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Hybridization methodology based on DP algorithm for hydraulic mobile machinery — Application to a middle size excavator

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HYBRIDIZATION METHODOLOGY BASED ON DP ALGORITHM FOR HYDRAULIC MOBILE MACHINERY – APPLICATION TO A MIDDLE SIZE EXCAVATOR

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Answer to Reviewers

II round

Editor:

In addition to the reviewers' comments below, please address the following:

- Highlights: the first one is too long.
- Figures: if at all possible (this may be the case for most of your figures), please submit images in vector format (i.e. where text is text and lines are lines, as opposed to everything being pixels). Examples of formats supporting vector content are PDF and EPS. MS Office files are also accepted by the journal (e.g. EMF).
- Conclusions: Your conclusions should include opening statements that review the limitations of the work reported in the manuscript and suggest areas for further research.

Answer: the first highlight has been corrected; figures has been prepared as required; conclusions have been improved as required, a sentence has been added at the end and little modifications at the beginning have been introduced.

Reviewer #2:

The authors have made substantial changes to this paper from their original submission. These changes have greatly improved the quality of this paper and it is now this reviewer's opinion that the paper be published in its current state.

The only minor suggestion, which if not addressed should not keep this paper from being published, is to include a zoomed-in view of Figure 30. This plot shows the command to a proportional valve but the time scale doesn't allow the reader to see if the valve is ever commanded to a position other than fully open or fully closed.

Answer: A figure has been added as required (fig. 31) . Figs. 30 and 33 have been little modified.

Reviewer #3:

Well written and well reworked. i have no further comments.

Answer: Thanks.

Highlights

1. A methodology to compare different hydraulic hybrid system layouts is presented.
2. The methodology takes advantage of the dynamic programming (DP) algorithm.
3. Optimal hybrid layout configuration and its optimal control policies can be defined

1 **Hybridization Methodology based on DP algorithm for**
2 **Hydraulic Mobile Machinery – Application to a Middle Size**
3 **Excavator**

4
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17

18 **Abstract**

19 Fuel consumption and pollutant emission reduction are and will continue to be the most important drivers in the
20 improvement of mobile machinery hydraulic system. Many different solutions and options are proposed in the literature
21 to improve the machinery fuel efficiency, and many of these are based on hybrid solutions. The aim of this paper is to
22 present a hybridization methodology which allows to compare different system layouts, to dimension the energy storage
23 devices, to define the optimal control policies, and finally to determine the more effective hybrid system layout. The
24 proposed methodology takes advantage of the dynamic programming (DP) algorithm. The machinery mathematical
25 model and information about working cycle have to be known “a priori” in order to take advantage of the presented
26 methodology.

27 The hybridization methodology has been applied to a hydraulic excavator as a guideline example, and the results are
28 reported in the last section of the paper.

29

30 **Keywords:** Hydraulic Hybrid Excavator; Hybridization Methodology; Dynamic Programming Optimization.

Nomenclature

Abbreviations

AUX	Auxiliary	VCO	Variable Control Orifice
DoE	Design of Experiment	ICE	Internal Combustion Engine
DP	Dynamic Programming	JCMAS	Japan Construction Machinery Association Standard
ECU	Electronic Control Unit	LS	Load Sensing

Symbol	Description	Unit	Symbol	Description	Unit
A_A	Hydraulic Actuator Piston Area	[m ²]	V	Volume	[m ³]
A_B	Hydraulic Actuator Piston Area	[m ²]	Vd	Volumetric Displacement	[m ³ /r]
c_d	Discharge Coefficient	[-]	w	System Disturbance	
F	Force	[N]	x	System State	
J	Cost Function		x_i	Spool Linear Position	[m]
mf	Fuel Burned Rate	[g/s]	y	System Output	
n	Angular Velocity	[r/min]	γ	Polytropic Index	
p	Pressure	[Pa]	η_{hm}	Hydraulic-Mechanical Efficiency	[-]
Q	Volumetric Flow Rate	[m ³ /s]	η_v	Volumetric Efficiency	[-]
T	Torque	[N·m]	π	Control Policy	
u	System Input		ρ	Hydraulic Density	[kg/m ³]
v	Linear Velocity	[m/s]	ω	Angular Velocity	[rad/s]

Superscript

* Optimal

31

32 1. Introduction

33 In the field of mobile machinery the increasing interest in the reduction of pollutant emissions, supported by more and
 34 more tight regulations, and of fuel consumption are leading the R&D activities towards new energy saving solutions.
 35 Focusing on hydraulic mobile machinery, especially on hydraulic excavators, there are different options to reduce fuel
 36 consumption. A first way is the improvement of single components efficiency, achieved by reducing friction losses
 37 and/or enhancing their flow dynamics characteristics. A second path concerns the adoption of new system layouts, e.g.,
 38 drive hydraulic actuators in closed loop hydraulic system using variable displacement machines (thus avoiding
 39 throttling losses in valves) [1], or adopting hydraulic transformers [2]. A further solution is the optimization of the
 40 matching between the internal combustion engine (ICE) and the hydraulic system taking account of the mission profile
 41 [3, 4]. Furthermore, new electro-hydraulic solutions are also under investigation, e.g., Electro Flow Matching (EFM) [5]
 42 and Electro Positive Control (EPC) [6]. Hybridization techniques, i.e., the use of two or more distinct power sources,
 43 combined with energy recovery systems, represent a further solution. Hybridization technology is widely used on on-
 44 road vehicles, where the ICE and an electric system (generator, motor, inverter and battery) are coupled, with

45 significant advantages in the improvement of fuel efficiency and pollutant emissions reduction, when optimized energy
46 management strategies and a proper design of components and overall architecture are adopted [7].

47 Concerning hydraulic excavators, during a typical working cycle the required power and torque vary periodically in a
48 wide range, influencing consequently the engine working conditions. Moreover, kinetic energy of the turret and
49 gravitational energy of the boom are typically dissipated as heat in the flow control valves. Thus, hybrid excavators
50 with energy recovery systems seem to be an effective solution to improve fuel efficiency, thanks to the possibility of a
51 better shaping of the load request to the ICE and the storage of otherwise wasted energy.

52 Developing an optimized hybrid system involves several topics such as the system layout definition, components size
53 specification and optimal energy management strategy definition. Many researchers have studied hybrid electric
54 excavators focusing on the optimal control strategy definition of some proposed electric hybrid excavator layouts [8, 9,
55 10, 11]. As previously mentioned, during the typical working cycle of a hydraulic excavator the required power and
56 torque change rapidly, periodically and in a very wide range. Furthermore, since boom and turret movements are very
57 fast, a hydraulic energy recovery system seems better than electrical one for this specific application [12], thanks to its
58 higher power density instead of a higher energy density.

59 The aim of this paper is to define a methodology to compare different proposed hybrid system layouts. The
60 methodology has been applied to a middle size (9 t) excavator. For this machinery, four different hydraulic hybrid
61 layouts were proposed and compared. For each layout it has been necessary to define both the new components required
62 for the energy recovery and the optimal energy management strategy in order to achieve the best fuel economy.

63 The proposed methodology takes advantage of the dynamic programming (DP) algorithm [13] in order to define the
64 optimal control policy (in this specific case minimizing fuel consumption) for the non-linear, time dependent problem
65 of excavator dynamics during a defined typical working cycle. This powerful optimization technique has been used
66 widely for defining optimal control strategy for hybrid on-road vehicles [14] and to investigate novel hydraulic hybrid
67 architectures for on-road vehicles [15]. Using the DP algorithm for each considered/proposed layout it is possible to
68 optimally size the components of interest through a DoE (Design of Experiment) procedure. Then it is finally possible
69 to determine the hybrid layout which guarantees the lower fuel consumption after comparing all the studied layouts in
70 their best energy saving configuration.

71 For applying the presented hybridization methodology some necessary steps have to be done:

72 - starting from the standard machinery mathematical model (with a direct causality), an inverse causality model has to
73 be defined and validated. The direct causality model is also exploited to define the inputs required by the inverse model;

- 74 - a representative working cycle for the considered machine has to be defined; for hydraulic excavators, a typical
75 working cycle has been identified in the JCMAS H020:2007 [16], where no interaction between bucket and ground is
76 considered;
- 77 - different hybrid layouts have to be defined for the considered machinery as well as their inverse causality model;
- 78 - the optimization problem has to be formulated;
- 79 - finally the hybrid layouts can be compared in order to define the best energy saving solution.

80 The paper is organized as follows: section 2 presents the standard machinery hydraulic layout previously analyzed and
81 modelled by the authors (assumed as reference for the current analysis), the standard machinery inverse modelling and
82 its validation on the selected reference working cycle; section 3 describes the optimization problem definition and the
83 dynamic programming algorithm; section 4 describes the proposed hydraulic hybrid layouts and their inverse causality
84 model schemes, as well as the DoE performed for the optimal components sizing; section 5 shows the optimization
85 results obtained applying the presented methodology.

86

87 **2. Physical Modeling**

88 **2.1 Direct causality model**

89 The machinery taken into account for this study is a middle size excavator whose simplified hydraulic circuit ISO
90 scheme is reported in Fig.1. The hydraulic layout of this kind of machinery can be divided in three parts [17]: the
91 generator section composed of the hydraulic pumps which transform the power output of the ICE into hydraulic power;
92 the conductive section which transfers pressurized fluid to actuators via pipes and valves; the motor related section
93 where fluid is used to move hydraulic cylinders, motors and auxiliaries.

94 The actuators considered in the mathematical model are those required for the defined working cycle [16]. As can be
95 seen in Fig.1, the arm, boom and bucket cylinders and the turret (or swing) and travels motors have been considered,
96 along with the related hydraulic flow control valves.

97 The main hydraulic circuit is a Load-Sensing (LS) system, where the highest load pressure level of parallel operating
98 hydraulic drives is detected and fed back to the pump regulators and to the valves pressure compensators. This allows to
99 control the inlet line pressure and keep a constant pressure drops across metering valves, even in flow saturation
100 conditions.

101

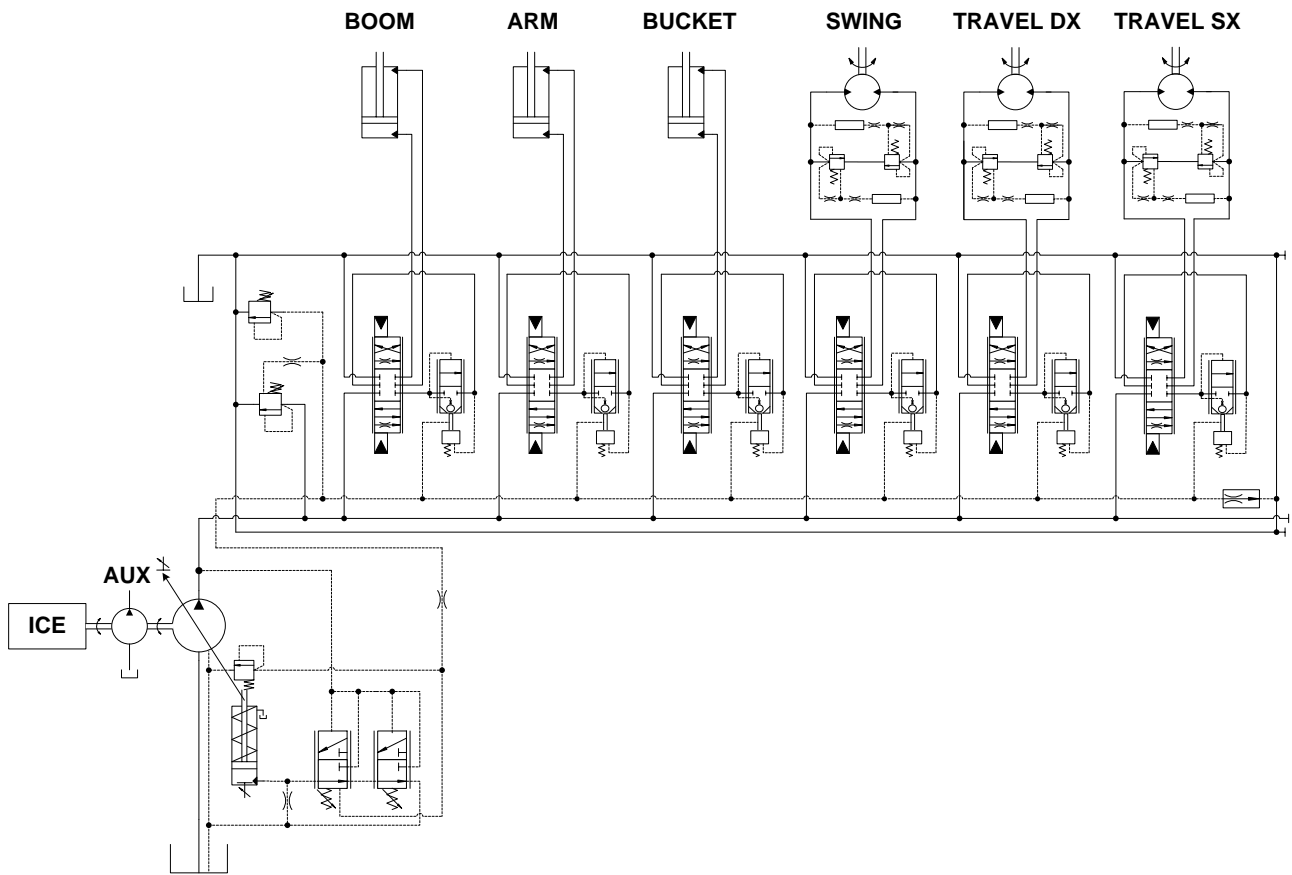


Fig. 1. ISO scheme of the standard excavator hydraulic circuit.

102

103

104

105 The main pump is a LS variable displacement axial piston pump and its mathematical model has been already
 106 developed and validated both in steady state and dynamic conditions [18, 19]. The main valve is a LS full flow sharing
 107 sectional valve. The valve block mathematical model has been already presented and validated with the comparison
 108 between numerical and experimental results [20]. The mathematical model considers also the front excavation tools, in
 109 order to reproduce the real forces acting on the hydraulic actuators during the implements movements. A detailed
 110 description of the standard machinery mathematical modelling is presented in [21]. The pilot circuit (AUX), powered
 111 by a fixed displacement external gear pump, operating at a constant pressure level, has been considered since it has a
 112 significant impact on the fuel consumption. The ICE is a Diesel engine and its mathematical model has been presented
 113 in [22]. The direct causality models are dynamic models based on differential equation. Pressures inside the chambers
 114 are calculated through the pressure rise rate equation applied to the chambers volume, the shaft speed is defined through
 115 the momentum equilibrium of the engine/pump shaft and forces acting on the hydraulic actuators are defined through
 116 the second Newton's law applied to the moving bodies of the kinematics. Figure 2 reports the direct causality
 117 mathematical model scheme.

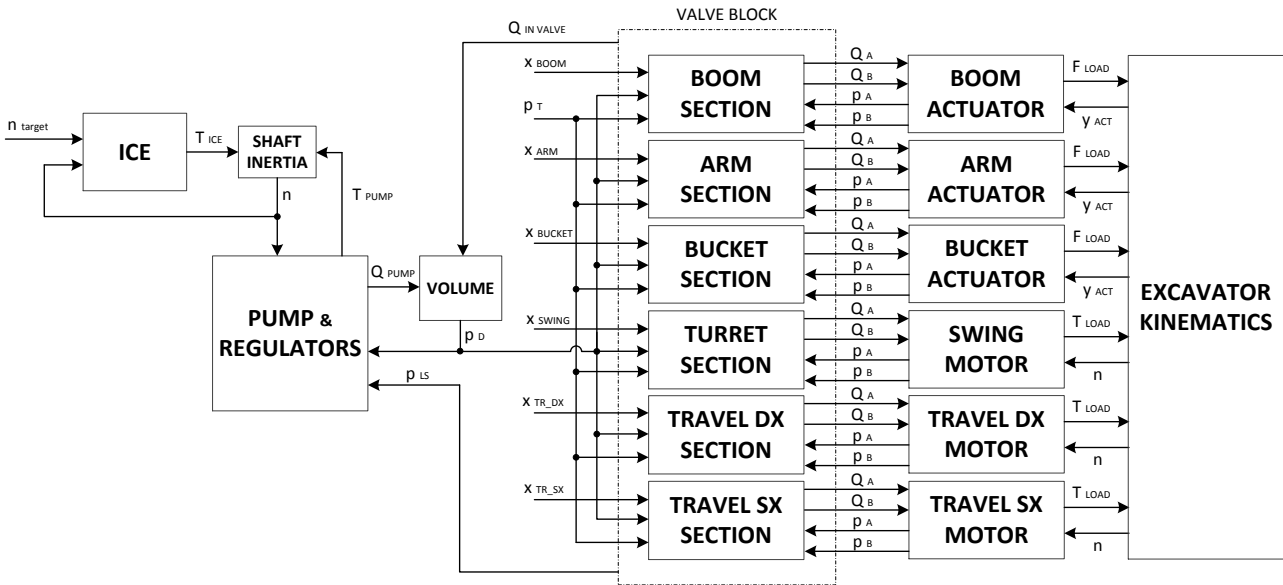


Fig. 2. Direct causality model of the standard excavator.

118

119

120

121 2.2 Inverse Causality Model

122 The presented optimization methodology, as previously mentioned, is based on the DP algorithm which has the
 123 characteristic of solving dynamic optimization problems backward-in-time. In order to solve the optimization problem
 124 the causality of the developed model has to be reverted. Due to the exponentially increasing complexity and
 125 computation time required to solve a DP problem with a large number of states (i.e. a model with many differential
 126 equations), all the non-essential dynamics of the model have been neglected. For this reason, when reversing the
 127 causality of the model, all sub-models have been reduced to pure algebraic models based on a Quasi-Steady
 128 formulation. This is justified by the fact that the time constants associated with the hydraulic components are very low
 129 compared to the characteristics time of a duty cycle. The comparison between excavator direct and inverse causality
 130 models presented in section 2.4 proves the validity of this assumption.

131 On the other hand, when introducing an energy recovery device, i.e. a hydraulic accumulator, it becomes clear that its
 132 dynamics cannot be neglected as they are considerably slower than the hydraulics one and greatly affects the behavior
 133 of the system.

134 Regarding the control of expander movements, in the direct causality case the forces applied to the moving parts depend
 135 on the inputs imposed by the driver (control valves positions) and the trajectories of the excavator components are
 136 calculated (i.e., boom, arm, bucket and swing); in this case, a driver model (namely a PI controller has been used to
 137 guarantee that the actual excavator tools trajectories match the desired profile set by the reference working cycle. In the

138 reverse causality case on the other hand, kinematics of the actuators are known a priori (being defined by the working
 139 cycle) and the forces acting on the actuators can be easily derived from the dynamic equilibrium equation.

140 2.2.1 Hydraulic Linear Actuator

141 Figure 3 shows the hydraulic linear actuator inverse model causality. The piston velocity (v) is assumed to be positive
 142 during the extension movement of the piston while the piston force (F) is positive when the piston pulls the connected
 143 kinematics element. The following assumptions were done for the modelling: no internal leakages are considered,
 144 friction forces are neglected and the cavitation of the fluid is not considered.



145 **Fig. 3.** Hydraulic Linear Actuator Inverse Model Causality.

146 According to the assumption done and to the defined conventions the flow rate at port A, Eq.(1), and at port B, Eq.(2),
 147 are calculated respectively as:

$$Q_A = A_A \cdot v \quad (1)$$

$$Q_B = -A_B \cdot v \quad (2)$$

148
 149 The chambers pressures are defined knowing the piston velocity (v) sign, which discriminates the utilized equations. If
 150 the piston velocity is greater or equal to zero, the exploited equations are (3) and (4); otherwise (5) and (6):

$$p_B = p_C \quad (3)$$

$$p_A = \frac{F}{A_A} + p_B \cdot \frac{A_B}{A_A} \quad (4)$$

151

$$p_A = p_C \quad (5)$$

$$p_B = -\frac{F}{A_B} + p_A \cdot \frac{A_A}{A_B} \quad (6)$$

152

153 The counter pressure (p_C) is due to the resistance introduced by the outlet orifice of the valve section controlling the
 154 user.

155

156 2.2.2 Hydraulic Motor

157 Figure 4 shows the hydraulic motor inverse model causality. No internal and external leakages are considered, friction
 158 forces are neglected and the fluid cavitation is not considered.



Fig. 4. Hydraulic Motor Inverse Model Causality.

159

160 If the turret angular velocity (ω) is greater or equal to zero (i.e. a clockwise turret rotation is performed) the flow rates at
 161 ports A and B as well as the pressures A and B are defined through Eq.s (7) – (10):

$$Q_A = \frac{\omega_{TURRET} \cdot V_d}{\eta_v} \quad (7)$$

$$Q_B = -Q_A \quad (8)$$

$$p_B = p_C \quad (9)$$

$$p_A = p_B + \frac{T_{TURRET} \cdot 2\pi}{V_d \cdot \eta_{hm}} \quad (10)$$

162

163 If a counter clockwise turret rotation is performed, i.e. the turret angular velocity (ω) is lower than zero, the flow rates
 164 and the pressures at ports A and B are calculated with Eq.s (11) – (14):

165

$$Q_A = -Q_B \quad (11)$$

$$Q_B = -\frac{\omega_{TURRET} \cdot V_d}{\eta_v} \quad (12)$$

$$p_A = p_C \quad (13)$$

$$p_B = p_A + \frac{T_{TURRET} \cdot 2\pi}{V_d \cdot \eta_{hm}} \quad (14)$$

166

167 The volumetric and the hydraulic-mechanical efficiencies are considered as constants.

168

169 2.2.3 Valve Section

170 Figure 5 shows the valve section inverse model causality assumed for the input/output variables. If an extension
 171 movement of the linear actuator or a clockwise rotation for the turret is performed Q_A will be positive, otherwise for a
 172 retraction movement of the linear actuator or a counter clockwise rotation of the turret Q_A will be negative.

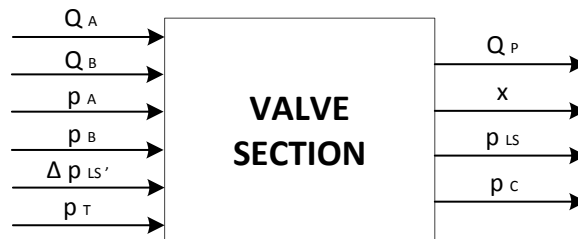


Fig. 5. Valve Section Inverse Model Causality.

173 If Q_A is greater or equal to zero the Eq.s (15) – (18) are exploited in order to evaluate the output variable from the valve
 174 section inverse model:

$$Q_P = Q_A \quad (15)$$

$$p_{LS} = p_A \quad (16)$$

$$A_{IN}(x) = A_A(x) = \frac{Q_P}{cd} \sqrt{\frac{\rho}{2 \cdot \Delta p_{LS}'}} \quad (17)$$

175 Form Eq.(17) the metering area is defined. The spool position (x) can be defined taking advantage of the correlation
 176 defined by the valve constructor. Always exploiting the valve constructor correlation the correspondent outlet area
 177 ($A_{OUT}(x) = A_B(x)$) can be defined.

$$p_C(x) = \frac{Q_B^2 \cdot \rho}{2 \cdot cd^2 \cdot A_{OUT}(x)^2} + p_T \quad (18)$$

178

179 If Q_A is lower than zero the Eq.s (19) – (22) are exploited in order to evaluate the output variable from the valve section
 180 inverse model:

$$Q_P = Q_B \quad (19)$$

$$p_{LS} = p_B \quad (20)$$

$$A_{IN}(x) = A_B(x) = \frac{Q_P}{cd} \sqrt{\frac{\rho}{2 \cdot \Delta p_{LS}'}} \quad (21)$$

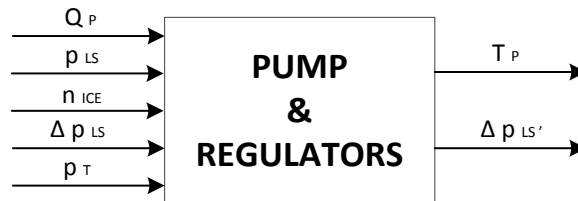
181 Form Eq.(21) the metering area is defined. The spool position (x) can be defined taking advantage of the correlation
 182 defined by the valve constructor. Always exploiting the valve constructor correlation the correspondent outlet area
 183 ($A_{OUT}(x) = A_A(x)$) can be defined.

$$p_C(x) = \frac{Q_A^2 \cdot \rho}{2 \cdot cd^2 \cdot A_{OUT}(x)^2} + p_T \quad (22)$$

184

185 2.2.4 Pump and Regulators

186 Figure 6 shows the pump and its regulators inverse model causality assumed for the input/output variables.



187 **Fig. 6.** Pump and Regulators Inverse Model Causality.

188 The required torque from the pump is defined taking advantage of a map based correlation defined with the aid of the
 189 pump direct causality mathematical model. The map inputs are the pump delivery flow rate (Q_p), the pump differential
 190 pressure (Δp_p) and the engine speed (n_{ICE}). The following Eq.s are utilized to define the required variables:

$$\Delta p_P = p_D - p_{Tank} \quad (23)$$

$$p_D = p_{LS} + \Delta p_{LS}' \quad (24)$$

191

192 Being the pump torque limited by its torque limiter, in order to avoid engine stall or shutdown, the instantaneous LS
 193 margin pressure ($\Delta p_{LS}'$) is defined according Eq.s (25) or (26):

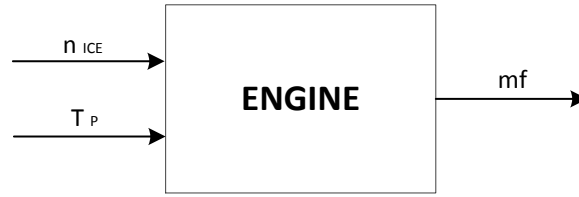
$$\Delta p_{LS}' = \Delta p_{LS} \quad \text{if } T_P < T_{ICE_{MAX}}(n) \quad (25)$$

$$\Delta p_{LS}' = \Delta p_{LS} - \Delta p_{LIM}(Q_p, p_{LS}, T_P) \quad \text{if } T_P \geq T_{ICE_{MAX}}(n) \quad (26)$$

194

195 2.2.5 Engine

196 Figure 7 shows the engine inverse model causality assumed for the input/output variables. The fuel mass flow rate (mf)
 197 is defined taking advantage of a map based correlation defined with the aid of the engine direct causality mathematical
 198 model. The map inputs are the engine speed (n_{ICE}) set to be constant and the pump required torque (T_P).



199 **Fig. 7.** Engine Inverse Model Causality.

200

200 2.2.6 Hydraulic Accumulator

201 A bladder type hydraulic accumulator has been considered in order to allow energy recovery during braking and load
 202 lowering phases in the working cycle. The accumulator (pre-charged with gaseous Nitrogen) is modeled assuming an
 203 adiabatic compression following a polytropic gas law (27):

$$p \cdot V^\gamma = const \quad (27)$$

204

205 Differentiating Eq. (27), pressure changes in the accumulator are obtained from the differential Eq. (28):

$$\frac{dp}{dt} = -\gamma \cdot \frac{p}{V} \cdot \frac{dV}{dt} \quad (28)$$

206

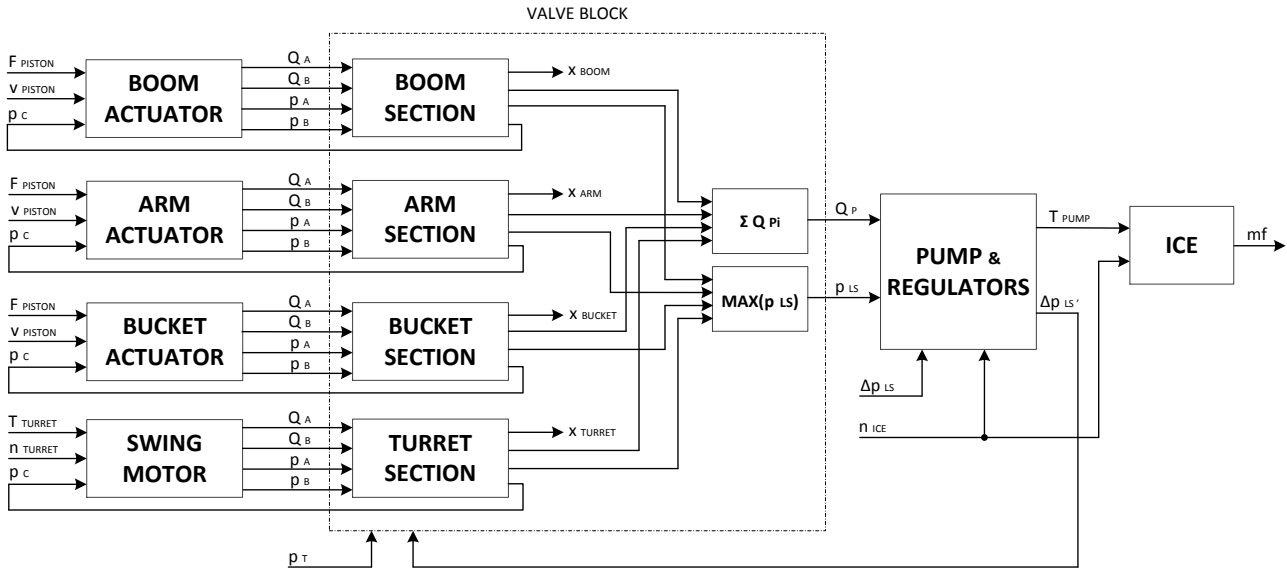
207 The volume variations depend on the inlet and outlet oil flow rate.

208

209

210 **2.2.7 Complete excavator inverse causality model**

211 Figure 8 depicts the complete model of the basic excavator layout (Figure 1) in the inverse causality representation.
 212 This model has been implemented in the Simulink[®] environment and will be used to perform the optimizations
 213 presented in the following sections.



214 **Fig.8. Inverse Causality Model of the Standard Excavator.**

214

215 **2.3 Reference Working Cycle**

216 Fuel consumption is strictly related to the machinery overall efficiency and is actually one of the main performance
 217 parameter used when comparing different machinery. Since fuel consumption strongly depends on the operating
 218 conditions as well as on the machinery layout and management strategy, the definition of a proper “benchmark” in
 219 terms of a significant working cycle is crucial. Many different working cycles have been defined, some commercial and
 220 others standardized. The Japan Construction Mechanization Association (JCMAS) has defined a standardized earth-
 221 moving machinery test procedure to evaluate the fuel consumption on hydraulic excavators, the JCMAS H20:2007 [16].
 222 This standardized working cycle does not involve any bucket-soil interaction, in order to not introduce stochastic effect
 223 on the measuring, and it is composed of four different operating modes: digging and loading motion; leveling motion;
 224 travelling motion and idling functioning. For each of them the JCMAS standard defines the sequential movements and
 225 the kinematics elements positioning, the engine rotational speed and the timing. Once fuel consumption has been
 226 calculated for each operating mode, the overall fuel consumption during a typical working hour can be estimated
 227 according with the procedure defined by the standard. The direct causality model of the standard excavator was
 228 exploited to performing the simulations of the previously mentioned standardized working cycles, defining the
 229 references (i.e. standard machine fuel consumption) for the comparisons.

230 Starting from the trajectories prescribed by the standard and knowing masses and geometries of the excavator
231 components, the velocities of the hydraulic actuators and, from dynamic equilibrium, the exerted forces to the actuators
232 can be obtained from the direct excavator model. This forces and velocities are also used for the simulations of the
233 reverse causality model.

234 Figures 9 – 12 show the boom, arm, bucket and turret inputs relative to the digging and loading motion of the actuated
235 user during the cycle.

236

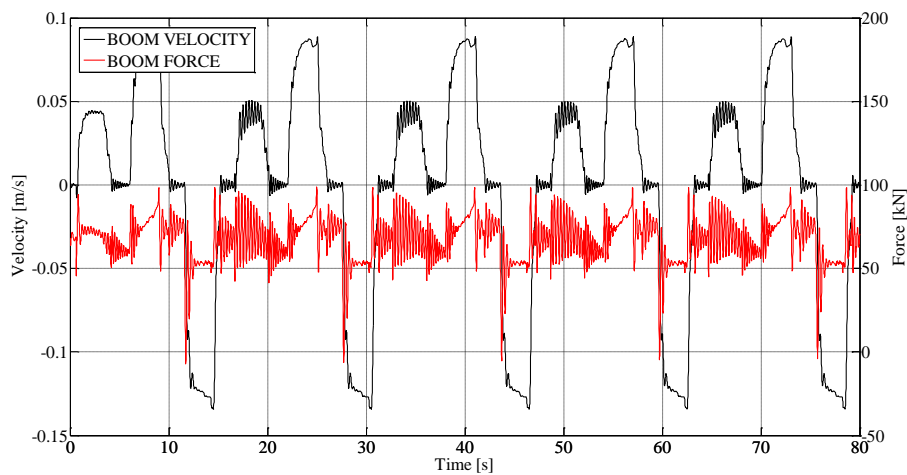


Fig.9. Boom Inputs – Digging and Loading motions.

237

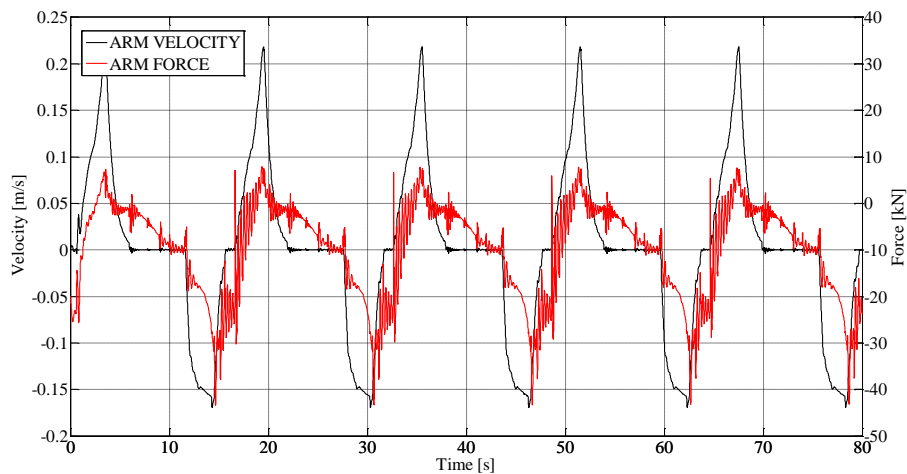


Fig.10. Arm Inputs – Digging and Loading motions.

238

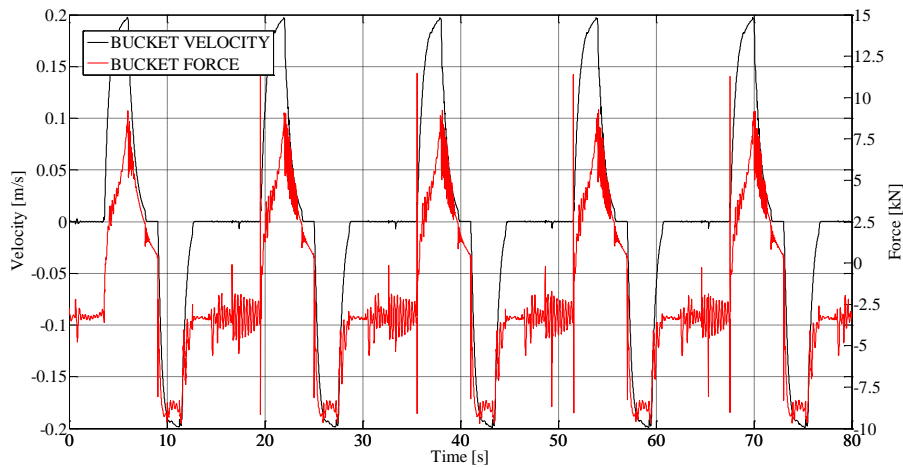


Fig.11. Bucket Inputs – Digging and Loading motions.

239

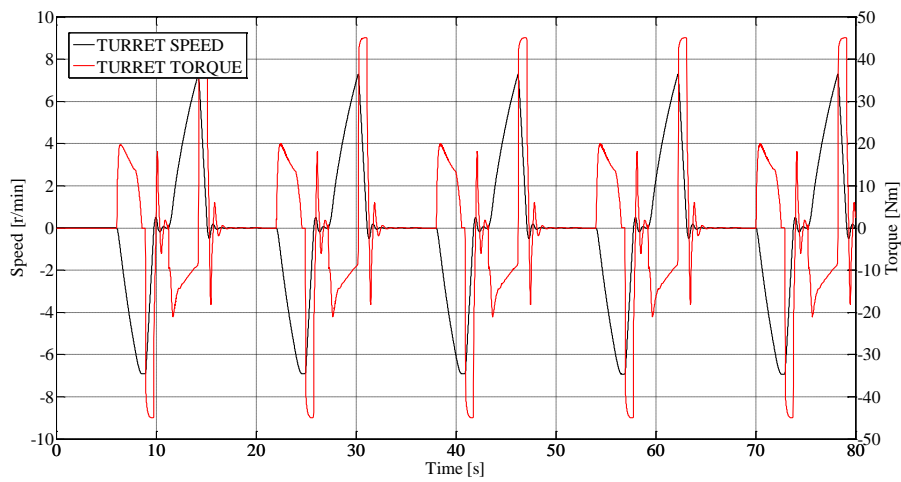


Fig.12. Turret Inputs – Digging and Loading motions.

240

241 Figures 13, 14 report the boom and arm defined inputs throughout the leveling motion respectively. During this
 242 operating mode the bucket and the swing are not actuated.

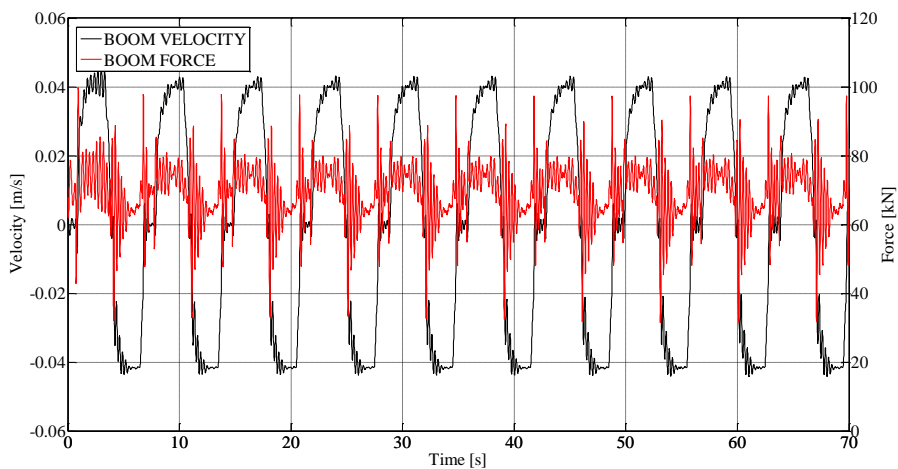


Fig.13. Boom Inputs – Leveling motion.

243

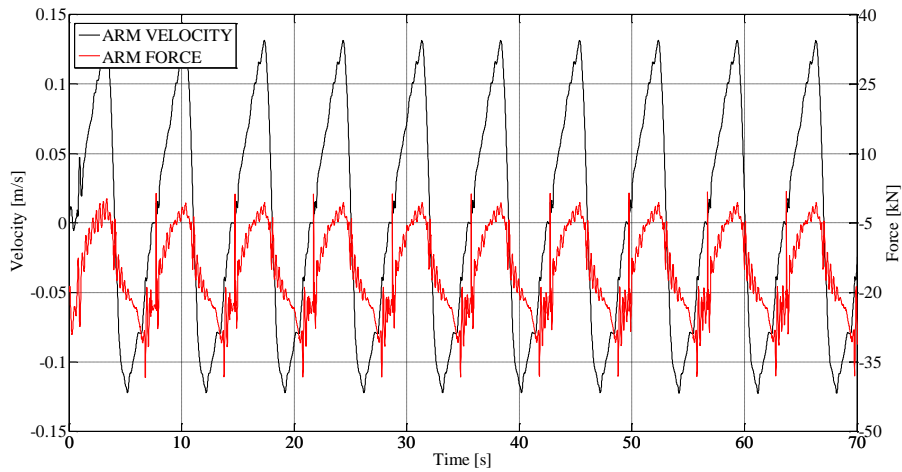


Fig.14. Arm Inputs – Leveling motion.

244

245 The travelling motion and the idling functioning were only performed with the direct causality model of the standard
 246 excavator because during these operating modes the energy recovery and reuse system will be never activated. Thus no
 247 differences between the proposed hybrid layouts will be pointed out.

248

249 **2.4 Inverse Model Validation**

250 To perform a comprehensive comparison between the proposed hybrid layouts and the reference one (the standard
 251 machinery layout), a validation of the inverse causality model of the standard excavator has to be performed firstly. The
 252 two models have been compared with reference to the previously described typical machinery working cycle. Starting
 253 from the illustrated profiles of forces and velocities of the various actuators, the remaining variables are calculated by
 254 means of the inverse causality model described in section 2.2.

255 Figures 15 – 18 report the comparison between the direct and the inverse causality models output variables during the
 256 simulations of the digging and loading motion of the JCMAS cycle.

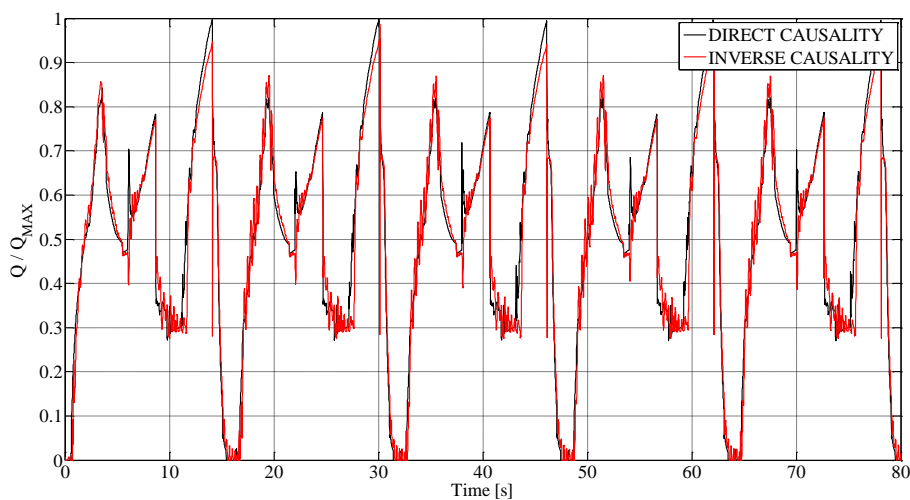
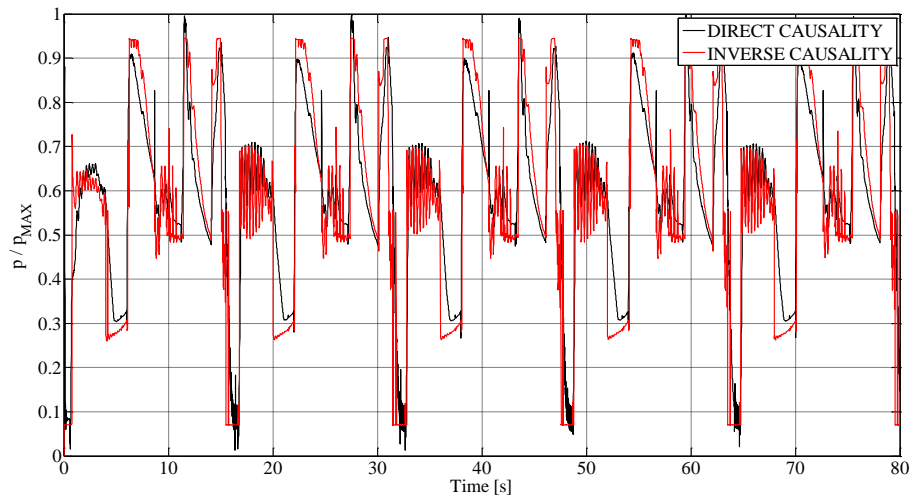
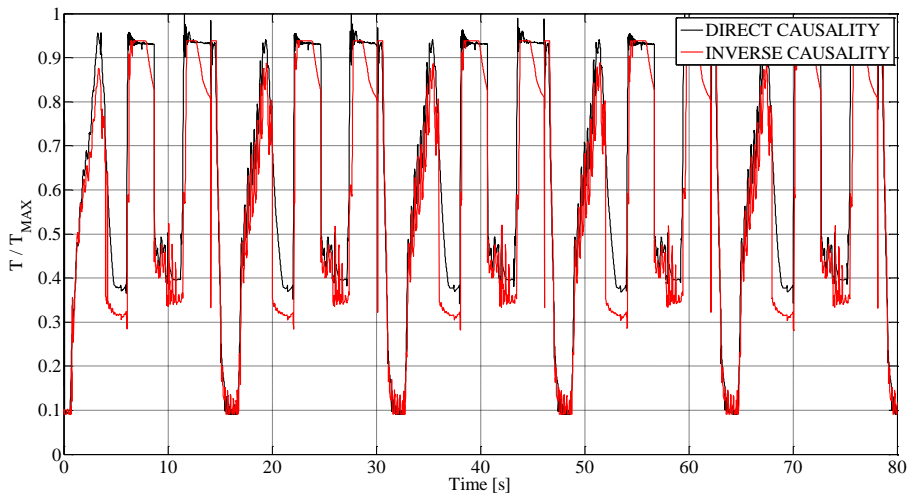


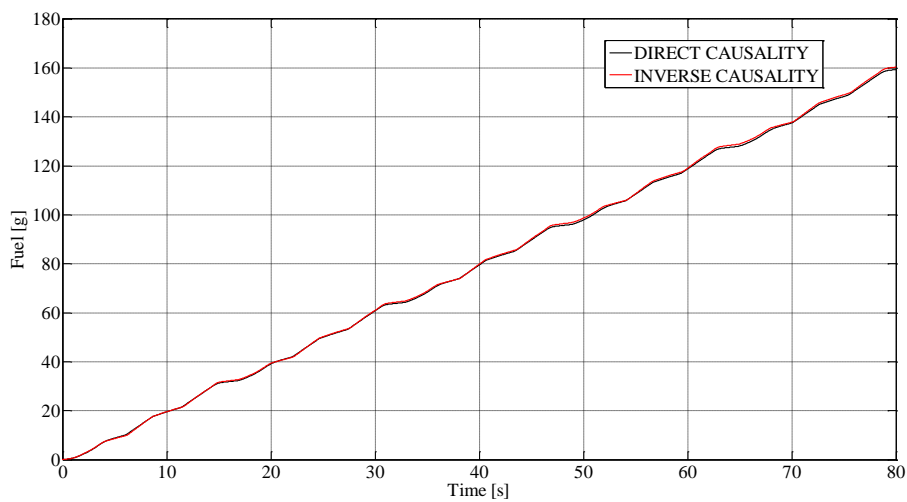
Fig.15. Comparison of pump outlet flow rate estimated from direct and inverse causality models.



257 **Fig.16.** Comparison of pump delivery pressure estimated from direct and inverse causality models.



258 **Fig.17.** Comparison of engine torque estimated from direct and inverse causality models.



259 **Fig.18.** Comparison of fuel consumption estimated from direct and inverse models.

260

261

It is noticeable that, despite of the quasi-steady assumption, there is a great agreement between inverse and direct causality models, with differences in the evaluation of fuel consumption lower than 1%.

262 3. Optimization problem definition

263 In order to apply the DP methodology to the fuel consumption minimization problem, system equations are rearranged
 264 in a discrete state space representation form, Eq.(29). Actuators solicitations are treated as known external disturbances,
 265 thus obtaining time invariant correlations f and g .

$$\begin{cases} x(k+1) = f(x(k), u(k), w(k)) \\ y(k) = g(x(k), u(k), w(k)) \end{cases} \quad (29)$$

266
 267 For the examined problem, the state variable $x(k)$ corresponds to the accumulator pressure p and the state update
 268 equation $[f(x(k), u(k), w(k))]$ is derived from Eq. (28) with the term $\frac{dV}{dt}$ accounting for the net inlet flow rate, function of
 269 external disturbances and control valves actuation. The term $u(k)$ is a vector representing the behavior of the set of
 270 controls valves regulating the flow to and from the accumulator, in the proposed hybrid layout under investigation. Its
 271 dimension is correlated to the specific adopted layout. Finally, $w(k)$ represents the external disturbances acting on the
 272 actuators. Its components are listed in Eq.(30).

$$[F_{BOOM}(k), v_{BOOM}(k), F_{ARM}(k), v_{ARM}(k), F_{BUCKET}(k), v_{BUCKET}(k), T_{TURRET}(k), \omega_{TURRET}(k)] \quad (30)$$

273
 274 where F, v, T, ω represent the force and velocity (torque and rotational speed) on the generic actuator, derived in the
 275 previous section, and whose profile is determined from the simulated working cycles with the aid of the direct causality
 276 mathematical model of the standard machinery.

277 The output $y(k)$ is the fuel mass flow rate burned by the ICE at every time step in order to guarantee the required power
 278 output. The time step Δt has been set to 0.01 s, which has been found to guarantee low computational time for problem
 279 resolution and good accuracy in the integration of the state space equation.

280 Some boundary constraints are added on accumulator pressure (to maintain safe operating conditions) and ICE engine
 281 torque (the torque request must be feasible):

$$x(k) \in [x_{min}, x_{max}] \quad (31)$$

$$T_{ICE}(k) \in [T_{ICE,min}(n_{ICE}), T_{ICE,max}(n_{ICE})] \quad (32)$$

282
 283
 284 where $x_{min} \in [p_{MIN 1}, \dots, p_{MIN i}, \dots, p_{MIN N}]$ bar (depending on the accumulator design), $x_{max} = 4 \cdot x_{min}$ and
 285 minimum/maximum engine torques $T_{ICE,min/max}$ are correlated to the instantaneous ICE speed (n_{ICE}).

286 The objective of the optimization is the minimization of the J cost corresponding to the fuel consumption during the
 287 working cycle:

$$J_{\pi}(x_0) = \sum_{k=1}^{N-1} y(k) \cdot \Delta t \quad (33)$$

288

289 Where $x_0 = x(p_{MIN i})$ is the initial accumulator pressure, chosen equal to x_{min} , and $\pi = \{\vec{u}_0, \vec{u}_1, \dots, \vec{u}_N\}$ is the generic
 290 control policy adopted on the valve controlling the energy storage. No additional terms are introduced regarding
 291 accumulator final state $x_f = x(N)$.

292 For the various proposed layouts and for the every combination of the bladder accumulator parameters (volume, initial
 293 pressure), the objective of the optimization is the determination of the optimal control policy π^* minimizing fuel
 294 consumption:

$$\pi^* = \arg \min_{\pi} J_{\pi}(x_0) \quad (34)$$

295

296 The corresponding cost $J^* = J_{\pi^*}(x_0)$ would be used in the comparison of different system layouts and pressure
 297 accumulator sizing.

298

299 3.1 DP algorithm

300 The evaluation of the optimal control policy is carried out by means of a DP algorithm which exploits Bellman principle
 301 of optimality [13] stating that an optimal policy has the property that whatever the initial state and initial decision are,
 302 the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision,
 303 which is equal to say that, given an optimal control policy $\pi^* = \{\vec{u}_0, \dots, \vec{u}_i, \dots, \vec{u}_N\}$, for the optimization problem
 304 $\min_{\pi} J_{\pi}(x_0)$, the truncated policy $\pi^*_K = \{\vec{u}_k, \vec{u}_{k+1}, \dots, \vec{u}_{k+i}, \dots, \vec{u}_N\}$ with $k > 1$ is still optimal for the "tail sub-
 305 problem" $\min_{\pi} J_{\pi}(x_k)$. Thus, the principle of optimality suggests that an optimal policy can be constructed in
 306 piecemeal fashion, first constructing an optimal policy for the "tail sub-problem" involving the last stage, then
 307 extending the optimal policy to the "tail sub-problem" involving the last two stages, and continuing in this manner
 308 backward until an optimal policy for the entire problem is constructed.

309 The implementation of the DP algorithm to the excavator model is done by means of a Matlab[®] function developed by
 310 [23].

311

312 4. Hydraulic Hybrid Excavator Layouts

313 4.1 Proposed Hybrid Layouts

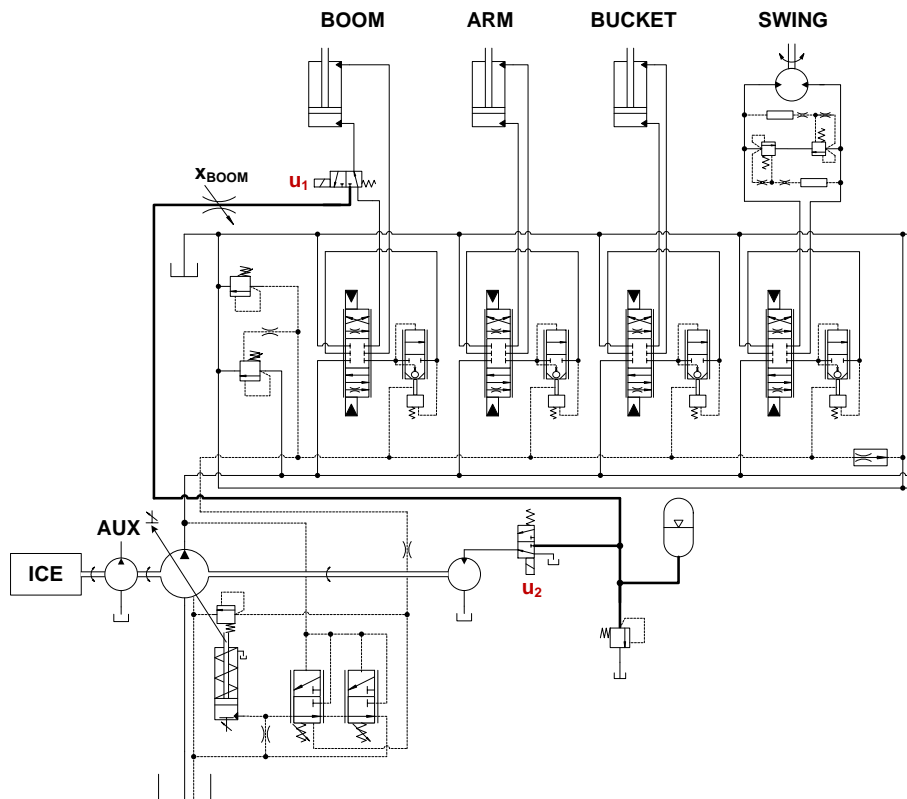
314 In this section the investigated hydraulic hybrid layouts of the excavator under study are presented. Although many
 315 different combinations of solution could be in general proposed and investigated for excavators, in this paper only

316 energy recovery from boom and turret have been considered. The proposed layouts features are reported in Tab.1. The
 317 actuators involved in energy recovery and the numbers of energy storage devices (bladder accumulators) are stated for
 318 each case.

Tab.1. Considered Layouts for the Hybrid System.

CIRCUIT LAYOUT	ACTUATOR(S)	NUMBER OF ACCUMULATOR(S)
A	Boom	1
B	Turret	1
C	Boom - Turret	1
D	Boom - Turret	2

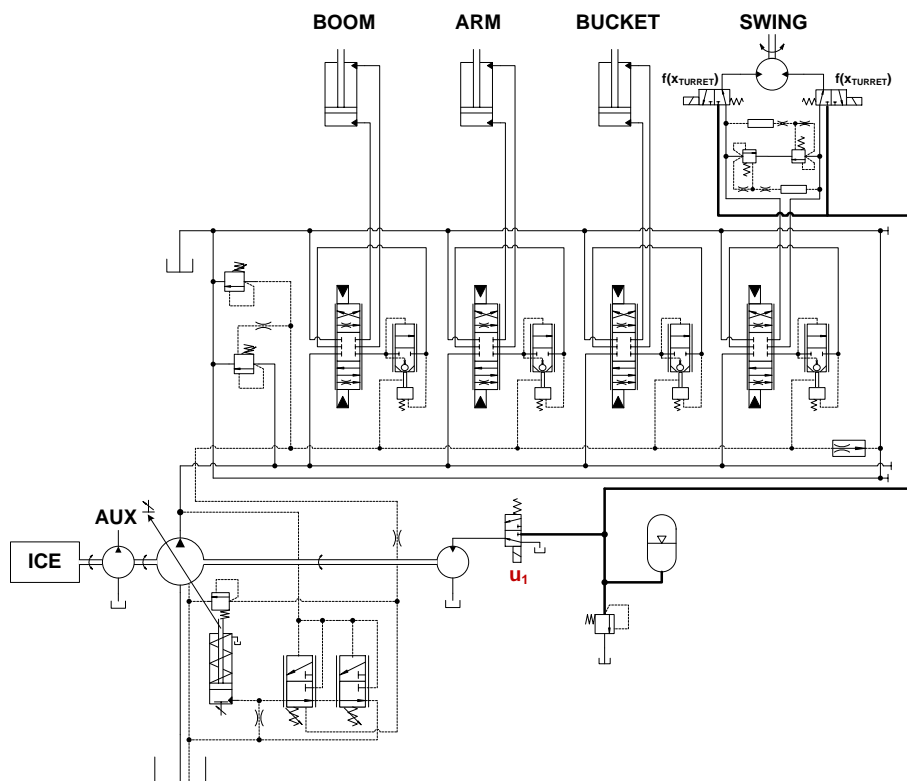
319
 320 The ISO schemes of the proposed layouts of Tab.1 are shown in Figs. 19 – 22. The travel motors and the relative flow
 321 control valve sections have not been reported for simplicity because the layouts do not involve those users for energy
 322 recovery. A common component for each of the layouts is a fixed displacement external gear hydraulic motor, installed
 323 to reuse the energy stored during the accumulation phases. The hybrid configurations exploiting boom lowering are also
 324 equipped with a variable flow control orifice (VCO), placed between the hydraulic actuator and the accumulator, which
 325 prevents cavitation into the actuator rod side and maintains control on the user velocity during the energy recovery
 326 phase. The VCO spool displacement is proportional and synchronized with the main spool displacement of the boom
 327 valve section. The proposed optimization methodology, based on the DP algorithm, has been exploited to define the
 328 optimal control laws for the introduced components (i.e. recovery and reuse valves) in the presented hybrid layouts. The
 329 recovery and reuse valves opening are assumed to be ON/OFF. Considering the solution A, Fig.19, the control laws (u_1 ,
 330 u_2) govern the opening of the corresponding valves, enabling or avoiding energy storage and/or recovery. For solution
 331 B, Fig.20, concerning the swing deceleration phases, the DP algorithm has been only used to define the recovery control
 332 law (u_1) which enables the energy reuse only, since the energy recovery phases are directly dependent by the user main
 333 valve spool positioning ($f(x_{TURRET})$). Solutions C and D, Figs. 21 – 22, are a combination of the solutions A and B, and
 334 for each of them the DP algorithm has been used to define (u_1), (u_2) and (u_1), (u_2), (u_3), (u_4) respectively.



335

336

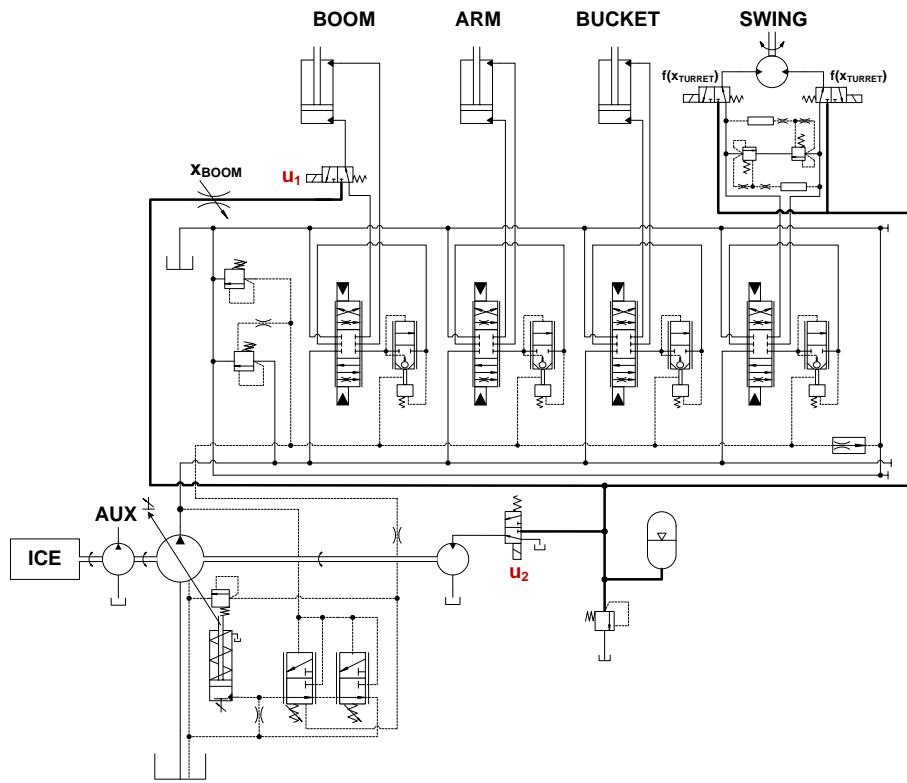
Fig.19. ISO Scheme of the Hybrid Layout A.



337

338

Fig.20. ISO Scheme of the Hybrid Layout B.

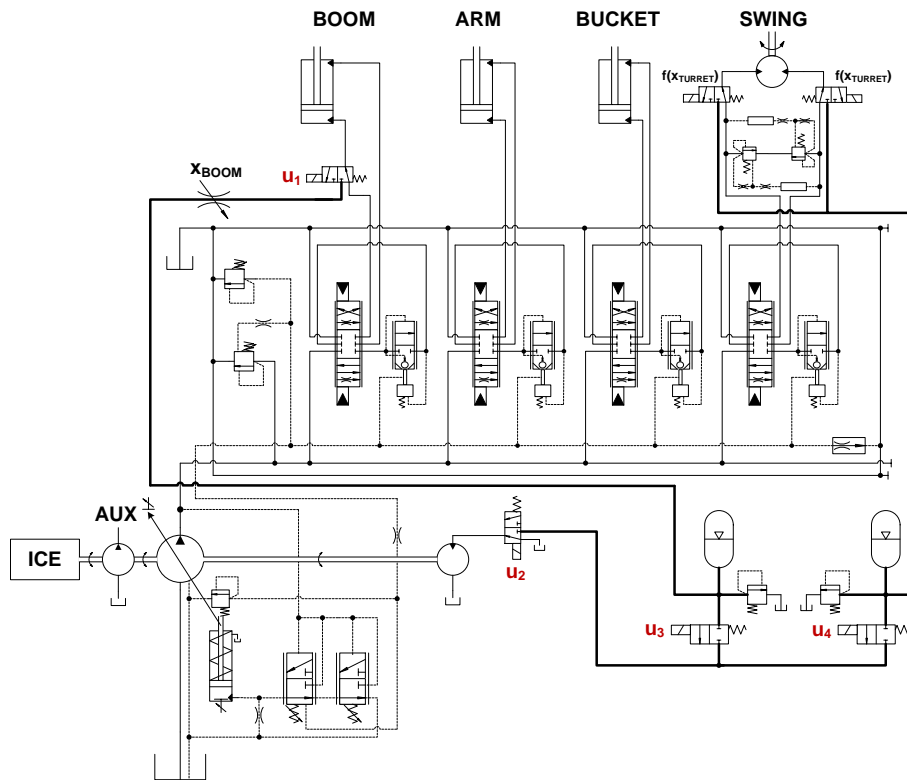


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340

341

Fig.21. ISO Scheme of the Hybrid Layout C.



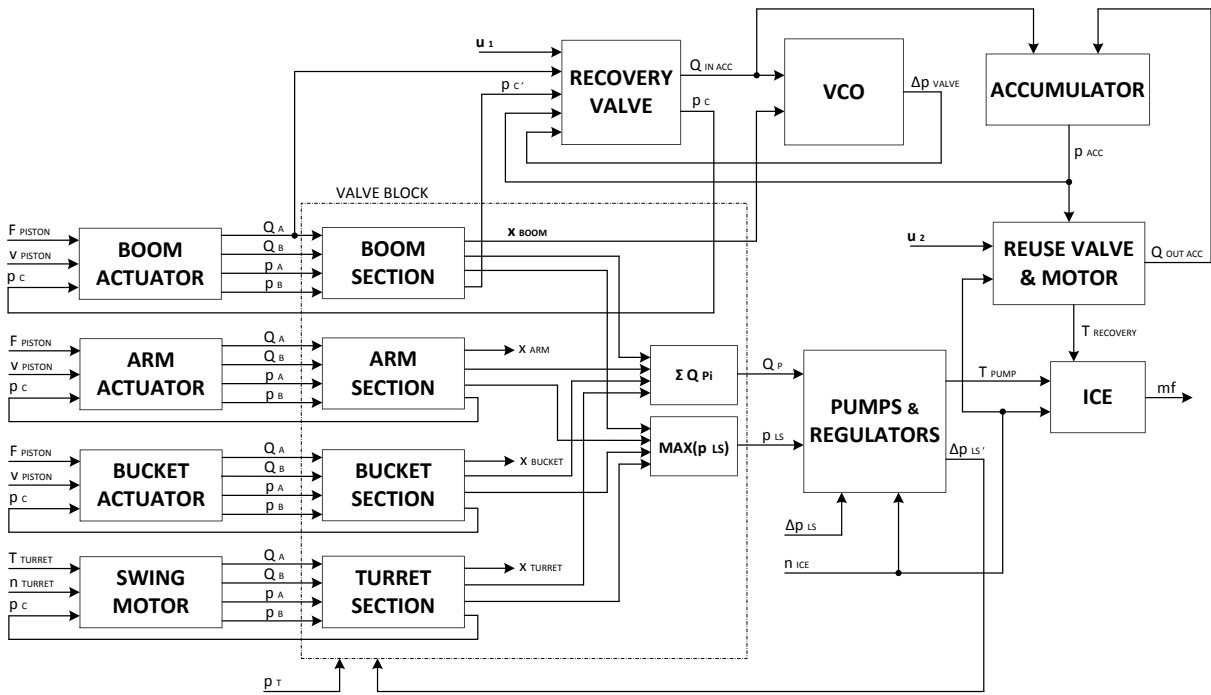
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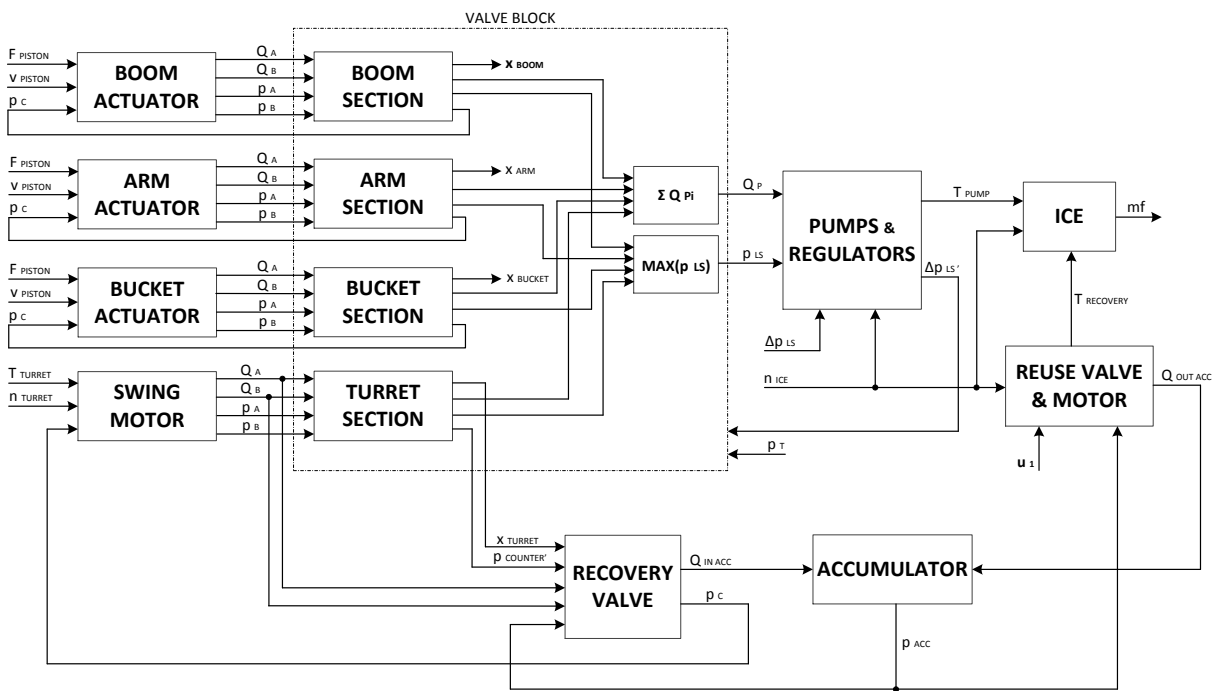
Fig.22. ISO Scheme of the Hybrid Layout D.

345 On the basis of the inverse causality model of the standard excavator, the inverse causality models of the proposed
 346 hybrid layouts under investigation were defined. As examples Figs. 23 – 24 depict the inverse causality models of the
 347 hybrid layouts A and B; layouts C and D are a combination of these.
 348



349
 350 **Fig.23.** Inverse Causality Scheme of the Hybrid Layout A.

351



352
 353 **Fig.24.** Inverse Causality Scheme of the Hybrid Layout B.

354 4.2 Hybrid Layouts Optimal Sizing

355 To perform a comprehensive comparison in order to define the solution which enables the best fuel saving performance,
356 the different proposed hybrid layouts have to be compared in their optimal configuration. This means that the
357 components introduced into the hybrid layouts have to be optimally sized.

358 The optimization target is that of minimize the fuel consumption during the performed working cycle. As for an
359 excavator the digging and loading movement are the most important phases during the typical working hour defined in
360 [16], this sub-cycle has been selected for the components optimization procedure.

361 The parameters of interest for the optimal dimensioning procedure are the accumulator minimum working pressure
362 (p_{MIN}) and the accumulator volume (V_0) for all the proposed hybrid layouts. When the energy recovery from the boom is
363 also an option, the equivalent diameter (d_{EQ}) of the VCO is an additional parameter to be optimally sized.

364 The dimensioning procedure has been done through a DoE (Design of Experiment) methodology, where a discrete grid
365 of possible values for the parameters of interest has been defined in order to explore the different parameter
366 combination still maintaining a reliable functioning of the system and the components. The variation ranges of the
367 parameters involved in the optimization are: $p_{MIN} \in [10; 50]$ bar with an increasing step of 5 bar for the layout A and
368 $p_{MIN} \in [10; 100]$ bar with an increasing step of 10 bar for the layout B; $V_0 \in [2.5, 4, 5, 6, 10]$ L according with the
369 available accumulators; $d_{EQ} \in [1; 8]$ mm with an increasing step of 0.5 mm (only for layout A, C, D). For each hybrid
370 layout and for each possible parameter combination the DP algorithm defines the optimal control strategy for governing
371 the recovery and reuse valves.

372

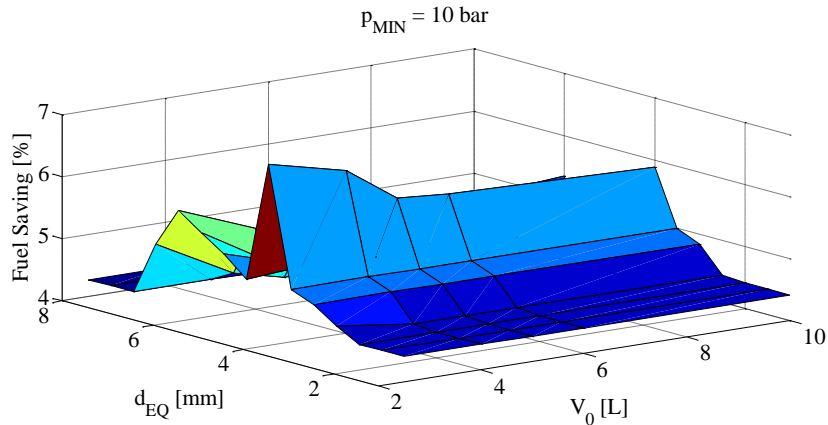
373 5. Hybrid Layout Comparison

374 This section present the results of the optimization performed with the aim of define the optimal combination of the
375 accumulator minimum pressure, accumulator volume and the equivalent diameter of the VCO of the boom (only for the
376 layouts that involve the energy recovery from it) as previously stated. The optimization has been performed for the
377 digging and loading motion defined in [16]. Sections 5.1 and 5.2 report the results concerning layouts A and B
378 respectively, while results about layouts C and D have not been reported for brevity. Finally once defined the optimal
379 sizing of the components of interest the complete JCMAS working cycle has been performed for all the proposed
380 layouts. The final results are reported in section 5.3 where a quantitative comparison based on fuel consumption is
381 presented. The fuel saving percentages are referred to the standard layout fuel consumption.

382

383 **5.1 DP Results: Layout A**

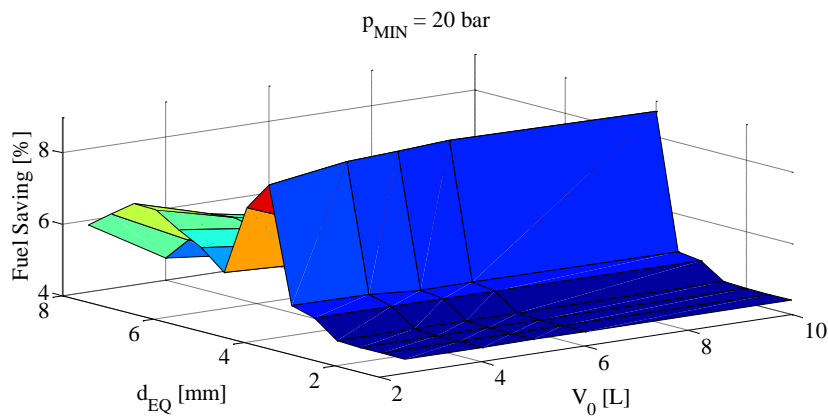
384 Figures 25 – 29 report some of the results obtained through the DoE methodology presented in section 4.2 and applied
385 to the layout A. Being the energy recovery from the boom considered in this configuration, the sizing procedure
386 involves the accumulator minimum pressure (p_{MIN}), the accumulator volume (V_0) and the equivalent diameter of the
387 VCO (d_{EQ}). The reported charts represent the fuel saving percentage, referring to the standard layout, during the
388 digging and loading motion defined in [16] at different accumulator minimum pressure levels.



389

390

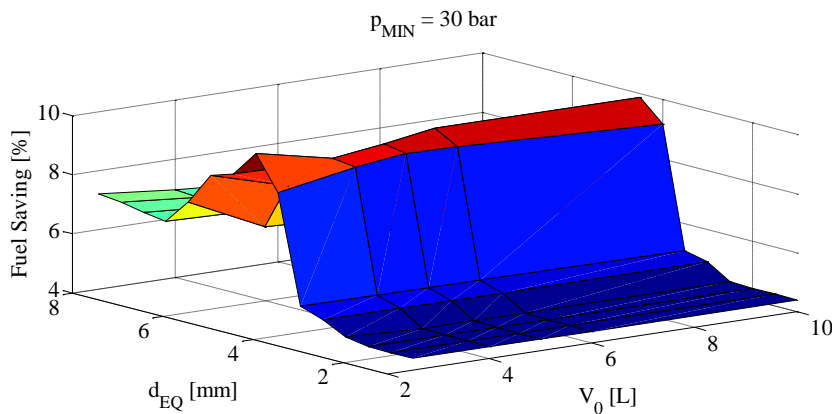
Fig. 25. Layout A Fuel Saving Percentage [$p_{MIN} = 10$ bar].



391

392

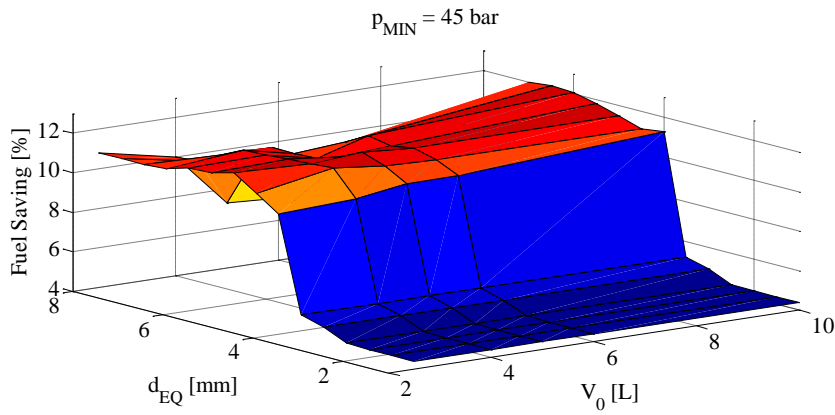
Fig. 26. Layout A Fuel Saving Percentage [$p_{MIN} = 20$ bar].



393

394

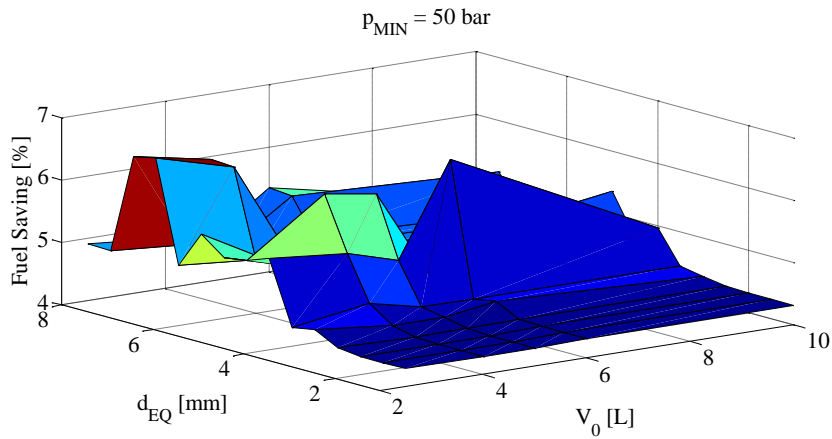
Fig. 27. Layout A Fuel Saving Percentage [$p_{MIN} = 30$ bar].



395

396

Fig. 28. Layout A Fuel Saving Percentage [$p_{MIN} = 45 \text{ bar}$].



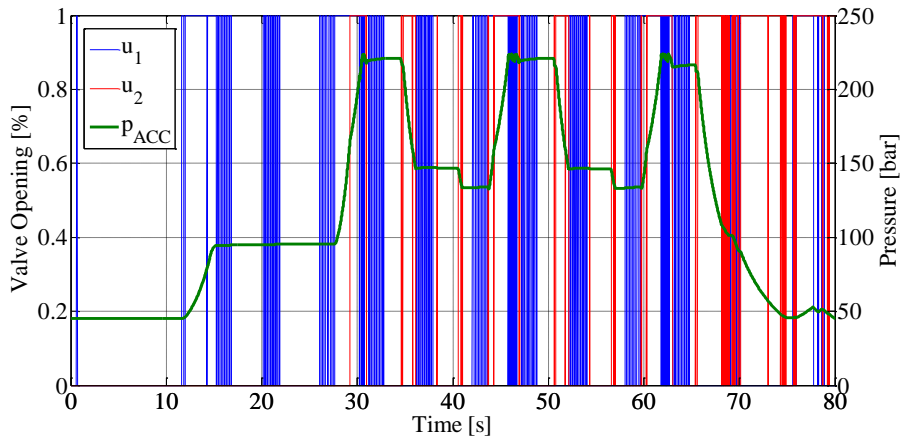
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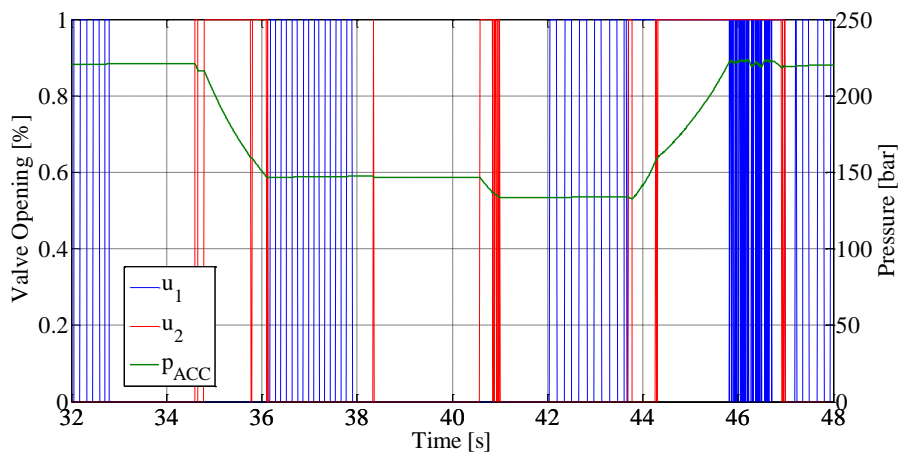
Fig. 29. Layout A Fuel Saving Percentage [$p_{MIN} = 50 \text{ bar}$].

399

400 As can be observed from the Figs. 25 – 29, for each combination of the considered parameters is possible to define a
 401 maximum percentage of fuel saving. Comparing the overall results obtained for the layout A, the best parameters
 402 combination which achieves the overall maximum fuel saving percentage could be identified: minimum accumulator
 403 working pressure $p_{MIN} = 45 \text{ bar}$, accumulator volume $V_0 = 10 \text{ L}$; VCO equivalent diameter $d_{EQ} = 6.5 \text{ mm}$. This
 404 parameter setting define the layout A best configuration while the DP optimal control laws (u_1, u_2) are applied
 405 governing the opening of the corresponding valves of Fig. 19 defining the instantaneous accumulator pressure. Figure
 406 30 reports the layout A optimal control laws and the correspondent accumulator pressure during a complete digging
 407 cycle (five sequential repetitions), while Fig.31 shows an intermediate digging cycle for a better understanding of the
 408 optimal control laws.



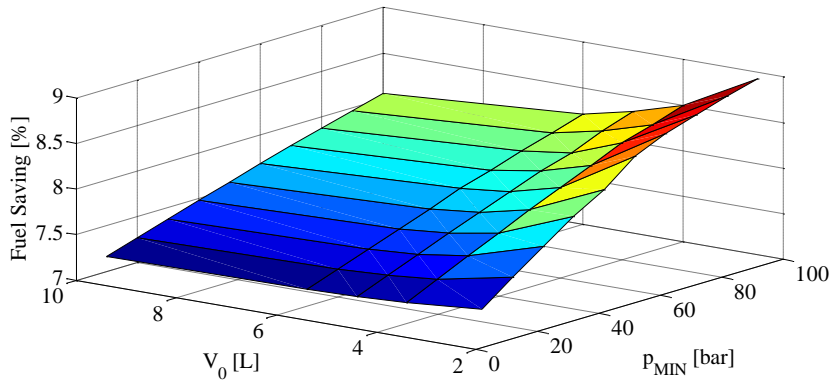
409
410
411 **Fig. 30.** Valves Optimal Control Policy (u_1, u_2) - Layout A



412
413
414 **Fig. 31.** Accumulator Pressure (p_{ACC}) of the Optimal Layout A Configuration.

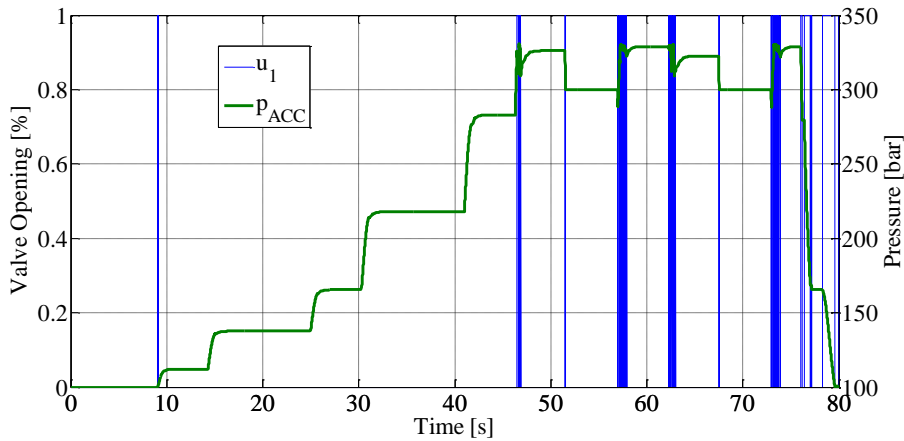
415 **5.2 DP Results: Layout B**

416 Figure 32 report the DoE results for the hybrid layout B, presented in section 4.1, where only the energy recovery from
417 the turret braking is considered. The sizing procedure only involves the accumulator minimum working pressure (p_{MIN})
418 and the accumulator volume (V_0). The reported chart represents the fuel saving percentage, referring to the standard
419 layout, during the digging and loading motion defined in [16]. The optimal combination of the accumulator parameters
420 which maximize the fuel saving percentage are: minimum accumulator working pressure $p_{MIN} = 100 \text{ bar}$, accumulator
421 volume $V_0 = 2.5 \text{ L}$.



422
423 **Fig. 32.** Layout B Fuel Saving Percentage Chart.

424 Figure 33 reports the optimal control law, associated to the optimal accumulator sizing parameter, governing the reuse
425 valve opening (u_1) and the correspondent accumulator pressure (p_{ACC}), referring to Fig. 24 layout, during the digging
426 and loading cycle performed.



427
428 **Fig. 33.** Valve Optimal Control Policy (u_1) and Accumulator Pressure (p_{ACC}) - Layout B

429 **5.3 Layout Comparison**

430 Once defined the optimal sizing parameters for the accumulator and for the VCO (where required), for each proposed
431 hybrid layout, it is possible to evaluate the fuel consumption over the complete working cycle defined in [16], with
432 reference to the baseline configuration (i.e., the original configuration), thus obtaining the percentage reduction in fuel
433 consumption per working hour. This final procedure makes possible to compare in a comprehensive manner the
434 proposed hybrid layout in their optimal configuration. Table 2 summarizes the overall fuel saving percentages.

Tab.2. Percentage of fuel savings from the baseline for the considered Hybrid Circuit Layouts.

CIRCUIT LAYOUT	ACTUATOR(S)	NUMBER OF ACCUMULATOR(S)	FUEL SAVED
A	Boom	1	10 – 11 %
B	Turret	1	2 – 3 %
C	Boom - Turret	1	11 – 12 %
D	Boom - Turret	2	13 – 14 %

435

436 Optimizations A and B yields an optimal accumulator size of respectively 10 L (energy recovery from boom lowering),
437 and 2.5 L (energy recovery from turret swing). As the two values are very different, when using a single common
438 accumulator for the two energy recovery tasks (layout C) the sizing of the accumulator is somehow a compromise
439 between the two and the resulting fuel consumption reduction is lower than what can be achieved in design D where the
440 two accumulators allow for a better exploitation of the available energy.

441

442 **6. Conclusion**

443 A hybridization comparison methodology based on DP algorithm has been presented in detail in this paper. The
444 methodology require some essential steps in order to be exploited: 1, the definition and the validation of the inverse
445 causality mathematical model for the considered system; 2, the definition of a reference working cycle, 3, the definition
446 of the different hybrid layout to be investigated and compared; 4, the definition of the solution of the control
447 optimization problem through a DP algorithm (for each proposed layout); 5, a DoE based optimization of the additional
448 components sizing.

449 The presented methodology defines for each proposed hybrid layout both the optimal components size and the optimal
450 control strategy. The optimal control strategy defined represents only a benchmark for a further definition of online
451 control strategies.

452 This hybridization comparison methodology has been successfully applied to a middle size hydraulic excavator as a
453 baseline example. The machinery hydraulic system mathematical model has been realized and validated in both direct
454 and inverse causality. Several different hybrid layouts of the excavator under study have been proposed and compared
455 by means of the presented hybridization methodology.

456 On the basis of the obtained results on the JCMAS working cycle for this machine size, it is understood that energy
457 recovery from the boom is far more effective then energy recovery from the turret.

458 Finally, this approach permits to obtain the best components sizing combination for the hybrid layouts considered, an
459 optimal control policy for each layout, and allows to choose the hybrid layout configuration able to yield the highest
460 fuel saving percentage with reference to the standard configuration.

461 Future works, based on the presented methodology, will concern the development of algorithms for the definition and
462 the implementation of online control strategy starting from the defined optimal control strategy of the selected hybrid
463 layout.

464

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