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**INFLUENCE OF NON-METALLIC INCLUSIONS ON THE
HIGH CYCLE FATIGUE STRENGTH OF STEELS**

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ABSTRACT

In the present paper, a criterion for fatigue assessment of high strength steels is proposed, where the influence of non-metallic inclusions on fatigue life is taken into account. To such an aim, the Carpinteri et al. criterion is used in conjunction with the $\sqrt{\textit{area}}$ -parameter model by Murakami and Yanase. An experimental campaign available in the literature is analysed to evaluate the criterion accuracy.

KEYWORDS: non-metallic inclusions; high strength steel; fatigue strength; infinite life; AISI 4140 steel

NOMENCLATURE

$\hat{1}\hat{2}\hat{3}$	averaged principal stress directions
C_a	amplitude of the shear stress component on the critical plane
HV	Vickers hardness
I	error index
N_a	amplitude of the normal stress component perpendicular to the critical plane
$N_{eq,a}$	equivalent normal stress amplitude on the critical plane
N_m	mean value of the normal stress component perpendicular to the critical plane
V	volume of the useful section
V_0	standard inspection volume
w	normal vector to the critical plane
β	phase shift between normal stress and shear stress
δ	off-angle defining the normal to the critical plane
ΔK_{th}	threshold value of stress-intensity factor range
$\sigma_{af,-1}$	material fatigue strength under fully reversed normal stress
$\sigma_{eq,a}$	equivalent uniaxial stress amplitude
σ_u	material ultimate tensile strength
σ_w	fatigue limit under normal loading
σ_{wl}	lower bound of the fatigue limit under normal loading
σ_{wu}	upper bound of the fatigue limit under normal loading
$\sigma_{x,a}$	amplitude of the applied normal stress
$\tau_{af,-1}$	material fatigue strength under fully reversed shear stress

τ_w fatigue limit under torsion
 τ_{wl} lower bound of the fatigue limit under torsion
 $\tau_{xy,a}$ amplitude of the applied shear stress

1. INTRODUCTION

Non-metallic inclusions (NMIs) are naturally occurring products which are formed during both the steel production and the manufacturing/treatment processes involving the metal in a liquid state [1]. NMIs are chemical compounds of metals such as, for example, iron, manganese, aluminium, and silicon with non-metals such as, for example, oxygen, carbon, and hydrogen, forming separated phases in the steel matrix. When NMIs contain more than one single compound, they are named complex NMIs.

Non-metallic inclusions are one of the leading cause of failure reported in the literature, regarding gearboxes [2-7], pipelines, railway wheels, crankshafts and so on [8-12]. Causes of premature fatigue failures related to roller element bearings in wind turbine gearboxes, ideally designed to last a lifetime of 20-25 years, are reported in the literature [2-5,10,11]. It is worth noting that fatigue failure counts for more than 60% of wind turbine gearbox failures, where bearing failure counts for about 70%. More precisely, it has been experimentally observed that the failure of such bearings is often associated with the occurrence of White Etching Cracks (WECs), where the crack initiation sites are in correspondence of non-metallic inclusions produced during the steel manufacturing process.

The main causes of corrosion and damage in oil- and gas-pipelines are hydrogen-induced cracking and sulphide stress cracking [12-14]. In more detail, the corrosion is produced by the penetration of hydrogen atoms (from the oil or gas) into the

steel which, accumulating into surface defects in steel matrix, cause the crack initiation due to the high internal pressure exerted by the formed hydrogen gas [15-17]. In fact, An et al. [15] experimentally investigated the effect of hydrogen environment on the fatigue crack growth of X80 pipeline steel, by observing that hydrogen facilitated the failure of the material in presence of defects, such as Mn₃TiSi inclusions. Therefore, non-metallic inclusions act as trapping sites to capture hydrogen [18-20], and several studies on their chemical compositions have been carried out with the aim to optimise the steel processing and to reduce their presence in the metallic matrix [21-23].

In rolling contact fatigue, characterising the railway wheels subjected to the cyclic rolling contact with rails, one type of crack initiation position is at the subsurface [24-26]. As a matter of fact, fatigue cracks may initiate and propagate from internal defects, such as non-metallic inclusions, that exist below the wheel tread surface [27-30], and significant plastic deformation may be present around such inclusions, as observed by Mahdavi et al. [31]. Moreover, rolling contact fatigue is a key feature reducing the service lives also of wind turbine gear, for which the influence of different inclusion sizes on failure risk has been investigated by Zhou et al. [32].

Numerous research studies have shown that fatigue is the dominant mechanisms for crankshaft failures [33], involving both web marine crankshafts [34] and diesel engine crankshafts used in

commercial vehicles [35-38], and that fatigue cracks are likely to initiate from non-metallic inclusions [39,40].

It is worth noting that fatigue failure mostly originates at non-metallic inclusions for high strength steels, whereas it originates by the formation of persistent slip bands at grain boundaries for low/medium strength steels. Several experimental studies on high strength steels have shown that both short and long lifetimes are affected by NMIs [41-45]. Therefore, it seems important to consider their influence in estimating the fatigue lifetime.

To such an aim, Machado et al. [46] proposed to use a classical critical plane-based criterion in conjunction with the concept of the \sqrt{area} -parameter by Murakami [47] and Yanase et al. [48]. Such a fatigue endurance methodology has been applied for the first time by employing both the Findley criterion [49] and the Modified Wöhler Curve Method (MWCM) [50].

The novelty of the present paper is to implement the \sqrt{area} concept in the Carpinteri et al. criterion [51-56], so that the criterion can be also used to estimate the fatigue strength of naturally defective high strength steels under multiaxial loading (steels for which NMIs strongly affect lifetime).

Such an extended criterion is employed to simulate an experimental campaign available in the literature, and performed on AISI 4140 steel specimens. Moreover, the volume associated to the useful cross-section, used to estimate the maximum inclusion

size, is optimised in order to minimise the mean value of the error index.

The paper is structured as follows: the effect of non-metallic inclusions on uniaxial fatigue strength is examined in **Section 2**. **Section 3** describes the fatigue endurance methodology here proposed for multiaxial loading. In **Section 4** the available experimental campaign used to assess the proposed methodology accuracy is examined and detailed. The results determined through the above methodology are presented in **Section 5**, and the main conclusions are summarised in **Section 6**.

2. EFFECT OF NON-METALLIC INCLUSIONS ON UNIAXIAL FATIGUE STRENGTH

2.1 Basic concepts

As is well-known, when the notch depth is of the same order as the size of the fatigue damage domain at the root of the notch (this is the case of shallow notches), the classical theories cannot be applied to estimate the fatigue limit.

In such a context, Murakami et al. [57] performed rotating bending fatigue tests on two types of steel (i.e. a low carbon and a medium carbon steel), by using specimens containing small artificial holes of different diameters (between 40 and 200 μm). They observed the presence of non-propagating cracks in all specimens at fatigue limit (that is, at the maximum nominal stress for which the life was at least 10^7 loading cycles), concluding that fatigue limit was the threshold condition for non-propagation

of cracks emanating from holes. Moreover, they observed that there were non-damaging defects, which did not decrease the fatigue strength. Thus, this study not only revealed that a critical defect size seemed to exist, but also that the size of the defect was crucial to determine the fatigue strength, mainly in high strength metals [57,58].

Therefore, it was recognised that the fatigue problem involving components with small defects could also be treated as a short crack problem, where the fatigue limit concept is associated with the presence of dormant cracks, and modified Stress Intensity Factor (SIF) based methodologies could be considered to tackle the problem.

The maximum value of the SIF along the front of a 3D crack has a strong correlation with the square root of the cracked area, \sqrt{area} [59], such a parameter being defined as the square root of the area determined by projecting the crack into the plane perpendicular to the maximum principal stress.

Murakami analysed fatigue limit results under rotating bending tests and tension-compression tests, performed on specimens made of ten different materials, containing very small drilled holes, very small and shallow notches, and very shallow circumferential cracks. The aim was to correlate the threshold value ΔK_{th} of stress-intensity factor range with the \sqrt{area} , where the \sqrt{area} was the projection of the above small defects and cracks into the plane perpendicular to the maximum principal stress [60]. The

Authors of such a paper found a linear relationship between ΔK_{th} and \sqrt{area} in a logarithmic scale, with slope equal to 1/3:

$$\Delta K_{th} \propto (\sqrt{area})^{\frac{1}{3}} \quad (1)$$

where ΔK_{th} was computed according to the relationship for the maximum value of the SIF along the front of a 3D surface crack [59]:

$$\Delta K_{th} = 0.65(2\sigma_w)\sqrt{\pi\sqrt{area}} \quad (2)$$

being σ_w the fatigue limit expressed in MPa, ΔK_{th} in MPa \sqrt{m} and \sqrt{area} in μm . Moreover, the Authors observed that materials having higher Vickers hardness, HV , showed highest values of ΔK_{th} , and the relationship between ΔK_{th} and HV could be expressed by:

$$\Delta K_{th} \propto (HV + C) \quad (3)$$

being C a material independent constant.

By combining Eq. (1) with Eq. (3) and by applying the least squares method, ΔK_{th} could be expressed as follows:

$$\Delta K_{th} = 3.3(10)^{-3}(HV + 120)(\sqrt{area})^{\frac{1}{3}} \quad (4)$$

with \sqrt{area} in μm , HV in kgf/mm^2 . By employing Eq. (2), the fatigue limit was given by:

$$\sigma_w = \frac{1.43(HV + 120)}{(\sqrt{area})^{\frac{1}{6}}} \quad (5)$$

2.2 Fatigue strength estimation equation under normal loading: the \sqrt{area} -parameter model

The fatigue fracture origin in low/medium strength steels is different from that in high strength steels.

Slip bands and/or grain boundary cracks, that nucleate in the matrix, represent the fatigue fracture origins typical of low/medium strength steels. In such a case, the size of inclusions, generally formed during the production process, are smaller than a critical value. The following linear empirical equation holds:

$$\sigma_w = 1.6HV \quad (6)$$

where σ_w is expressed in MPa and HV in kgf/mm^2 .

White spots (or fish eyes), in the vicinity of non-metallic inclusions and in the form of white circular areas on the fracture surface, represent the fatigue fracture origins typical of high strength steels, where the fatigue crack: (a) nucleates at the

interface between the inclusion and the matrix and/or the inclusion itself is cracked, (b) propagates by a small amount, and (c) then stops propagating. In such a case, the relationship between fatigue limit and hardness, given by Eq. (6), is no longer valid since σ_w depends on the hardness of the microstructure close to the fracture origin and not on the average hardness. Moreover, it was experimentally observed that the σ_w values fell below the straight line represented by Eq. (6), and a large scatter was registered by varying *HV* [47,61].

It is worth noting that, when the crack nucleates, the stress within the inclusion is relieved, and the inclusion may be regarded as mechanically equivalent to a small defect. In such a case which is typical of high strength steels, the sizes of inclusions, generally formed during heat treatment, are greater than a critical value. The non-metallic inclusions can be treated as small defects, where Eq. (5) holds for a surface inclusion, whereas the coefficient 1.43 in Eq. (5) is replaced by either 1.41 for an inclusion just in contact with the free surface or 1.56 for an internal inclusion.

Note that \sqrt{area} in Eq.(5) is the square root of the "area" of an inclusion into a plane perpendicular to the maximum principal stress.

Therefore, low cycle fatigue limit values in high strength steels are due to the presence of inclusions larger than a critical size and, since both the location and sizes of inclusions

influence the σ_w value, a wide scatter band of the experimental data was observed [47,61].

Due to the fact that inclusions are more than a single one, it is needed to consider a statistical distribution of the inclusions, providing for σ_w not a unique value but an upper and a lower bound. The upper bound is provided by Eq.(6), which holds when inclusions do not affect σ_w ($\sigma_{wu} = 1.6HV$). The lower bound is computed by assuming that: (i) the inclusion with the maximum area, $area_{max}$, contained in the selected cross-section of the specimen is expected to become the fatigue fracture origin, and (ii) such an inclusion is just below the cross-section surface. Such a lower bound is given by:

$$\sigma_{wl} = \frac{1.44(HV + 120)}{(\sqrt{area_{max}})^{\frac{1}{6}}} \quad (7)$$

Different methods are available in the literature to estimate $\sqrt{area_{max}}$. According to Murakami [47], such a fatigue strength for high strength steels can be assumed to occur in correspondence of $1 \cdot (10)^7$ loading cycles.

2.3 Fatigue strength estimation equation under shear loading: the \sqrt{area} -parameter model

In view of Eq.(5), Yanase et al. [48] proposed a simple yet reasonably accurate equation to estimate fatigue limit under torsion:

$$\tau_w = \frac{1.21(HV + 120)}{(\sqrt{area})^{\frac{1}{6}}} \quad (8)$$

Therefore, according to (i) and (ii) assumptions above, the torsion fatigue limit is given by:

$$\tau_{wl} = \frac{1.19(HV + 120)}{(\sqrt{area_{max}})^{\frac{1}{6}}} \quad (9)$$

where such an equation is simply obtained from Eq.(8) by replacing \sqrt{area} with $\sqrt{1.137} \cdot \sqrt{area_{max}}$.

3. MULTIAXIAL FATIGUE STRENGTH ESTIMATION: THE CARPINTERI CRITERION COUPLED WITH THE \sqrt{area} -PARAMETER MODEL

The criterion proposed by Carpinteri et al. [51-56] uses the following non-linear relationship to assess the fatigue damage:

$$\sigma_{eq,a} = \sqrt{N_{eq,a}^2 + \left(\frac{\sigma_{af,-1}}{\tau_{af,-1}}\right)^2} C_a^2 \leq \sigma_{af,-1} \quad (10)$$

where $\sigma_{af,-1}$ and $\tau_{af,-1}$ are the fatigue strengths (at a given number of loading cycles, N_0 , generally assumed equal to $2 \cdot (10)^6$) for fully reversed normal stress and for fully reversed shear stress, respectively. The values $N_{eq,a}$ and C_a are computed on the critical plane, where its normal \mathbf{w} is defined by means of both the direction $\hat{\mathbf{1}}$ of maximum principal stress, at the time instant when such a stress attains its maximum value, and the angle δ given by:

$$\delta = \frac{3}{2} \left[1 - \left(\frac{\tau_{af,-1}}{\sigma_{af,-1}} \right)^2 \right] 45^\circ \quad (11)$$

More precisely, the $\hat{\mathbf{1}}$ direction is rotated by δ in the $\hat{\mathbf{1}}\hat{\mathbf{3}}$ plane [51-56]. The value of $N_{eq,a}$ is computed as follows:

$$N_{eq,a} = N_a + \sigma_{af,-1} \left(\frac{N_m}{\sigma_u} \right) \quad (12)$$

where σ_u is the ultimate tensile strength of the material, N_a and N_m are the amplitude and the mean value of the stress normal to the critical plane, respectively. The value of C_a , which is the amplitude of the shear stress lying on the critical plane, is computed according to the Maximum Rectangular Hull method [62].

Since the aim of the present work is the coupling of the above multiaxial criterion with the \sqrt{area} -parameter model (outlined in **Section 2.2** and **2.3**), the fatigue strengths $\sigma_{af,-1}$ and $\tau_{af,-1}$ in Eqs (10-12) are replaced with σ_{wl} and τ_{wl} , respectively. In such a way, the Carpinteri et al. criterion can be used to also estimate the fatigue strength of naturally defective high strength steels under multiaxial loading, which are steels for which NMIs strongly affect lifetime.

4. EXPERIMENTAL WORK EXAMINED

4.1 Multiaxial fatigue testing

The experimental campaign here examined is available in the literature [**46,63,64**].

The specimens were cut from a broken power generator crankshaft which had experienced failure under service. The material was AISI 4140 steel, oil quenched and tempered at 600°C, whose mechanical properties are listed in **Table 1**.

Table 1

The specimen manufacturing process consisted of cutting prisms from the block of the crankshaft material. That was followed by machining such prisms to the designed specimen geometry, shown in **Figure 1**, and then polishing the specimens with sandpaper to a final roughness lower than $0.2\mu m$.

Figure 1

The fatigue tests were performed under load control according to the international standard ASTM E466-15, by using a servo-hydraulic axial-torsional fatigue testing machine (100 kN and 1100 Nm load capacity, respectively). Fully reversed sinusoidal signals were considered. Test frequencies were between 5 and 15 Hz, depending on the loading level. The values of the ratio between shear stress amplitude $\tau_{xy,a}$ and normal stress amplitude $\sigma_{x,a}$ were (a) 0.0, 0.5, 1.0, 2.0 and ∞ with a phase shift β equal to 0° , and (b) 0.5, 1.0 and 2.0 with a phase shift $\beta = 90^\circ$ [46, 63, 64]. Two tests were performed for each loading condition.

The specimen complete rupture was used as the failure criterion, whereas the run-out condition was in correspondence of $2 \cdot (10)^6$ loading cycles.

Under uniaxial loading, the S-N (normal stress against number of loading cycles) and T-N (shear stress against number of loading cycles) curves obtained were expressed by:

$$S = 1202N^{-0.083} \quad (13)$$

$$T = 679N^{-0.062} \quad (14)$$

with S and T in MPa and N is the number of loading cycles.

The results obtained under multiaxial loading are reported in detail in Ref. [46].

4.2 Estimation of the maximum inclusion size \sqrt{area}_{\max}

In order to compute σ_{wl} (see Eq.(7)) and τ_{wl} (see Eq.(9)), an inclusion content analysis was performed [46,63,64], and the main steps and results are here reported.

Two cross-sections of a specimen (described in **Section 4.1**) were examined: one is perpendicular to the longitudinal axis of the specimen (representing the direction of the maximum principal stress under fully reversed axial loading), and the other one is forming 45° with the longitudinal axis of the specimen. Each cross-section was observed by a Scanning Electron Microscope (SEM, JEOL JSM-7100F) with an inspection area equal to $0.41mm^2$ (standard inspection area); the biggest inclusion was identified and its area was computed as that enclosed by a smooth boulder line surrounding the inclusion; then, the value of $\sqrt{area}_{max,i}$ was obtained. Such an operation was repeated 60 times for each cross-section ($i=1,\dots,60$).

Then, the trend of a reduced variable y against the variable \sqrt{area}_{max} was plotted, and the following inclusion distributions were computed:

$$\sqrt{area}_{max} = \frac{1}{0.0963} y + \frac{1.0671}{0.0963} \quad (15)$$

for the cross-section perpendicular to the longitudinal axis of the specimen, and:

$$\sqrt{area}_{max} = \frac{1}{0.1184} y + \frac{1.4751}{0.1184} \quad (16)$$

for the cross-section forming an angle equal to 45° with respect to the above axis, with the reduced variable y given by:

$$y = -\ln \left[-\ln \left(\frac{T-1}{T} \right) \right] \quad (17)$$

being T the return period.

In order to compute T , two volumes have to be identified:

$$T = \frac{V}{V_0} \quad (18)$$

where V is the volume associated to the useful cross-section, whereas V_0 is the standard inspection volume, computed as the product of the inspection area, S_0 , and the thickness of the observed cross-section by SEM, h . V was set equal to the volume of the specimen gauge region, that is, $V_1 = 2400 \text{mm}^3$, whereas V_0 was computed by using the data coming from the microscope observations, that is, equal to 0.00617mm^3 for the cross-section perpendicular to the longitudinal axis of the specimen ($S_0 = 0.41 \text{mm}^2$, $h = 1.51(10)^{-2} \text{mm}$) and 0.00643mm^3 for the cross-section forming an angle equal to 45° with respect to the above axis ($S_0 = 0.41 \text{mm}^2$, $h = 1.57(10)^{-2} \text{mm}$). More details may be found in Ref. [46]. It is

worth noting that, by increasing V , the probability to find bigger inclusions within this volume increases.

The computed values of \sqrt{area}_{max} were $145\mu m$ and $121\mu m$ for the cross-sections perpendicular to and forming an angle of 45° with the longitudinal axis of the specimen, respectively.

4.3 Fatigue strength affected by non-metallic inclusions

By employing Eqs (7) and (9), the fatigue strength σ_{wl} was equal to 271MPa, whereas τ_{wl} was equal to 235MPa.

Since the run-out condition in the experimental campaign examined was taken in correspondence of $2 \cdot (10)^6$ loading cycles, it was assumed that the fatigue strength affected by non-metallic inclusions could be estimated at $2 \cdot (10)^6$. Therefore, assuming the same slopes of the experimental S-N and T-N curves reported in Eqs (13) and (14), the new values of σ_{wl} and τ_{wl} , corresponding to $2 \cdot (10)^6$ cycles were equal to 309MPa and 260MPa, respectively.

5. RESULTS

5.1 Uniaxial and multiaxial fatigue results

The multiaxial criterion outlined in **Section 3** is applied to simulate uniaxial and multiaxial fatigue tests, where $\sigma_{af,-1} = \sigma_{wl}$ and

$$\tau_{af,-1} = \tau_{wl} \cdot$$

More precisely, Eq.(10) is graphically represented by an ellipse in the C_a against $N_{eq,a}$ diagram (**Figure 2**). Fatigue failure occurs if the point associated with a given experimental test lies out of the elliptical domain, whereas the safe points are those located inside such a domain.

Figure 2

In **Figure 2**, the error band equal to $\pm 10\%$ is represented by dashed lines, whereas that equal to $\pm 20\%$ is represented by dash-dot lines. The full symbols refer to specimens that failed, whereas the empty ones refer to run-outs. Moreover, empty symbols in red represent loading conditions in correspondence of which both failure and run-out were experimentally observed. The in-phase loading conditions are plotted in **Figure 2(a)**, whereas the out-of-phase loading conditions are plotted in **Figure 2(b)**.

The following remarks can be outlined from **Figure 2(a)**:

- under fully reversed axial loading (that is, $\frac{\tau_{xy,a}}{\sigma_{x,a}} = 0$), the criterion estimates the failure of all test specimens, that is, the results are too conservative although most of the results falls into the $+20\%$ scatter band;
- under fully reversed torsion loading (that is, $\frac{\tau_{xy,a}}{\sigma_{x,a}} = \infty$), the estimation are in complete agreement with the experimental results;

- under multiaxial loading, although the criterion is generally not able to correctly capture failure and run-out conditions, it provides conservative estimates, being the above points out or very close to the elliptical domain.

The following remarks can be outlined from **Figure 2(b)** :

- under multiaxial loading and $\frac{\tau_{xy,a}}{\sigma_{x,a}} = 0.5$, the estimations (except one) are in agreement with the experimental data;
- under multiaxial loading and $\frac{\tau_{xy,a}}{\sigma_{x,a}} = 1.0$ or $\frac{\tau_{xy,a}}{\sigma_{x,a}} = 2.0$, the estimations are totally in agreement with the experimental data.

The accuracy of the criterion is evaluated through the error index, I , given by:

$$I = \frac{\sigma_{eq,a} - \sigma_{af,-1}}{\sigma_{af,-1}} \cdot 100 \quad (19)$$

The mean value \bar{I} of the error index is equal to 1.4% for all loading conditions examined. A comparison in terms of \bar{I} is performed applying the Findley criterion [49] and the MWCM [50], by separating the computation for in-phase loading from that for out-of-phase loading, as is listed in **Table 2**. The comparison shows that a more satisfactory accuracy is obtained by applying the Carpinteri et al. criterion, being the $|\bar{I}|$ value lower than 10% for both in- and out-of-phase loading.

Table 2

5.2 Optimisation of the volume associated to the useful section

As is described in **Section 4.2**, the value of the volume V has to be set. Different values of V are considered and listed in **Table 3**: $V_2 = 10V_1$, $V_3 = 30V_1$, and V_4 equal to the crankshaft volume, where $V_1 = 2400\text{mm}^3$ is the volume employed in **Section 5.1**.

Table 3

By performing the aforementioned procedure (**Section 5.1**) for each volume value above, the mean value of I is computed and listed in **Table 3** together with the corresponding values of σ_{wl} and τ_{wl} .

The procedure is also repeated by using the experimental values of the fatigue strength under both fully reversed axial loading (equal to 375 MPa [46]) and fully reversed torsion loading (equal to 282MPa [46]). The mean value of I is computed and listed in **Table 3**.

For all the above cases, the $C_a - N_{eq,a}$ plots are shown in **Figures** from 3 to 6.

Figure 3

Figure 4

Figure 5

Figure 6

In **Figure 7**, \bar{I} (the mean value of I) is plotted against σ_{wl} . Such points can be interpolated by a parabolic curve:

$$\bar{I} = 250.62 - 1.31\sigma_{wl} + 0.016\sigma_{wl}^2 \quad (20)$$

where \bar{I} is equal to zero when $\sigma_{wl} = 313\text{MPa}$, value assumed in correspondence of $2 \cdot (10)^6$ loading cycles (**Figure 7**). The corresponding value of σ_{wl} at $1 \cdot (10)^7$ cycles is equal to 274MPa.

Figure 7

By exploiting Eq.(7), the value of $\sqrt{\text{area}_{max}}$ results to be equal to $135.16\mu\text{m}$ and, consequently, the values of y (by Eq.(15), $y=11.95$) and T (by Eq.(17), $T=154649.85$) can be computed. Then, by employing Eq.(18), the value of the 'optimum' volume associated to the useful cross-section that determines $\bar{I}=0$ can be computed: $V_{opt} = 954.19\text{mm}^3$.

The value of τ_{wl} for such a return period is computed: it is equal to 238MPa at $1 \cdot (10)^7$ loading cycles, and 263MPa at $2 \cdot (10)^6$ loading cycles.

By employing the values of fatigue strength at $2 \cdot (10)^6$ cycles, that is, $\sigma_{wl} = 313 \text{MPa}$ and $\tau_{wl} = 263 \text{MPa}$ in Eq.(10), the results obtained by applying the multiaxial criterion by Carpinteri et al. in conjunction with the $\sqrt{\text{area}}$ -parameter model by Murakami and Yanase are shown in **Figure 8**.

Figure 8

6. CONCLUSIONS

In the present paper, the authors have proposed to extend the Carpinteri et al. criterion to fatigue assessment of high strength steels characterised by a fatigue life strongly influenced by non-metallic inclusions.

To such an aim, the above multiaxial criterion has been used in conjunction with the $\sqrt{\text{area}}$ concepts proposed by Murakami and Yanase.

The extended criterion has been applied to an experimental campaign available in the literature and performed on AISI 4140 steel specimens.

The results have been quite satisfactory, being the criterion accuracy higher than that of two other criteria available in the literature.

On the basis of such results, it has been observed that the volume of the specimen gauge-length region is suitable, since a very low value of \bar{I} is obtained. Moreover, an optimisation

procedure has been employed in order to find the volume corresponding to the error index mean value equal to zero.

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**INFLUENCE OF NON-METALLIC INCLUSIONS ON THE
HIGH CYCLE FATIGUE STRENGTH OF STEELS**

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FIGURES AND TABLES

Table 1. Mechanical properties of AISI 4140 steel.

MATERIAL	σ_y [MPa]	σ_u [MPa]	ε_u [%]	$HV \left[\frac{kgf}{mm^2} \right]$
AISI4140	689	900	178	320

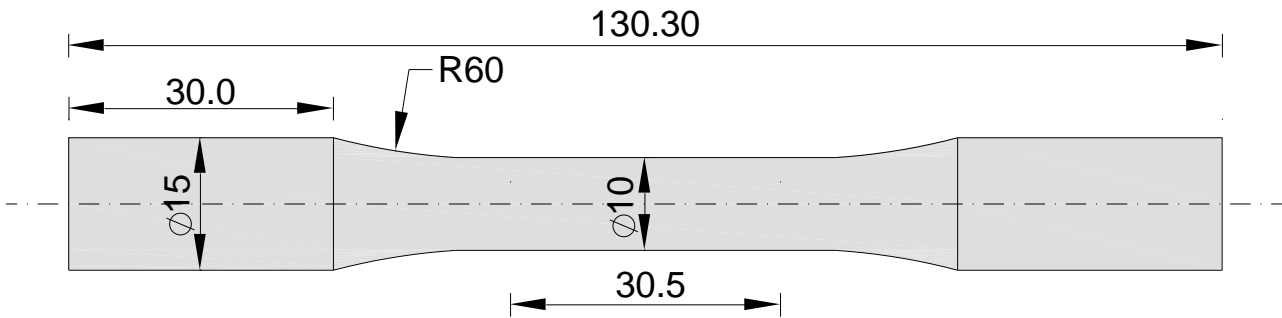


Figure 1. Geometry of the specimen according to the ASTM International E466-15 (sizes in mm).

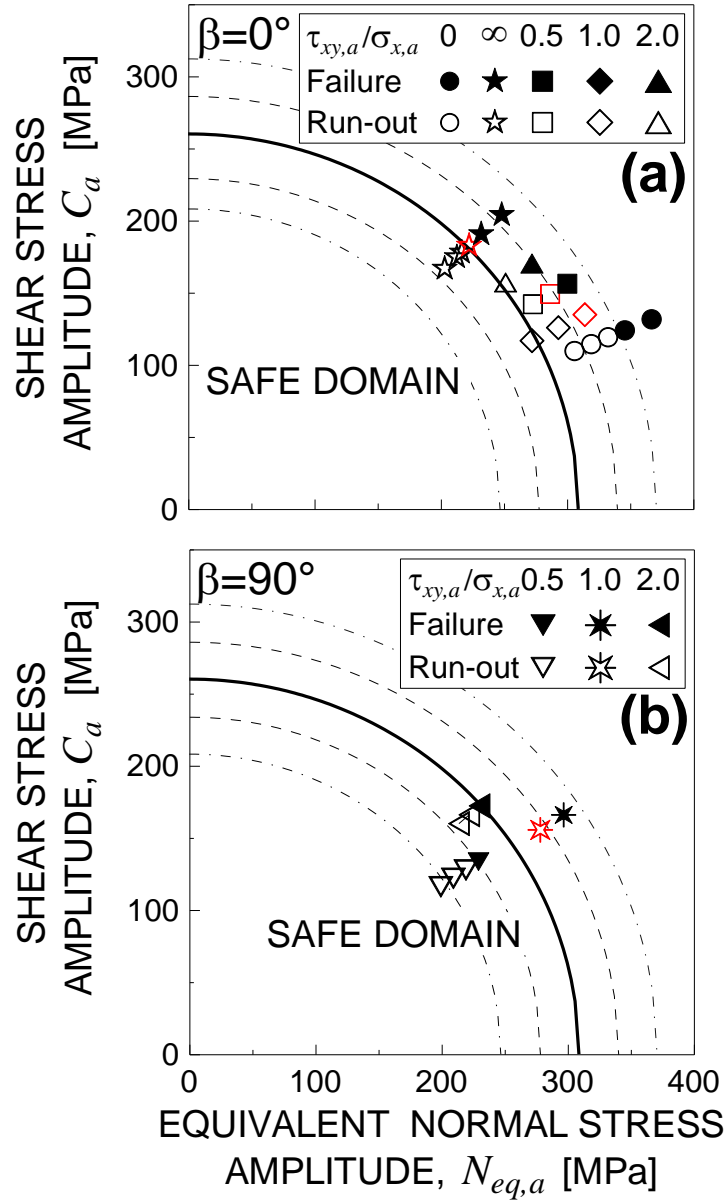


Figure 2. Shear stress amplitude vs. equivalent normal stress amplitude acting on the critical plane: theoretical predictions and experimental data. The volume associated to the useful cross-section is equal to $V_1 = 2400\text{mm}^3$.

Table 2. Mean value of the error index, \bar{I} , by employing the Findley criterion [49], the MWCM [50] and the present methodology.

	Findley [49]		MWCM [50]		Present study	
Phase shift, β [°]	0	90	0	90	0	90
Error index, \bar{I} [%]	15.3	18.4	12.0	22.7	8.2	-5.4

Table 3. Fatigue strength under normal and shear loading and mean value of the error index by varying the volume associated to the useful cross-section.

Volume, V	V value [mm^3]	σ_{wl} [MPa]	τ_{wl} [MPa]	\bar{I} [%]
V_1	2400	309	260	1.4
V_2	24000	302	254	3.4
V_3	72000	298	251	4.5
$V_{crankshaft}$	7.9E+09	274	231	13.9
V_{opt}	954.19	313	263	0
<i>Exp. tests</i>	-	375	282	-11.8

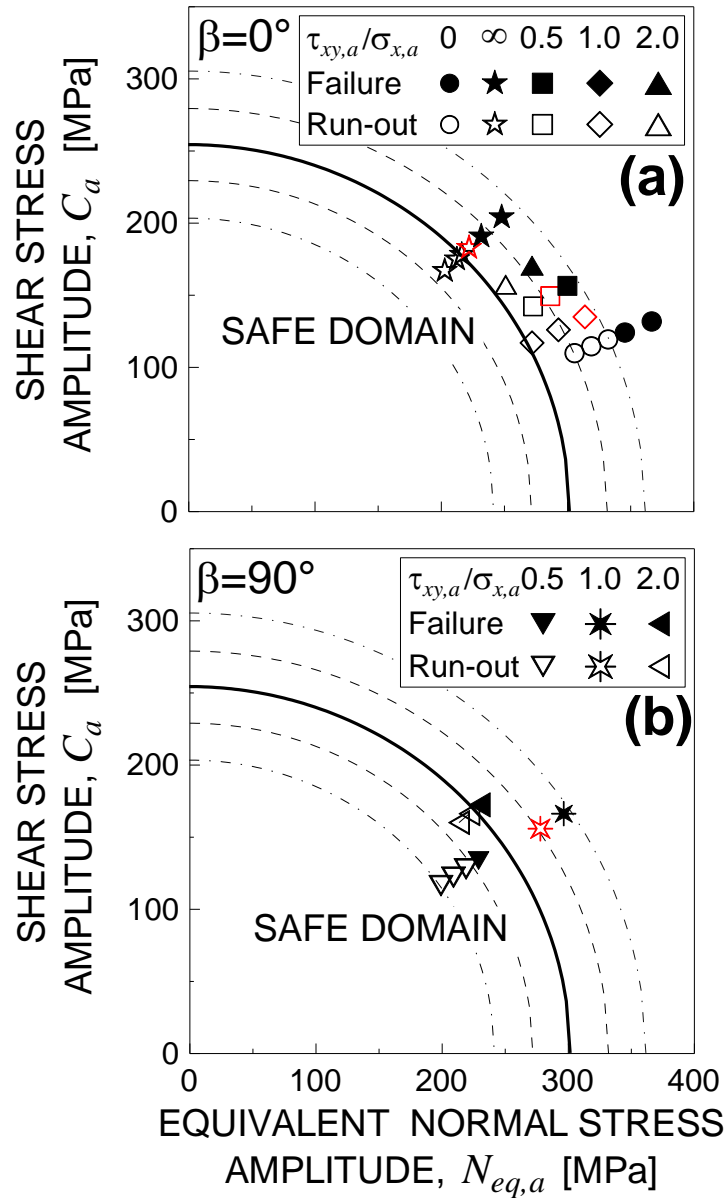


Figure 3. Shear stress amplitude vs. equivalent normal stress amplitude acting on the critical plane: theoretical predictions and experimental data. The volume associated to the useful cross-section is equal to $V_2 = (10) \cdot 2400 \text{mm}^3$.

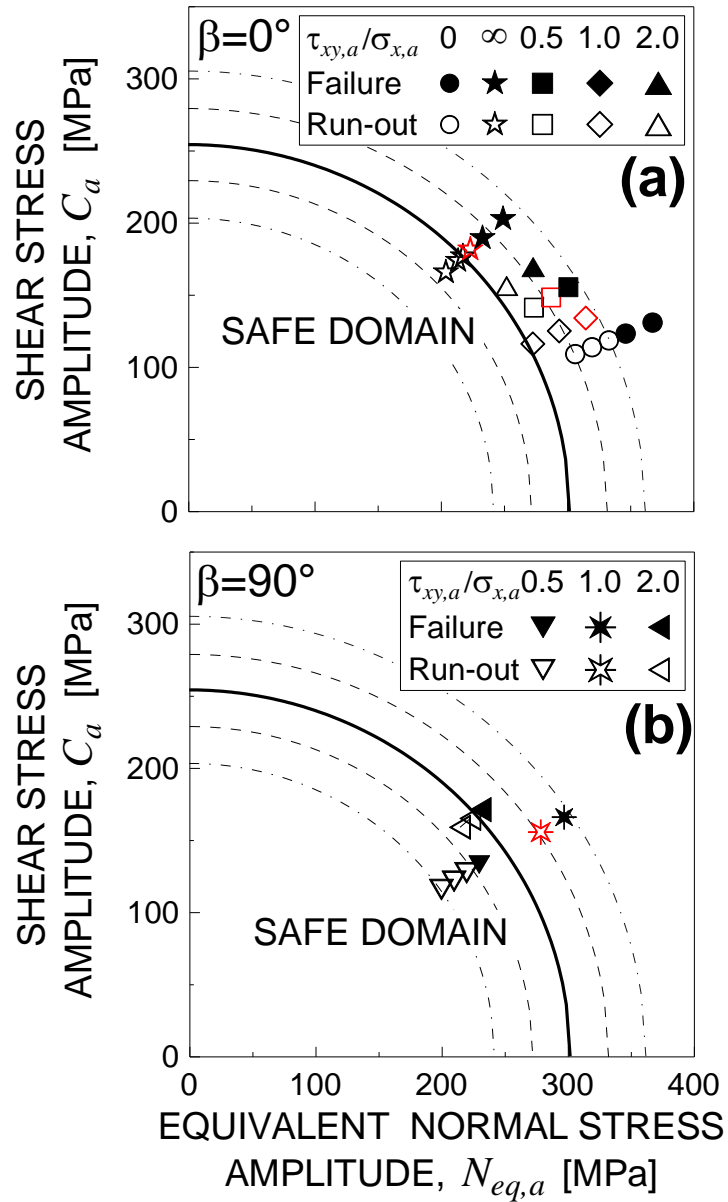


Figure 4. Shear stress amplitude vs. equivalent normal stress amplitude acting on the critical plane: theoretical predictions and experimental data. The volume associated to the useful cross-section is equal to $V_3 = (30) \cdot 2400 \text{mm}^3$.

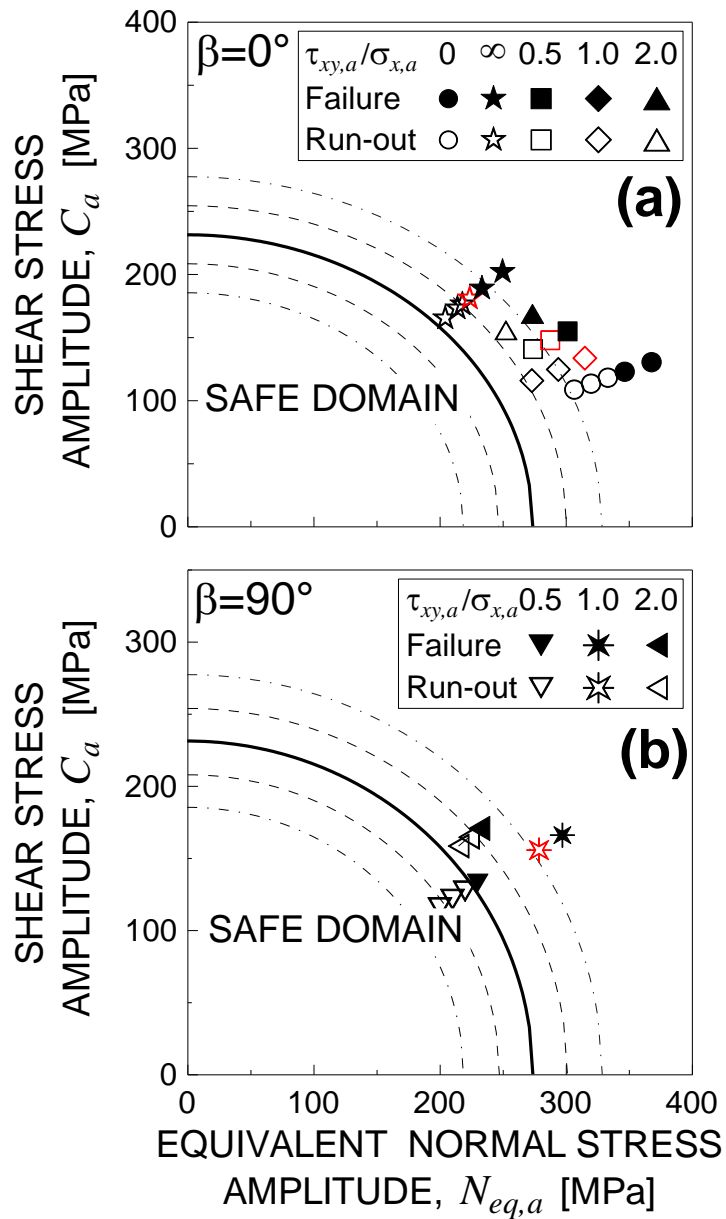


Figure 5. Shear stress amplitude vs. equivalent normal stress amplitude acting on the critical plane: theoretical predictions and experimental data. The volume associated to the useful cross-section is equal to that of the examined crankshaft,

$$V_{crankshaft} \cdot$$

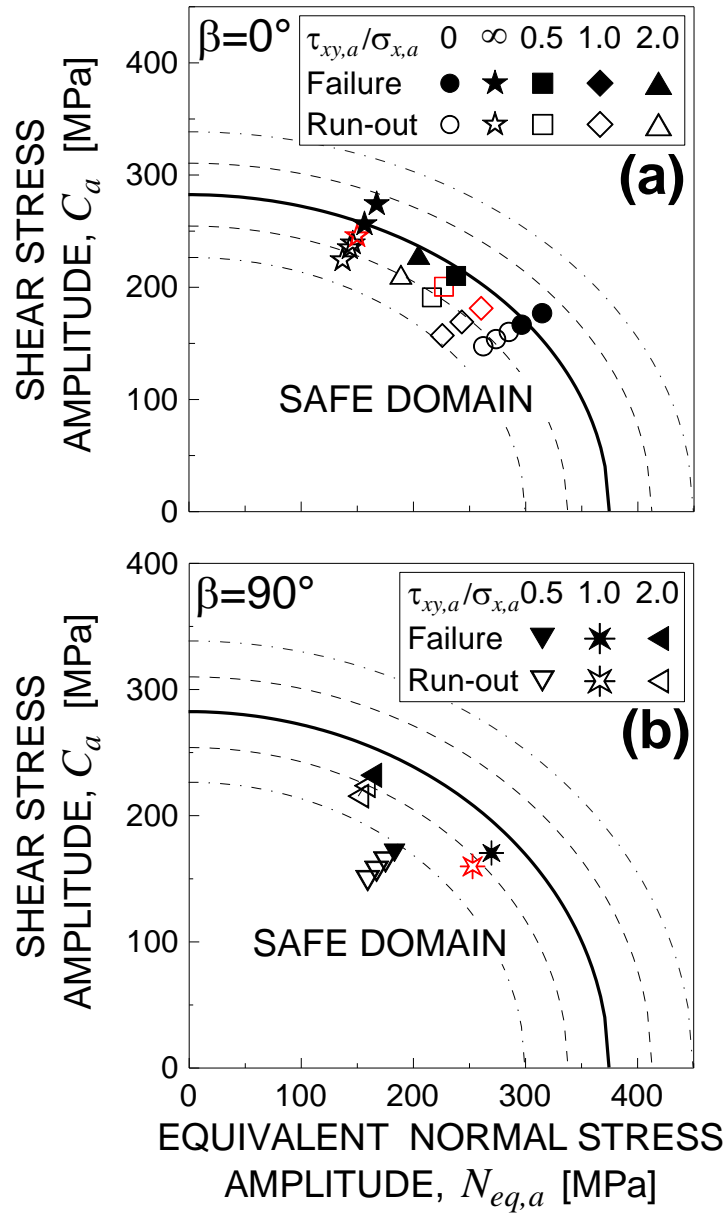


Figure 6. Shear stress amplitude vs. equivalent normal stress amplitude acting on the critical plane: theoretical predictions ($\sigma_{wl} = 375\text{MPa}$, $\tau_{wl} = 282\text{MPa}$) and experimental data.

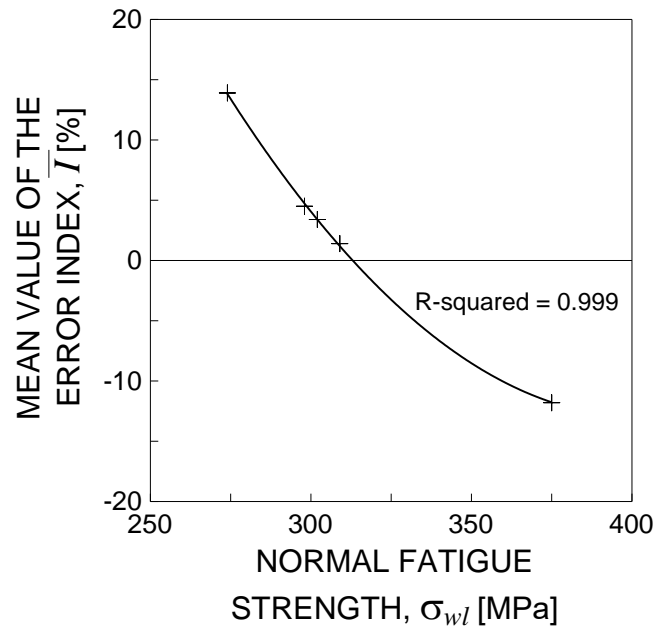


Figure 7. Mean value of the error index vs fatigue strength under normal loading for different values of the volume associated to the useful cross-section.

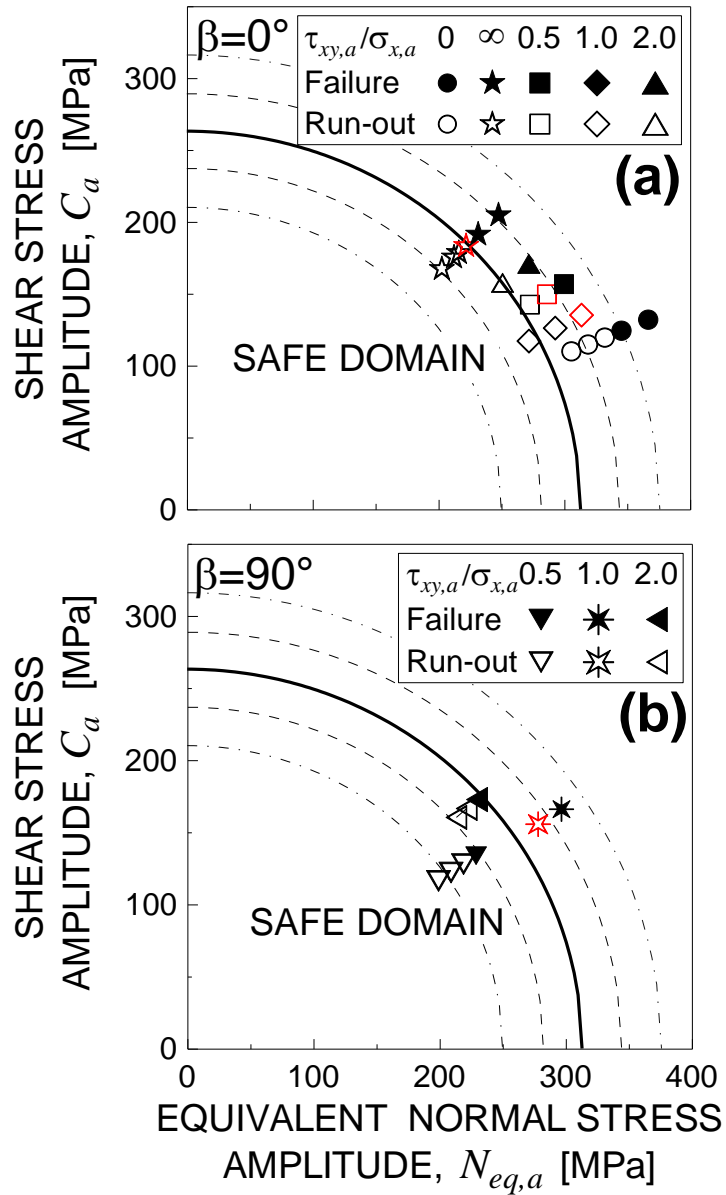


Figure 8. Shear stress amplitude vs. equivalent normal stress amplitude acting on the critical plane: theoretical predictions and experimental data. The volume associated to the useful cross-section is equal to the optimised volume, V_{opt} .