A fuzzy lot-sizing problem with two-stage composite human learning

A fuzzy lot-sizing problem with two-stage composite human learning

Abstract

Due to the repetitive nature of inventory planning over the planning horizon, the operator in charge has to perform planning tasks repetitively, and consequently s/he becomes more familiar with the tasks over time. Familiarity with the tasks suggests that learning takes place in inventory planning. Even though the operator's learning over time might improve his/her efficiency, prior research on fuzzy lot-sizing problems mostly overlooked the effect of human learning in their models and its impact on the operator's performance. To close the research gap in this area, this paper models the operator learning in a fuzzy economic order quantity (EOQ) model with backorders. The paper models a situation where the operator applies the acquired knowledge over the cycles in setting the fuzzy parameters at the beginning of every planning cycle, where his/her learning ability includes the cognitive and motor capabilities of a human being. Subsequently, a mathematical model which takes account of two-stage human learning over the planning cycles is developed, which is then analytically investigated using sample data sets. The results indicate that both operator's capabilities, cognitive and motor, affect the efficiency of the fuzzy lot-sizing inventory model, but the influence of the cognitive capability is more profound, which in turn suggests the importance of training programs for the workforces. The results of the sensitivity analysis also draw some managerial insights for the case that some model parameters vary over the planning horizon.

Keywords: Fuzzy EOQ model, backorders, lot-sizing, human learning, cognitive/motor capabilities.

1. Introduction

Since the early decades of the 20th century, when the lot-sizing problem, also known as the economic order/production (EOQ/EPQ) quantity model, was introduced, it has become the central model in the inventory management community, attracting the attention of both academics and practitioners. Although the original lot-sizing models are very popular and easy to use, they have not been immune from criticism given the fact that the basic models have several shortcomings with regard to the real-world applications (Jaber et al., 2008). To overcome these drawbacks and make the models more realistic, variant forms of the basic models have rapidly appeared in the literature, which have included, but are not limited to: imperfect quality products (Salameh and Jaber, 2000; Jaber et al., 2014), perishable products (Taleizadeh et al., 2013; Bottani et al., 2014), joint inventory models (Jaber and Goyal, 2008; Glock, 2011; Glock, 2012b; Cárdenas-Barrón and Sana, 2014; Mudak et al., 2015), human factors (Jaber et al., 2009) and so on.

One of the areas in which the classical models do not fit well with real applications is the case when the information for an inventory system is uncertain, not stable or incomplete over the planning horizon. Nevertheless, owing to today's turbulent business environment, it is a common occurrence in inventory planning that much relevant input data are neither known precisely, nor found easily due to the lack of sufficient information (Soni and Joshi, 2013; Fallahpour et al., 2015). For instance, inventory cost for a product is likely to vary during the planning horizon because of the variable costs like repairing, financial interest, storage costs, and transportation cost (Soni and Joshi, 2013; Lukinskiy et al., 2015). In line with the other extensions, one of the research streams that has been receiving increasingly favorable attention by researchers during recent years is the extension of the classical models to take account of uncertainty (see Guiffrida, 2009). In economics, production and operations management literature, fuzzy set theory is recognized as an appropriate methodology to deal quantitatively with the uncertain decision making problems (Kazemi et al., 2010; Gholizadeh and Shekarian, 2012; Shekarian and Gholizadeh, 2013; Shekarian et al., 2014b). Unlike deterministic inventory models, in which the decision maker tries to assign a unique and constant value to each inventory parameter, or stochastic models that use the randomness and probability concepts to quantify uncertain parameters, fuzzy set theory helps in modelling imprecise data mathematically, which deterministic and stochastic inventory models often fail to deal with (Kazemi et al., 2014; Kazemi et al., 2015a).

Another issue that is identified by many researchers is the human factor. Human factor elements can be defined as the interactions in a human–system relationship (Grosse et al., 2015). Many researchers emphasized that human factors have to be taken into account when planning production and operations management activities (Boudreau et al., 2003; Gino and Pisano, 2008; Lodree et al., 2009; Neumann and Dul, 2010; Neumann and Village, 2012; Grosse and Glock, 2013; Grosse et al., 2015). Similar to other operations management areas, inventory management is usually a human-dependent task in practice and the human, as a decision maker or an operator, plays a pivotal role in activities like collecting, retrieving, analyzing and processing information. On the basis of the role that the human plays in inventory systems, one might suppose that a remarkable number of research articles in fuzzy inventory management have already been published to investigate the effects of human factors on the inventory system. Even though the impact of human factors on production and operations management models is well-documented in the literature (e.g., Jaber, 2006; Anzanello and Fogliatto, 2011; Jaber and Bonney, 2011), a meticulous overview of the works that studied the fuzzy inventory management shows that, surprisingly, just a few papers accounted for human capabilities in their models (see Section 3). As is apparent from the real practice and in the light of the evidence from the production and operations management literature, in an inventory management process where the human repeats the planning over cycles, he becomes more familiar with the planning tasks through time. Here, familiarity with the planning tasks suggests learning. On the other hand, there is permanently an interaction between the individual and inventory planning process. Since human performance changes over time, his performance could subsequently affect the planning outcome. Hence, it follows that the individual characteristics, in particular those which are not stable during the planning horizon (e.g., learning from previous cycles or forgetting the information acquired in the prior cycles), affect the efficiency of the planning and the outcome as a result. Therefore, taking the individual characteristics of a planner into account in an inventory process is indispensable.

A closer look at the literature reveals that the existing literature falls short in considering the cognitive and motor abilities of humans in fuzzy lot-sizing problems. The cognitive and motor abilities of the inventory planning operator, however, are inherent human traits and are of special importance for the efficiency of the fuzzy inventory system. To model and investigate the effect of human learning with cognitive and motor capabilities on a fuzzy lot-sizing problem and contribute to covering the identified research gap, this paper develops a fuzzy lot-sizing problem by modeling the effect of two-stage composite human learning in adjusting fuzzy parameters. The model of this paper is an extension of the model of Björk (2009). The analytical model developed in this paper assists managers or decision makers to take human learning into account when making decisions on fuzzy optimal lot-sizing, particularly in environments where inventory operations are mostly dependent on the human workforce.

The remainder of this paper is structured as follows: the next section briefly reviews the literature related to the fuzzy lot-sizing problem and learning. Section 3 proposes the problem that this paper aims to study, followed by the necessary assumptions required to formulate it along with the notations. In Section 4, the analytical model of the paper is developed. The paper then provides some numerical studies in Section 5, and finally concludes in Section 6.

2. Literature review

As this paper includes two different research streams, the literature review is divided into two different sections, and each section gives an overview of the most important studies in that stream.

The final part of this section reviews the papers in the literature that have consolidated both factors into a single model, and then positions the paper in the literature.

2.1. Fuzzy inventory management

Papers on fuzzy inventory management can be divided into several different classes, among which the main ones are: EOQ models, EPQ models, and joint lot-sizing models. The primary objective of the papers in these categories was to develop the classical models to account for fuzziness, and then to compare the fuzzy model's outcome with that of the classical models. For a review of the application of fuzzy set theory in inventory management, readers are referred to Guiffrida (2009).

Incorporating fuzziness into inventory models was initially addressed in EOQ models in the literature. Park (1987) is believed to be the first researcher who addressed the fuzzy EOQ problem. In this paper, an EOQ model with fuzzy ordering and holding costs was proposed, where the fuzzy parameters were modeled as triangular fuzzy numbers. Another earlier work in this research stream is due to Vujošević et al. (1996), who considered the same parameters in Park (1987) as fuzzy numbers. After these two primary works, variant forms of model evolved in the literature, with a variety of schemes of fuzzy parameters. For instance, the EOQ model with backorders was studied by Björk (2009), Kazemi et al. (2010), and Milenkovic and Bojovic (2014); the EOQ model with imperfect quality was developed by Chang (2004), Mahata and Gowasami (2013) and Sharifi et al. (2015); multi-item EOQ model by Jana et al. (2014), and the EOQ model with deteriorating items was investigated by Mahata and Goswami (2007) and Mahata and Mahata (2011).

Another important and distinctive research stream is the extension of the basic EPQ model using fuzzy set theory. One of the first models dealing with the fuzzy EPQ problem is due to Lee and Yao (1998), who proposed an EPQ model with fuzzy demand and fuzzy production rate. Another treatment of the formulation of the fuzzy EPQ model can be found in Chang (1999), Lee and Yao (1999), Lin and Yao (2000) and Hsieh (2002), who developed a fuzzified version of the classical model assuming that the parameters of the original model are fuzzy numbers. A number of extensions of the EPQ model into the fuzzy case can be tracked in the literature. A few recent works, out of many, may be named: fuzzy EPQ models for defective products developed by Shekarian et al. (2014a) and Mahata (2014); multi-item fuzzy EPQ models offered by Jana et al. (2013) and Mezei and Björk (2015); a fuzzy EPQ model with deteriorating products provided by Pal et al. (2014); a multi-period model offered by De and Sana (2014).

Besides the aforesaid categories, fuzzy set theory was also incorporated into the integrated inventory models, generally referred to as joint economic lot size (JELS) models. JELS models aim at finding the coordination policy among the members of an echelon in the supply chain that benefits each member in the whole chain rather than taking the optimal policy individually (see Ben-Daya et al., 2008; Khouja and Goyal, 2008; Glock, 2012a). Lam and Wong (1996) were the first researchers to address the JELS model; they analyzed the effect of fuzzy single and multiple price discount on a crisp model in the literature. Following the work of Lam and Wong (1996), Das et al. (2004) developed a fuzzy multi-objective model for deteriorating items with price discounts, which was an extension of the model in Yang and Wee (2000). After the primary works on the fuzzy JELS model, a couple of papers appeared that examined the coordination mechanism for supply chain actors when the uncertainty is a part of decision making. Mahata et al. (2005), Mandal and Khan (2014), Kumar et al. (2014), Soni and Patel (2015), Chakraborty et al. (2015) and Sadeghi and Niaki (2015) are just a few works out of many to address this problem.

The models discussed so far just studied one aspect in the fuzzy models, i.e. the fuzziness associated with the inventory models and the uncertainty modelling using fuzzy set theory, but ignored the effect that human characteristics could have on the models and the planning outcomes accordingly. Due to a high proportion of the tasks that an inventory planner is involved in, his capabilities are of major importance for the system's efficiency. Therefore, more studies are needed to contribute to close this research gap and to develop models that present a better picture of reality.

2.2. Learning

Learning is one of the human characteristics that has been studied in the literature extensively. In production and operations management, learning is usually defined as the improvement in performance when a person or an organization is involved in a repetitive task (Jaber, 2006; Jaber and Bonney, 2011). For example, when a worker at a working station performs a task repetitively, s/he could learn over time, causing a decrease in the time required to perform the task. The attempts made to predict and monitor the performance of individuals or groups performing a task resulted in the development of the learning and forgetting curves, which are usually a mathematical relationship between the time and the number of the produced products. The first one who observed the learning phenomenon in an industrial setting was Wright (1936), who found that there existed a relation between the individual task and the unit production cost. The results of Wright's (1936) study ascertained that the unit production cost decreases proportionally to the cumulative number of units produced by a worker, in conformance with a power-form learning curve. Fig. 1 illustrates the learning phenomenon of Wright (1936), along with the forgetting phenomenon if there are interruptions between production cycles.

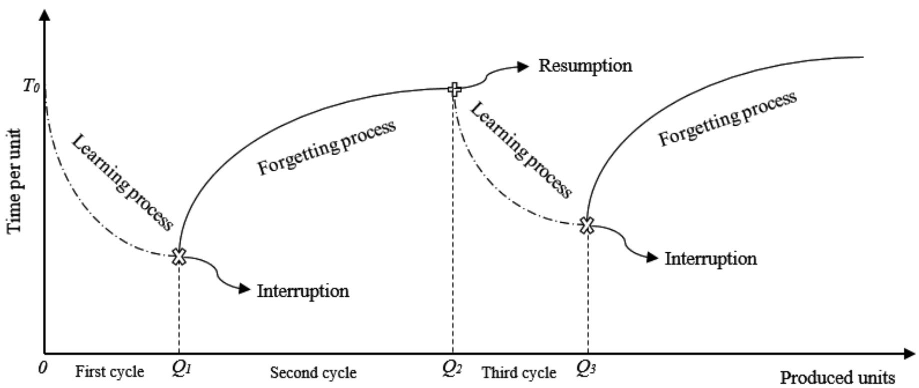

Fig. 1. Illustration of learning and forgetting process

Even though Wright's (1936) model provides a simple learning curve in terms of mathematics and computations and could be fitted with a wide range of data fairly well, it has been criticized because of the fact that it could not be applied in many industrial contexts. This is mostly due to the different specifications of the working conditions. Therefore, the model of Wright (1936) has frequently been extended to account for different learning processes, different workers' characteristics or different tasks the workers are involved in (Dar-El, 2000). Learning and its application in various industrial contexts has been a subject investigated widely by researchers. As a large number of learning models could be tracked in the literature, readers are referred to Jaber (2006) and Anzanello and Fogliatto (2011) for an extensive analysis of the learning models and their applications.

2.3. Synthesis of both research streams

A deep analysis of the literature in fuzzy inventory management and human learning illustrates that both research streams were studied frequently in the past. It is, however, surprising that researchers dealt with both topics mostly independently. As described before, production and operations activities are inherently performed in a manned environment, where human characteristics may influence the decision outcome and the system's efficiency. Therefore, it is worth mentioning here that if human factors are not considered in modelling the production and operations processes, then this could lead to an erroneous or perhaps costly decision making. On the other hand, given the fact that human capabilities are not constant over time, treating them as constant factors in inventory decision making is, undoubtedly, in contrast to real situations, which trivially makes the available models in the literature impractical. The studies by Glock et al. (2012), Kazemi et al. (2015b) and Kazemi et al. (2015c) are the only research papers that addressed the problem of human learning in a fuzzy lot-sizing problem considering human learning as a factor affecting the fuzzy inventory system. Glock et al. (2012) studied the basic fuzzy EOQ model with fuzzy demand and modelled a situation where the decision maker learns with every order. In the second study, Kazemi et al. (2015b) investigated a fuzzy EOQ model with imperfect quality in which the decision maker learns through time with the number of shipments. In the third work, Kazemi et al. (2015c) developed a fuzzy EOQ model with backorders to take account of human learning over the planning horizon. However, none of the reviewed models considered human cognitive and motor abilities in their models. To fill this research gap, this paper models a different learning process into a fuzzy lot-sizing problem to deal with a circumstance where the inventory planning operator (hereafter, 'the operator') learns through time using both his cognitive and motor capabilities.

3. Problem statement

In an inventory planning process where the operator has to operate under uncertainty, he often requires information from previous planning cycles that helps him to shape his knowledge base and estimate the uncertain (fuzzy) inventory parameters more precisely. Therefore, he should devote time to obtain, process and analyze information during the execution of the planning. A portion of the time required to obtain information and build-up knowledge constitutes the first phase of the planning, where the operator prepares the necessary knowledge for the next phase. After the phase where the knowledge is acquired by the operator and the prerequisite knowledge is built up, the planning process starts, which is the second phase of the planning. This phase is referred to as the knowledge retrieval step (see Dar-El, 2000). As stated by Jaber and Glock (2013), learning may occur in the two steps, but the learning of the operator in both steps of the planning

Commentato [e.b.1]: Check the spelling. The same word is written with the hyphenation in the previous row (build-up). I also saw a different spelling in another part of the paper.

is different. After the operator becomes familiar with the planning information by doing it several times, less time is required for looking up the information, and the planning steps could be performed faster. In the learning literature, the first stage is called “cognitive” learning, whilst the second one is termed as “motor” learning. It is obvious that human learning with cognitive and motor capabilities has a broad application in various fields of inventory management. In order to show the application of this phenomenon, we apply this concept to a fuzzy lot-sizing problem, following the EOQ policy.

3.1. Definition

The following notations will be used throughout the paper.

D	demand during planning period
M	maximum inventory level
Q	order quantity
r	reorder point
L	lead time
C_o	fixed ordering cost
C_h	holding cost per unit per planning period
C_p	shortage penalty cost per unit per planning period
n	number of orders per planning period
$\Delta_{\boldsymbol{l}}^{\boldsymbol{D}}$	lower fuzzy deviation value for demand
$\Delta_{\boldsymbol{u}}^{\boldsymbol{D}}$	upper fuzzy deviation value for demand
$\Delta_{\boldsymbol{l}}^{M}$	lower fuzzy deviation value for maximum inventory
$\Delta_{\boldsymbol{u}}^{M}$	upper fuzzy deviation value for maximum inventory
P_t	performance in tth unit of the time
P_1	performance at the beginning of the planning period
α	the weight representing the percentage that P_1 could be divided into cognitive and motor components
l_c	learning exponent for cognitive task
l_m	learning exponent for motor task
i	order counter
T	planning period per unit time
$\Delta_{l,i}^{j}$	the lower fuzzy deviation value of ith order for $j = D, M$ in the planning horizon
$\Delta_{u,i}^{j}$	the upper fuzzy deviation value of ith order for $j = D, M$ in the planning horizon
$\Delta_{\boldsymbol{l},\boldsymbol{1}}^{\boldsymbol{j}}$	the first lower fuzzy deviation value for $j = D, M$ in the planning horizon
$\Delta_{\boldsymbol{u},\boldsymbol{1}}^{\boldsymbol{j}}$	the first upper fuzzy deviation value for $j = D, M$ in the planning horizon
TCU_C	crisp total cost function
TCU_F	fuzzy total cost function without learning
TCU_{FL_i}	fuzzy total cost function with learning for ith order
TCU_{FL}	fuzzy total cost function with learning for n orders

3.2. Assumptions

To investigate the effect of two-stage composite learning with cognitive and motor elements on the optimal policy of a fuzzy inventory model, a fuzzy EOQ model with backorders (termed as EOQ-S) constitutes the basic model of this paper. In this paper, similar to Björk (2009), we assume that demand and lead times are uncertain parameters over the planning cycles, but it is possible to describe them with fuzzy numbers. It is necessary to note that the impact of the uncertain lead time could not be instantly found in the formulation; however, due to the equation $M = r - LD + Q$, it affects the maximum inventory level in the formulation of the problem (see Björk, 2009). The problem of uncertain demand and lead times is a common occurrence in real-world inventory management, as these two parameters are usually very difficult to estimate precisely. Here, it is assumed that the only available information at the initial stage of the inventory planning is the lead time for the first cycle and the size of the first order, yet the total demand and lead times over the planning horizon are not known precisely. In this situation, the operator prefers to specify tolerance values for the uncertain parameters to reflect the range over which the parameters are most likely to be. As more orders are issued over the planning horizon, the operator could acquire more information and build up knowledge about the demand and lead times, and could analyze the variation of the quantities to achieve a better estimate of the parameters. In addition to the assumption made before, the following assumptions are made hereafter:

1- The operator learns with every order over the planning horizon, and constitutes his/her knowledge base by analyzing information gained from the earlier planning cycles;
2- The operator uses the acquired knowledge at the beginning of every planning cycle to set the relevant upper and lower specified values for fuzzy parameters;
3- The knowledge build-up helps the operator improve the performance over time, which leads to decreased uncertainty;
4- The upper and lower values of the demand and maximum inventory level change according to the same learning curve;
5- The learning rates are identical for all parameters;
6- The learning is fully transferred within the cycles; that is, the inventory planners do not forget the information gained from the earlier cycles;
7- The learning in every cycle occurs in two phases as described in Section 3.

4. Model development

Björk (2009) developed a fuzzy EOQ model by assuming that maximum inventory levels and demand can be described using triangular fuzzy numbers as $\widetilde{M} = (M - \Delta_l^M, M, M + \Delta_u^M)$ and $D = (D - \Delta_l^D, D, D + \Delta_u^D)$. The reader is referred to Björk (2009) for a detailed description of the model and its properties. According to this author, the expected total cost function of an EOQ-S model with fuzzy demand and lead times is given by:

$$TCU_F(Q,\widetilde{M}) = \frac{C_o D}{Q} + \frac{C_o \Delta_h^D}{4Q} - \frac{C_o \Delta_l^D}{4Q} + \frac{M^2 C_h}{2Q} + \frac{\Delta_l^{M^2} C_h}{12Q} + \frac{\Delta_u^{M^2} C_h}{12Q} + \frac{QC_p}{2} + \frac{M^2 C_p}{2Q}$$
$$+ \frac{\Delta_l^{M^2} C_p}{12Q} + \frac{\Delta_u^{M^2} C_p}{12Q} + \frac{M\Delta_u^M C_h}{4Q} - \frac{M\Delta_l^M C_h}{4Q} + \frac{M\Delta_u^M C_p}{4Q} - \frac{M\Delta_l^M C_p}{4Q} + \frac{\Delta_l^M C_p}{4} - \frac{\Delta_u^M C_p}{4} - C_p M \qquad (1)$$

Since we assumed that that the operator learns with every order, the relevant decision variable in the model should be the number of orders, but not the order quantity. Therefore, Eq. (1) can be modified to account for the number of orders as a decision variable. By replacing $n = \frac{D}{Q}$ in Eq. (1) and after rearrangement, the total cost function will be given by:

$$TCU_F(n,\widetilde{M}) = nC_o - C_p M + \frac{DC_p}{2n} + \frac{nM^2 C_h}{2D} + \frac{nM^2 C_p}{2D} + \frac{nC_o \Delta_u^D}{4D} - \frac{nC_o \Delta_l^D}{4D} + \frac{n\Delta_l^{M^2} C_h}{12D}$$
$$+ \frac{n\Delta_u^{M^2} C_h}{12D} + \frac{n\Delta_l^{M^2} C_p}{12D} + \frac{n\Delta_u^{M^2} C_p}{12D} + \frac{nM\Delta_u^M C_h}{4D} - \frac{nM\Delta_l^M C_h}{4D} + \frac{nM\Delta_u^M C_p}{4D} - \frac{nM\Delta_l^M C_p}{4D} + \frac{\Delta_l^M C_p}{4} - \frac{\Delta_u^M C_p}{4} \qquad (2)$$

4.1. Learning effect on fuzzy parameters

As stated before, this paper assumes that a two-stage composite learning with full transfer of learning between cycles occurs in fuzzy inventory planning, which influences the fuzzy parameters. As the learning affects the Δ values, their values change over the planning period in conformance with a learning curve. To define cognitive and motor capabilities of the operator, this paper applies a learning curve developed by Jaber and Glock (2013), which is an extension form of the dual learning curve proposed by Dar-el et al. (1995). This learning curve was proved to capture the cognitive and motor characteristics of humans properly and fitted empirical data very well. The learning curve with the cognitive and motor capabilities developed by Jaber and Glock (2013), adapted to the problem defined in this paper, is of the form:

$$P_t = \alpha P_1 t^{-l_c} + (1-\alpha) P_1 t^{-l_m} = P_1[\alpha(t^{-l_c} - t^{-l_m}) + t^{-l_m}], \quad (3)$$

where P_t is the performance in the tth unit of the time, P_1 is the performance in the first unit of the time (i.e., the beginning of the planning period in the problem defined), α is the percentage (weight) applied to P_1 to divide it into the cognitive and motor components, and l_c and l_m represent the learning exponents for the cognitive and motor tasks, respectively. If full transfer of learning occurs between the cycles, considering that the EOQ-S model follows an equal lot-size replenishment policy in which a lot of size Q is received in equal intervals over the planning horizon, the jth Δ value at the time of ith order will be as follows:

$$\Delta_{l,i}^{j} = \begin{cases} \Delta_{l,1}^{j} & i = 1 \\ \alpha \Delta_{l,1}^{j}[(i-1)\frac{T}{n}]^{-l_c} + (1-\alpha)\Delta_{l,1}^{j}[(i-1)\frac{T}{n}]^{-l_m} & \\ = \Delta_{l,1}^{j}\{\alpha\{[(i-1)\frac{T}{n}]^{-l_c} - [(i-1)\frac{T}{n}]^{-l_m}\} + [(i-1)\frac{T}{n}]^{-l_m}\} & i > 1 \end{cases} \quad (4)$$

$$\Delta_{u,i}^{j} = \begin{cases} \Delta_{u,1}^{j} & i = 1 \\ \alpha \Delta_{u,1}^{j}[(i-1)\frac{T}{n}]^{-l_c} + (1-\alpha)\Delta_{u,1}^{j}[(i-1)\frac{T}{n}]^{-l_m} & \\ = \Delta_{u,1}^{j}\{\alpha\{[(i-1)\frac{T}{n}]^{-l_c} - [(i-1)\frac{T}{n}]^{-l_m}\} + [(i-1)\frac{T}{n}]^{-l_m}\} & i > 1 \end{cases} \quad (5)$$

In Eqs. (4) and (5), the first term in the summations represents the cognitive component of the planning task, while the second expression indicates the motor component. α represents a mechanism under which planning could be broken down into two elements. Due to the natural capability of humans, who tend to recall the process faster with every repetition, the cognitive component in Eqs. (4) and (5) reduces at a faster rate compared to the motor element (Jaber and Glock, 2013). At the beginning of the planning horizon, when the first cycle starts, the Δ values adopt their largest possible limit over the planning horizon, which are set by the planner. As the process continues with every repetition, caused by order shipments, the Δ values diminish according to the learning curve, with the effect of both cognitive and motor characteristics of the operator.

4.2. Incorporating human learning into fuzzy EOQ-S model

In this subsection, the total cost function for the EOQ-S model with the effect of two-stage composite human learning is derived. To do so, first, the cost of every planning cycle is computed, and then the total cost of the inventory system over the planning horizon is determined by integrating the total costs over n cycles. Considering the definition of the Δ values in Eqs. (4) and (5), the cost function of the first cycle will be as follows:

$$TCU_{FL_1}(n,\widetilde{M}) =$$
$$C_o + \frac{DC_p}{2n^2} - \frac{MC_p}{n} + \frac{M^2C_h}{2D} + \frac{M^2C_p}{2D} + \frac{C_o\Delta_{u,1}^{D}}{4D} - \frac{C_o\Delta_{l,1}^{D}}{4D} + \frac{\Delta_{l,1}^{M^2}C_h}{12D} + \frac{\Delta_{u,1}^{M^2}C_h}{12D} + \frac{\Delta_{l,1}^{M^2}C_p}{12D} + \frac{\Delta_{u,1}^{M^2}C_p}{12D}$$
$$+ \frac{M\Delta_{u,1}^{M}C_h}{4D} - \frac{M\Delta_{l,1}^{M}C_h}{4D} + \frac{M\Delta_{u,1}^{M}C_p}{4D} - \frac{M\Delta_{l,1}^{M}C_p}{4D} + \frac{\Delta_{l,1}^{M}C_p}{4n} - \frac{\Delta_{u,1}^{M}C_p}{4n} \tag{6}$$

Following the definitions proposed in Eqs. (4) and (5), the cost function of the inventory system for ith cycle, $i \in [2,n]$, would be as follows:

$$TCU_{FL_i}(n,\widetilde{M}) = C_o + \frac{DC_p}{2n^2} - \frac{MC_p}{n} + \frac{M^2C_h}{2D} + \frac{M^2C_p}{2D}$$
$$+\frac{C_o}{4D}[(\alpha\Delta_{u,1}^{D}((i-1)\frac{T}{n})^{-l_c} + (1-\alpha)\Delta_{u,1}^{D}((i-1)\frac{T}{n})^{-l_m}) - (\alpha\Delta_{l,1}^{D}((i-1)\frac{T}{n})^{-l_c}) - (1-\alpha)\Delta_{l,1}^{D}((i-1)\frac{T}{n})^{-l_m})]$$
$$+\frac{C_h}{12D}[(\alpha\Delta_{u,1}^{M}((i-1)\frac{T}{n})^{-l_c} + (1-\alpha)\Delta_{u,1}^{M}((i-1)\frac{T}{n})^{-l_m})^2 + (\alpha\Delta_{l,1}^{M}((i-1)\frac{T}{n})^{-l_c} + (1-\alpha)\Delta_{l,1}^{M}((i-1)\frac{T}{n})^{-l_m})^2]$$
$$+\frac{C_p}{12D}[(\alpha\Delta_{u,1}^{M}((i-1)\frac{T}{n})^{-l_c} + (1-\alpha)\Delta_{u,1}^{M}((i-1)\frac{T}{n})^{-l_m})^2 + (\alpha\Delta_{l,1}^{M}((i-1)\frac{T}{n})^{-l_c} + (1-\alpha)\Delta_{l,1}^{M}((i-1)\frac{T}{n})^{-l_m})^2]$$
$$+\frac{MC_h}{4D}[(\alpha\Delta_{u,1}^{M}((i-1)\frac{T}{n})^{-l_c} + (1-\alpha)\Delta_{u,1}^{M}((i-1)\frac{T}{n})^{-l_m}) - (\alpha\Delta_{l,1}^{M}((i-1)\frac{T}{n})^{-l_c}) - (1-\alpha)\Delta_{l,1}^{M}((i-1)\frac{T}{n})^{-l_m})]$$
$$+\frac{MC_p}{4D}[(\alpha\Delta_{u,1}^{M}((i-1)\frac{T}{n})^{-l_c} + (1-\alpha)\Delta_{u,1}^{M}((i-1)\frac{T}{n})^{-l_m}) - (\alpha\Delta_{l,1}^{M}((i-1)\frac{T}{n})^{-l_c} - (1-\alpha)\Delta_{l,1}^{M}((i-1)\frac{T}{n})^{-l_m})]$$
$$+\frac{C_p}{4n}[(\alpha\Delta_{l,1}^{M}((i-1)\frac{T}{n})^{-l_c} + (1-\alpha)\Delta_{l,1}^{M}((i-1)\frac{T}{n})^{-l_m}) - (\alpha\Delta_{u,1}^{M}((i-1)\frac{T}{n})^{-l_c} - (1-\alpha)\Delta_{u,1}^{M}((i-1)\frac{T}{n})^{-l_m})]$$
$$\tag{7}$$

The total cost function of the inventory system is determined by summing Eqs. (6) and (7) over the entire n cycles, resulting in the following expression:

$$TCU_{FL}(n,\widetilde{M}) = \sum_{i=1}^{n} TCU_{FL_i}(n,\widetilde{M}) = TCU_{FL_1}(n,\widetilde{M}) + \sum_{i=2}^{n} TCU_{FL_i}(n,\widetilde{M}) = nC_o + \frac{DC_p}{2n} - C_pM + \frac{nM^2C_h}{2D} + \frac{nM^2C_p}{2D} +$$
$$+\frac{C_o}{4D}\Delta_{u,1}^{D}\left(\left[1+\alpha\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_c}\right] + \left[1+(1-\alpha)\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_m}\right]\right)$$

$$-\frac{C_o}{4D}\Delta_{l,1}^{D}\left(\left[1+\alpha\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_m}\right]\right)$$

$$+\frac{C_h}{12D}{\Delta_{u,1}^{M}}^2\left(\left[1+\alpha^2\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-2l_c}\right]+\left[1+(1-\alpha)^2\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-2l_m}\right]+\left[1+2\alpha(1-\alpha)\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_c-l_m}\right]\right)$$

$$+\frac{C_h}{12D}{\Delta_{l,1}^{M}}^2\left(\left[1+\alpha^2\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-2l_c}\right]+\left[1+(1-\alpha)^2\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-2l_m}\right]+\left[1+2\alpha(1-\alpha)\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_c-l_m}\right]\right)$$

$$+\frac{C_p}{12D}{\Delta_{u,1}^{M}}^2\left(\left[1+\alpha^2\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-2l_c}\right]+\left[1+(1-\alpha)^2\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-2l_m}\right]+\left[1+2\alpha(1-\alpha)\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_c-l_m}\right]\right)$$

$$+\frac{C_h}{12D}{\Delta_{l,1}^{M}}^2\left(\left[1+\alpha^2\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-2l_c}\right]+\left[1+(1-\alpha)^2\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-2l_m}\right]+\left[1+2\alpha(1-\alpha)\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_c-l_m}\right]\right)$$

$$+\frac{MC_h}{4D}\Delta_{u,1}^{M}\left(\left[1+\alpha\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_m}\right]\right)$$

$$-\frac{MC_h}{4D}\Delta_{l,1}^{M}\left(\left[1+\alpha\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_m}\right]\right)$$

$$+\frac{MC_p}{4D}\Delta_{u,1}^{M}\left(\left[1+\alpha\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_m}\right]\right)$$

$$-\frac{MC_p}{4D}\Delta_{l,1}^{M}\left(\left[1+\alpha\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_m}\right]\right)$$

$$+\frac{C_p}{4n}\Delta_{l,1}^{M}\left(\left[1+\alpha\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_m}\right]\right)$$

$$-\frac{C_p}{4n}\Delta_{u,1}^{M}\left(\left[1+\alpha\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_m}\right]\right) \tag{8}$$

After rearrangement, Eq. (8) reduces to the following:

$$TCU_{FL}(n,\widetilde{M})=nC_o+\frac{DC_p}{2n}-C_pM+\frac{nM^2C_h}{2D}+\frac{nM^2C_p}{2D}+$$

$$\left(\left[1+\alpha\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_m}\right]\right)(\frac{C_o}{4D}\Delta_{u,1}^{D}-\frac{C_o}{4D}\Delta_{l,1}^{D}+\frac{MC_h}{4D}\Delta_{u,1}^{M}-\frac{MC_h}{4D}\Delta_{l,1}^{M}$$

$$+\frac{MC_p}{4D}\Delta_{u,1}^{M}-\frac{MC_p}{4D}\Delta_{l,1}^{M}+\frac{C_p}{4n}\Delta_{l,1}^{M}-\frac{C_p}{4n}\Delta_{u,1}^{M})$$

$$+\left(\left[1+\alpha^2\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-2l_c}\right]+\left[1+(1-\alpha)^2\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-2l_m}\right]+\left[1+2\alpha(1-\alpha)\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_c-l_m}\right]\right)(\frac{C_h}{12D}{\Delta_{u,1}^{M}}^2$$

$$+\frac{C_h}{12D}{\Delta_{l,1}^{M}}^2+\frac{C_p}{12D}{\Delta_{u,1}^{M}}^2+\frac{C_p}{12D}{\Delta_{l,1}^{M}}^2)$$

(9)

In Eq. (9), one variable, n, appears in the upper limit of the series, and thus conducting the convexity test using a partial derivative with respect to this variable is not an easy task. However, using the second variable, M, helps us to check the convexity of the total cost function. Before examining the convexity condition, the following Lemma helps in analyzing the problem.

Lemma 1. Finding a global optimal solution for $TCU_{FL}(n,\widetilde{M})$ can be reduced to search for finding a local optimal solution for n.

To prove Lemma 1, it is required to show that $TCU_{FL}(n,\widetilde{M})$ is convex in M. For a given value of n, the first and second order partial derivatives of Eq. (9) with respect to M are given by:

$$\frac{\partial TCU_{FL}(n,\widetilde{M})}{\partial M}=-C_p+\frac{nMC_h}{D}+\frac{nMC_p}{D}$$
$$+\left(\left[1+\alpha\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_m}\right]\right)(\frac{C_h}{4D}\Delta_{u,1}^{M}-\frac{C_h}{4D}\Delta_{l,1}^{M}+\frac{C_p}{4D}\Delta_{u,1}^{M}-\frac{C_p}{4D}\Delta_{l,1}^{M}) \quad (10)$$

and

$$\frac{\partial^2 TCU_{FL}(n,\widetilde{M})}{\partial^2 M}=\frac{nC_h}{D}+\frac{nC_p}{D} \quad (11)$$

Eq. (11) is obviously greater than zero. This implies that $TCU_{FL}(n,\widetilde{M})$ is convex in M, and that for a fixed value of n, there exists a unique M that minimizes $TCU_{FL}(n,\widetilde{M})$. For a given value of n, the optimal solution for M is derived by setting Eq. (10) to zero, giving the following result:

$$M^*=$$
$$\frac{pD-\left(\left[1+\alpha\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l_m}\right]\right)\left[\frac{h}{4}(\Delta_{h,1}^{M}-\Delta_{l,1}^{M})+\frac{p}{4}(\Delta_{h,1}^{M}-\Delta_{l,1}^{M})\right]}{n(h+p)} \quad (12)$$

According to Lemma 1 and given the fact that n adopts only integer values, an optimal solution for $TCU_{FL}(n,M)$ could be obtained by applying a linear search for a unique n over the interval $[2,\infty)$ such that the inequality $TCU_{FL}(n^*-1,M)\geq TCU_{FL}(n^*,M)\leq TCU_{FL}(n^*+1,M)$ is satisfied. Although the aforementioned steps taken in finding the optimal solution for the problem developed are straightforward, we prefer to provide the reader with the step-by-step algorithm that aids in better understanding of the solution procedure. The solution procedure can be summarized in the following algorithm.

Step 1. Start the algorithm by setting $n = 2$.

Step 2. Compute $M_{(n)}$ as in Eq. (12).

Step 3. Compute $TCU_{FL}(n, M_{(n)})$ in Eq. (9).

Step 4. *If* $TCU_{FL}(n, M_{(n)}) < TCU_{FL}(n-1, M_{(n)})$ then set $n = n + 1$ and go to step 2, otherwise go to the next step.

Step 5. $n^* = n - 1$ and $M^* = M_{(n-1)}$ and compute $TCU^*{}_{FL}(n^*, M^*)$.

The algorithm's loop examines the successive values of n over the interval $[2,\infty)$ to identify for which n the total cost function changes from descending to ascending. This is tested by step 4 in the proposed algorithm. As soon as the total cost function starts to go up for a specific value of n, this value will be recorded as the optimal one, and then the optimal solution will be derived by substituting the optimal value of n in M and TCU.

5. Numerical examples

To evaluate the effect of cognitive and motor learning on the fuzzy EOQ-S model, a numerical analysis is conducted whereby the model developed in section 4 is investigated using different rates of learning and varying weights for both cognitive and motor components. Consider a fuzzy inventory management process for a company that heavily depends on the workforce for inventory-related operations. The person in charge should refer to the knowledge of how accurately to perform the operations (cognitive process), especially when the cycle starts, and to the procedure of how to use the acquired knowledge for completing the operations (motor process). In our paper, for the case of fuzzy data, where the deviation parameters should be determined for the initial cycle, it is assumed that these parameters could be principally taken from the history of the inventory operations. In the analysis given henceforward, except for the values of learning exponent and weights of the cognitive and motor components, the other relevant data are obtained from Björk (2009). Fig.2 represents the behavior of the total cost function described by Eq. (9) for four different learning rates and different weights for cognitive tasks (and as a result motor tasks), which are varied over the interval [0, 1]. As it was empirically observed by Dar-el et al. (1995)

and emphasized later by some works such as Jaber and Kher (2002) and Jaber and Glock (2013), it is common in the real cases that, with every repetition, humans call up the process or steps learnt before at a faster rate. Thus, it is logical that the learning rate for the cognitive part of a task is higher than that of the motor part. For this purpose, the assumption $l_c > l_m$ is made throughout the numerical analysis. The pattern of the total cost in Fig.2 shows that, irrespective of the learning rate, as the task becomes more cognitive, the system's performance in terms of the total cost improves. Nevertheless, the total inventory cost showed more profound change by faster learning in the cognitive part rather than faster learning in the motor part. According to the result, bringing in workforce learning ability into consideration in fuzzy lot-sizing gives clear managerial and operational implications highlighting the importance that training programs for the workforce have on the total cost of the inventory system. For instance, if the company's policy is to give higher priority on reducing total inventory cost, then more investment in workforce training programs has reasonable justification. If this is the issue, it is then indispensable to make a trade-off between the amount of the investment the company is willing to spend on the training and the reduction in the total inventory cost that would be achieved. This, in turn, could help the company to prioritize worker's training programs. Another impression from Fig.2 is the change observed by faster learning. That is, the inventory system benefits from faster learning in either cognitive or motor form as the total inventory cost decreases in both cases. However, the reduction caused by faster learning in the motor part of the task is much more significant than the one resulted from the cognitive part. The effect of human learning on reducing the total cost of the inventory in deterministic models has already been reported in the literature (see e.g. Jaber et al., 2008; Zanoni et al., 2012; Grosse and Glock, 2013). Our finding in this paper is in line with the literature on deterministic models and shows that for both deterministic and fuzzy lot-sizing problems, personal training for the purpose of fostering learning among workforces could be a useful tool for managers, who wish to decrease the total cost of the system.

To gain further insights into the optimal policy of the model, we address the question of how the two-stage learning discussed in section 4.1 affects the maximum inventory level. Fig. 3 depicts the variation of the maximum inventory by varying the amount of α stepwise over an interval from 0 to 1, with the learning rates set the same as in Fig. 2. Similar to the total annual inventory cost, which is observed to be lower when the operation is more cognitive, the same could be perceived for the maximum inventory level. The finding implies that the higher the cognitive task, the more the maximum inventory could be reduced. Moreover, through faster learning, obtained by an increasing learning exponent from 0.514 to 0.814 for l_c and from 0.152 to 0.452 for l_m, the inventory system tends to hold less inventory. Nevertheless, the impact of learning on the cognitive and motor elements is found to be unequal. Strictly speaking, while faster learning leads to a slight change in the maximum stock level when the cognitive part of the task is considered, this phenomenon results in a dramatic drop in the maximum stock level when the motor part of the task is taken into consideration (blue line vs. green, red line vs. purple).

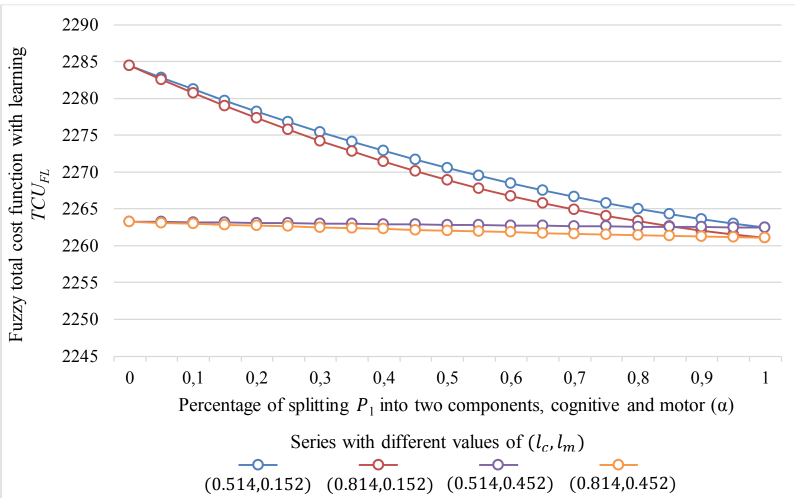

Fig. 2. Comparing the effect of different values of weigh (α), and learning exponents of cognitive and motors on the total cost function (TCU_{FL})

To further illustrate the effect of human learning on the fuzzy lot-sizing model, a numerical analysis with different sets of data is conducted, with the results provided in Table 1. The sets of data applied here are: $D = 200{,}000$ units/year with lower and upper deviation of 7,200 units/year and 8,400 units/year, respectively; C_o= 250 \$/order; C_h= 1 \$/unit/year; $C_p = 4$ \$/unit/year. In addition, the results are computed for two different groups of data, where the first group assumes the task to be more cognitive ($\alpha = 0.8$) and the second group considers the task to be more motor ($\alpha = 0.2$). Looking at Table 1 it should be noted that introducing two-stage human learning into the fuzzy lot-sizing model brings in a noticeable cost saving for the inventory system compared to the base fuzzy model (i.e., the model of Björk, 2009). Furthermore, the inventory system can benefit more from the learning if the planning task becomes more cognitive, which confirms what we obtained with the previous sets of data. Another important finding is that the optimal policy when learning in planning is considered is to decrease the frequency of orders, which, consequently, leads to increasing the lot-size. The result also shows that the inventory system tends to hold less inventory over the planning horizon when learning occurs.

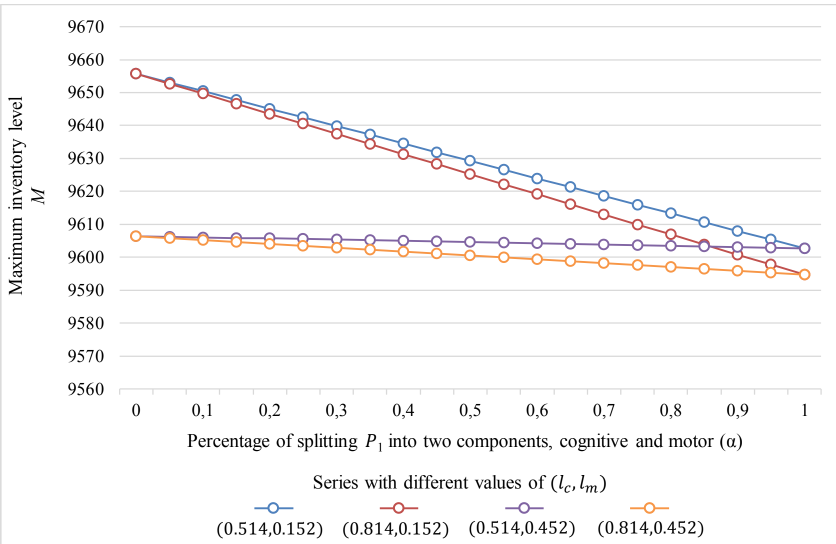

Fig. 3. Comparing the effect of different values of weigh (α), and learning exponents of cognitive and motors on the maximum inventory level (M)

6. Sensitivity analysis

To further analytically investigate the performance of the model presented in section 4.2, a sensitivity analysis was performed, with the results summarized in Tables 2 and 3. To better compare the behavior of the model, we numerically studied the model for two scenarios. In the first one, the motor part of the learning is dominant ($\alpha = 0.20$), whereas in the second case the learning is much more cognitive ($\alpha = 0.8$). It should be noted that the learning curve in all the analysis presented below is assumed to be $(l_c, l_m) = (0.514, 0.152)$, consistent with the learning exponent observed by (Dar-el et al., 1995). Looking at Tables 2 and 3, it can be noticed that the model is fairly sensitive to changes in demand, order cost and holding cost. However, it is discovered that changing the shortage penalty cost does not have much effect on the optimal policy comparing to the remaining three parameters. This section of the numerical study sheds some light

Commentato [e.b.2]: Remove the bracket on the main aspects of the model. When the learning occurs in the planning and the demand and/or holding cost are on the rise, it is recommended to order more frequently with smaller lot-sizes, which tend to increase the maximum inventory in hand. If the same scenario happens for order cost, the recommended policy is the opposite, which suggests decreasing the frequency of the orders.

Table 1. Comparing the optimal policy of the fuzzy models with and without learning

Commentato [e.b.3]: Maybe it is better to present table one previously (?). I noticed that it is recalled earlier in the manuscript.

Fuzzy model with two-stage learning						**Fuzzy model**			
(l_c, l_m)	α	n^*	Q^*	M^*	TCU^*	n^*	Q^*	M^*	TCU^*
(0.514,0.152)	0.2	17	11,765	9,747	9,368	16	12702.427	10846.891	10161.941
(0.814,0.152)	0.2	18	11111.11	9212.342	9,356				
(0.514,0.452)	0.2	18	11111.11	9212.342	9193.798				
(0.814,0.452)	0.2	18	11111.11	9026.19	9190.352				
(0.514,0.152)	0.8	18	11111.11	9069.886	9211.069				
(0.814,0.152)	0.8	18	11111.11	9037.53	9193.191				
(0.514,0.452)	0.8	18	11111.11	9023.348	9189.524				
(0.814,0.452)	0.8	18	11111.11	8990.992	9180.647				

On the other hand, to understand the pattern of the total cost of inventory when one parameter starts going up, the total inventory cost of the four analyzed parameters can be compared in both Tables.

Table 2. Summary of sensitivity analysis of parameters for $(l_c, l_m) = (0.514, 0.152)$ and $\alpha = 0.2$

$\boldsymbol{D}$	50000	55000	60000	65000	70000	75000	80000	85000	90000
$\boldsymbol{n^*}$	5	6	6	6	6	7	7	7	7
$\boldsymbol{Q^*}$	10000	9166.67	10000	10833.33	11666.67	10714.29	11428.57	12142.86	12857.14
$\boldsymbol{M^*}$	9645.14	8853.74	9658.63	10463.51	11268.40	10363.02	11053.89	11744.75	12435.62
$\boldsymbol{TCU^*_{FL}}$	2278.21	2389.62	2494.65	2599.91	2705.35	2804.32	2896.08	2987.94	3079.89
$\boldsymbol{C_o}$	200	400	600	800	1000	1200	1400	1600	1800
$\boldsymbol{n^*}$	5	4	3	3	2	2	2	2	2
$\boldsymbol{Q^*}$	10000	12500	16666.67	16666.67	25000	25000	25000	25000	25000
$\boldsymbol{M^*}$	9645.14	12039.65	16030.64	16030.64	24012.98	24012.98	24012.98	24012.98	24012.98
$\boldsymbol{TCU^*_{FL}}$	2278.21	3186.47	3889.18	4502.99	5095.74	5507.62	5919.50	6331.38	6743.26
$\boldsymbol{C_h}$	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
$\boldsymbol{n^*}$	5	7	10	10	11	12	13	13	14
$\boldsymbol{Q^*}$	10000	7142.86	5555.56	5000	4545.45	4166.67	3846.15	3846.15	3571.43
$\boldsymbol{M^*}$	9645.14	6599.46	4928.41	4261.21	3728.50	3295.27	2937.46	2835.71	2550.07
$\boldsymbol{TCU^*_{FL}}$	2278.21	3124.91	3727.53	4208.73	4607.89	4949.08	5248.48	5507.38	5741.89
$\boldsymbol{C_p}$	5	10	15	20	25	30	35	40	45
$\boldsymbol{n^*}$	5	5	5	5	5	5	5	5	5
$\boldsymbol{Q^*}$	10000	10000	10000	10000	10000	10000	10000	10000	10000
$\boldsymbol{M^*}$	9645.14	9877.43	9957.40	9997.88	10022.32	10038.69	10050.41	10059.22	10066.08
$\boldsymbol{TCU^*_{FL}}$	2278.21	2373.94	2450.62	2522.37	2592.11	2660.85	2729.00	2796.79	2864.33

Referring to Table 2, it is evident that, for all parameters discussed, the total inventory cost tends to increase with the increasing value of the parameter. However, the results in Table 3 suggest that if the planning task becomes more cognitive, it could counter the increase in the total inventory cost. Following the conclusions reached above, this part highlights the prominent role that workforce training programs have on the total inventory cost and reemphasizes fostering learning among the workforce to benefit from the cost saving.

Table 3. Summary of sensitivity analysis of parameters for $(l_c, l_m) = (0.514, 0.152)$ and $\alpha = 0.80$

D	50000	55000	60000	65000	70000	75000	80000	85000	90000
n^*	5	6	6	6	6	7	7	7	7
Q^*	10000	9166.67	10000	10833.33	11666.67	10714.29	11428.57	12142.86	12857.14
M^*	9613.33	8817.25	9618.82	10420.39	11221.96	10311.82	10999.27	11686.73	12374.18
TCU^*_{FL}	2265.00	2372.24	2476.61	2581.14	2685.80	2779.48	2870.19	2960.96	3051.79
C_o	200	400	600	800	1000	1200	1400	1600	1800
n^*	5	4	3	3	2	2	2	2	2
Q^*	10000	12500	16666.67	16666.67	25000	25000	25000	25000	25000
M^*	9613.33	12009.86	16004.2	16004.21	23993.24	23993.24	23993.24	23993.24	23993.24
TCU^*_{FL}	2265.00	3173.34	3878.39	4489.89	5088.30	5499.03	5909.76	6320.49	6731.22
C_h	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
n^*	5	8	9	10	11	12	13	13	14
Q^*	10000	6250	5555.56	5000	4545.45	4166.67	3846.15	3846.15	3571.43
M^*	9613.33	5748.15	4892.99	4225.35	3692.27	3258.74	2900.67	2798.92	2513.07
TCU^*_{FL}	2265.00	3102.39	3698.04	4174.35	4568.37	4904.18	5197.95	5455.50	5684.01
C_p	5	10	15	20	25	30	35	40	45
n^*	5	6	6	6	6	6	6	6	6
Q^*	10000	8333.33	8333.33	8333.33	8333.33	8333.33	8333.33	8333.33	8333.33
M^*	9613.33	8209.26	8275.90	8309.63	8330.00	8343.64	8353.41	8360.75	8366.47
TCU^*_{FL}	2265.00	2351.46	2418.97	2482.37	2544.10	2604.98	2665.39	2725.49	2785.39

7. Conclusion

In this paper, a fuzzy lot-sizing problem with backorders is studied, where the operator, who is in charge of planning, experiences learning over the planning period. Both demand and lead time during the planning period are assumed to be fuzzy. Therefore, a fuzzy EOQ model with backorders and a two-stage composite human learning is formulated. This work is motivated by the earlier papers in the literature, which reported that the human cognitive and motor capabilities are highly relevant in operations management in practice. Thus, it is assumed that the cognitive and motor capabilities of the operator helps him catch up with the planning process, and constitutes the required knowledge for performing the planning process cycle by cycle. Beside the study of Glock et al. (2012), Kazemi et al. (2015b) and Kazemi et al. (2015c), who studied a simple learning process in the fuzzy lot-sizing problem, this paper is one of the first to integrate human learning in fuzzy lot-sizing. In particular, this paper is the first one that formulates a two-stage human learning in a fuzzy lot-sizing problem in the literature, which helps to make the model more realistic.

Some numerical analyses are provided to test the impact of the operator's learning rate and his learning capabilities on the developed model. The results indicate that, no matter the rate by which the operator learns, his cognitive capabilities could improve the performance of the inventory system in terms of the total costs, particularly when the operator learns faster with his cognitive ability. When it comes to the maximum inventory that should be stocked during the planning horizon, still the effect of the cognitive ability of the operator is found to be considerable, which in turn suggests that the more the planning is cognitive the less inventory should be kept. The benefits that the inventory system could gain through either cognitive process or quickening the learning should recommend managers to pay especial attention to investment on training programs for workforces. Additionally, the model developed in this paper is compared with the model in the literature without learning and it is observed that human learning has the potential to decrease the total cost of the inventory system considerably. Finally, the model is exemplified using a sensitivity analysis on the demand and inventory costs over the planning horizon and some optimal policies are recommended.

We have encountered some limitations when formulating the model. Firstly, owing to the lack of experimental studies in the fuzzy lot-sizing problem, the parameters of this study are taken from the literature. It is clear that more studies are required to observe more evidence from real cases and to ensure that the estimated parameters are close to what happens in practice. Secondly, as for most of the learning models, the result of our paper is highly dependent on the assumptions that were made in formulating the problem. Hence, it is interesting for future research to investigate whether any change in the model assumptions would lead to new results. The research in this paper may be developed in a number of ways. First and foremost, we modelled merely one factor in the studied fuzzy lot-sizing problem; however, more human factors, e.g. human error, could be integrated into the model to provide an analytical basis for investigating the interaction between human factors and the fuzzy inventory system. Another possible extension of the work could be to formulate relationships between organizational criteria and learning. This enables the study of organizational measures and helps to identify how they could influence promoting learning in inventory planning. The eventual aim is the evaluation of what would be the consequences of fostering learning with the help of organizational measures on the optimal policies. Besides, the concept discussed in this study could conceptually be integrated to other decision making models (Bottani et al. 2015; Shekarian, 2015) to improve decision making performance. Finally, it is obvious that further studies are still required to account for different learning processes in different fuzzy inventory models.

References

Anzanello, M.J., Fogliatto, F.S., 2011. Learning curve models and applications: Literature review and research directions. International Journal of Industrial Ergonomics 41, 573-583.

Ben-Daya, M., Darwish, M., Ertogral, K., 2008. The joint economic lot sizing problem: Review and extensions. European Journal of Operational Research 185, 726-742.

Björk, K.-M., 2009. An analytical solution to a fuzzy economic order quantity problem. International Journal of Approximate Reasoning 50, 485-493.

Bottani, E., Ferretti, G., Montanari, R., Rinaldi, M., 2014. Analysis and optimisation of inventory management policies for perishable food products: a simulation study. International Journal of Simulation and Process Modelling 9, 16-32.

Bottani, E., Montanari, R., Rinaldi, M., 2015. How many to produce? The impact of machine flexibility on the performance of a supply chain. International Journal of Supply Chain and Inventory Management 1, 62-90.

Boudreau, J., Hopp, W., McClain, J.O., Thomas, L.J., 2003. On the interface between operations and human resources management. Manufacturing & Service Operations Management 5, 179-202.

Cárdenas-Barrón, L.E., Sana, S.S., 2014. A production-inventory model for a two-echelon supply chain when demand is dependent on sales teams′ initiatives. International Journal of Production Economics 155, 249-258.

Chang, S.-C., 1999. Fuzzy production inventory for fuzzy product quantity with triangular fuzzy number. Fuzzy Sets and Systems 107, 37-57.

Chang, H.-C., 2004. An application of fuzzy sets theory to the EOQ model with imperfect quality items. Computers & Operations Research 31, 2079-2092.

Dar-El, E.M., 2000. Human learning: from learning curves to learning organizations. Boston/Dordrecht/London: Kluwer Academic Publishers.

Dar-el, E.M., Ayas, K., Gilad, I., 1995. A dual-phase model for the individual learning process in industrial tasks. IIE transactions 27, 265-271.

Commentato [e.b.4]: The surname of the first author is written in a different manner in these two references. Please check the correctness

Das, K., Roy, T., Maiti, M., 2004. Buyer–seller fuzzy inventory model for a deteriorating item with discount. International Journal of Systems Science 35, 457-466.

De, S.K., Sana, S.S., 2014. A multi-periods production–inventory model with capacity constraints for multi-manufacturers–A global optimality in intuitionistic fuzzy environment. Applied Mathematics and Computation 242, 825-841.

Fallahpour, A., Olugu, E.U., Musa, S.N., Khezrimotlagh, D., Wong, K.Y., 2015. An integrated model for green supplier selection under fuzzy environment: application of data envelopment analysis and genetic programming approach. Neural Computing and Applications, 1-19.

Gino, F., Pisano, G., 2008. Toward a theory of behavioral operations. Manufacturing & Service Operations Management 10, 676-691.

Glock, C.H., 2011. A multiple-vendor single-buyer integrated inventory model with a variable number of vendors. Computers & Industrial Engineering 60, 173-182.
Glock, C.H., 2012a. The joint economic lot size problem: A review. International Journal of Production Economics 135, 671-686.
Glock, C.H., 2012b. Lead time reduction strategies in a single-vendor–single-buyer integrated inventory model with lot size-dependent lead times and stochastic demand. International Journal of Production Economics 136, 37-44.
Glock, C.H., Schwindl, K., Jaber, M.Y., 2012. An EOQ model with fuzzy demand and learning in fuzziness. International Journal of Services and Operations Management 12, 90-100.
Golizadeh, A.A., Shekarian, E., 2012. A new approach on housing choice using fuzzy logic. Tahghighat-E-Eghtesadi 47, 65-84. (In Persian)
Grosse, E.H., Glock, C.H., 2013. An experimental investigation of learning effects in order picking systems. Journal of Manufacturing Technology Management 24, 850-872.
Grosse, E.H., Glock, C.H., Jaber, M.Y., Neumann, W.P., 2015. Incorporating human factors in order picking planning models: framework and research opportunities. International Journal of Production Research 53, 695-717.
Guiffrida, A.L., 2009. Chapter 8. Fuzzy Inventory Models. Inventory Management: Non-Classical Views, 173-190.
Hsieh, C.H., 2002. Optimization of fuzzy production inventory models. Information Sciences 146, 29-40.
Jaber, M.Y., Bonney, M., 2011. The lot sizing problem and the learning curve: A review. Learning Curves: Theory, Models, and Applications, CRC Press (Taylor and Francis Group), 267-293.
Jaber, M.Y., Bonney, M., Moualek, I., 2009. Lot sizing with learning, forgetting and entropy cost. International Journal of Production Economics 118, 19-25.
Jaber, M.Y., Goyal, S., 2008. Coordinating a three-level supply chain with multiple suppliers, a vendor and multiple buyers. International Journal of Production Economics 116, 95-103.
Jaber, M.Y., Goyal, S., Imran, M., 2008. Economic production quantity model for items with imperfect quality subject to learning effects. International Journal of Production Economics 115, 143-150.
Jaber, M.Y., Kher, H., 2002. The dual-phase learning–forgetting model. International Journal of Production Economics 76, 229-242.
Jaber, M.Y., 2006. Learning and forgetting models and their applications. Handbook of industrial and systems engineering, 30.31-30.27.
Jaber, M.Y., Glock, C.H., 2013. A learning curve for tasks with cognitive and motor elements. Computers & Industrial Engineering 64, 866-871.
Jaber, M.Y., Zanoni, S., Zavanella, L.E., 2014. Economic order quantity models for imperfect items with buy and repair options. International Journal of Production Economics 155, 126-131.
Jana, D.K., Maity, K., Das, B., Roy, T.K., 2013. A fuzzy simulation via contractive mapping genetic algorithm approach to an imprecise production inventory model under volume flexibility. Journal of Simulation 7, 90-100.
Kazemi, N., Ehsani, E., Glock, C.H., 2014. Multi-objective supplier selection and order allocation under quantity discounts with fuzzy goals and fuzzy constraints. International Journal of Applied Decision Sciences 7 (1), 66-96.
Kazemi, N., Ehsani, E., Glock, C.H., Schwindl, K., 2015a. A mathematical programming model for a multi-objective supplier selection and order allocation problem with fuzzy objectives. International Journal of Services and Operations Management 21 (4), 435-465.

Kazemi, N., Ehsani, E., Jaber, M.Y., 2010. An inventory model with backorders with fuzzy parameters and decision variables. International Journal of Approximate Reasoning 51 (8), 964-972.

Kazemi, N., Olugu, E.U., Abdul-Rashid, S.H., Ghazilla, R.A.B.R., 2015b. Development of a fuzzy economic order quantity model for imperfect quality items using the learning effect on fuzzy parameters. Journal of Intelligent and Fuzzy Systems 28, 2377–2389.

Kazemi, N., Shekarian, E., Cárdenas-Barrón, L.E., Olugu, E.U., 2015c. Incorporating human learning into a fuzzy EOQ inventory model with backorders. Computers & Industrial Engineering 87, 540–542.

Khouja, M., Goyal, S., 2008. A review of the joint replenishment problem literature: 1989–2005. European Journal of Operational Research 186, 1-16.

Kumar, R.S., Tiwari, M., Goswami, A., 2014. Two-echelon fuzzy stochastic supply chain for the manufacturer–buyer integrated production–inventory system. Journal of Intelligent Manufacturing, 1-14.

Lam, S.M., Wong, D.S., 1996. A fuzzy mathematical model for the joint economic lot size problem with multiple price breaks. European Journal of Operational Research 95, 611-622.

Lam, S.M., Wong, D.S., 1996. A fuzzy mathematical model for the joint economic lot size problem with multiple price breaks. European Journal of Operational Research 95, 611-622.

Lee, H.-M., Yao, J.-S., 1998. Economic production quantity for fuzzy demand quantity, and fuzzy production quantity. European Journal of Operational Research 109, 203-211.

Lee, H.-M., Yao, J.-S., 1999. Economic order quantity in fuzzy sense for inventory without backorder model. Fuzzy Sets and Systems 105, 13-31.

Lin, D.-C., Yao, J.-S., 2000. Fuzzy economic production for production inventory. Fuzzy Sets and Systems 111, 465-495.

Lodree, E.J., Geiger, C.D., Jiang, X., 2009. Taxonomy for integrating scheduling theory and human factors: Review and research opportunities. International Journal of Industrial Ergonomics 39, 39-51.

Lukinskiy, V.S., Valeryevich, L.V., Zamaletdinova, D.A., 2015. Integrated method of analysing logistics costs in supply chain. International Journal of Supply Chain and Inventory Management 1, 48-61.

Mahata, G.C., 2014. A production-inventory model with imperfect production process and partial backlogging under learning considerations in fuzzy random environments. Journal of Intelligent Manufacturing, 1-15.

Mahata, G., Goswami, A., 2007. An EOQ model for deteriorating items under trade credit financing in the fuzzy sense. Production Planning and Control 18, 681-692.

Mahata, G., Goswami, A., Gupta, D., 2005. A joint economic-lot-size model for purchaser and vendor in fuzzy sense. Computers & Mathematics with Applications 50, 1767-1790.

Mahata, G.C, Goswami, A., 2013. Fuzzy inventory models for items with imperfect quality and shortage backordering under crisp and fuzzy decision variables. Computers & Industrial Engineering 64, 190-199.

Mahata, G.C., Mahata, P., 2011. Analysis of a fuzzy economic order quantity model for deteriorating items under retailer partial trade credit financing in a supply chain. Mathematical and Computer Modelling 53, 1621-1636.

Mandal, S., Khan, D.A., 2014. Integrated model in one vendor-one customer situation when costs are fuzzy and lot sizes are deterministic. International Journal of Logistics Systems and Management 18, 49-71.

Mezei, J., Björk, K.-M., 2015. An economic production quantity problem with backorders and fuzzy cycle times. Journal of Intelligent & Fuzzy Systems: Applications in Engineering and Technology 28, 1861-1868.
Milenkovic, M., Bojovic, N., 2014. Fuzzy modeling approach to the rail freight car inventory problem. Transportation Planning and Technology 37, 119-137.
NM Modak, S Panda, SS Sana, In press. Three-echelon supply chain coordination considering duopolistic retailers with perfect quality products, International Journal of Production Economics.
Neumann, W.P, Village, J., 2012. Ergonomics action research II: a framework for integrating HF into work system design. Ergonomics 55, 1140-1156.
Neumann, W.P, Dul, J., 2010. Human factors: spanning the gap between OM and HRM. International Journal of Operations & Production Management 30, 923-950.
Pal, S., Mahapatra, G., Samanta, G., 2014. An EPQ model of ramp type demand with Weibull deterioration under inflation and finite horizon in crisp and fuzzy environment. International Journal of Production Economics 156, 159-166.
Park, K., 1987. Fuzzy-set theoretic interpretation of economic order quantity. Systems, Man and Cybernetics, IEEE Transactions on 17, 1082-1084.
Sadeghi, J., Niaki, S.T.A., 2015. Two parameter tuned multi-objective evolutionary algorithms for a bi-objective vendor managed inventory model with trapezoidal fuzzy demand. Applied Soft Computing 30, 567-576.
Salameh, M., Jaber, M.Y., 2000. Economic production quantity model for items with imperfect quality. International Journal of Production Economics 64, 59-64.
Sharifi, E., Shabani, S., Sobhanallahi, M.A., Mirzazadeh, A., 2015. A fuzzy economic order quantity model for items with imperfect quality and partial backordered shortage under screening errors. International Journal of Applied Decision Sciences 8, 109-126.
Shekarian, E., 2015. A novel application of the VIKOR method for investigating the effect of education on housing choice. International Journal of Operational Research 24, 161-183.
Shekarian, E., Gholizadeh, A.A., 2013. Application of adaptive network based fuzzy inference system method in economic welfare. Knowledge-Based Systems 39, 151-158.
Shekarian, E., Glock, C.H., Amiri, S.M.P., Schwindl, K., 2014a. Optimal manufacturing lot size for a single-stage production system with rework in a fuzzy environment. Journal of Intelligent and Fuzzy Systems 27, 3067-3080.
Shekarian, E., Jaber, M.Y., Kazemi, N., Ehsani, E., 2014b. A fuzzified version of the economic production quantity (EPQ) model with backorders and rework for a single-stage system. European Journal of Industrial Engineering 8, 291-324.
Shekarian, E., Olugu, E. U., Salwa Hanim, A.-R., Kazemi, N. In press. An economic order quantity model considering different holding costs for imperfect quality items subject to fuzziness and learning. Journal of Intelligent and Fuzzy Systems.
Soni, H.N., Joshi, M., 2013. A fuzzy framework for coordinating pricing and inventory policies for deteriorating items under retailer partial trade credit financing. Computers & Industrial Engineering 66, 865-878.
Taleizadeh, A.A., Mohammadi, B., Cárdenas-Barrón, L.E., Samimi, H., 2013. An EOQ model for perishable product with special sale and shortage. International Journal of Production Economics 145, 318-338.
Vujošević, M., Petrović, D., Petrović, R., 1996. EOQ formula when inventory cost is fuzzy. International Journal of Production Economics 45, 499-504.

Wright, T., 1936. Factors affecting the cost of airplanes. Journal of Aeronautical Science 3, 122-128.
Yang, P.-C., Wee, H.-M., 2000. Economic ordering policy of deteriorated item for vendor and buyer: an integrated approach. Production Planning & Control 11, 474-480.
Zanoni, S., Jaber, M.Y., Zavanella, L.E., 2012. Vendor managed inventory (VMI) with consignment considering learning and forgetting effects. International Journal of Production Economics 140, 721-730.