Spectral fatigue life estimation for non-proportional multiaxial random loading

Original

Spectral fatigue life estimation for non-proportional multiaxial random loading / Carpinteri, Andrea; Fortese, Giovanni; Ronchei, Camilla; Scorza, Daniela; Vantadori, Sabrina. - In: THEORETICAL AND APPLIED FRACTURE MECHANICS. - ISSN 0167-8442. - 83:(2016), pp. 67-72. [10.1016/j.tafmec.2015.12.019]

Publisher:

Elsevier

Published

DOI:10.1016/j.tafmec.2015.12.019

Special Issue ICEAF 4
Theoretical and Applied Fracture Mechanics – November 2015
Revised version - December 2015

Spectral fatigue life estimation for non-proportional multiaxial random loading

Andrea Carpinteri, Giovanni Fortese, Camilla Ronchei,
Daniela Scorza, Sabrina Vantadori

Dept. of Civil-Environmental Engineering and Architecture,
University Of Parma, Parco Area delle Scienze 181/A
43124 Parma – Italy
Corresponding author:
Sabrina Vantadori, email: sabrina.vantadori@unipr.it

ABSTRACT

A frequency-domain High-Cycle Fatigue (HCF) criterion based on the critical plane approach is here proposed to estimate fatigue life of smooth metallic structural components under multiaxial random loading. The procedure consists of the following three steps: (a) definition of the critical plane; (b) PSD evaluation of an equivalent normal stress; (c) computation of the fatigue life. A new formulation to define the critical plane is adopted in order to improve the lifetime estimation. The criterion is validated through comparison of the obtained numerical results with experimental fatigue data available in the literature for structural steel specimens subjected to a combination of random non-proportional bending and torsion.

KEYWORDS: critical plane-based criterion, frequency-domain criterion, multiaxial fatigue, random loading

NOMENCLATURE

$\mathbf{C} = \mathbf{C}(\phi, \theta, \psi)$	rotation matrix from PXYZ to $\mathrm{PX'Y'Z'}$
$\tilde{\mathbf{C}} = \tilde{\mathbf{C}}(\phi, \theta, \psi)$	rotation matrix from $\mathrm{P\hat{1}\hat{2}\hat{3}}$ to Puvw
$E[D]$	expected fatigue damage per unit time
$p_a(s)$	marginal amplitude distribution of the counted equivalent stress cycles
PXYZ	fixed reference system
PX'Y'Z'	rotated reference system
$\mathrm{P\hat{1}\hat{2}\hat{3}}$	reference system of the weighted mean principal stress axes
Puvw	reference system attached to the critical plane
$R_{i,j}(\tau)$	auto/crosscorrelation functions
$S_{eq}(\omega)$	equivalent PSD function
$\mathbf{s}_{\mathrm{xyz}}(t)$	stress vector referred to the reference system PXYZ
$\mathbf{s}_{\mathrm{x'y'z'}}(t)$	stress vector referred to the reference system $\mathrm{PX'Y'Z'}$
$\mathbf{s}_{\mathrm{uvw}}(t)$	stress vector referred to the reference system Puvw
$\mathbf{S}_{\mathrm{xyz}}(\omega)$	Power Spectral Density (PSD) matrix of $\mathbf{s}_{\mathrm{xyz}}(t)$
$S_{i,j}(\omega)$	coefficients of the $\mathbf{S}_{\mathrm{xyz}}(\omega)$ matrix
$\mathbf{S}_{\mathrm{x'y'z'}}(\omega)$	Power Spectral Density (PSD) matrix of $\mathbf{s}_{\mathrm{x'y'z'}}(t)$
$S_{i',j'}(\omega)$	coefficients of the $\mathbf{S}_{\mathrm{x'y'z'}}(\omega)$ matrix
$S_{3',3'}$	PSD function of the normal stress $\sigma_{z'}$
$S_{6',6'}$	PSD function of the shear stress $\tau_{y'z'}$
$\mathbf{S}_{\mathrm{uvw}}(\omega)$	Power Spectral Density (PSD) matrix of $\mathbf{s}_{\mathrm{uvw}}(t)$
$S_{3'',3''}$	PSD function of the normal stress σ_{w}
$S_{6'',6''}$	PSD function of the shear stress τ_{vw}
t	time
T	observation time interval
T_{cal}	calculated fatigue life

T_{exp}	experimental fatigue life
γ	rotation about $\mathbf{w}$-axis
δ	angle between the averaged direction $\hat{\mathbf{1}}$ and the normal $\mathbf{w}$ to the critical plane (Fig. 2b)
$\delta_{L,i}$	angles between the averaged direction $\hat{\mathbf{1}}$ and the normal $\mathbf{w}$ to the critical plane proposed by Łagoda et al., with $i = 1,\ldots 4$
$\tilde{\delta}$	off-angle according to one of the expressions reported in Eqs (11) and (12)
λ_n	n-th spectral moment, where n is a positive real number
ν_a	expected rate of occurrence of the counted equivalent stress cycles
ν_p	expected rate of occurrence of peaks of the counted equivalent stress cycles
ν_0^+	expected rate of zero-upcrossings of $\sigma_{Z'}$
$\sigma_{af,-1}$	normal stress fatigue limit for fully reversed normal stress (loading ratio $R = -1$)
$\tau_{af,-1}$	shear stress fatigue limit for fully reversed shear stress (loading ratio $R = -1$)
ϕ, θ, ψ	Euler angles
$\hat{\phi}, \hat{\theta}, \hat{\psi}$	averaged Euler angles
ω	pulsation

1. INTRODUCTION

The type of loading experienced by many engineering structural components during their service life may be random and non-proportional. This is the case for metallic structures such as pressure vessels, nuclear and pressure water reactors, gas turbines, and automobile crankshafts [1,2]. Fatigue analysis under multiaxial random loading is a research topic still open [3-10]: as a matter of fact, the multiaxial state of stress under random loading requires the use of cycle counting techniques and models which are significantly more complex than those used to estimate life under uniaxial loading.

Multiaxial fatigue criteria have historically been formulated according to the time-domain approach [11-14]. Time-domain procedures are based on cycle counting methods and damage accumulation rules and, furthermore, require the knowledge of the time histories of the local stress tensor components. Note that many records are needed in order to obtain reliable statistical parameters of the loading process.

On the other hand, multiaxial fatigue criteria formulated according to the frequency-domain approach (often named spectral methods) [3,9,15-21] are alternative tools to treat loading of random type [22]. Frequency-domain procedures require no cycle counting methods but they need damage accumulation rules and, starting from the Power Spectral Density (PSD) function of local stress tensor components, an estimation of damage is directly obtained. Such features make the above criteria much more computationally efficient than the time domain ones [15], still providing high levels of accuracy.

Hybrid frequency-time domain methods for predicting multiaxial fatigue life are also available in the literature [4].

Some multiaxial fatigue criteria originally developed in time domain have been reformulated in frequency domain as multiaxial spectral methods [6], and may be based on either an equivalent uniaxial stress [15-21] or stress invariants [23,24].

Spectral fatigue life estimation for non-proportional multiaxial random loadings is the subject of the present paper. For such a purpose, the authors here propose a fatigue criterion based on an equivalent uniaxial stress evaluated on the critical plane [25-27]. This criterion is a frequency-based reformulation of the original time-domain method presented in Refs [28-30].

The procedure consists of the following three steps:

(a) definition of the critical plane;

(b) PSD evaluation of an equivalent normal stress on the critical plane;

(c) computation of the fatigue life.

The orientation of the critical plane is connected to both averaged principal stress directions related to critical points of the structural component being examined and the fatigue properties of material. The latter dependence is taken into account through a rotational angle, δ, whose expression depends on material fatigue limits under fully reversed normal stress ($\sigma_{af,-1}$) and shear stress ($\tau_{af,-1}$) [28].

Recently, Łagoda et al. [31-33] have proposed some relationships different from the original δ expression, and have estimated fatigue life of construction materials under *cyclic loading*. In the present paper, these relationships are implemented in the proposed criterion in order to estimate fatigue life of materials under *random loading*. The scope is to verify whether such expressions are able to improve the criterion in terms of lifetime evaluation.

Firstly, the theoretical framework of the criterion is presented. Note that the background theory on the frequency-based characterization of uniaxial and multiaxial random stresses as well as on uniaxial spectral methods for fatigue damage assessment can be found in Ref.[25]. Then, an application related to fatigue tests on steel round specimens subjected to a combination of random non-proportional bending and torsion [34] is discussed.

2. THEORETICAL FRAMEWORK OF THE PROPOSED CRITERION

Figure 1 shows the algorithm of the stress-based critical plane criterion here proposed for the frequency-based analysis related to High-Cycle Fatigue (HCF) random multiaxial loading. All the reported steps are discussed in the following Sections.

Figure 1.

2.1 Determination of the PSD matrix with respect to the PXYZ reference system

Let us consider the stress tensor $\mathbf{s}_{\mathrm{xyz}}(t)=\{s_1, s_2, s_3, s_4, s_5, s_6\}^T=\{\sigma_{\mathrm{x}},\sigma_{\mathrm{y}},\sigma_{\mathrm{z}},\tau_{\mathrm{xy}},\tau_{\mathrm{xz}},\tau_{\mathrm{yz}}\}^T$ with respect to the fixed frame XYZ (Fig. 2a), at point P in the structural component exposed to a general time-varying random stress state. By assuming that the random features can be described by a six-dimensional ergodic stationary Gaussian stochastic process with zero mean values, the PSD matrix with respect to PXYZ (*Step 1* in Fig.1) is here displayed:

$$\mathbf{S}_{\mathrm{xyz}}(\omega)=\begin{bmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} & S_{1,5} & S_{1,6} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} & S_{2,5} & S_{2,6} \\ S_{3,1} & S_{3,2} & S_{3,3} & S_{3,4} & S_{3,5} & S_{3,6} \\ S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4} & S_{4,5} & S_{4,6} \\ S_{5,1} & S_{5,2} & S_{5,3} & S_{5,4} & S_{5,5} & S_{5,6} \\ S_{6,1} & S_{6,2} & S_{6,3} & S_{6,4} & S_{6,5} & S_{6,6} \end{bmatrix} \qquad (1)$$

where ω is the pulsation. The coefficients, $S_{i,j}(\omega)$, of such a matrix are defined by means of the auto/crosscorrelation functions, $R_{i,j}(\tau)$:

$$R_{i,j}(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_0^T s_i(t)\, s_j(t+\tau)\,dt \qquad i,j=1,\ldots 6 \qquad (2)$$

$$S_{i,j}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}R_{i,j}(\tau)e^{-i\omega\tau}d\tau \qquad i,j=1,\ldots 6 \tag{3}$$

being t = time and T = observation period.

Figure 2.

2.2 Determination of the PSD matrix with respect to the PX′Y′Z′ reference system

Let us consider the rotated reference system PX'Y'Z' (Fig. 2a), defined by the three Euler angles ϕ,θ,ψ, and the corresponding stress tensor is $\mathbf{s}'_{\mathrm{xyz}}(t)=\{s'_1,s'_2,s'_3,s'_4,s'_5,s'_6\}^T=\{\sigma'_{\mathrm{x}},\sigma'_{\mathrm{y}},\sigma'_{\mathrm{z}},\tau'_{\mathrm{xy}},\tau'_{\mathrm{xz}},\tau'_{\mathrm{yz}}\}^T$. By employing the rotation matrix $\mathbf{C}=\mathbf{C}(\phi,\theta,\psi)$:

$$\mathbf{C}=\begin{bmatrix}1&0&0&0&0&0\\0&c_\psi^2&s_\psi^2&2c_\psi s_\psi&0&0\\0&s_\psi^2&c_\psi^2&-2c_\psi s_\psi&0&0\\0&-2c_\psi s_\psi&2c_\psi s_\psi&c_\psi^2-s_\psi^2&0&0\\0&0&0&0&c_\psi&-s_\psi\\0&0&0&0&s_\psi&c_\psi\end{bmatrix}\begin{bmatrix}c_\theta^2&0&s_\theta^2&0&2c_\theta s_\theta&0\\0&1&0&0&0&0\\s_\theta^2&0&c_\theta^2&0&-2c_\theta s_\theta&0\\0&0&0&c_\theta&0&-s_\theta\\-c_\theta s_\theta&0&c_\theta s_\theta&0&c_\theta^2-s_\theta^2&0\\0&0&0&s_\theta&0&c_\theta\end{bmatrix}\begin{bmatrix}1&0&0&0&0&0\\0&c_\phi^2&s_\phi^2&2c_\phi s_\phi&0&0\\0&s_\phi^2&c_\phi^2&-2c_\phi s_\phi&0&0\\0&-2c_\phi s_\phi&2c_\phi s_\phi&c_\phi^2-s_\phi^2&0&0\\0&0&0&0&c_\phi&-s_\phi\\0&0&0&0&s_\phi&c_\phi\end{bmatrix} \tag{4}$$

with $c_\psi=\cos\psi$, $s_\psi=\sin\psi$, $c_\theta=\cos\theta$, $s_\theta=\sin\theta$, $c_\phi=\cos\phi$ and $s_\phi=\sin\phi$, the PSD matrix with respect to such a rotated reference system can be computed as follows (*Step 2* in Fig.1):

$$\mathbf{S}_{\mathrm{x'y'z'}}(\omega)=\mathbf{C}\ \mathbf{S}_{\mathrm{xyz}}(\omega)\ \mathbf{C}^T \tag{5}$$

2.3 Determination of the critical plane orientation

The critical plane is assumed to be linked to averaged principal stress directions (*Step 3* in Fig.1) by varying the angles ϕ,θ so that the direction Z' is experiencing the maximum $\sigma_{Z'} \equiv s_{3',3'}$ in a statistical sense [35]:

$$E\left[\max_{0 \le t \le T} \sigma_{z'}(t)\right] \cong \sqrt{\lambda_0}\,\sqrt{2\ln(\nu_0^+ T)} + \frac{0.5772}{\sqrt{2\ln(\nu_0^+ T)}} \tag{6}$$

where λ_0 is the spectral moment of order 0 of the PSD function, $S_{3',3'}$:

$$\lambda_0 = \int_{-\infty}^{+\infty} S_{3',3'}(\omega)\, d\omega \tag{7}$$

and ν_0^+ is the expected rate of mean zero-upcrossings of $\sigma_{Z'} \equiv s_{3',3'}$, given by the following expression:

$$\nu_0^+ = \frac{1}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0}} \tag{8}$$

being

$$\lambda_2 = \int_{-\infty}^{+\infty} |\omega|^2 S_{3',3'}(\omega)\, d\omega \tag{9}$$

The angle ψ (related to the Y' axis) is made to vary in order to maximize the variance of $\tau_{y'z'} \equiv s_{6',6'}$:

$$\max_{0 \le \psi \le 2\pi}\left[\sigma^2_{6',6'}\right] = \left[\max_{0 \le \psi \le 2\pi} \int_{-\infty}^{+\infty} S_{6',6'}(\omega,\psi)\, d\omega\right] \tag{10}$$

The directions Z' and Y' are regarded as the averaged principal directions $\hat{1}$ and $\hat{3}$, respectively, and the direction X' is consequently the averaged principal direction $\hat{2}$ (Fig. 2b). The corresponding averaged Euler angles are named $\hat{\phi}, \hat{\theta}, \hat{\psi}$ in the following.

The normal **w** to the critical plane is defined by the off-angle δ (clockwise rotation) about the axis $\hat{2}$ (Fig. 2b), being δ a function of the fully reversed shear ($\tau_{af,-1}$) and normal ($\sigma_{af,-1}$) stress fatigue limits [28]:

$$\delta = \frac{3}{8}\left[1-\left(\frac{\tau_{af,-1}}{\sigma_{af,-1}}\right)^2\right] 45° \tag{11}$$

Recently, Łagoda and co-authors have proposed some relationships [31-33] different from the original δ expression, in order to estimate fatigue life of construction materials under cyclic loading. Such relationships are empirical expressions deduced by both interpolating experimental data and taking into account the observed dependence of the critical plane orientation on the ratio between the normal and shear fatigue limits.

Such proposals are pertinent to the original expression (Eq.11) as far as the limit conditions are concerned, that is, the off-angle is equal to 0° for hard metals (this is the case of fatigue limit ratio $\tau_{af,-1}/\sigma_{af,-1}$ = 1), whereas it is equal to 45° for borderline mild/hard metals (this is the case of $\tau_{af,-1}/\sigma_{af,-1}$ = $1/\sqrt{3}$).

Such proposals related to the off-angle are implemented in the proposed criterion:

$$\delta_{L,1} = \frac{9}{8}\left[1-\left(\frac{\tau_{af,-1}}{\sigma_{af,-1}}\right)^4\right] 45° \tag{12a}$$

$$\delta_{L,2} = \frac{3\sqrt{3}}{3\sqrt{3}-1}\left[1-\left(\frac{\tau_{af,-1}}{\sigma_{af,-1}}\right)^3\right] 45° \tag{12b}$$

$$\delta_{L,3} = \frac{3\sqrt{3}}{3\sqrt{3}-3}\left[1-\left(\frac{\tau_{af,-1}}{\sigma_{af,-1}}\right)\right] 45° \tag{12c}$$

$$\delta_{L,4} = \frac{3}{\left(\sqrt{3}-1\right)^2}\left[1-\left(\frac{\tau_{af,-1}}{\sigma_{af,-1}}\right)\right]^2 45° \tag{12d}$$

The diagram shown in Fig. 3 represents a graphical interpretation of Eqs (11) and (12), related to the $\tilde{\delta}$ off-angle against the ratio between fully reversed shear ($\tau_{af,-1}$) and normal ($\sigma_{af,-1}$) stress fatigue limits.

Figure 3.

2.4 Determination of the PSD matrix with respect to the Puvw reference system

Now we consider the reference system Puvw attached to the critical plane (Fig. 2b), the corresponding stress tensor being $\mathbf{s}_{\mathrm{uvw}}(t)=\{s_{1''},s_{2''},s_{3''},s_{4''},s_{5''},s_{6''}\}^T=\{\sigma_{\mathrm{u}},\sigma_{\mathrm{v}},\sigma_{\mathrm{w}},\tau_{\mathrm{uv}},\tau_{\mathrm{uw}},\tau_{\mathrm{vw}}\}^T$. The u-axis is defined by the angle γ, that represents a counterclockwise rotation about the w-axis (Fig. 2b), so that the u-axis defines the direction that maximizes the variance of τ_{vw}:

$$\max_{0\leq\gamma\leq 2\pi}\left[\sigma^2_{6'',6''}\right]=\left[\max_{0\leq\gamma\leq 2\pi}\int_{-\infty}^{+\infty}S_{6'',6''}(\omega,\gamma)\,d\omega\right] \tag{13}$$

The PSD matrix with respect to such a reference system (*Step 4* in Fig.1) can be computed as follows:

$$\mathbf{S}_{\mathrm{x'''y'''z'''}}(\omega)=\mathbf{S}_{\mathrm{uvw}}(\omega)=\tilde{\mathbf{C}}\;\mathbf{S}_{\mathrm{xyz}}(\omega)\;\tilde{\mathbf{C}}^T \tag{14}$$

where

$$\tilde{\mathbf{C}}=\begin{bmatrix} c_\gamma^2 & s_\gamma^2 & 0 & 0 & 0 & 2c_\gamma s_\gamma \\ s_\gamma^2 & c_\gamma^2 & 0 & 0 & 0 & -2c_\gamma s_\gamma \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_\gamma & s_\gamma & 0 \\ 0 & 0 & 0 & -s_\gamma & c_\gamma & 0 \\ -c_\gamma s_\gamma & c_\gamma s_\gamma & 0 & 0 & 0 & c_\gamma^2-s_\gamma^2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{\tilde{\delta}}^2 & s_{\tilde{\delta}}^2 & -2c_{\tilde{\delta}}s_{\tilde{\delta}} & 0 & 0 \\ 0 & s_{\tilde{\delta}}^2 & c_{\tilde{\delta}}^2 & 2c_{\tilde{\delta}}s_{\tilde{\delta}} & 0 & 0 \\ 0 & c_{\tilde{\delta}}s_{\tilde{\delta}} & -c_{\tilde{\delta}}s_{\tilde{\delta}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{\tilde{\delta}} & s_{\tilde{\delta}} \\ 0 & 0 & 0 & 0 & -s_{\tilde{\delta}} & c_{\tilde{\delta}} \end{bmatrix}\mathbf{C} \tag{15}$$

$\mathbf{C}$ is the rotation matrix (see Eq. (4)) evaluated by considering the averaged Euler angles $\hat{\phi}\hat{\theta}\hat{\psi}$, defining the averaged coordinate system $\hat{1}\,\hat{2}\,\hat{3}$, and $\tilde{\delta}$ is the off-angle according to one of the expressions reported in Eqs (11) and (12). Further: $c_\gamma = \cos\gamma$, $s_\gamma = \sin\gamma$, $c_{\tilde{\delta}} = \cos\tilde{\delta}$ and $s_{\tilde{\delta}} = \sin\tilde{\delta}$.

In order to reduce the multiaxial stress state to an equivalent uniaxial stress, we propose to determine the PSD function of an equivalent normal stress, related to the critical plane, by means of the linear combination of PSD functions here reported:

$$S_{eq}(\omega) = S_{3'',3''} + \left(\frac{\sigma_{af,-1}}{\tau_{af,-1}}\right) S_{6'',6''} = S_{\mathrm{w,w}} + \left(\frac{\sigma_{af,-1}}{\tau_{af,-1}}\right) S_{\mathrm{vw,vw}} \tag{16}$$

Note that, by employing such a combination, the equivalent normal stress is assumed to be represented by a stationary Gaussian stochastic process with zero mean value, that is, the statistical properties of the input signal, $\mathbf{s}_{\mathrm{xyz}}(t)$, are conserved.

2.5 Fatigue damage evaluation

Let us evaluate the expected fatigue damage per unit time (*Step 5* in Fig.1) by employing a linear cumulative damage rule:

$$E[D] = \nu_a\, C^{-1} \int_0^{+\infty} s^k\, p_a(s)\, ds \tag{17}$$

where ν_a is the expected rate of occurrence of the counted equivalent stress cycles, k and C are the parameters of the bending stress S-N curve, and $p_a(s)$ is the marginal amplitude distribution of the counted equivalent stress cycles.

Let us consider the Rain-Flow Counting (RFC) procedure [36] which is a 'complete counting' procedure, that is, each peak is paired with a lower or equal valley and, therefore, the expected rate of loading cycles is equal to the expected rate of peaks. In other words, ν_a in Eq. (17) is equal to the expected rate of occurrence, ν_p, related to the peaks of the equivalent normal stress:

$$\nu_p = \frac{1}{2\pi}\sqrt{\frac{\int_{-\infty}^{+\infty} |\omega|^4 S_{eq}(\omega)d\omega}{\int_{-\infty}^{+\infty} |\omega|^2 S_{eq}(\omega)d\omega}} \tag{18}$$

For the RFC methods, an analytical solution for $p_a(s)$ is not available in the literature and, therefore, Benasciutti et al. [16] addressed the problem of the RFC damage evaluation as the search for the proper intermediate point between the lower and the upper bounds of the above damage (see also Ref. [25]). As a matter of fact, such a procedure is 'cross-consistent', that is, the upper bound of the expected damage per unit time is the damage evaluated by employing the narrow-band approximation, whereas the lower bound is the damage evaluated by employing the range-mean counting procedure.

Note that the expected fatigue damage in Eq. (17) is constant in a stationary process and, considering a critical damage equal to the unity, the calculated fatigue life is:

$$T_{cal} = 1/E[D] \tag{19}$$

3. EXPERIMENTAL TESTING

Now the proposed criterion is applied to some relevant random fatigue experimental results available in the literature [34]. The smooth specimens are made of 18G2A steel. The loading histories are non-proportional random independent bending and torsion stresses following a Gaussian probability distribution with zero mean values:

the dominant frequency is 28.3 Hz for bending and 30 Hz for torsion (Fig. 4), the duration is equal to 820 s, and the sampling frequency is equal to 250 Hz. Thirteen loading combinations are processed by varying the value of the ratio $\tau_{\max}/\sigma_{\max}$ [34].

Figure 4.

In Figure 5, the numerical results determined by employing the proposed frequency-domain criterion are compared with the experimental data [34], by expressing $\tilde{\delta}$ through one of the Eqs (11) or (12).

Figure 5.

Since one of the aims of the present work is to verify whether the relationships proposed by Łagoda and Co-authors [31-33] are able to improve the present criterion in terms of lifetime estimation, the results obtained by employing Eq. (11) are compared (in Fig.5) to those achieved by using the expressions in Eqs (12a) to (12d).

To interpret the results from a statistical point of view, the root mean square error method can be used in order to identify which expression of $\tilde{\delta}$ (see Eqs (11) and (12)) is able to produce better results in terms of theoretical estimations. The value of the root mean square logarithmic error is given by:

$$e_{RMS} = \sqrt{\frac{\sum_{i=1}^{n} \log^2\left(\frac{T_{exp,i}}{T_{cal,i}}\right)}{n}} \tag{20}$$

where n is the total number of the processed loading combinations, T_{exp} is the experimental fatigue life, and T_{cal} is the calculated fatigue life. Finally, the mean square error T_{RMS} of the scatter ~~is~~ can be determined as follows:

$$T_{RMS} = 10^{e_{RMS}} \tag{21}$$

Figure 6 shows the mean square error obtained by alternatively applying the above expressions of $\tilde{\delta}$. On the basis of the analysis of such results, it can be concluded that the higher conformity is gained by employing Eq. (12a).

Figure 6.

4. DISCUSSION AND CONCLUSIONS

In the present paper, a frequency-domain criterion to evaluate the fatigue life of smooth metallic structures subjected to multiaxial random loading has been proposed. The PSD function of an equivalent normal stress related to the critical plane is evaluated by combining the PSD functions of stress components (normal and tangential) acting on the critical plane, and is adopted as the parameter to quantify the fatigue damage.

The influence of the orientation of the critical plane on the fatigue life evaluation has been examined by alternatively applying different expressions of the off-angle $\tilde{\delta}$. Such relationships are empirical expressions proposed by Lagoda et al. by both interpolating experimental data and taking into account the observed dependence of the critical plane orientation on the ratio between the normal and shear fatigue limits.

The effectiveness of the present criterion is evaluated by means of experimental data available in the literature, related to fatigue tests on steel round specimens subjected to a combination of random non-proportional bending and torsion. Theoretical results are deduced for different values of $\tilde{\delta}$, in order to assess its influence on the fatigue life evaluation.

The comparison with such experimental tests has been satisfactory, especially by employing $\delta_{L,1}$, being the mean square error T_{RMS} equal to 2.14 in this case, whereas T_{RMS} is equal to 2.27 by employing the expression δ in Eq.(11).

Acknowledgements

The authors gratefully acknowledge the financial support of the Italian Ministry of Education, University and Research (MIUR).

References

[1] A. Fissolo, C. Gourdin, Y. Chen, G. Perez, J.M. Stelmaszyk, *Investigations into the cumulative fatigue life of an AISI 304L austenitic stainless steel used for pressure water reactors: Application of a double linear damage rule*, Int. J. Fatigue 77 (2015) 199–215.
[2] R. Tanegashima, I. Ohara, H. Akebono, M. Kato, A. Sugeta, *Cumulative fatigue damage evaluations on spot-welded joints using 590 MPa-class automobile steel*, Fatigue Fract. Engng. Mater. Struct. 38 (2015) 870-879.
[3] C. Braccesi, F. Cianetti, G. Lori, D. Pioli, *Random multiaxial fatigue: A comparative analysis among selected frequency and time domain fatigue evaluation methods*, Int. J. Fatigue 74 (2015) 107–118.
[4] J. Ge, Y. Sun, S. Zhou, L. Zhang, Y. Zhang, Q. Zhang, *A hybrid frequency-time domain method for predicting multiaxial fatigue life of 7075-T6 aluminium alloy under random loading*, Fatigue Fract. Engng. Mater. Struct. 38 (2015) 247-256.
[5] A. Altamura, D. Straub, *Reliability assessment of high cycle fatigue under variable amplitude loading: Review and solutions*, Engng Fract. Mech. 121-122 (2014) 40-66.
[6] D. Benasciutti, *Some analytical expressions to measure the accuracy of the "equivalent von Mises stress" in vibration multiaxial fatigue*, Sound and Vibration 333 (2014) 4326-4340.
[7] E. Habtour, W. Connon, M.F. Pohland, S.C. Stanton, M. Paulus, A. Dasgupta, *Review of response and damage of linear and nonlinear systems under multiaxial vibration*, Shock and Vibration 2014 (2014), Article ID 294271, 21 pages, http://dx.doi.org/10.1155/2014/294271.
[8] Z. Marciniak, D. Rozumek, E. Macha, *Verification of fatigue critical plane position according to variance and damage accumulation methods under multiaxial loading*, Int. J. Fatigue 58 (2014) 84–93.
[9] D. Benasciutti, A. Cristofori, R. Tovo, *Analogies between spectral methods and multiaxial criteria in fatigue damage evaluation*, Probabilistic Engineering Mechanics 31 (2013) 39-45.
[10] M. Paulus, A. Dasgupta, E. Habtour, *Life estimation model of a cantilevered beam subjected to complex random vibration*, Fatigue Fract. Engng. Mater. Struct. 35 (2012) 1058-1070.

[11] N.E. Dowling, *Fatigue failure prediction for complicated stress-strain histories,* J. Mater. 7 (1972) 71–87.
[12] T. Lagoda, E. Macha, *Estimated end experimental fatigue life of 30CrNiMo8 steel under in- end out-of-phase combined bending and torsion with variable amplitudes,* Fatigue Fract. Engng. Mater. Struct. 17 (1994) 1307–1318.
[13] G. Petrucci, B. Zuccarello, *Fatigue life prediction under wide band random loading,* Fatigue Fract. Eng. Mater. Struct. 27 (2004) 1183–1195.
[14] T. Łagoda, E. Macha, A. Niesłony, *Fatigue life calculation by means of the cycle counting and spectral methods under multiaxial random loading*, Fatigue Fract. Engng. Mater. Struct. 28 (2005) 409–420.
[15] X. Pitoiset, A. Preumont, *Spectral Methods for Multiaxial Random Fatigue Analysis of Metallic Structures*, Int. J. Fatigue 22 2000 541-550.
[16] D. Benasciutti, R. Tovo, Comparison of spectral methods for fatigue analysis of broad-band Gaussian random processes, Probabilistic Engineering Mechanics 21 (2006) 287-299.
[17] A. Niesłony, E. Macha, *Spectral Method in Multiaxial Random Fatigue*, Springer, Berlin, Germany, 2007.
[18] K. Bel Knani, D. Benasciutti, A. Signorini, R. Tovo, *Fatigue Damage Assessment of a Car Body-In-White Using a Frequency-Domain Approach*, Int. J. Mater. Prod. Technol. 30 (2007) 172-198.
[19] D. Benasciutti, A. Cristofori, *A Frequency-Domain Formulation of MCE Method for Multi-Axial Random Loadings*, Fatigue Fract. Eng. Mater. Struct. 31 (2008) 937-948.
[20] L. Susmel, R. Tovo, D. Benasciutti, *A Novel Engineering Method Based on the Critical Plane Concept to Estimate the Lifetime of Weldments Subjected to Variable Amplitude Multiaxial Fatigue Loading*, Int. J. Fatigue 32 (2009) 441-459.
[21] D. Benasciutti F. Sherratt, A. Cristofori, *Basic principles of spectral multi-axial fatigue analysis*, Procedia Engineering 101 (2015) 34-42.
[22] N. Nguyen, M. Bacher-Höchst, C.M. Sonsino, *A frequency domain approach for estimating multiaxial random fatigue life*, Materialwissenschaft und Werkstofftechnik 42 (2011) 904–908.
[23] A. Cristofori, D. Benasciutti, R. Tovo, A stress invariant based spectral method to estimate fatigue life under multiaxial random loading, Int. J. Fatigue 33 (2011) 887–899.
[24] A. Cristofori, D. Benasciutti, "Projection-by-Projection" approach: a spectral method for multiaxial random fatigue, SAE Technical Paper 2014-01-0924 (2014) doi:10.4271/2014-01-0924.
[25] A. Carpinteri, A. Spagnoli, S. Vantadori. *Reformulation in the frequency domain of a critical plane-based multiaxial fatigue criterion,* Int. J. Fatigue 67 (2014) 55-61.
[26] A. Carpinteri, A. Spagnoli, C. Ronchei, D. Scorza, S. Vantadori, *Critical plane criterion for fatigue life calculation: time and frequency domain formulations,* Procedia Engineering 101 (2015) 518-523.
[27] A. Carpinteri, A. Spagnoli, C. Ronchei, S. Vantadori, *Time and frequency domain models for multiaxial fatigue life estimation under random loading*, Frattura ed Integrità Strutturale 9 (2015) 376-381.

[28] A. Carpinteri, A. Spagnoli, S. Vantadori, *A multiaxial fatigue criterion for random loading*, Fatigue Fract. Eng. Mater. Struct. 26 (2003) 515-522.
[29] A. Carpinteri, A. Spagnoli, S. Vantadori, *Fatigue life estimation under multiaxial random loading using a critical plane-based criterion.* In: Proceedings of the 2nd International Conference on Material and Component Performance under Variable Amplitude Loading, 2009, Vol. 1, pp. 475-484, Darmstadt, Germany.
[30] A. Carpinteri, A. Spagnoli, S. Vantadori, C. Bagni, *Structural integrity assessment of metallic components under multiaxial fatigue: the C-S criterion and its evolution,* Fatigue Fract. Eng. Mater. Struct. 36 (2013) 870-883.
[31] Walat K., Łagoda T., *Lifetime of semi-ductile materials through the critical plane approach,* Int. J. Fatigue 67 (2013) 73-77.
[32] M. Kurek, T. Łagoda, S. Vantadori, *Estimation of fatigue life of selected construction materials under cyclic loading,* Frattura ed Integrità Strutturale 9 (2015) 302-308.
[33] C. Ronchei, A. Carpinteri, G. Fortese, A. Spagnoli, S. Vantadori, M. Kurek, T. Łagoda, *Life estimation by varying the critical plane orientation in the modified Carpinteri-Spagnoli criterion,* Frattura ed Integrità Strutturale 9 (2015) 74-79.
[34] Z. Marciniak, D. Rozumek, E. Macha, *Fatigue lives of 18G2A and 10HNAP steels under variable amplitude and random non-proportional bending with torsion loading,* Int J Fatigue 30 (2008) 800-813.
[35] A.G. Davenport, *Note on the distribution of the largest value of a random function with application to gust loading*, Proc Inst Civil Eng 28 (1964) 187-196.
[36] R.J. Anthes, *Modified rainflow counting keeping the load sequence*, Int. J. Fatigue 19 (1997) 529-535.