at  $1.064 \mu\text{m}$  with peak power  $P_0$ . Solid lines represent the input polarization, whereas dashed lines represent the orthogonal (quantum-noise-seeded) polarization. Only the two  $\text{HE}_{12}$  polarization states are included in the simulation.

In Fig. 3 two-mode output spectra for the 1C, 7C2R and 7C3R designs are shown. For a fair comparison, the peak power  $P_0$  of the input pulse was adjusted to just below the level where 1% of the output power appeared in the orthogonal polarization. For the 1C design, this limit was reached around  $P_0=100 \text{ kW}$ , whereas for the 7C2R and 7C3R designs pumped in the  $\text{HE}_{12}$  mode,  $P_0$  levels of 1.5 and 1.9 MW, respectively, were reached. Thus, the use of the  $\text{HE}_{12}$  mode enables a power scaling of more than an order of magnitude, corresponding to the effective-area scaling. The intensity and fluence level are therefore similar to those obtained in single-mode ANDi fibers [2], and well within the  $\sim 1\text{-}2 \text{ J/cm}^2$  fluence limit for sub-100 fs pulses which can be estimated from studies of dielectric breakdown in silica [22]. It is also interesting to note that the 7C designs provide a broader continuum for comparable levels of cross-polarization coupling. The  $P_0$  difference of  $\sim 25\%$  between 7C2R and 7C3R is roughly consistent with the difference in  $A_{\text{eff}}$  at the pump wavelength. The notable differences in  $A_{\text{eff}}$  and dispersion properties on the long-wavelength side of the pump appear to have little impact on the power scaling and spectral shape.

The output spectrum for full MM propagation with  $P_0=1.5 \text{ MW}$  in the 7C2R structure, averaged over 30 different quantum-noise seeded simulations [7], is shown in Fig. 4. The spectrum in the  $\text{HE}_{12x}$  (pump) mode compares well with the two-mode simulation. Also the  $\text{HE}_{12y}$  spectra compare reasonably well, and the coupling to the orthogonal polarization state of the  $\text{HE}_{12}$



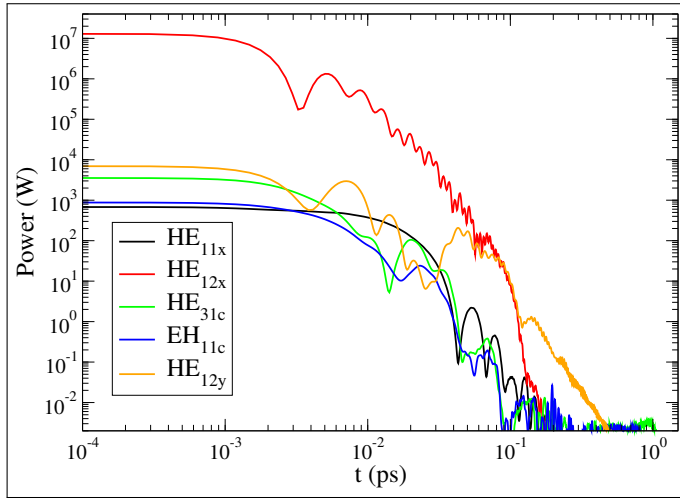
**Fig. 4.** Output spectrum and coherence function of a full MM simulation with  $P_0=1.5 \text{ MW}$  in the 7C2R structure.

mode is seen to be dominant. Three other modes have noticeable populations, with all other mode types being below  $10^{-5} \text{ nJ/nm}$  over the entire spectral range. The  $\text{HE}_{11x}$  mode has the same azimuthal symmetry and polarization as the pump mode, but the two modes are widely separated in effective index. The other higher-order modes have mixed polarizations, but can be distinguished according to the  $E_z$  field component having either  $\cos(m\phi)$  (c) or  $\sin(m\phi)$  (s) azimuthal dependence. The  $\text{HE}_{31c}$  and  $\text{EH}_{11c}$  modes share this angular dependence with the pump mode, and are also close in effective index. The nature of the mode couplings differ in that the  $\text{HE}_{12x}$ - $\text{HE}_{12y}$  coupling is a continuous parametric process, and so the output  $\text{HE}_{12y}$  energy fraction grows with fiber length. Conversely, the coupling to  $\text{HE}_{11x}$ ,  $\text{EH}_{11c}$  and  $\text{HE}_{31c}$  happens over the first cm of propagation, in a seemingly deterministic process. To quantify the noise properties, the frequency-resolved coherence function  $G_{12}^{(m)}(\omega)$  for mode  $m$  is defined as

$$G_{12}^{(m)}(\omega) = \frac{1}{n(n-1)} \sum_{kl} \frac{A_m^{(k)*}(\omega) A_m^{(l)}(\omega)}{|A_m^{(k)}(\omega) A_m^{(l)}(\omega)|} \quad (1)$$

where  $n$  is the number of simulations in the ensemble, and  $A_m^{(k)}(\omega)$  is the field amplitude for mode  $m$  in simulation  $k$ . The red curve in Fig. 4 shows  $G_{12}$  for the pump mode. A value of 1 indicates perfect coherence. In the region where the spectrum has significant weight  $G_{12}$  is seen to be very close to 1, with only a small dip to  $\sim 0.987$  occurring in the frequency band where the  $\text{HE}_{12y}$  spectrum peaks. As expected, this indicates that the upper boundary of low-noise operation with respect to input power has been identified.

Another important issue is whether the presence of other modes than the pump mode affects the femto- and picosecond contrast of compressed pulses. To investigate this in a simple way, we calculated transform-limited (flat phase) pulse profiles in all modes. This is a conservative assessment since in reality one would optimize compression of the pump mode only. The trailing half of the (symmetric) compressed pulse is shown in Fig. 5. The largest satellite peak is in the pump mode and  $\sim 10\%$  of the main peak, whereas all other modes are suppressed by a factor  $10^{-3}$ - $10^{-4}$  or higher. This is then the upper bound on temporal contrast degradation due to the presence of other modes, even if the other mode orders shift temporally relative to the  $\text{HE}_{12x}$  mode. In the case of perfect alignment, the  $\text{HE}_{12y}$



**Fig. 5.** Double-log plot of dechirped output pulses from the spectra in Fig. 4. Only half of the symmetric pulse profile is shown.

mode begins to dominate beyond  $\sim 100$  fs, at a suppression level of  $\sim 10^{-5}$ , whereas the  $\text{EH}_{11c}$  and  $\text{HE}_{31c}$  modes only become dominant beyond  $\sim 500$  fs at a suppression level of  $10^{-10}$ .

Our nonlinear simulations do not take mode conversions from linear scattering into account. However, the smallest effective-index difference between the  $\text{HE}_{12}$  and other mode groups is  $> 5 \cdot 10^{-4}$  over the whole  $\text{HE}_{12x}$  30-dB bandwidth in Fig. 4. Microdeformations coupling modes with an effective-index separation  $\Delta n_{\text{eff}}$  at a wavelength  $\lambda$  should have a spatial period corresponding to the beat length  $\frac{\lambda}{\Delta n_{\text{eff}}}$ . The impact of short-period fluctuations on the waveguide core is limited by the fiber outer diameter,  $D$ , and the stiffness of fiber and coating. For typical fiber coatings, one can estimate that only modes having  $\Delta n_{\text{eff}} \lesssim \frac{0.1\lambda}{D}$  are susceptible to coupling [23]. This implies that an outer diameter of 2-300  $\mu\text{m}$  should suffice to inhibit intermodal scattering in the fiber structure considered here. Other authors have suggested an even lower critical value of  $\Delta n_{\text{eff}} < 1.5 \cdot 10^{-4}$  for HOM stability in step-index fibers [24]. In addition, the short fiber lengths required for supercontinuum generation further reduces the impact of linear mode couplings. By a similar reasoning, we do not expect significant issues with macrobending losses, and in any event it is not a practical problem to keep a 10-cm piece of fiber straight if necessary [25].

The HOM approach to power scaling is in principle open-ended, as HOMs of higher order may be leveraged to engineer dispersion in still larger cores. Ultimately, this scaling will be limited by linear and/or nonlinear mode couplings due to decreasing effective-index differences between different mode groups. In addition, it needs to be determined from simulations whether a useful curvature and bandwidth of the dispersion curve can also be achieved for higher mode orders.

In conclusion, we have presented a class of multimode microstructured fiber designs for fs-pumped coherent supercontinuum generation in the  $\text{HE}_{12}$  mode. Linear and nonlinear simulations demonstrate that an order-of-magnitude power scaling compared to single-mode designs can be achieved without detrimental effects of nonlinear mode coupling.

**Disclosures.** The author declares no conflicts of interest.

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