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Inverse estimation of the local heat transfer coefficient in curved tubes: a numerical validation

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Abstract. Wall curvature represents one of the most used passive techniques to enhance convective heat transfer. The effectiveness of wall curvature is due to the fact that it gives origin to the centrifugal force: this phenomenon induces local maxima in the velocity distribution that locally increase the temperature gradients at the wall by then maximizing the heat transfer. This fact brings to a significant variation of the wall temperature and of the wall heat flux along the circumferential coordinate. The convective heat transfer coefficient is consequently not uniformly distributed along the tube's perimeter and is characterized by higher values at the extrados wall surface in comparison to the ones at the intrados wall surface. Therefore, for predicting the overall performance of heat transfer apparatuses that involve the use of curved tubes, it becomes important to know the local distribution of the convective heat transfer coefficient not only along the axis of the heat transfer section, but also on the internal tube's surface along the cross section circumference. The present paper is intended to the assessment of a procedure developed to evaluate the local convective heat transfer coefficient, along the circumferential coordinate, at the internal wall of a coiled pipe.

1. Introduction

Among the most used passive techniques to enhance convective heat transfer, wall curvature is found. The effectiveness of wall curvature is due to the fact that it gives origin to the centrifugal force: this phenomenon induces local maxima in the velocity distribution that locally increase the temperature gradients at the wall by then maximizing the heat transfer [1-3]. The asymmetrical distribution of the velocity field over the tube's cross-section leads to a significant variation of the wall temperature and of the wall heat flux along the circumferential coordinate: the convective heat transfer coefficient is then not uniformly distributed along the tube's perimeter but it presents higher values at the extrados wall surface in comparison to the ones registered at the intrados wall surface.

This irregular distribution could be critical in some industrial application, such as in the ones that involve a thermal process. For instance, in food pasteurization, the irregular temperature field induced by the wall curvature could reduce the bacteria heat-killing or could locally overheat the product. Therefore, in order to predict the overall performance of heat transfer apparatuses that involve the use of curved tubes it is necessary to know the local distribution of the convective heat transfer coefficient not only along the axis of the heat transfer section, but also on the internal tube's surface along the cross section circumference.

Although many Authors have investigated the forced convective heat transfer in coiled tubes, most of them have presented the results only in terms of the Nusselt number averaged along the wall circumference: only few Authors have studied the phenomenon locally. Jayakumar et al.[4] numerically analyzed the turbulent heat transfer in helically coiled tubes and presented the local Nusselt number at various cross sections along the curvilinear coordinate. The results showed that, on any cross section, the highest Nusselt number is on the outer side of the coil, while the lowest one is expected on the inner sides. Moreover, the Authors proposed a correlation for predicting the local Nusselt number as a function of average Nusselt number and angular location for both constant temperature and constant heat flux boundary conditions. Bai et al. [5] studied experimentally the turbulent heat transfer in helically coiled tubes using deionized water as working fluid. As expected they found out that the local heat transfer coefficient was not evenly distributed along the periphery on the cross section and that in particular, at the outside surface of the coil it was up-to four times than that at the inside surface. One of the most promising way to estimate the local convective heat transfer coefficient on the interior wall surface of a solid domain is found in the solution of the inverse heat conduction problem (IHCP) in the wall, starting from the temperature distribution acquired on the external wall surface [6]. However IHCPs generally present some complications due to the fact that they are ill-posed: this entails that they show a great sensitiveness to small variations in the input data. In order to find a solution to this problem many methods based on a numerical approach have been improved: among these techniques the conjugate gradient iterative method based on the adjoint problem [7], the Laplace transform method [6], the direct sensitivity coefficient method [8], the space marching method [9], the sequential function specification method [10], the maximum entropy method [11], the Tikhonov regularization method [12] and the filtering technique [13-14] are found.

Tikhonov method is perhaps the most common used regularization technique: it is based on the minimization of an object function in order to reformulate the IHCP as a well-posed problem. The object function is expressed by the sum of the squared difference between measured and estimated temperature discrete data and of a regularization parameter times a term that expresses the smoothness of the unknown quantity. This regularization scheme, originally suggested by Tikhonov and Arsenin [12], enables to overcome the problem's instability in case of particularly critical signal to noise ratio and it proved to be very successful, although the selection of the regularization parameter requires some care.

2. Problem's definition

A typical condition in which curved tubes are tested is the one in which the convective heat transfer within the fluid that flows inside the tube occurs under the uniform heat flux boundary conditions, such as the one considered by Rainieri et al. [1,15] in their experimental investigations where a heat flux was dissipated by Joule effect directly within the tube wall.

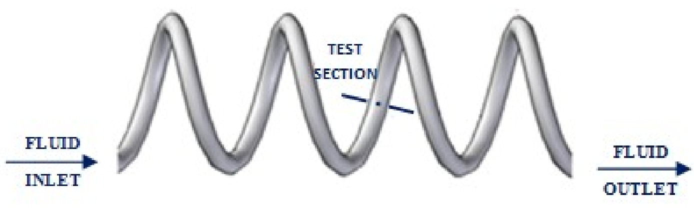

Figure 1: Coiled tube.

In order to evaluate the local actual value of the convective heat transfer coefficient at the fluid internal wall interface on a given cross section (as highlighted in figure 1) the following procedure could be followed: the temperature distribution is acquired on the external wall surface on a given test section and then the IHCP in the wall domain is solved by considering the convective heat transfer coefficient distribution on the internal wall surface to be unknown.

The temperature on the external side could be acquired by using thermocouples, located along the circumference of the text section or more suitably by adopting an infrared thermographic system.

In order to test the above described parameter estimation procedure a simplified 2-D numerical model of the test section (sketched in figure 2) was formulated by assuming that along the axis of the tube the temperature gradient is almost negligible.

In the 2-D solid domain the steady state energy balance equation is expressed in the form:

$$\frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{k}{r^2}\frac{\partial}{\partial \phi}\left(\frac{\partial T}{\partial \phi}\right)+q_g=0 \tag{1}$$

where q_g is the heat generated per unit volume and k is the tube wall thermal conductivity.

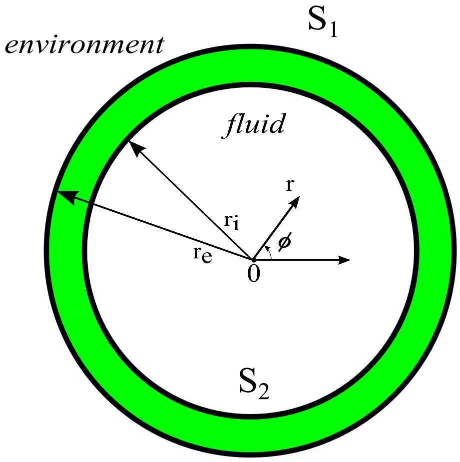

Figure 2. Geometrical domain with coordinate system.

The following two boundary conditions completed the energy balance equation:

$$k\frac{\partial T}{\partial r}=\alpha_e\cdot\left(T_e-T(\phi)\right) \tag{2}$$

that is applied on surface S_1 and where α_e is the overall heat transfer coefficient between the tube wall and the surrounding environment with the temperature T_e ;

$$-k\frac{\partial T}{\partial r}=h_{int}\cdot\left(T_b-T(\phi)\right) \tag{3}$$

that is applied on surface S_2 and where T_b is the bulk fluid temperature and h_{int} is the local convective heat transfer coefficient at the fluid- internal wall interface.

From the knowledge of the total wall heat flux and by measuring the temperature of the fluid at the inlet section, it is possible to calculate the bulk fluid temperature on the tube's cross-section by imposing that the energy balance is satisfied.

3. Parameter estimation procedure

The parameter estimation procedure is embedded in the inverse solution of the problem expressed by equations (1-3). The temperature distribution on the external surface of the section, $T(\phi, r = r_e, h_{int})$, can be easily computed numerically, by imposing a trial convective heat transfer coefficient distribution on the internal wall side h_{int}.

In the inverse formulation of problem, this computed temperature distribution is forced to match the experimental temperature distribution $\Theta(\phi)$, by tuning the convective heat transfer coefficient distribution on the internal wall side, $h_{int}(\phi)$ .The matching of the two temperature distributions (the computed and the experimentally acquired) could be easily performed under a least square approach. To overcome the ill-posed nature of the problem and in particular its ill-conditioned character the Tikhonov regularization method is adopted [12]. This approach, successfully applied in literature [16] allows to reformulate the problem as a well-posed problem by minimizing the following objective function:

$$J\left(h_{int}(\phi)\right) = \|T(\phi, r = r_e, h_{int}) - \Theta(\phi)\|_2^2 + \lambda^2 \|D^n\, h_{int}\|_2^2 \tag{4}$$

where λ is the regularization parameter, *D* is the derivative operator and $T(\phi, r = r_e, h_{int})$ is the distribution of external surface temperature derived from direct numerical solution of the problem obtained by imposing a convective heat transfer coefficient distribution on the internal wall side h_{int}. Often *D* is the zero, first or second derivative operator: in this work the second derivative formulation was chosen in order to preserve local variation of heat transfer coefficient distribution. Under this assumption the object function becomes:

$$J\left(h_{int}(\phi)\right) = \|T(\phi, r = r_e, h_{int}) - \Theta(\phi)\|_2^2 + \lambda^2 \left\|\frac{d^2 h_{int}(\phi)}{d\phi^2}\right\|_2^2 \tag{5}$$

The above function represents a trade-off between two optimization processes: agreement between the data and solution and smoothness or stability of the solution. Thus, an appropriate choice of λ should give an optimal balance. The importance of the choice of the regularization parameter was analyzed by Bazàn and Borges [17]. Choosing an high regularization parameter means to impose too much regularization to the solution prejudicing the fitting of the data and obtaining a great residual; the absence of regularization or a too small regularization parameter will bring to a good fitting but also to a solution affected by the data errors. This idea has led to the development of the L-curve, which was first proposed by Hansen and O'Leary [18]. This method defines the ideal regularization parameter by locating the 'corner' on a plot of the function of the norm of the second derivative of computed convective heat transfer coefficient,$\|d^2 h_{int}(\phi)/d\phi^2\|_2$, versus the norm of the difference between experimental and computed temperature values, $\|T(\phi, r = r_e, h_{int}) - \Theta(\phi)\|_2$.

In the present analysis in order to limit the number of degrees of freedom in the inverse problem approach, the convective heat transfer distribution was simplified by considering a continuous piecewise linear function composed of six sections as follows:

$$h_{int}(\phi) = \begin{cases} b_0 + \phi \cdot \left(\frac{b_1 - b_0}{\pi/3}\right), & 0 \le \phi < \frac{\pi}{3} \\ b_1 + \left(\phi - \frac{\pi}{3}\right) \cdot \left(\frac{b_2 - b_1}{\pi/3}\right), & \frac{\pi}{3} \le \phi < \frac{2\pi}{3} \\ b_2 + \left(\phi - \frac{2\pi}{3}\right) \cdot \left(\frac{b_3 - b_2}{\frac{\pi}{3}}\right), & \frac{2\pi}{3} \le \phi \le \pi \\ b_3 + (\phi - \pi) \cdot \left(\frac{b_4 - b_3}{\frac{\pi}{3}}\right), & \pi \le \phi \le \frac{4\pi}{3} \\ b_4 + \left(\phi - \frac{4\pi}{3}\right) \cdot \left(\frac{b_5 - b_4}{\frac{\pi}{3}}\right), & \frac{4\pi}{3} \le \phi \le \frac{5\pi}{3} \\ b_5 + \left(\phi - \frac{5\pi}{3}\right) \cdot \left(\frac{b_0 - b_5}{\frac{\pi}{3}}\right), & \frac{5\pi}{3} \le \phi \le 2\pi \end{cases} \tag{6}$$

where b_0 b_1 b_2 b_3 b_4 b_5 values become the six unknowns of the inverse problem.

4. Results and discussion

The objective of this paper was to validate the above described procedure of estimation of the local convective heat transfer coefficient. The physical parameters used in this work correspond to the ones of a stainless steel type AISI 304. For what concern the geometrical parameters an internal radius r_i of 7 mm and an external radius r_e of 8 mm were considered. The validation of the procedure was performed by adopting synthetic data. By imposing a known distribution of h_{int} and by solving the governing equations (1-3), a synthetic temperature distribution on the external wall surface was obtained. For the local convective heat transfer coefficient, two different distributions $h_{int}{}^{(A)}$ and $h_{int}{}^{(B)}$ according to the data of Jayakumar et al. [4] and shown in figures 3-4, were considered.

The two distributions were derived by numerical simulations performed under the turbulent flow regime and they have been chosen in this work because they represent two extreme cases: the former distribution, that corresponds to a representative axial location in the thermal entry region, is almost flat and it is characterized by a maximum to minimum value ratio of about 1.3, the latter, that corresponds to the thermally fully developed region, shows a significant variation along the curvilinear coordinate and it is characterized by a maximum to minimum value ratio of about 5. Then the synthetic temperature distribution on the external wall surface, deliberately spoiled by random noise, was used as the input data of the inverse problem. In particular, a white noise characterized by a standard deviation of 0.1 was considered. By solving the inverse problem, i.e. by forcing the external surface temperature $T(\phi, r = r_e, h_{int})$ to match the synthetic temperature distribution $\Theta(\phi)$, it's possible to restore the convective heat transfer coefficient distribution on the internal wall side $h_{int}(\phi)$, that is the only term unknown.

The $h_{int}(\phi)$ distribution found was then compared to the distribution used to obtain the synthetic temperature $\Theta(\phi)$ in order to evaluate the effectiveness of the parameter estimation procedure.

Given the symmetry of the two distributions of the convective heat transfer coefficient, the number of the unknown variables reduces to four, being $b_2 = b_4$ and $b_1 = b_5$. This enabled to further reduce the computational cost of the minimization algorithm.

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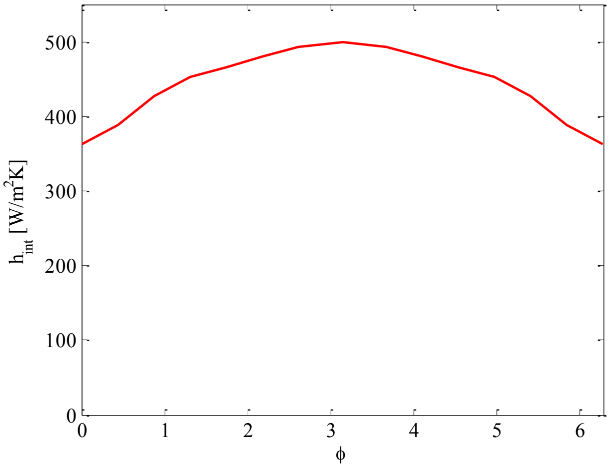

Figure 3. First distribution of the local convective heat transfer coefficient $h_{int}{}^{(A)}$ (Jayakumar et al. [4]).

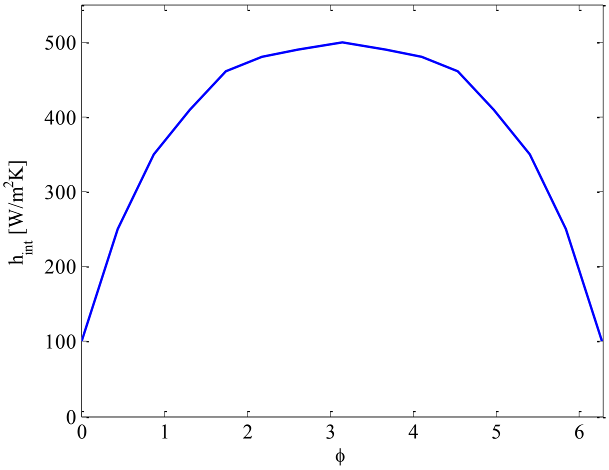

Figure 4. Second distribution of the local convective heat transfer coefficient $h_{int}{}^{(B)}$ (Jayakumar et al. [4]).

The direct problem was solved by finite element method implemented in Comsol Multiphysics® environment with a mesh of about 2400 triangular elements while the minimization of Tikhonov target function was run within the Matlab Optimization Toolbox® by using as stopping criterion a relative tolerance on object function lower than 1e-004. The algorithm adopted in the minimization process is the Nelder-Mead algorithm [19] that represents one of the best known solution for multidimensional unconstrained optimization.

In figure 5 the L-curve for the distribution $h_{int}{}^{(A)}$ is shown. Following the above described L-curve method, the optimal value $\lambda = 0.0003$, corresponding to the 'corner' of the curve, was identified.

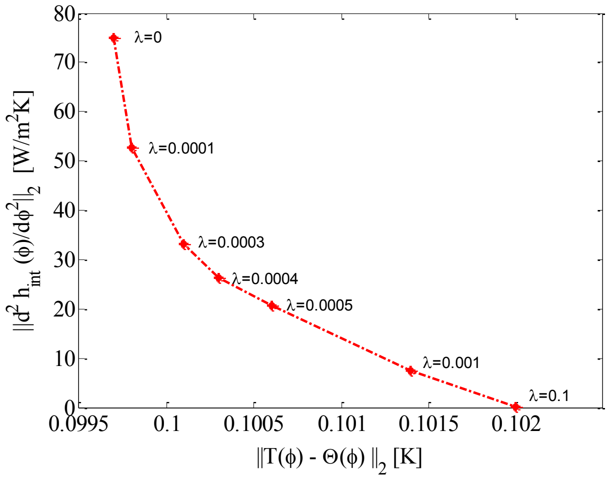

Figure 5. L-curve for distribution $h_{int}{}^{(A)}$.

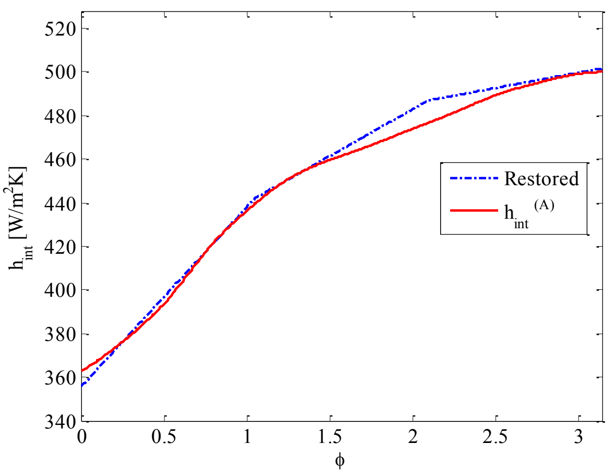

Figure 6. Restored and exact local heat transfer coefficient distribution ($h_{int}{}^{(A)}$, $\lambda = 0.0003$).

In figure 6 the heat transfer coefficient distribution, restored by adopting the regularization parameter $\lambda = 0.0003$, is reported and compared to the exact value.

The restored values matched with a good approximation the original local heat transfer distribution $h_{int}\ (A)$, by demonstrating the robustness of the procedure presented in this paper.

In figure 7 the restored temperature distribution resulting from the minimization procedure, implemented by considering λ = 0.0003, is shown. The data confirm that, by adopting the optimal regularization parameter, the Tikhonov scheme enables to filter out the noise from the raw input signal by restoring the smoothest approximate solution compatible with the experimental data within a given noise level [13,14].

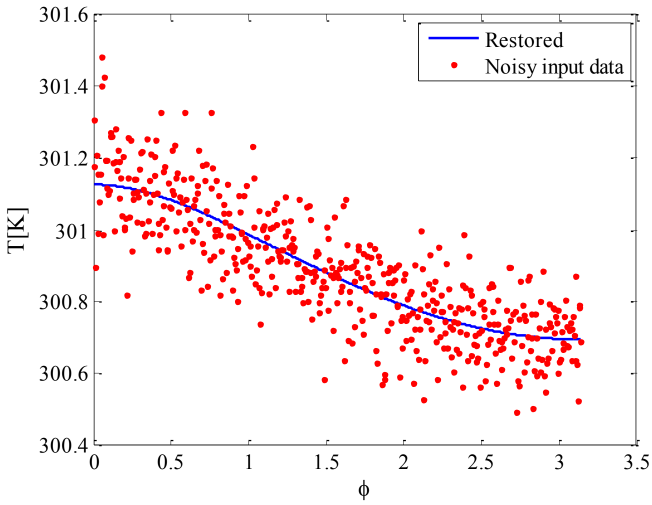

Figure 7. Restored temperature distribution ($h_{int}\ (A)$, λ = 0.0003).

It is well known that the smoothness is controlled by the choice of the regularization parameter: if the regularization parameter is not the correct one the problem's ill-conditioned character could lead to an incorrect solution. For instance by considering the values λ = 0 and λ = 0.1, corresponding to the two extreme values of the L-curve, the local heat transfer coefficient obtained from minimization of the function given by equation (4) doesn't match the exact distribution as it is shown in figures 8 and 9.

In both cases (λ = 0 and λ = 0.1) the parameter estimation procedure fails by proving that the selection of the regularization parameter is a critical task within Tikhonov regularization: if it is too small the filtering of the raw signal is too weak, while if it is too large the regularized solution is over-smoothed.

The same procedure was applied to the second local heat transfer coefficient distribution ${h_{int}}^{(B)}$: the L-curve is reported for this case in figure 10 and the optimal value of λ chosen, corresponding to the 'corner' of the curve resulted λ = 0.0002.

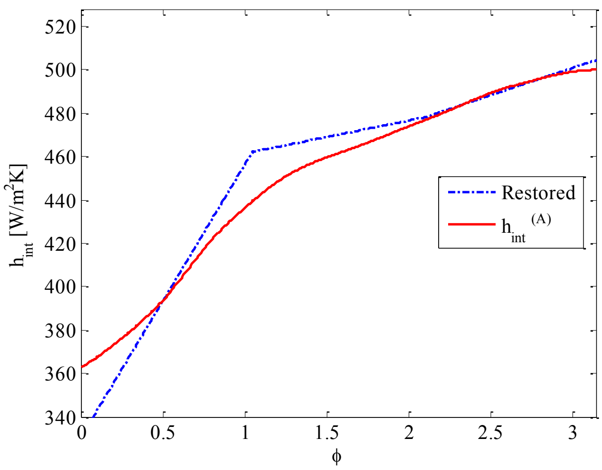

Figure 8. Restored and exact local heat transfer coefficient ($h_{int}{}^{(A)}$, $\lambda = 0$)

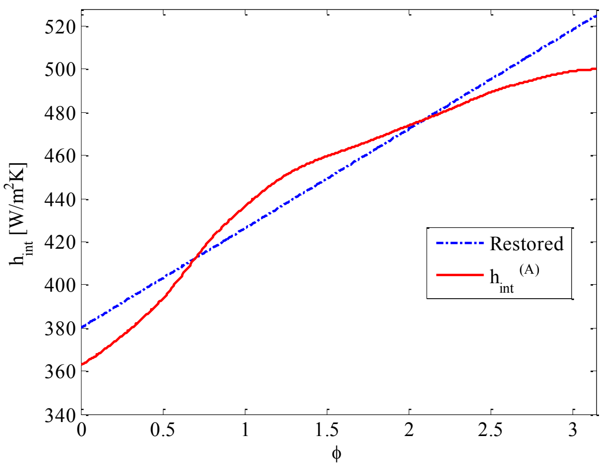

Figure 9. Restored and exact local heat transfer coefficient ($h_{int}{}^{(A)}$, $\lambda = 0.1$).

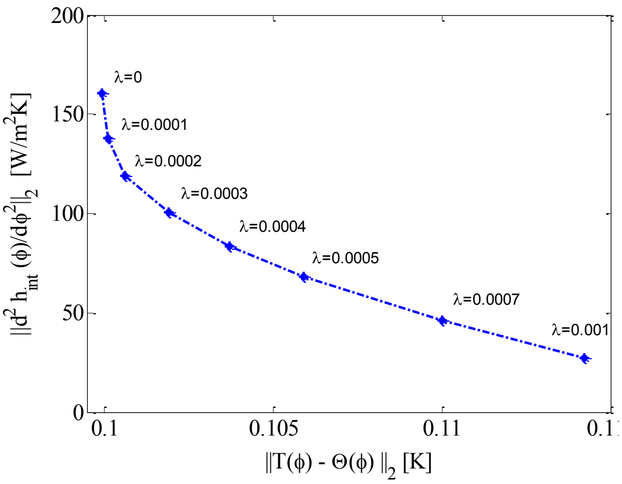

Figure 10. L-curve for distribution $h_{int}{}^{(B)}$

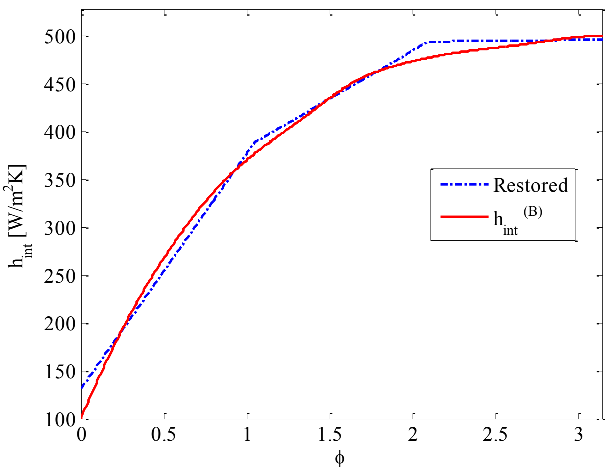

Figure 11. Restored and exact local heat transfer coefficient ($h_{int}{}^{(B)}$, $\lambda = 0.0002$).

The restored values of local heat transfer coefficient are reported in figure 11. Also in this case there is a good matching between the restored and the exact distributions when the regularization parameter is identified by means of the L-curve method. In order to highlight the effect of the choice of the regularization parameter a residual analysis could be performed by plotting the estimation error, defined as follows:

$$E = \frac{\left\| \left(h_{int}(\phi)\right)_{restored} - \left(h_{int}(\phi)\right)_{exact} \right\|_2^2}{\left\| \left(h_{int}(\phi)\right)_{exact} \right\|_2^2} \tag{7}$$

versus the regularization parameter λ.

The estimation error is reported in figure 12 against the regularization parameter λ for the representative case corresponding to the distribution $h_{int}{}^{(B)}$: as expected, the data show that the minimum error corresponds to the optimal λ value, that for this case was identified equal to 0.0002 by the L-curve method. Moreover, these data confirm the necessity of the regularization scheme for enhancing the parameter estimation accuracy.

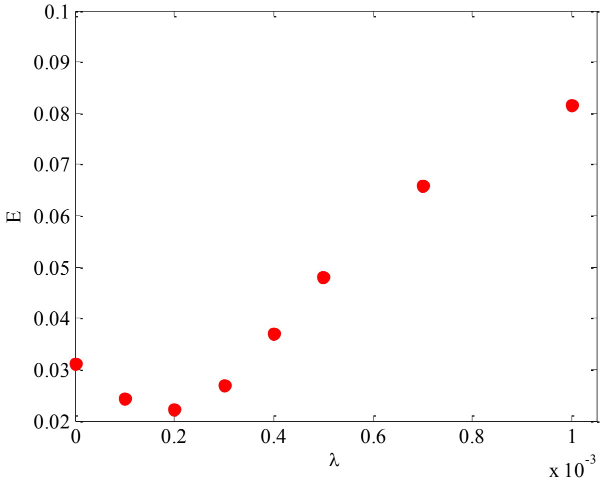

Figure 12. Estimation error for the distribution $h_{int}{}^{(B)}$.

5. Conclusions

In the present paper a procedure to estimate the local convective heat transfer coefficient along the circumferential coordinate at the internal wall of a coiled pipe and based on the solution of the Inverse Heat Conduction Problem (IHCP) was presented and validated. This problem could be particularly interesting in applications that require the knowledge of the local heat transfer distribution at the fluid-wall interface, such as in food industries.

The validation of the procedure was here performed throughout its application to numerical data.

A simplified 2-D numerical model of the test section was formulated by assuming that along the axis of the tube the temperature gradient is almost negligible. A synthetic temperature distribution on the external wall surface was then obtained by imposing a given distribution of the local convective heat transfer coefficient on the internal side of the wall. Regarding the distribution of the local convective heat transfer coefficient, the results of Jayakumar et al. [4] were considered. The synthetic temperature distribution on the external wall surface, deliberately spoiled by random noise, was then used as the input data of the inverse problem. The IHCP solution was derived by the Tikhonov regularization technique coupled to the L-curve method.

The results showed that the restored values of the convective heat transfer coefficient match with a good approximation with the exact distribution, by demonstrating the robustness of the here addressed parameter estimation procedure. The results enabled also to highlight the criticality associated to the choice of the regularization parameter successfully identified by the L-curve method. A necessary further step of the research is the application of the estimation procedure to real experimental data.

Nomenclature

Symbol	Quantity	SI Unit
E	Estimation error, equation (7)	-
h	Convective heat transfer coefficient	W/m^2K
k	Thermal conductivity	W/mK
q_g	Internal heat generation per unit volume	W/m^3
r	Radial coordinate	m
T	Temperature	K

α	Overall heat transfer coefficient	W/m^2K
ϕ	Angular coordinate	rad
λ	Regularization parameter	-

Subscripts

b	bulk
e	external
i,int	internal

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