PROPAGATION OF GRAVITY CURRENTS OF NON-NEWTONIAN POWER-LAW FLUIDS IN POROUS MEDIA

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PROPAGATION OF GRAVITY CURRENTS OF NON-NEWTONIAN POWER-LAW FLUIDS IN POROUS MEDIA

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ABSTRACT

A comprehensive analytical and experimental framework is presented to describe gravity-driven motions of rheologically complex fluids through porous media. These phenomena are relevant in geophysical, environmental, industrial and biological applications.
The fluid is characterized by an Ostwald-DeWaele constitutive equation with behaviour index *n*. The flow is driven by the release of fluid at the origin of an infinite porous domain. In order to represent several possible spreading scenarios, we consider: i) different domain geometries: plane, radial, and channelized, with the channel shape parameterized by β; ii) instantaneous or continuous injection, depending on the time exponent of the volume of fluid in the current, α; iii) horizontal or inclined impermeable boundaries. Systematic heterogeneity along the streamwise and/or transverse direction is added to the conceptualization upon considering a power-law permeability variation governed by two additional parameters ω and β. Scalings for current length and thickness are derived in self similar form coupling the modified Darcy's law accounting for the fluid rheology with the mass balance equation. The speed, thickness, and aspect ratio of the current are studied as a function of model parameters; several different critical values of α emerge and govern the type of dependency, as well as the tendency of the current to accelerate or decelerate and become thicker or thinner at a given point. The asymptotic validity of the solutions is limited to certain ranges of model parameters.
Experimental validation is performed under constant volume, constant and variable flux regimes in tanks/channels filled with transparent glass beads of uniform or variable diameter, using shear-thinning suspensions and Newtonian mixtures. The experimental results for the length and profile of the current agree well with the self-similar solutions at intermediate and late times.

THEORETICAL BACKGROUND

- Fluid constitutive equation in simple shear $\tau = m\dot{\gamma}\left|\dot{\gamma}\right|^{n-1}$

m consistency index, *n* flow behavior index

- Darcy law for flow in p.m. $\nabla p - \rho \mathbf{g} = -\frac{1}{\Lambda k^{(1+n)/2}}\left|\mathbf{u}\right|^{n-1}\mathbf{u},$

u Darcy velocity
p pressure
k, ϕ permeability, porosity

$$\Lambda = \Lambda(\phi, m, n) = \frac{8^{(n+1)/2}}{2}\left(\frac{n}{3n+1}\right)^n \frac{\phi^{(n-1)/2}}{m}$$

- Motion driven by density difference $\Delta\rho$ between heavy intruding fluid and light fluid saturating the medium and gravity and channel slope
- Sharp interface
- Current height is thin compared to length and porous medium thickness
- Negligible surface tension effects
- Under previous assumptions, vertical velocities in the intruding fluid are neglected, the pressure within is hydrostatic, ambient fluid is taken to be at rest; the height of the intrusion is to be determined as *h*(*x*,*t*)
- Current volume introduced at the system boundary $\propto t^\alpha$

(α = 0 constant volume, α = 1 constant flux injection)

- Zero height at the front $x_N(t)$, $r_N(t)$

Radial geometry

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\int_0^h u\,dz\right) = -\phi\frac{\partial h}{\partial t} \qquad 2\pi\phi\int_0^{r_N(t)} rh(r,t)\,dr = Qt^\alpha \qquad h(r_N,t) = 0$$

Channel of constant cross section

$$\phi\frac{\partial}{\partial t}\left(A_c h^{(\beta+1)/\beta}\right) + \frac{\partial}{\partial x}\left(u_x A_c h^{(\beta+1)/\beta}\right) = 0;\ A_c = \frac{2\beta}{\beta+1}\frac{a^{(\beta-1)/\beta}}{h_c^{1/\beta}},$$

$$\phi\int_0^{x_N(t)} A_c h^{F_1}\,dx = Qt^\alpha \qquad h(x_N,t) = 0$$

Equations are rendered dimensionless with $T = t/t_0;\ X = x/x_0;\ H = h/x_0$

$$t_0 = (Q/(\phi v_0^3))^{1/(3-\alpha)};\ x_0 = v_0 t_0;\ v_0 = \frac{(\Lambda\Delta\rho g)^{1/n} k^{(1+n)/2n}}{\phi}$$

Inspection of resulting PDEs (horizontal currents) reveals fundamental scaling properties of solution

$$X_N \approx T^{F_2} \qquad H \approx T^{F_3}$$

Such scaling suggest the adoption of self-similar solutions for the long-time evolution of the current.

Radial currents in homogeneous or vertically graded media

Power-law permeability variation along the vertical $k(z) = k_0(z/x_0)^{\omega-1}$
Problem parameters:
n flow behavior index
ω permeability variation
α rate of growth of current volume

Self-similar variable and solution form, horizontal current

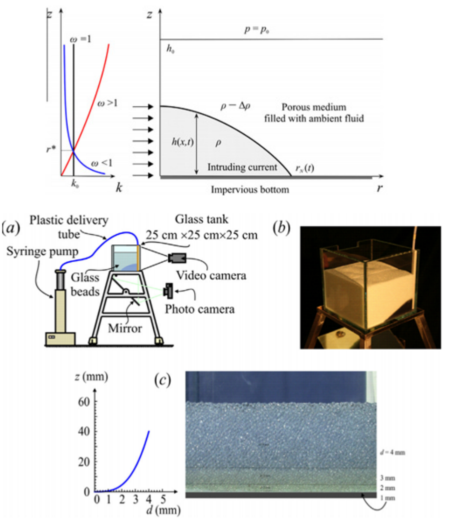
$$\eta = F_1^{F_4}\frac{R}{T^{F_2}} \qquad \eta_N = \eta(R_N) \qquad \zeta = \frac{\eta}{\eta_N}$$

$$H(R,T) = F_1^{F_4}\eta_N^{F_5}T^{F_3}\Psi(\zeta) \qquad F_i = F_i(n,\omega,\alpha), i = 1,\ldots,5$$

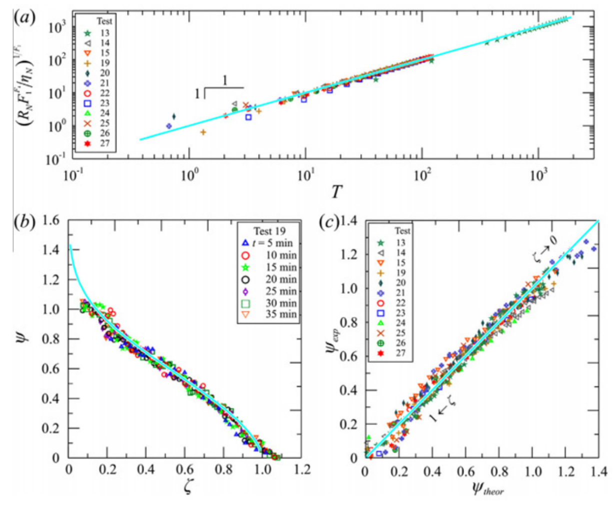
Solution is given by ODE; analytical solution α=0, numerical $\alpha \neq 0$

$$\frac{d}{d\zeta}\left[\zeta\Psi^{F_1}\left(-\frac{d\Psi}{d\zeta}\right)^{1/n}\right] - F_2\zeta^2\frac{d\Psi}{d\zeta} + F_3\zeta\Psi = 0 \qquad \Psi(1) = 0$$

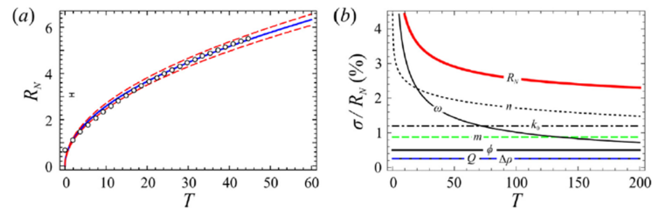
$$\eta_N = \left(2\pi\int_0^1 \zeta\Psi(\zeta)\,d\zeta\right)^{-1/(F_5+2)}$$



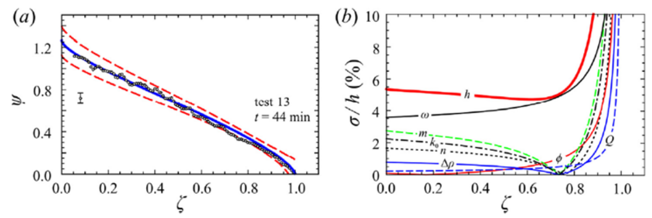
(a) Experimental setup; (b) Glass tank; (c) Stratification of glass beads, ω = 1.63



(a) Position of current front; (b) Experimental vs. theoretical shape function at different times, test #19; (c) Experimental vs. theoretical shape function, all tests.



Test #27: (a) Experimental/theoretical position of current front vs. time and 95% confidence limits (b) Coefficient of variation of front position, *m*=0.14±3:5% Pa·s, *n*=1.0±3:5%; Δp=1241±1% kg·m³; Q_0=0.888±1.0% ml·s⁻¹; α=1; ϕ=0.37±1.0%; ω=1.63± 4.3%; k_0=9.51·10⁻⁹±4.8% m²



Test #13: (a) Experimental/theoretical shape function vs. similarity variable and 95% confidence limits (b) Coefficient of variation of front position, *m*=0.60±3:5% Pa·s, *n*=0.33±3:5%; Δp=1175±1% kg·m³; Q_0=0.40±0.5% ml·s⁻¹; α=1; ϕ=0.37±1.0%; ω=1.63± 4.3%; k_0=4.99·10⁻⁹±4:8% m²

Currents in porous channels

Channel wall given by $b(y) = h_c a(y/a)^\beta$
Parameter β describes cross sectional shape: β = 1 corresponds to triangle, while $\beta \to \infty$ to a rectangular channel of half-width *a*.
Problem parameters:
n flow behavior index
β channel shape
α rate of growth of current volume

Self-similar variable and solution form, horizontal channels

$$\eta = A_c^{F_4}\frac{X}{T^{F_2}} \qquad \eta_N = \eta(X_N) \qquad \zeta = \frac{\eta}{\eta_N}$$

$$H(R,T) = A_c^{-(n+1)F_4}\eta_N^{F_5}T^{F_3}\Psi(\zeta)$$

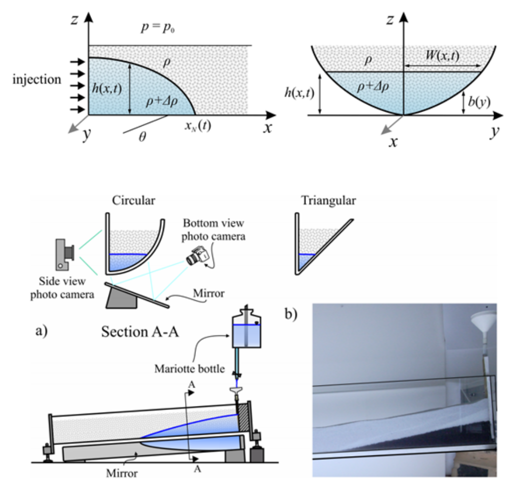
$$F_i = F_i(n,\beta,\alpha), i = 1,\ldots,5$$

Solution is given by ODE; analytical solution α=0, numerical $\alpha \neq 0$

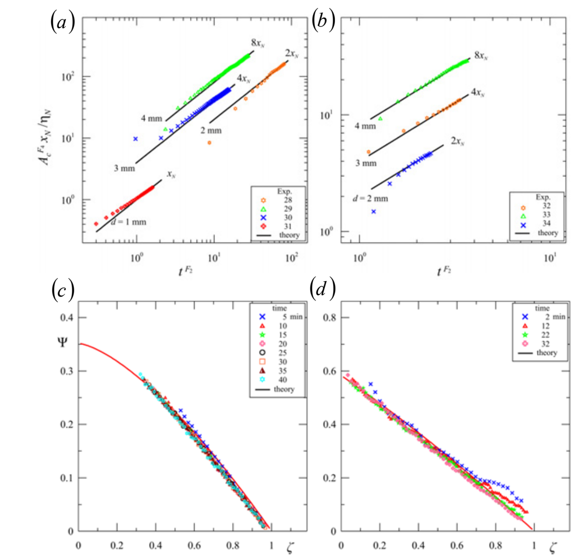
$$\frac{d}{d\zeta}\left[\Psi^{F_1}\left(-\frac{d\Psi}{d\zeta}\right)^{1/n}\right] - F_1\Psi^{F_1-1}\left(F_2\frac{d\Psi}{d\zeta} - F_3\Psi\right) = 0 \qquad \Psi(1) = 0$$

$$\eta_N = \left(\int_0^1 \Psi^{F_1}(\zeta)\,d\zeta\right)^{-F_4}$$

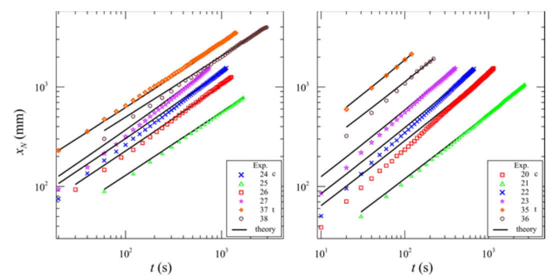
For inclined channels, a numerical solution in self-similar form is obtained when the product between the channel inclination and the slope of the free-surface is much smaller than unity.



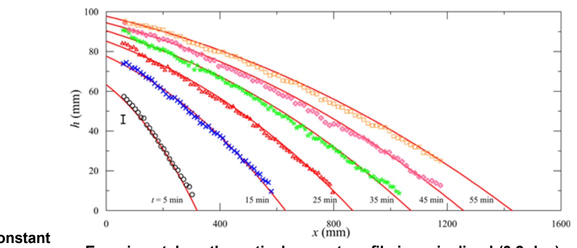
(a) Experimental setup; (b) Gravity current for α=1, β=1, *n*=0.75



Position of current front for a triangular cross section, horizontal channel : (a) constant volume flux (α = 1); (b) constant volume (α = 0); Experimental vs. theoretical shape function at different times for: (c) test #34 (α = 1); (d) test #29 (α = 0)



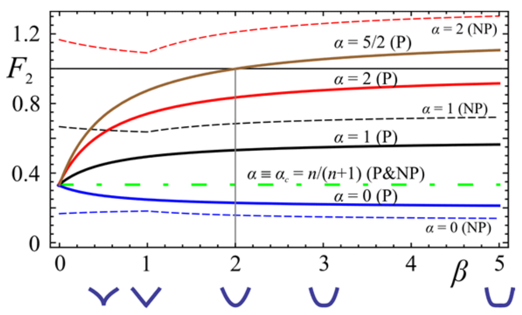
Position of current front for inclined semi-circular and triangular channels, constant volume flux (α = 1) : (a) 3.2 deg; (b) 5.2 deg



Experimental vs. theoretical current profile in an inclined (3.2 deg) triangular channel test #38 (α = 1)

DISCUSSION AND FUTURE WORK

- F_2, F_3, $F_3 - F_2$ key time exponents govern rate change of current length, thickness, and aspect ratio with time
- F_2, is always positive, while F_3 and $F_3 - F_2$ may be positive, null or negative
- Critical values of α (horizontal currents) govern the dependence of key time exponents F_3, $F_3 - F_2$ on fluid rheology *n* and channel shape/permeability variation β/ω
- For values of α larger than the critical values, validity of model assumptions is not respected asymptotically, e.g. aspect ratio and thickness increase with time (violating thin current assumption and finiteness of porous domain)
- Results for porous channels may be compared with free-surface viscous currents; critical α_c is the same (governed by mass balance) but dependency is different (governed by flow equation, Darcy vs. Stokes)
- Future work includes:
- Adoption of more complex rheological models (yield stress, …)
- Inclusion of capillarity effects
- Mixture of axial and channelized flow
- Inclusions of different permeability
- Random permeability variations



Exponent of current front vs. channel shape β, critical value of $\alpha = \alpha_c = n/(n+1)$, porous (P) and nonporous (NP) channels

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